Title: Simplifying general relativity

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Series: Cosmology & Gravitation

Date: June 11, 2019 - 11:00 AM

URL: http://pirsa.org/19060001

Abstract: Modified gravity theories typically feature numerous additional parameters and functions as compared to general relativity, which are unmotivated by observations and challenging to meaningfully constrain. We instead propose a new theory of gravity with the startling property of having *fewer* degrees of freedom than general relativity with a cosmological constant, by invoking a duality property within a first-order formulation that supports torsion. In this theory the cosmological constant becomes a space-time variable without introduction of kinetic terms, but its behaviour is tied to that of matter. I will describe the main properties of the theory, including its implications for cosmology. While there are strong indications that the simplest incarnation of the theory is not observationally viable, the theory can be used as a starting point for various extensions which appear more promising.

Based on arXiv:1905.10380 and arXiv:1905.10382, with Alexander, CortÃªs, Magueijo, Sims, and Smolin.

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Simplifying general relativity

Andrew Liddle June 2019

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Stephon Alexander, Marina Cortês, João Magueijo, Robert Sims, and Lee Smolin

Modifying gravity

The Standard Cosmological Model assumes general relativity plus a cosmological constant. It is fantastic at fitting observations.

Why would you want to modify it?

Modifying gravity

- Because you want to solve the cosmological constant problem.
- Because you want to explain the observed acceleration of the Universe.
- Because you want to solve the coincidence problem.
- Because you want alternatives to the Standard Cosmological Model to help judge how good it is.
- Because you believe the current `tensions' are real and require explanation.
- Because you want to eliminate dark matter.
- Because you want a better starting point to try to quantise gravity.
- Because you have to publish to get your next job.

How might you modify it?

There are endless options. A tiny subset includes

- Making Lambda dynamical by making it the potential of some new (probably scalar) degree of freedom: quintessence, k-essence etc.
- Scalar-(vector-)tensor gravity theories: Horndeski/galileon etc.
- Higher-order gravity: $f(R)$, $f(T)$, bimetric etc.
- Phenomenological models: growth rate, modified Poisson, parameterised post Friedmann, effective field theory, etc.

Principally, these are characterised by the introduction of numerous extra free parameters and functions into the theory.

Table 2. Candidate parameters: those which might be relevant for cosmological observations, but for which there is presently no convincing evidence requiring them. They are listed so as to take the value zero in the base cosmological model. Those above the line are parameters of the background homogeneous cosmology, and those below describe the perturbations.

Ω_k	spatial curvature
$N_\nu - 3.04$	effective number of neutrino species (CMBFAST definition)
m_{ν_i}	neutrino mass for species 'i'
	[or more complex neutrino properties]
$m_{\rm dm}$	(warm) dark matter mass
$w+1$	dark energy equation of state
dw/dz	redshift dependence of w
	[or more complex parametrization of dark energy evolution]
$c_{\rm S}^2 - 1$	effects of dark energy sound speed
$1/r_{\rm top}$	topological identification scale
	[or more complex parametrization of non-trivial topology]
$d\alpha/dz$	redshift dependence of the fine structure constant
dG/dz	redshift dependence of the gravitational constant
$n-1$	scalar spectral index
$dn/d\ln k$	running of the scalar spectral index
r	tensor-to-scalar ratio
$r + 8n_{\rm T}$	violation of the inflationary consistency equation
$dn_{\rm T}/d\ln k$	running of the tensor spectral index
$k_{\rm cut}$	large-scale cut-off in the spectrum
$A_{\rm feature}$	amplitude of spectral feature (peak, dip or step) ...
$k_{\rm feature}$	... and its scale
	[or adiabatic power spectrum amplitude parametrized in N bins]
$f_{\rm NL}$	quadratic contribution to primordial non-gaussianity
	[or more complex parametrization of non-gaussianity]
$\mathcal{P}_S$	CDM isocurvature perturbation ...
n_S	... and its spectral index ...
$\mathcal{P}_{S\mathcal{R}}$	... and its correlation with adiabatic perturbations ...
$n_{S\mathcal{R}} - n_S$	... and the spectral index of that correlation
	[or more complicated multi-component isocurvature perturbation]
$G\mu$	cosmic string component of perturbations

From Liddle 2004

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I hadn't anticipated the huge range of modified gravity parameters that would be proposed.

