Title: Gravitational thermodynamics of causal diamonds

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Series: Quantum Fields and Strings

Date: May 21, 2019 - 2:30 PM

URL: http://pirsa.org/19050036

Abstract: Black hole (more generally, horizon) thermodynamics is a window into quantum gravity. Can horizon thermodynamics---and ultimately quantum gravity---be quasi-localized? A special case is the static patch of de Sitter spacetime, known since the work of Gibbons and Hawking to admit a thermodynamic equilibrium interpretation. It turns out this interpretation requires that a negative temperature is assigned to the state. I'll discuss this example, and its generalization to all causal diamonds in maximally symmetric spacetimes. This story includes a Smarr formula and first law of causal diamonds, analogous to those of black hole mechanics. Iâ€™ll connect this first law to the statement that generalized entropy in a small diamond is maximized in the vacuum at fixed volume.

Black hole entropy

$$S_{\rm gen} = A_{\rm H}/4\hbar G + S_{\rm out}$$

General relativity and quantum field theory ensure that Bekenstein's generalized entropy *locally* satisfies the second law, despite the fact that entropy can be tossed into a black hole.

More precisely, if we include outside field modes up to a cutoff energy Λ,

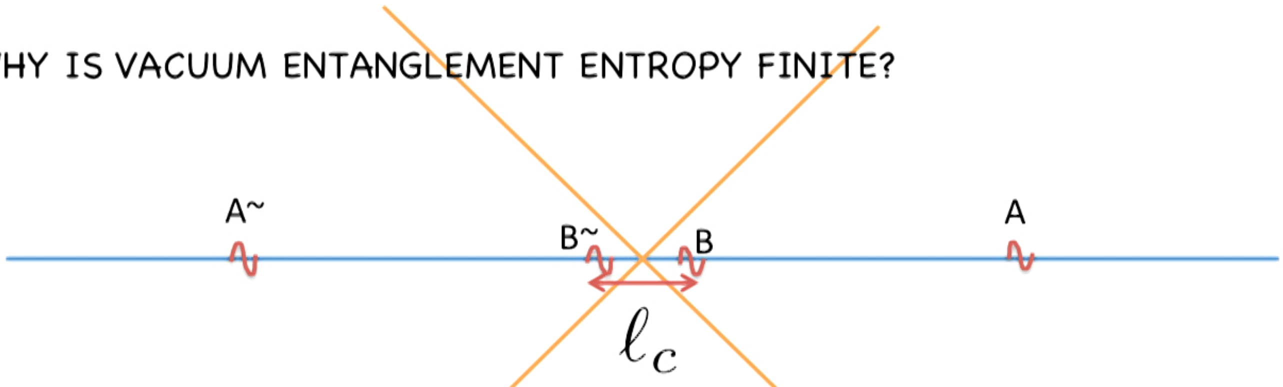
$$S_{\rm gen} = A_{\rm H}/4\hbar G_\Lambda + S_{{\rm out},<\Lambda}$$

...which strongly suggests that black hole entropy is, in some UV completed sense, entanglement entropy of the vacuum, and that area is a measure of entanglement.

Moreover, requiring the Clausius relation $dS_H = dQ/T_U$ for all local acceleration horizons, or, equivalently, stationarity of S_{gen},, implies the Einstein equation.

The vacuum entanglement entropy area density is 1/(4hbar G);
more entanglement implies smaller G, i.e. increased spacetime "stiffness".

WHY IS VACUUM ENTANGLEMENT ENTROPY FINITE?



At ℓ_c scale, energy uncertainty is $\Delta E \sim \hbar/\ell_c$.

Gravity is strong at this scale when $\ell_c \lesssim r_g \sim G\,\Delta E \sim \hbar G/\ell_c$

$$\text{i.e. when } \ell_c \lesssim \ell_P$$

Causal structure fluctuates, blurs subsystem, cutting off entanglement entropy at the Planck scale.

N.B. Cutoff on proper separation of *pairs,* which is Lorentz invariant.

