Title: Classical Spinning Black Holes From Scattering Amplitudes

Speakers: Alfredo Guevara

Series: Strong Gravity

Date: May 30, 2019 - 1:00 PM

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Abstract: Following the advent of LIGO measurements, it has been recently observed that QFT amplitudes can be used to derive observables appearing in the scattering of two black holes, to very high orders in perturbation theory. Such framework easily fits into the Post-Newtonian and Post-Minkowskian expansions appearing in the treatment of the binary inspiral. In this talk we will review recent progress in this direction for the case of spinning black holes, focusing on radiation and the multipole expansion. From the QFT point of view these are in close relation to long-studied Soft Theorems.

Scattering Amplitudes & Spinning Black Holes

Alfredo Guevara (PI)

30/05/19

1812.06895 w/ A. Ochirov (ETH) & J. Vines (AEI)
1903.12419 w/ F. Bautista (York/PI)
1906.XXXX

GW Catalogue (1811.12907)

Event	$m_1/\mathrm{M}_\odot$	$m_2/\mathrm{M}_\odot$	$\mathcal{M}/\mathrm{M}_\odot$	χ_{eff}	$M_\mathrm{f}/\mathrm{M}_\odot$	a_f	$E_\mathrm{rad}/(\mathrm{M}_\odot c^2)$	$\ell_\mathrm{peak}/(\mathrm{erg\,s}^{-1})$	d_L/Mpc	z	$\Delta\Omega/\mathrm{deg}^2$
GW150914	$35.6^{+4.8}_{-3.0}$	$30.6^{+3.0}_{-4.4}$	$28.6^{+1.6}_{-1.5}$	$-0.01^{+0.12}_{-0.13}$	$63.1^{+3.3}_{-3.0}$	$0.69^{+0.05}_{-0.04}$	$3.1^{+0.4}_{-0.4}$	$3.6^{+0.4}_{-0.4} \times 10^{56}$	430^{+150}_{-170}	$0.09^{+0.03}_{-0.03}$	180
GW151012	$23.3^{+14.0}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	$15.2^{+2.0}_{-1.1}$	$0.04^{+0.28}_{-0.19}$	$35.7^{+9.9}_{-3.8}$	$0.67^{+0.13}_{-0.11}$	$1.5^{+0.5}_{-0.5}$	$3.2^{+0.8}_{-1.7} \times 10^{56}$	1060^{+540}_{-480}	$0.21^{+0.09}_{-0.09}$	1555
GW151226	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.6}$	$8.9^{+0.3}_{-0.3}$	$0.18^{+0.20}_{-0.12}$	$20.5^{+6.4}_{-1.5}$	$0.74^{+0.07}_{-0.05}$	$1.0^{+0.1}_{-0.2}$	$3.4^{+0.7}_{-1.7} \times 10^{56}$	440^{+180}_{-190}	$0.09^{+0.04}_{-0.04}$	1033
GW170104	$31.0^{+7.2}_{-5.6}$	$20.1^{+4.9}_{-4.5}$	$21.5^{+2.1}_{-1.7}$	$-0.04^{+0.17}_{-0.20}$	$49.1^{+5.2}_{-3.9}$	$0.66^{+0.08}_{-0.10}$	$2.2^{+0.5}_{-0.5}$	$3.3^{+0.6}_{-0.9} \times 10^{56}$	960^{+430}_{-410}	$0.19^{+0.07}_{-0.08}$	924
GW170608	$10.9^{+5.3}_{-1.7}$	$7.6^{+1.3}_{-2.1}$	$7.9^{+0.2}_{-0.2}$	$0.03^{+0.19}_{-0.07}$	$17.8^{+3.2}_{-0.7}$	$0.69^{+0.04}_{-0.04}$	$0.9^{+0.05}_{-0.1}$	$3.5^{+0.4}_{-1.3} \times 10^{56}$	320^{+120}_{-110}	$0.07^{+0.02}_{-0.02}$	396
GW170729	$50.6^{+16.6}_{-10.2}$	$34.3^{+9.1}_{-10.1}$	$35.7^{+6.5}_{-4.7}$	$0.36^{+0.21}_{-0.25}$	$80.3^{+14.6}_{-10.2}$	$0.81^{+0.07}_{-0.13}$	$4.8^{+1.7}_{-1.7}$	$4.2^{+0.9}_{-1.5} \times 10^{56}$	2750^{+1350}_{-1320}	$0.48^{+0.19}_{-0.20}$	1033
GW170809	$35.2^{+8.3}_{-6.0}$	$23.8^{+5.2}_{-5.1}$	$25.0^{+2.1}_{-1.6}$	$0.07^{+0.16}_{-0.16}$	$56.4^{+5.2}_{-3.7}$	$0.70^{+0.08}_{-0.09}$	$2.7^{+0.6}_{-0.6}$	$3.5^{+0.6}_{-0.9} \times 10^{56}$	990^{+320}_{-380}	$0.20^{+0.05}_{-0.07}$	340
GW170814	$30.7^{+5.7}_{-3.0}$	$25.3^{+2.9}_{-4.1}$	$24.2^{+1.4}_{-1.1}$	$0.07^{+0.12}_{-0.11}$	$53.4^{+3.2}_{-2.4}$	$0.72^{+0.07}_{-0.05}$	$2.7^{+0.4}_{-0.3}$	$3.7^{+0.4}_{-0.5} \times 10^{56}$	580^{+160}_{-210}	$0.12^{+0.03}_{-0.04}$	87
GW170817	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	$1.186^{+0.001}_{-0.001}$	$0.00^{+0.02}_{-0.01}$	≤ 2.8	≤ 0.89	≥ 0.04	$\geq 0.1 \times 10^{56}$	40^{+10}_{-10}	$0.01^{+0.00}_{-0.00}$	16
GW170818	$35.5^{+7.5}_{-4.7}$	$26.8^{+4.3}_{-5.2}$	$26.7^{+2.1}_{-1.7}$	$-0.09^{+0.18}_{-0.21}$	$59.8^{+4.8}_{-3.8}$	$0.67^{+0.07}_{-0.08}$	$2.7^{+0.5}_{-0.5}$	$3.4^{+0.5}_{-0.7} \times 10^{56}$	1020^{+430}_{-360}	$0.20^{+0.07}_{-0.07}$	39
GW170823	$39.6^{+10.0}_{-6.6}$	$29.4^{+6.3}_{-7.1}$	$29.3^{+4.2}_{-3.2}$	$0.08^{+0.20}_{-0.22}$	$65.6^{+9.4}_{-6.6}$	$0.71^{+0.08}_{-0.10}$	$3.3^{+0.9}_{-0.8}$	$3.6^{+0.6}_{-0.9} \times 10^{56}$	1850^{+840}_{-840}	$0.34^{+0.13}_{-0.14}$	1651