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Melting Lambda with torsion

The GR action (Palatini and forms)

$$S^{\mathrm{GR}}[e^a, \omega^{ab}, \psi] = \frac{1}{8\pi G} \int_{\mathcal{M}} \epsilon_{abcd} \left(e^a \wedge e^b \wedge R^{cd}(\omega) - \frac{\Lambda}{6} e^a \wedge e^b \wedge e^c \wedge e^d \right) + S^{\mathrm{matter}}[e^a, \psi] \,.$$

Here we have

- e^a: one-form frame field
- ω^{ab}: one-form spin connection
- R^{cd}: curvature two-form
- ϵ_{abcd}: antisymmetric tensor
- Λ: cosmological `constant'

Latin indices are in the internal flat Lorentzian space-time and Greek indices (normally suppressed) in coordinate space-time. The forms are already densities hence no $\sqrt{-g}$.

Torsion

We are in first-order form because we want to include torsion. The torsion two-form is

$$T^a \equiv \mathcal{D}e^a$$

with

$$\mathcal{D}e^a \equiv de^a + \omega^a{}_c \wedge e^c$$

and has 24 components. Varying the action wrt the connection gives

$$0 = \frac{\delta S^{\mathrm{GR}}}{\delta \omega_{ab}} \Longrightarrow \mathcal{D}\left(e^a \wedge e^b\right) = 2T^{[a} \wedge e^{b]} = 0\,,$$

This imposes 24 constraints on the torsion, forcing it to vanish.

Einstein equation

This comes from variation wrt the frame fields

$$0 = \frac{\delta S^{\mathrm{GR}}}{\delta e^a} \implies$$
$$\epsilon_{abcd} e^b \wedge \left(R^{cd} - \frac{\Lambda}{3} e^c \wedge e^d \right) = -16\pi G \tau_a$$

where τ_a is the energy-momentum three-form. It's handy to define the term in brackets as the self-dual current

$$\mathcal{J}^{cd} \equiv R^{cd} - \frac{\Lambda}{3} e^c \wedge e^d$$

If $\mathcal{J}^{cd} = 0$ the vacuum Einstein equations are solved.
De Sitter space has this property.

Derivative of Einstein, and the Bianchi identity

Differentiating Einstein, temporarily forgetting we have shown the torsion is zero and that we think Λ is constant, gives

$$\epsilon_{abcd}\left\{e^b \wedge \left[\frac{d\Lambda}{3} \wedge e^c \wedge e^d + \frac{2\Lambda}{3} T^c \wedge e^d\right] + T^b \wedge J^{cd}\right\} + \mathcal{D}\tau_a = 0$$

where we used the Bianchi identity $\mathcal{D}R^{ab} = 0$.

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where we used the Bianchi identity $\mathcal{D}R^{ab} = 0$.

In GR, energy-momentum conservation plus vanishing torsion implies $d\Lambda = 0$, ie. a cosmological constant. Vanishing torsion also makes the first- and second-order formulations equivalent.

Non-varying Λ (in a different sense)

$$S^{\mathrm{GR}}[e^a, \omega^{ab}, \psi] = \frac{1}{8\pi G} \int_{\mathcal{M}} \epsilon_{abcd} \Big(e^a \wedge e^b \wedge R^{cd}(\omega) - \frac{\Lambda}{6} e^a \wedge e^b \wedge e^c \wedge e^d \Big) + S^{\mathrm{matter}}[e^a, \psi] \, .$$

In the GR action, Λ is treated as a parameter which can only be fixed through comparison to observations.

The action is ***not*** extremized wrt variations in Λ, which would generate only unphysical zero-volume solutions.

What can we do to change this?

Our core ideas

- The presence of torsion could allow Λ to have space-time variation.
- Minimal disruption to Einstein gravity occurs if we adopt the `quasi-topological principle': introduce new terms in Λ which are topological when Λ is constant.
- Such terms can be strongly restricted by imposing duality symmetries, especially if we require self-dual solutions.
- The above causes the argument that the action cannot be varied wrt Λ to break down, ***reducing*** the degrees of freedom of the system.