AdS/CFT appears to provide a realization of these dreams:

The Ryu-Takayanagi formula (& its time-dependent generalization) relates CFT entanglement entropy to bulk acceleration horizon entropy, with a *nonzero* Newton constant, 1/G ~ # fields of CFT < ∞.

...and the bulk Einstein equation can be derived from RT formula together with CFT entropy properties.

- *Is the AdS boundary essential?*

- How locally can notions of black hole thermodynamics be applied?

- Thermodynamics of dS static patch?

- A small causal diamond in *any* spacetime is a small deformation of a maximally symmetric causal diamond, and the Einstein equation is equivalent to the first law for such diamonds. Is this because entanglement entropy is maximized in vacuum?

- Can this shed light on the cosmological constant problem?

Origin of the first law of black hole mechanics

$$dM = \frac{\kappa}{8\pi G}\, dA + \Omega_H\, dJ + \Phi\, dQ$$

1972, *Bekenstein*: Varied parameters in the Kerr-Newman solution.

1972, *Bardeen, Carter & Hawking*: Varied the *Smarr Formula*

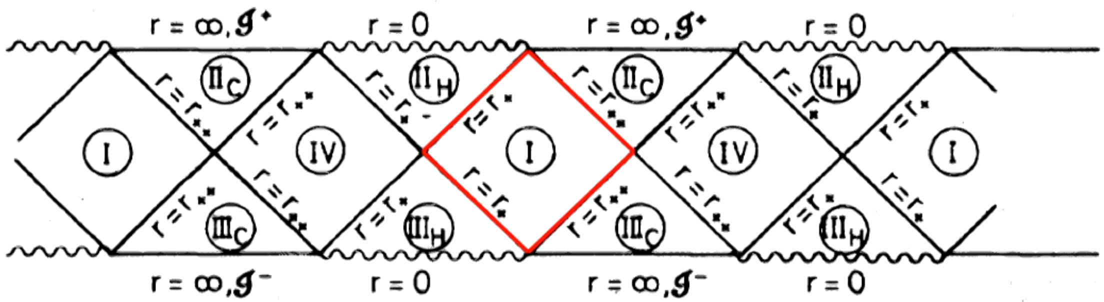
$$M = 2\Omega_H J + \frac{1}{4\pi G}\kappa A + \text{matter terms}$$

which they obtained from a Killing vector identity and the Einstein equation. Showed that κ is an *intensive* variable, the surface gravity, which (like Ω and Φ) is constant on the horizon. (Carter wanted to call it the "decay constant".)

1992, *Sudarsky & Wald*; 1993, *Wald*; 1994, *Iyer & Wald*:
Derived the first law for *any* diffeomorphism invariant theory, with the role of entropy played by the horizon *Noether charge* for the horizon Killing vector; valid for *all* perturbations (not just stationary ones).

The first law in Kerr-de Sitter spacetime

Gibbons & Hawking, 1977



The first law of event horizons.

$$\int \delta T_{ab} K^a d\Sigma^b = -\kappa_c \delta A_C (8\pi)^{-1} - \kappa_H \delta A_H (8\pi)^{-1} - \Omega_H \delta J_H$$

GH obtained this by deriving a Smarr formula, then varying.

The first law in de Sitter spacetime

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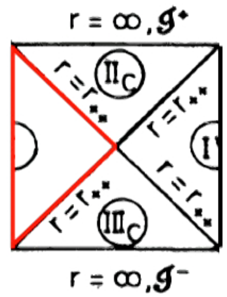

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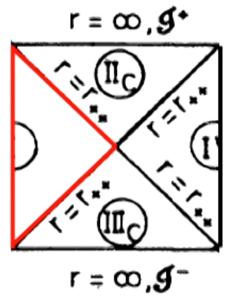

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Negative temperature!
(suggested by
Klemm & Vanzo, 2004);
picked up by nobody…

Negative temperature requires an upper bound to the energy

…and there are independent reasons to think the dS Hilbert space is finite dimensional: finite entropy (Banks & Fischler,…)

But isn't the Gibbons-Hawking temperature of dS *positive*??