$$\chi_{eff} = \frac{\left(m_1\vec{\chi}_1 + m_2\vec{\chi}_2\right)\cdot\vec{L}_N}{m_1+m_2}$$

GW Catalogue (1811.12907)

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- **As more precise measurements will take place (i.e. LISA), more accurate templates are needed (i.e. through EOB)**

Quantum Field Theory

Black Hole Coalescence

Quantum Field Theory

CFT, Defects, SUSY, etc...

Black Hole Coalescence

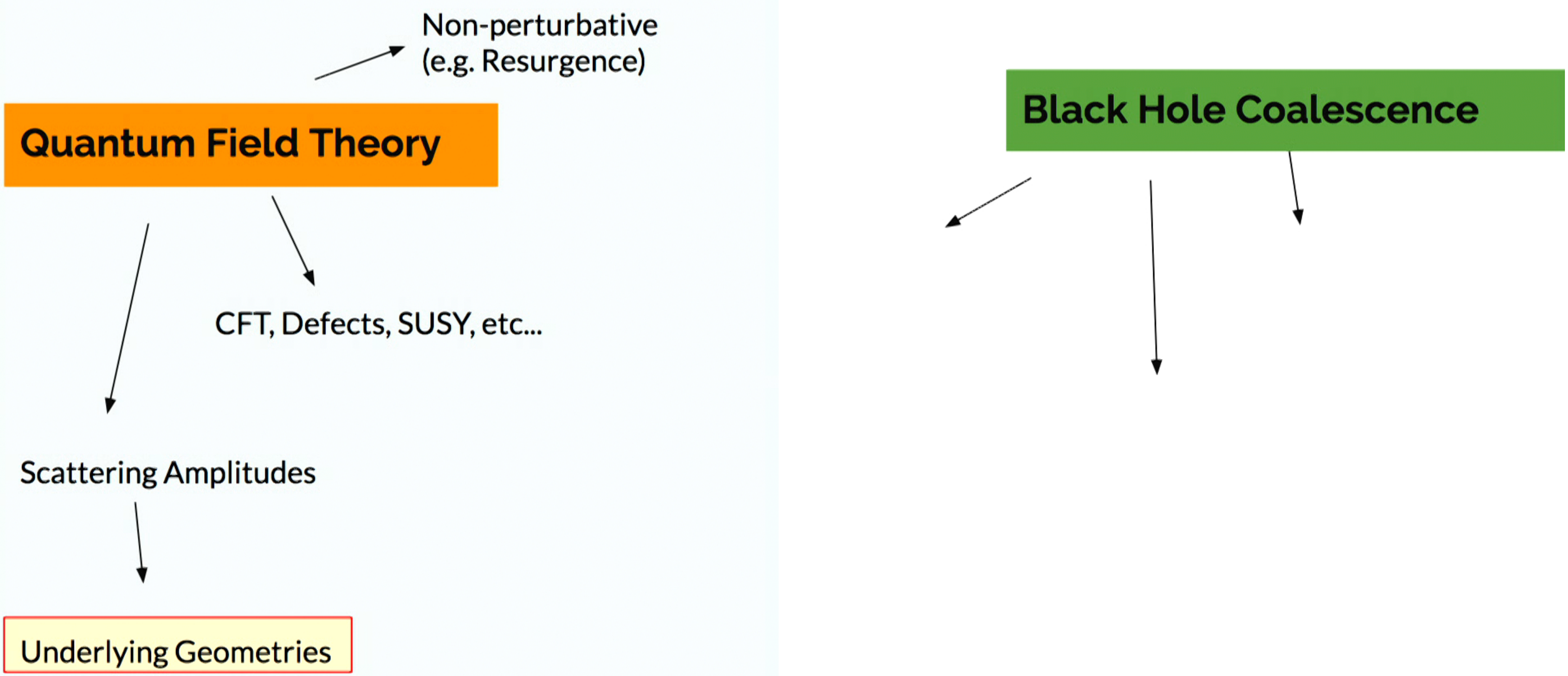
Non-perturbative
(e.g. Resurgence)