Phenomenological torsion for self-dual solutions

Consider self-dual solutions $\mathcal{J}^{cd} = 0$, which hold only in vacuum. Then

$$\epsilon_{abcd}\left\{e^b \wedge \left[\frac{d\Lambda}{3} \wedge e^c \wedge e^d + \frac{2\Lambda}{3} T^c \wedge e^d\right] + T^b \wedge J^{cd}\right\} + \mathcal{D}\tau_a = 0$$

is automatically solved provided

$$T^a = -\frac{d\Lambda}{2\Lambda} \wedge e^a$$

Now there are self-dual vacuum solutions for Λ with arbitrary space-time variation $\Lambda(x,t)$, with the corresponding torsion given by the above formula.

But these `solutions' cannot be solutions to GR, because it does not support torsion. What theory are they solutions to?

GR again

$$S^{\mathrm{GR}}[e^a, \omega^{ab}, \psi] = \frac{1}{8\pi G} \int_{\mathcal{M}} \epsilon_{abcd} \left(e^a \wedge e^b \wedge R^{cd}(\omega) - \frac{\Lambda}{6} e^a \wedge e^b \wedge e^c \wedge e^d \right) + S^{\mathrm{matter}}[e^a, \psi] \, .$$

GR narrowly fails to exhibit a duality symmetry between the frame fields and the curvature, of the form

$$R^{ab} \leftrightarrow \frac{\Lambda}{3} e^a \wedge e^b$$

This is particularly evident in the Plebański formulation of GR where the frame fields are replaced by an area two-form

$$\Sigma^{AB} \stackrel{\mathrm{def}}{=} e^A \wedge e^B .$$

Our new theory

We can instate this duality by adding an extra term to the action

$$S^{\mathrm{GR+new}} = \frac{1}{8\pi G}\int_{\mathcal{M}} \epsilon_{abcd}\Big(e^a \wedge e^b \wedge R^{cd}(\omega) - \frac{\Lambda}{6} e^a \wedge e^b \wedge e^c \wedge e^d - \frac{3}{2\Lambda} R^{ab}(\omega) \wedge R^{cd}(\omega)\Big)$$

We now do have invariance under

$$R^{ab} \leftrightarrow \frac{\Lambda}{3} e^a \wedge e^b$$

which moreover makes the self-dual solutions self-dual (in a different sense).

Notice that the new term is precisely specified - there are no new free parameters.

Our new theory

$$S^{\mathrm{GR+new}} = \frac{1}{8\pi G}\int_{\mathcal{M}} \epsilon_{abcd}\Big(e^a\wedge e^b\wedge R^{cd}(\omega) - \frac{\Lambda}{6}e^a\wedge e^b\wedge e^c\wedge e^d - \frac{3}{2\Lambda}R^{ab}(\omega)\wedge R^{cd}(\omega)\Big)$$

The new term is related to the Gauss-Bonnet invariant

$$I^{\mathrm{GB}} = -\int_{\mathcal{M}} \epsilon_{abcd} R^{ab}\wedge R^{cd}$$

If Λ were constant, this term is purely topological and GR is recovered. But we will not assume that.
Nevertheless the Einstein equations are unchanged as the new term does not feature the frame fields.

Our new theory

$$S^{\mathrm{GR+new}} = \frac{1}{8\pi G}\int_{\mathcal{M}} \epsilon_{abcd}\Big(e^a \wedge e^b \wedge R^{cd}(\omega)$$
$$-\frac{\Lambda}{6}e^a \wedge e^b \wedge e^c \wedge e^d - \frac{3}{2\Lambda}R^{ab}(\omega)\wedge R^{cd}(\omega)\Big)$$

The connection equation of motion now gives

$$T^{[a} \wedge e^{b]} = -\frac{3}{2\Lambda^2} d\Lambda \wedge R^{ab}$$

which for self-dual solutions matches the phenomenological torsion we invoked earlier!

This is the theory that supports our new varying Λ solutions with torsion.