Yes, indeed. The dS vacuum of a quantum field is thermal wrt the Hamiltonian generating time translation on the static patch. This was found in the original Gibbons-Hawking paper, and it is a dS analog of the Unruh effect in the Rindler wedge of Minkowski spacetime.

$$T_{\rm GH} = \hbar\kappa_c/2\pi, \quad \kappa_c = H = \sqrt{\Lambda/3}$$

So doesn't this contradict the first law of dS?

No! The matter *entropy adds to the dS horizon entropy*, so that the *generalized entropy is stationary:*

$$\int \delta T_{ab} K^a d\Sigma^b = -\kappa_c \delta A_c (8\pi)^{-1}$$

$$\updownarrow \qquad\qquad \updownarrow$$

$$T_{\rm GH}dS_{\rm m} = -T_{\rm GH}dS_{\rm BH} \implies dS_{\rm gen} = 0 \; !$$


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$$T_{\rm GH} dS_{\rm m} = -T_{\rm GH} dS_{\rm BH} \implies dS_{\rm gen} = 0 \ !$$

Suggests that entropy of the dS vacuum is *maximal*.

Supporting arguments:

1. Schwarzschild-dS $A_C + A_H$ is maximized for $A_H = 0$
   a. at fixed Λ
   b. at fixed volume
   c. at fixed "energy", i.e. fixed Noether charge and $\kappa_c = 1$.
2. Matter has less entropy than a black hole for the same mass, so adding matter doesn't increase entropy.
3. GSL implies that maximal entropy in the cosmological horizon of asymptotically dS is the entropy of the static patch (Bousso's "D-bound")

Related to the *maximal vacuum entanglement hypothesis*, that the generalized entropy of small geodesic balls is maximal at fixed volume in Minkowski spacetime, wrt variations of the state away from the Minkowski vacuum (TJ, 2015).

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Smarr formula from Noetherology

$$\delta L = E\,\delta\phi + d\theta(\phi, \delta\phi)\,,$$

where L is the Lagrangian d-form, E is the equation of motion d-form, and tensor indices are suppressed. From the symplectic potential form θ one constructs the *Noether current* $(d-1)$-form associated with any vector field χ:

$$j_\chi := \theta(\phi, \mathcal{L}_\chi \phi) - \chi \cdot L\,.$$

When L is diffeomorphism covariant, and the equation of motion $E = 0$ holds,

$$j_\chi = dQ_\chi,$$

where the $(d-2)$-form Q_χ is constructed from the dynamical fields together with χ and its first derivative, and is called the *Noether charge* form. The integral yields a generalized Smarr formula,

$$\boxed{\oint_{\partial\mathcal{R}} Q_\chi = \int_{\mathcal{R}} j_\chi.}$$

If χ is a Killing field, and if $L = 0$ on shell, then $j_\chi = 0$, and this leads to the original Smarr formula when applied to a hypersurface bounded by the black hole horizon on one side and the sphere at spatial infinity on the other.

Smarr formula for Schwarzschild-de Sitter

In Schwarzschild-dS we have a Killing field, but $L \neq 0$ on shell because $R \propto \Lambda \neq 0$. Taking the hypersurface Σ between the black hole and cosmological horizons, the extra term is

$$-\int_\Sigma \chi \cdot L = -\frac{\Lambda}{(d-2)4\pi G} V_\chi, \quad \text{where} \quad V_\chi := \int_\Sigma \chi \cdot \epsilon$$

This was present in Gibbons and Hawking and described as "the (negative) contribution of the Λ term to the mass within the cosmological horizon." That is, it contributes as does a matter stress tensor to the "gravitating mass". Another interpretation is that it is proportional to the *pressure* $p_\Lambda = -\Lambda/8\pi G$ times the *thermodynamic volume* V_χ, a quantity that was named by Kastor, Ray, and Traschen in the AdS setting.