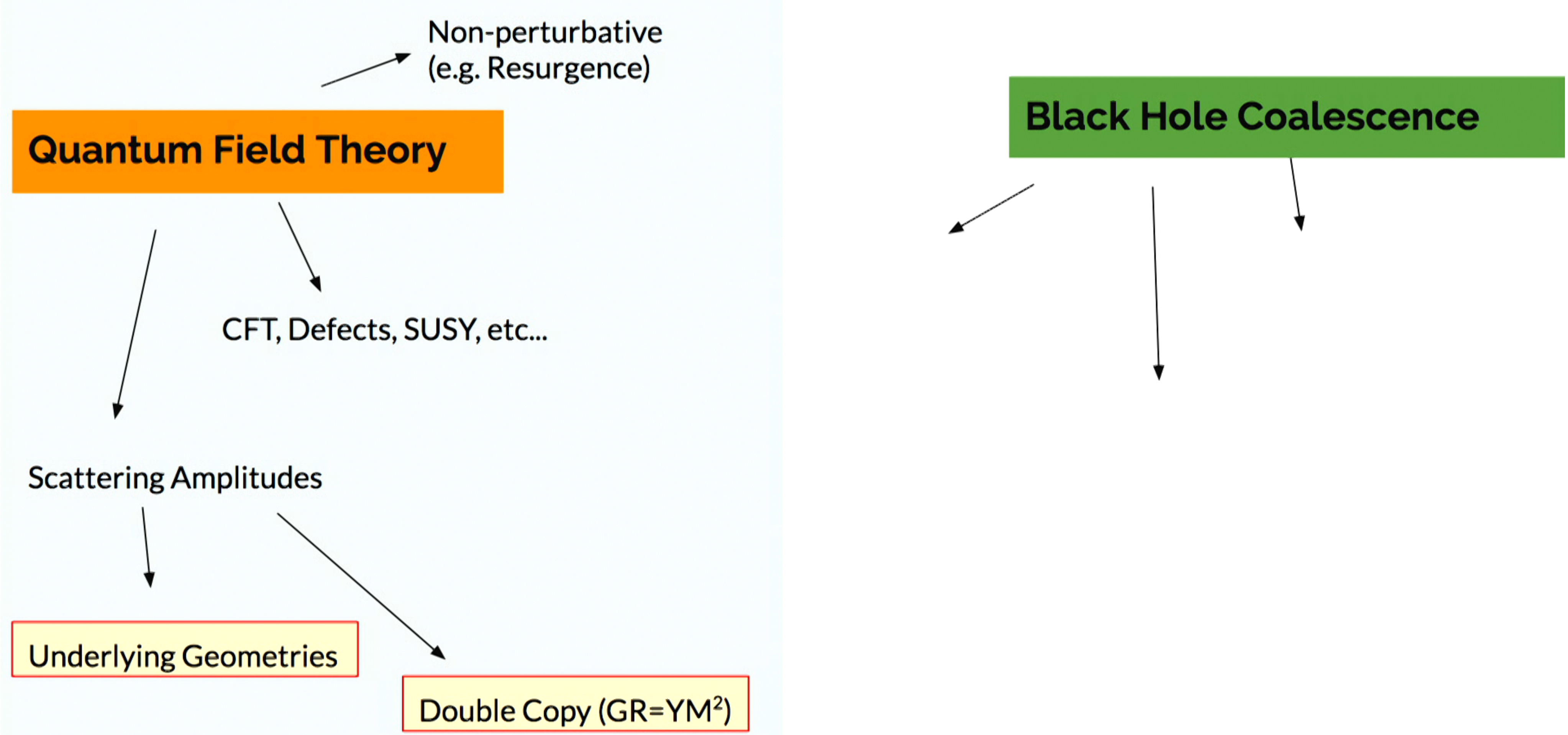
Quantum Field Theory

CFT, Defects, SUSY, etc...