The Λ equation of motion

$$S^{\mathrm{GR+new}} = \frac{1}{8\pi G}\int_{\mathcal{M}} \epsilon_{abcd}\Big(e^a \wedge e^b \wedge R^{cd}(\omega) - \frac{\Lambda}{6} e^a \wedge e^b \wedge e^c \wedge e^d - \frac{3}{2\Lambda} R^{ab}(\omega) \wedge R^{cd}(\omega)\Big)$$

A new ingredient is that we can now vary the action wrt Λ. This gives

$$\frac{\Lambda^2}{9} = \frac{\epsilon_{abcd} R^{ab} \wedge R^{cd}}{e^4}$$ with $$e^4 \equiv \epsilon_{abcd}\left(e^a \wedge e^b \wedge e^c \wedge e^d\right)$$

For self-dual vacuum solutions this equation is automatically satisfied. But otherwise it ties Λ to the other sources of gravity determining R^{ab}.

Hence the Λ degree of freedom is eliminated, simplifying the theory with respect to GR+Λ!!

The (lack of a) GR limit

$$S^{\mathrm{GR+new}} = \frac{1}{8\pi G}\int_{\mathcal{M}} \epsilon_{abcd}\Big(e^a\wedge e^b\wedge R^{cd}(\omega) - \frac{\Lambda}{6}e^a\wedge e^b\wedge e^c\wedge e^d - \frac{3}{2\Lambda}R^{ab}(\omega)\wedge R^{cd}(\omega)\Big)$$

Choosing to vary the action wrt Λ makes the GR limit subtle.

- There is no new parameter to set to zero in order to restore GR.
- In the constant Λ limit the new term becomes topological, but one still needs to additionally specify that the action will no longer be varied wrt Λ.
- If we do insert an adjustable constant pre-factor θ to the new term (see later), the same additional condition is required in the θ → 0 limit.

Cosmology of the simplest theory

Reduction to (flat) FRW

Frame fields:
$$e^0 = dt$$
$$e^i = a dx^i$$

Torsion ansatz:
$$T^0 = 0$$
$$T^i = -T(t) e^0 \wedge e^i$$

[The torsion ansatz is the general parity-even choice consistent with the FRW symmetries, but there is also a parity-odd option that merits investigation.]

The net effect as far as the connection is concerned is
$$H \rightarrow H + T$$

The Friedmann/fluid equations

$$
\begin{aligned}
\left(\frac{\dot a}{a} + T\right)^2 &= \frac{\Lambda + \kappa\rho}{3} && \text{Friedmann} \\
T &= \frac{\dot\Lambda}{2\Lambda}\left(1 + \frac{\kappa\rho}{\Lambda}\right) && \text{Connection} \\
\dot\rho + 3\frac{\dot a}{a}(\rho + p) &= 0 && \text{E-M conservation} \\
(\Lambda + \kappa\rho)\left(\Lambda - \frac{\kappa}{2}(\rho + 3p)\right) &= \Lambda^2. && \text{Lambda}
\end{aligned}
$$

Notice that the fluid energy-momentum conservation is with respect to the torsion-free (metric-compatible) connection. This follows both from general principles and from tedious algebra.

The Λ equation is algebraic and ties Λ to the matter content.

Solutions: single fluid

$$\left(\frac{\dot a}{a} + T\right)^2 = \frac{\Lambda + \kappa\rho}{3} \qquad \text{Friedmann}$$

$$T = \frac{\dot\Lambda}{2\Lambda}\left(1 + \frac{\kappa\rho}{\Lambda}\right) \qquad \text{Connection}$$

$$\dot\rho + 3\frac{\dot a}{a}(\rho + p) = 0 \qquad \text{E-M conservation}$$

$$(\Lambda + \kappa\rho)\left(\Lambda - \frac{\kappa}{2}(\rho + 3p)\right) = \Lambda^2. \qquad \text{Lambda}$$

The last equation implies Λ will track the fluid density, c.f. quintessence tracking solutions. With fluid equation of state *w*

$$\Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho + \rho_\Lambda} = \frac{1 + 3w}{2}$$

which can be positive or negative. But it is disastrous for $w = 1/3$.