First Law from Noetherology

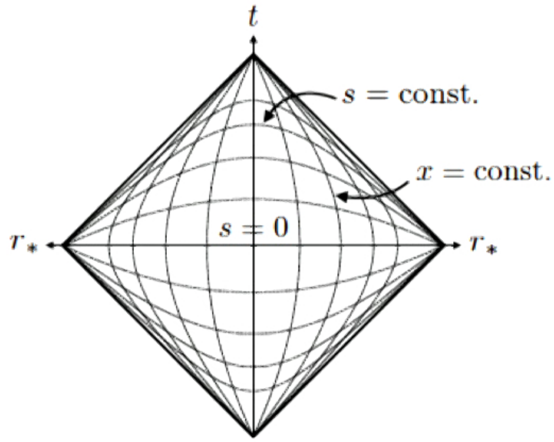
Varying the Noether current away from a background satisfying the equation of motion, and assuming the linearized initial value constraints hold for the variation (i.e. varying to any other point in phase space), one finds (Wald, 1993)

$$\delta H_\chi = \oint_{\partial\Sigma} \left[\delta Q_\chi - \chi \cdot \theta(\phi, \delta\phi)\right] . \tag{1}$$

If χ is a true Killing vector of the background metric and matter fields, then $\delta H_\chi = 0$, so the variational identity reduces to a relation between the boundary integrals. This is how the first law of black hole mechanics arises. In a dS static patch there is only one boundary, where $\chi = 0$, so the variation of that one Noether charge must vanish. Adding classical matter and/or a variable cosmological constant, one picks up volume contributions, because the description involves potentials that are not invariant under the Killing flow.

Maximally symmetric causal diamonds

(TJ, 2015; TJ & Manus Visser, 2018)



Except in certain limits, they admit only a *conformal* Killing vector. The metric can be presented as a conformal factor times (hyperbolic space) x (time):

$$ds^2 = C^2(s,x)[-ds^2 + dx^2 + \rho^2(x)d\Omega^2_{d-2}]$$

$$C^{-1} = \frac{\cosh s + \cosh(x)\cosh(R_*/L)}{L\sinh(R_*/L)}, \qquad \rho = \sinh x$$

$$\zeta = \partial_s$$

is a conformal Killing vector, with unit surface gravity, and is an "instantanous" true Killing vector at s = 0.

Remarkably, the slices of constant s form a CMC foliation:

$$K = \frac{1-d}{L\sinh(R_*/L)}\sinh s = (d-1)\dot{\alpha}|_{s=0}\sinh s$$

First law for maximally symmetric causal diamonds

(TJ, 2015; TJ & Manus Visser, 2018)

A Smarr formula and a First Law can be derived using the diff. Noether current à la Wald. First Law has an additional term, since ckv not a kv:

$$\delta H_\zeta^{\mathrm{matter}} = \frac{1}{8\pi G}\Big(-\kappa(\delta A - k\,\delta V) - V_\zeta\,\delta\Lambda\Big)$$

V is the ball's volume, k is the outward extrinsic curvature of its edge. In dS, k = 0.

The volume term is $-\delta H_\zeta^{\mathrm{grav}}$; it has this geometric form thanks to a minor miracle: d(div ς) has constant norm ~ κk on Σ (i.e. at s = 0).
That it is proportional to the *volume* variation is presumably related to the *York time* (-K) Hamiltonian being the volume.

The last term is the *thermodynamic volume* $V_\zeta = \int_\Sigma \zeta\cdot\epsilon$ times the pressure variation.

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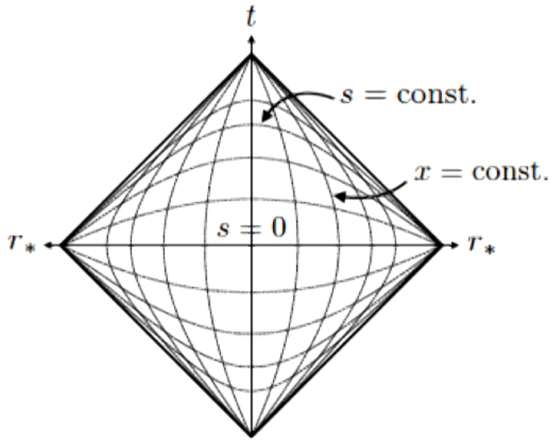

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Comments on the First Law

- The diamond has negative temperature, like the dS static patch.
- The volume of the ball can be defined in a gauge invariant way as the volume of the *maximal slice* with fixed boundary. This might be important when extending this relation to a second order variation.
- The variation of area at fixed volume has a *deficit*, while the variation of volume at fixed area has an *excess.*
- A "small" diamond in an arbitrary spacetime can be viewed as a variation of a maximally symmetric space, and this variation must satisfy the first law if the spacetime is a solution to Einstein's eqn. Conversely, these first laws imply the Einstein equation.