Scattering Amplitudes

Black Hole Coalescence


Non-perturbative
(e.g. Resurgence)
Quantum Field Theory
CFT, Defects, SUSY, etc...
Scattering Amplitudes
Underlying Geometries
Black Hole Coalescence


Non-perturbative
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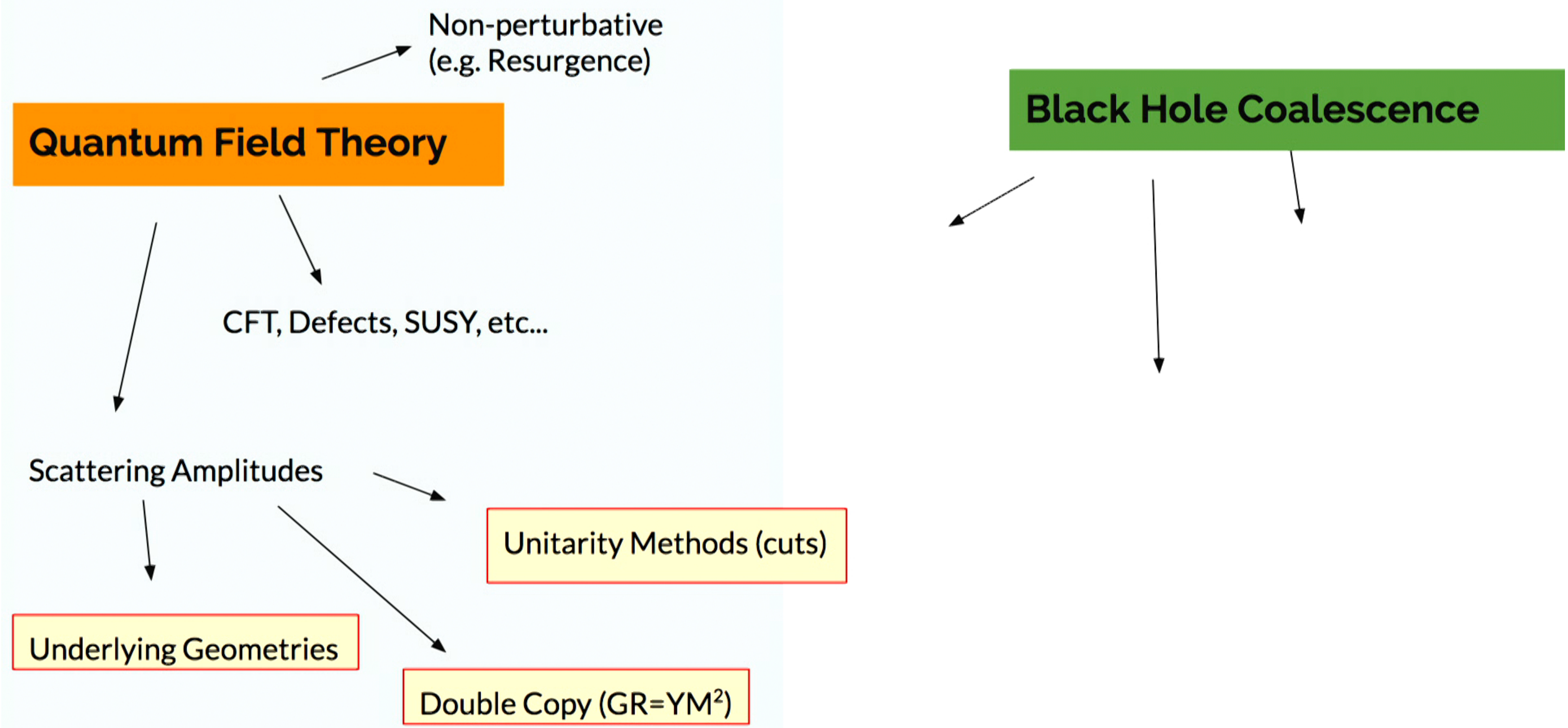