Solutions: multiple fluids

$$
\begin{aligned}
\left(\frac{\dot a}{a}+T\right)^2 &= \frac{\Lambda+\kappa\rho}{3} && \text{Friedmann}\\
T &= \frac{\dot\Lambda}{2\Lambda}\left(1+\frac{\kappa\rho}{\Lambda}\right) && \text{Connection}\\
\dot\rho+3\frac{\dot a}{a}(\rho+p) &= 0 && \text{E-M conservation}\\
(\Lambda+\kappa\rho)\left(\Lambda-\frac{\kappa}{2}(\rho+3p)\right) &= \Lambda^2. && \text{Lambda}
\end{aligned}
$$

With matter and radiation together we have

$$
\Lambda = \kappa\frac{2\rho_r^2+\rho_m^2+3\rho_m\rho_r}{\rho_m}
$$

with Λ always dominating. Also not good!

Solutions: vacuum energy

$\left(\frac{\dot a}{a}+T\right)^2 = \frac{\Lambda+\kappa\rho}{3}$	Friedmann
$T = \frac{\dot\Lambda}{2\Lambda}\left(1+\frac{\kappa\rho}{\Lambda}\right)$	Connection
$\dot\rho + 3\frac{\dot a}{a}(\rho+p) = 0$	E-M conservation
$(\Lambda+\kappa\rho)\left(\Lambda-\frac{\kappa}{2}(\rho+3p)\right) = \Lambda^2.$	Lambda

If the Universe is dominated by a separate vacuum energy (eg from quantum fields), we find $\Lambda = -\rho_{vac}/2$, i.e a partial cancellation.

Extending the theory

$$S^\theta = \frac{1}{8\pi G}\int_{\mathcal{M}} \epsilon_{abcd}\Big(e^a\wedge e^b\wedge R^{cd}(\omega)$$
$$-\frac{\Lambda}{6}e^a\wedge e^b\wedge e^c\wedge e^d - \frac{3\theta}{2\Lambda}R^{ab}(\omega)\wedge R^{cd}(\omega)\Big)$$

We can generalise the theory by putting a coefficient θ in front of the new term, returning us to the same degrees of freedom as GR+Λ.

$$\left(\frac{\dot a}{a}+T\right)^2 = \frac{\Lambda+\kappa\rho}{3} \qquad \text{Friedmann}$$
$$T = \frac{\theta\dot\Lambda}{2\Lambda}\left(1+\frac{\kappa\rho}{\Lambda}\right) \qquad \text{Connection}$$
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$$\theta(\Lambda+\kappa\rho)\left(\Lambda-\frac{\kappa}{2}(\rho+3p)\right) = \Lambda^2. \qquad \text{Lambda}$$

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Extending the theory

This greatly increases the flexibility of the theory; the algebraic equation for Λ becomes quadratic with a complicated root structure.

$$\frac{\kappa\rho}{\Lambda} = \frac{1}{2}\left[\frac{1-3w}{1+3w} \pm \sqrt{\left(\frac{1-3w}{1+3w}\right)^2 + \frac{8(\theta-1)}{\theta(1+3w)}}\right]$$

Tracking solutions can be reinterpreted as a rescaling of $\kappa = 8\pi G$ (or equivalently as early dark energy), e.g. in a radiation era

$$\bar{\kappa} = \kappa \frac{1+\sqrt{\frac{\theta}{\theta-1}}}{\left[1-2\theta\left(1+\sqrt{\frac{\theta-1}{\theta}}\right)\right]^2}$$

This is unchanged (hence meeting BBN constraints) if $\theta = 9/8$.

Other extensions

If indeed the simplest theory or its θ-extension are unable to match observations, there are other options:

- Replace the Gauss-Bonnet term with the other available topological invariant, the parity-odd Pontryagin invariant

$$S^{\mathrm{new-Pont}} = -\frac{1}{8\pi G}\int_{\mathcal{M}} \frac{3}{2\Lambda} R^{ab} \wedge R_{ab}$$

  which might link Λ to an imbalance of chiral particle creation rates at a mass scale 3×10^{-3} eV.

- Add torsion-squared terms to the action seeking to induce kinetic energy for lnΛ to give it more conventional dynamics.