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Comments on the First Law, contd.

- For *conformal* matter, Hamiltonian variation can be traded for an entanglement entropy variation, which combines with the area to make the *generalized entropy* variation.

- The trade works also for *non*-conformal matter in *small* diamonds*, if a particular variation of the (local) cosmological constant is included.*

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- The trade works also for *non*-conformal matter in *small* diamonds, *if a particular variation of the (local) cosmological constant is included.*

$$\delta\langle H\rangle_\zeta^{\mathrm{matter}} = T_H \delta S^{\mathrm{matter}} - V_\zeta\, \delta X$$

This relation was checked by Speranza, and by Casini, Galante & Myers. In the last term X depends on the size of the ball. Using this,

$$T\delta S_{\mathrm{gen}} = \frac{1}{8\pi G}\left[-\kappa k \delta V + V_\zeta \underbrace{(\delta\lambda - 8\pi G \delta X)}_{\delta\Lambda}\right]$$

If $\delta\Lambda = 0$, S_{gen} is stationary: "entanglement equilibrium"

Second order variation

$$\delta^2 H_\zeta = \oint_{\partial\Sigma} \delta^2 Q_\zeta = -\frac{\kappa}{8\pi G}\delta^2 A - \Psi$$

$$\Psi = \frac{\kappa}{64\pi G}\oint_{\partial\Sigma}(u^a u^b \delta g_{ab} + r^a r^b \delta g_{ab})^2 \geq 0$$

[Jacobson, Senovilla, Speranza; 2017]

In thermodynamic language:

$$\delta^2 E - T\delta^2 S \leq 0$$

It seems free energy is maximized!

True for negative temperature systems ✓

Quantum Corrections

- $\Psi = 0$ for variations that keep ζ tangent to the lightcone. Degeneracy?
- Conformal matter: H_ζ^m = modular Hamiltonian. $\delta\langle H_\zeta^m\rangle = \frac{\kappa}{8\pi G}\delta S_m$
- Quantum corrected first law $\delta H_\zeta^{g+\Lambda} = T\delta S_{\text{gen}}$

  (Also valid for non-conformal matter [Jacobson, Visser; 2018])

  Generalized entropy $S_{\text{gen}} \equiv \frac{1}{4G}A + S_m$
- "Energy" term gets contributions from geometry and Λ.
- Clearer connection to entanglement equilibrium setup: fixed volume and cosmological constant $\Rightarrow$ Einstein equations.

Concavity of conformal free energy

$$\delta^2 H_\zeta^{g+\Lambda} = \delta^2 H_\zeta - \delta^2 H_\zeta^m = -\frac{\kappa}{8\pi G}\delta^2 A - \Psi - \delta^2 \langle H_\zeta^m \rangle$$

For conformal matter:

$$\delta^2 \langle H_\zeta^m \rangle = \frac{\kappa}{2\pi}\delta^2 S_m + \Phi \quad , \quad \Phi > 0$$

e.g. [Kelly, Kuns, Marolf; 2015]

$$\Rightarrow \delta^2 H_\zeta^{g+\Lambda} = T\delta^2 S_{\text{gen}} - \underbrace{(\Psi + \Phi)}_{>0}$$

$$F \equiv H_\zeta^{g+\Lambda} - T\,S_{\text{gen}} \qquad \delta^2 F < 0$$

Clue to thermodynamic stability!

Remainder Questions

- Non-conformal matter.
- "Cross-terms" in second variation of total entropy; higher order?
  - change in matter entropy due to variations of geometry.
  - change in geometry entropy due to variations of quantum states.
- Correct thermodynamic ensemble for diamond: path integral.

Thank you.

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