Double Copy (GR=YM²)
Black Hole Coalescence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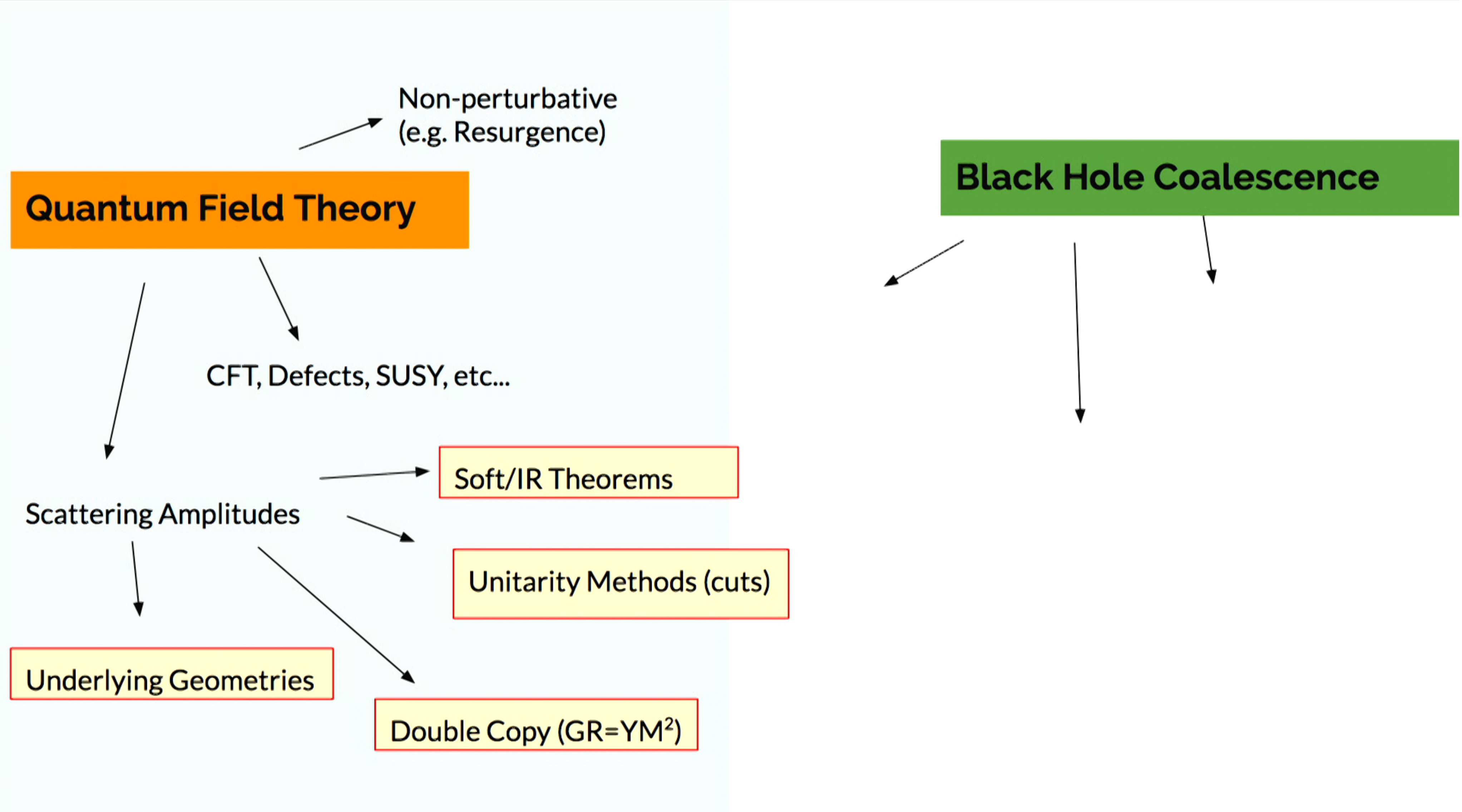
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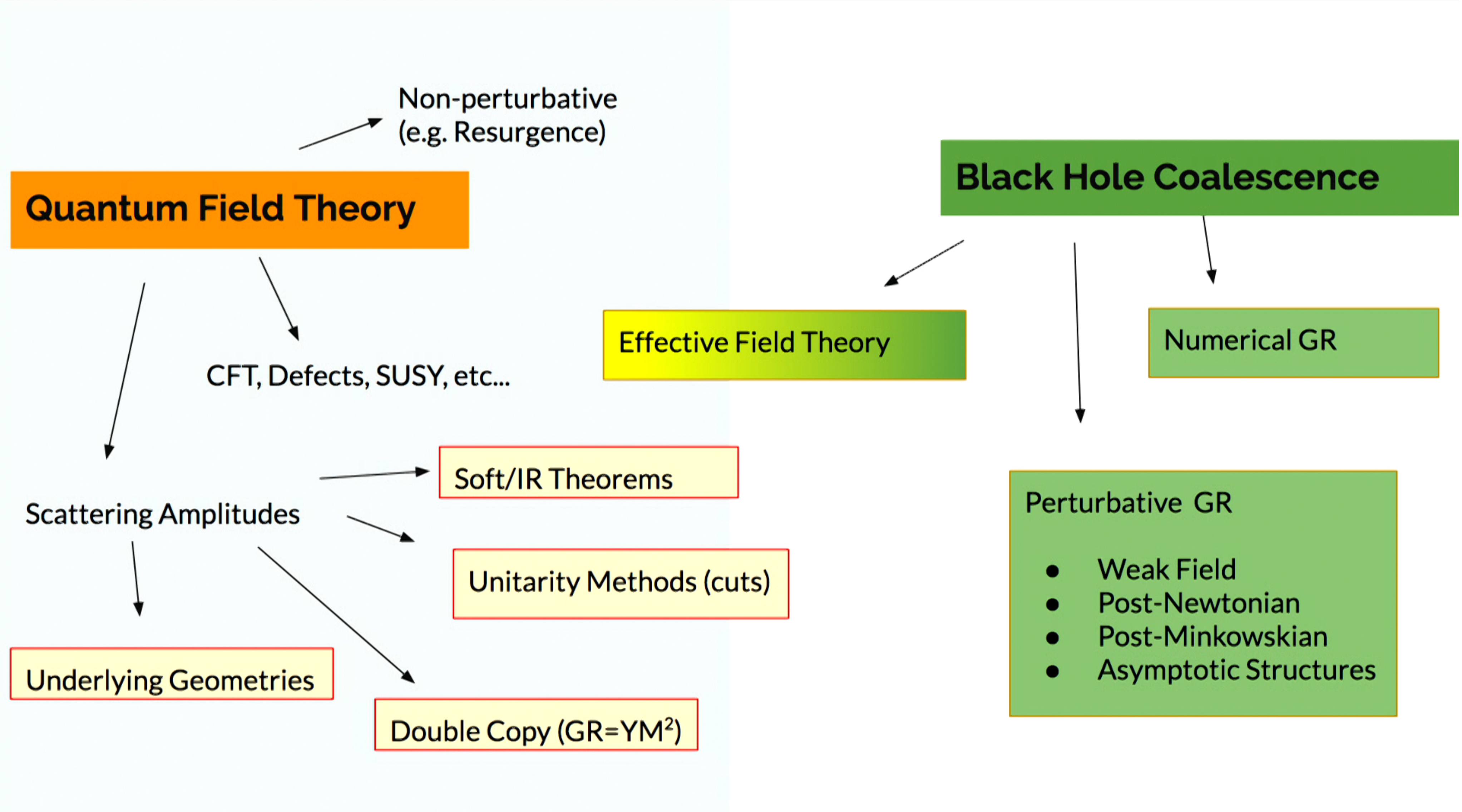
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Effective Field Theory