Other environments

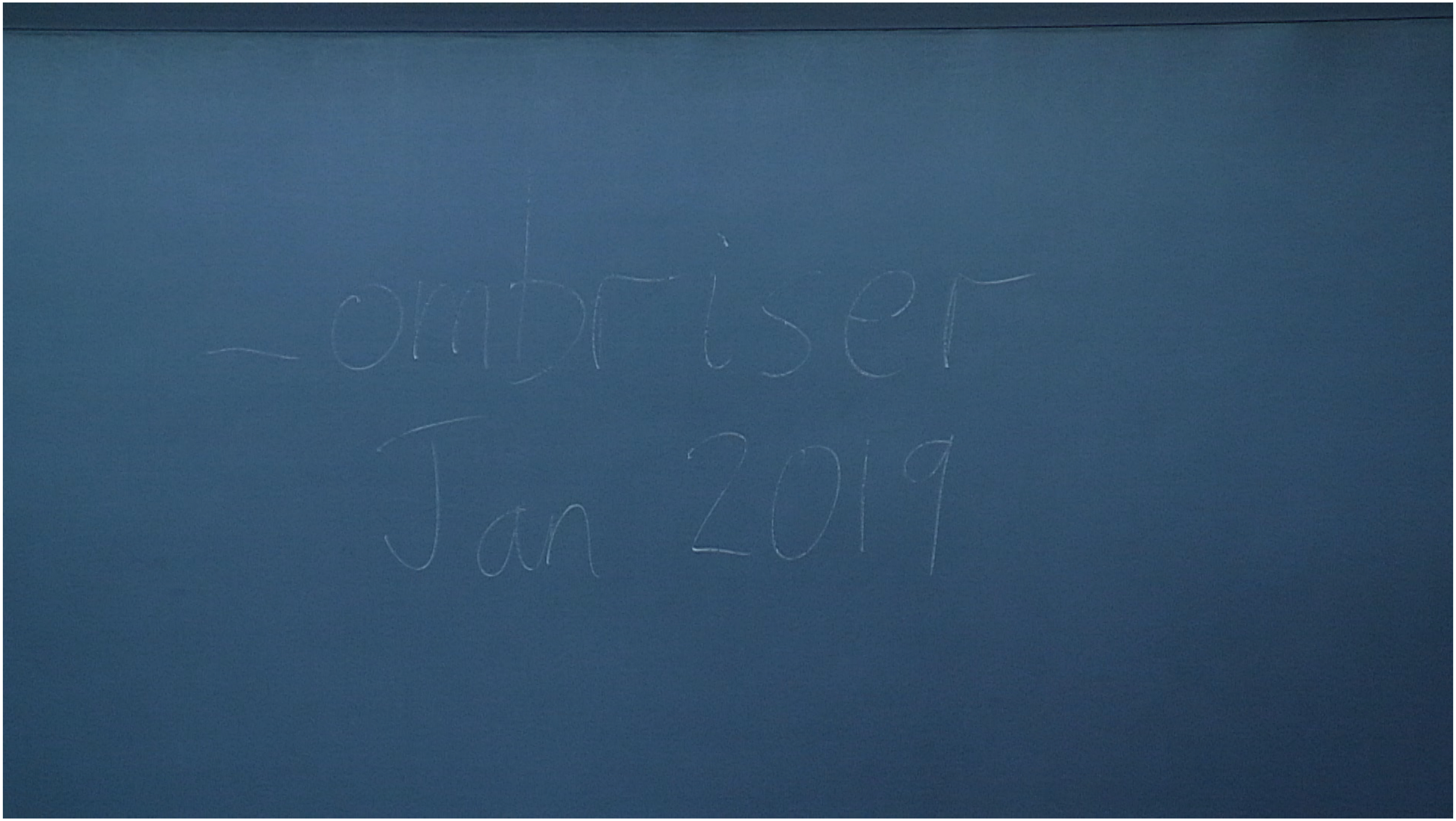
While we continue to investigate cosmological implications of this class of theories, it is important to also develop predictions for

- The Schwarzschild-like solution and Solar System tests.
- Gravitational waves.

In vacuum, the Λ equation of motion imposes a non-trivial constraint on the Weyl tensor, hence we expect modifications to each of these.

Conclusions

- We have proposed a new theory of gravity which in its simplest form has less freedom than GR+Λ!
- This sharply contrasts with usual modified gravity approaches which introduce new parameters and functions.
- Being so specific, it is highly predictive …
- … which however may mean it is already excluded.
- But it is a new starting point from which to develop new theories which might rival GR for simplicity and freedom.
- The general concept of introducing a Λ equation of motion has much wider applicability and may impact the cosmological constant problem.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Jan 2019