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Soft/IR Theorems
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Perturbative GR
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Post-Newtonian
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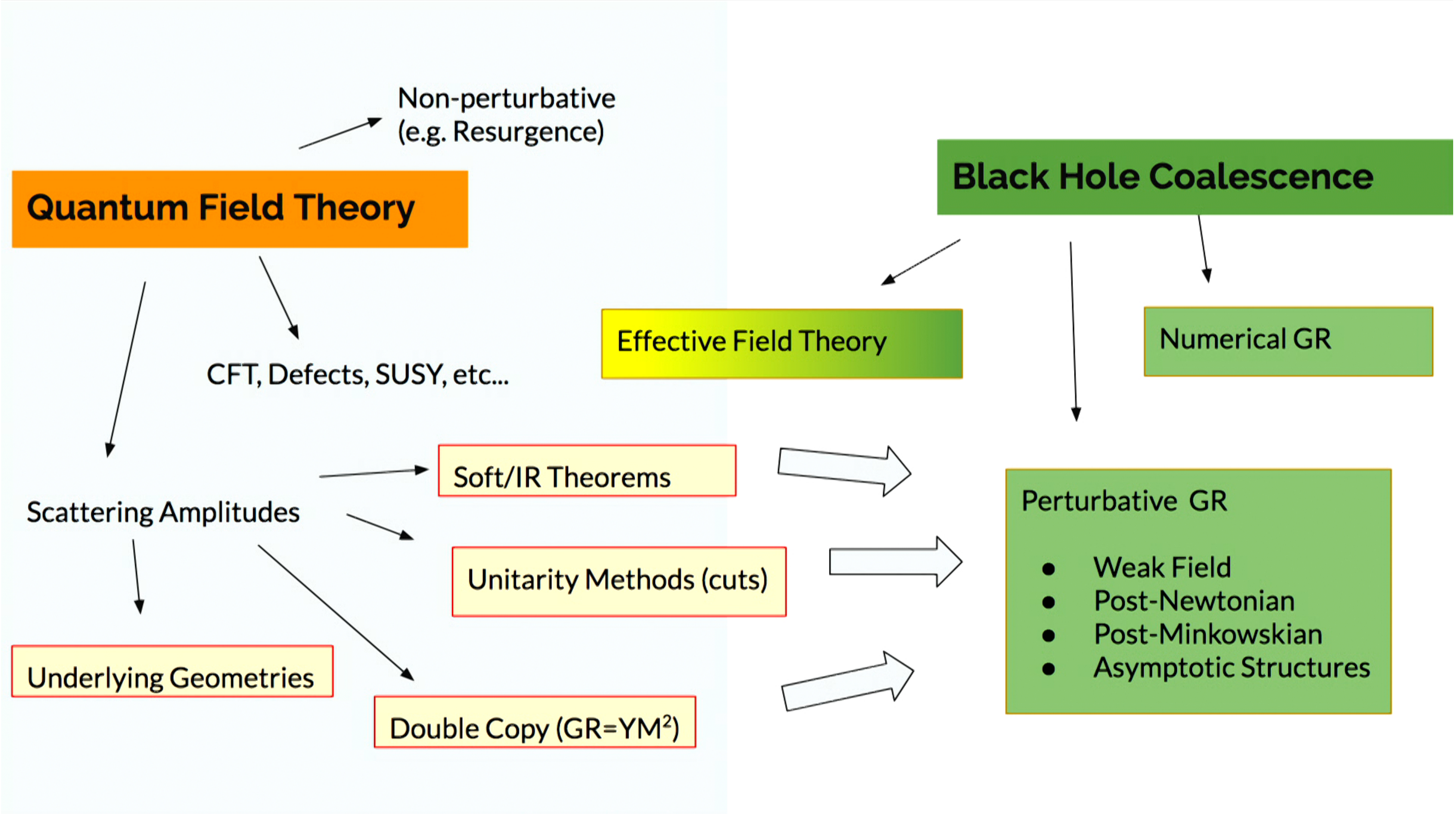
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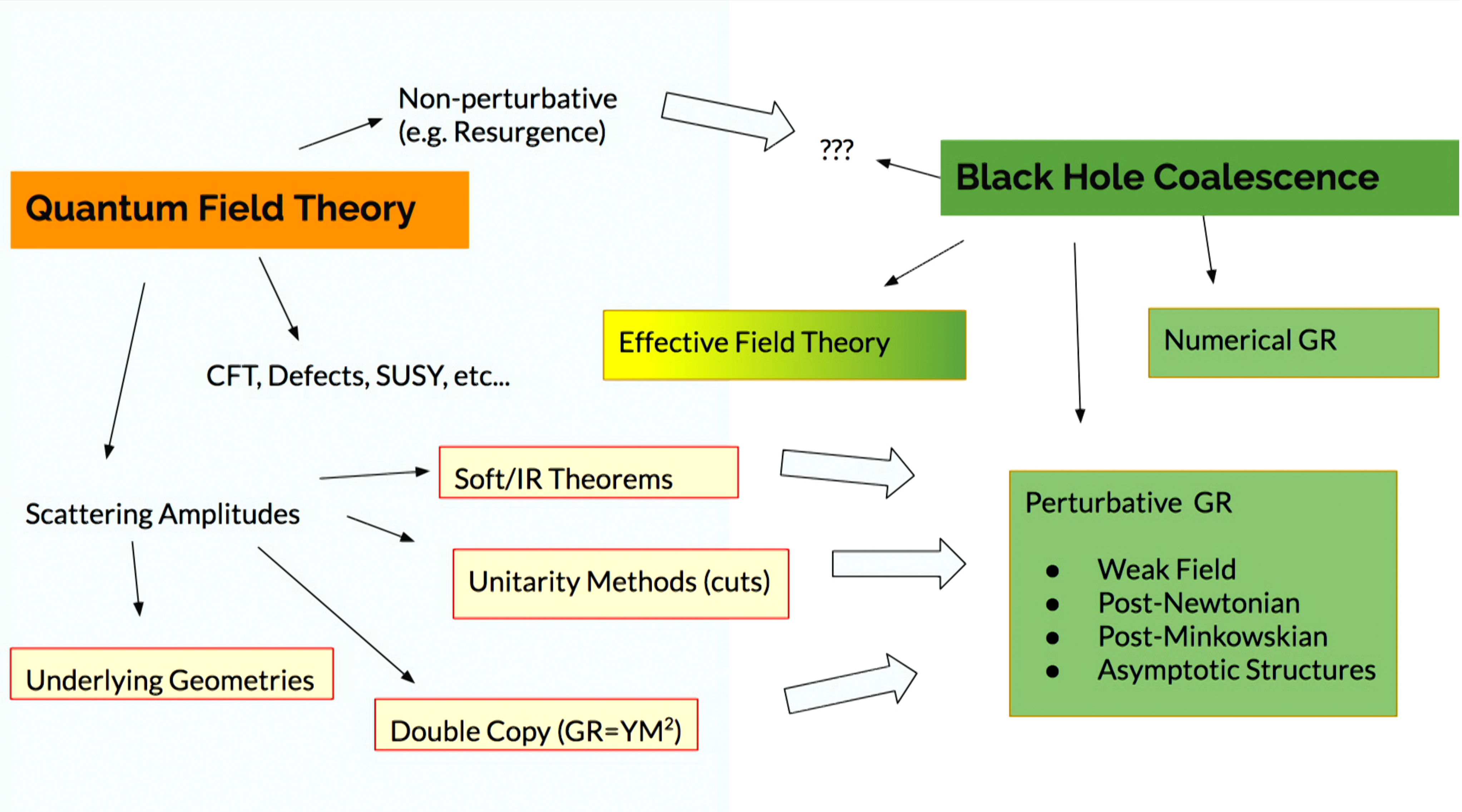
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???
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From 2-Body Problem to $2 \to 2$ Scattering

- In the PN case one looks for the potential (off-shell) modes, i.e.

$$\ell \sim (v/r, 1/r)$$

leading to an instantaneous long-range potential valid in the near zone $r \leq \lambda_{\rm rad}$

- **Post-Minkowskian** expansion gives
  - The radiation field in far zone region $r \gg \lambda$
  - A resummation of PN orders

Limit	Perturbation theory
Newtonian gravity $c \to \infty$	post-Newtonian $\frac{m_1}{m_2} \sim 1, \quad \frac{Gm}{rc^2} \sim \frac{v^2}{c^2} \ll 1$
special relativity $G \to 0$	post-Minkowskian $\frac{m_1}{m_2} \sim 1, \quad \frac{Gm}{rc^2} \ll \frac{v^2}{c^2} \sim 1$

		0PN	1PN	$\frac{3}{2}$PN	2PN	$\frac{5}{2}$PN	3PN	$\frac{7}{2}$PN	4PN	$\frac{9}{2}$PN	5PN
spin^0:	0PM:	v^2	v^4		v^6		v^8		v^{10}		v^{12}
	1PM:	$1/r$	v^2/r		v^4/r		v^6/r		v^8/r		v^{10}/r
	2PM:		$1/r^2$		v^2/r^2		v^4/r^2		v^6/r^2‡		v^8/r^2‡
	3PM:				$1/r^3$		v^2/r^3		v^4/r^3		v^6/r^3
	4PM:						$1/r^4$		v^2/r^4		v^4/r^4
	…								…		…
spin^1:	1PM:			va/r^2		v^3a/r^2		v^5a/r^2		v^7a/r^2	
	2PM:					va/r^3		v^3a/r^3		av^5/r^3	
	3PM:							va/r^4		v^3a/r^4	
	…									…	

→ Known! Bern et al. 1901.04424

PN information. PM information. PN-PM overlap, and match! Unknown. ‡‡Tail terms.

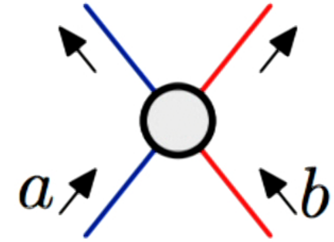
From J. Vines (AEI)

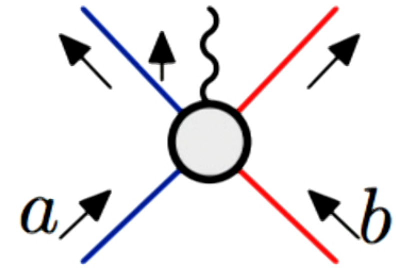
From 2-Body Problem to $2 \to 2$ Scattering

- Compute perturbative observables from a 2 body scattering amplitude
- Classical scaling follows from

  $$J = r \times p \gg 1\,, \quad \text{where} \quad r \sim \frac{1}{\hbar q}$$

- Easy to obtain observables, i.e. gauge/frame independent quantities.
  - Scattering deflection $\Delta p_a^\mu = -\Delta p_b^\mu$
  - Radiation field $\lim_{r\to\infty} h_{\mu\nu}(r)$
- These can be mapped to binary system data (same potential, same EOM)

$M_4 =$ 

$M_5 =$ 

Kosower, Maybee, O'connell 19
(see also Goldberger & Ridgway 17, Shen 18)

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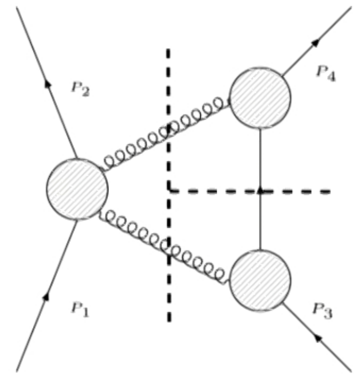
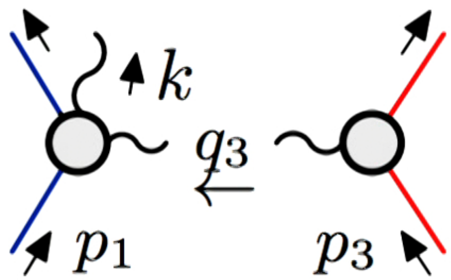


1705.10262 w/ F. Cachazo



1903.12419 w/ F. Bautista

2PM Effective Potential $\Longleftrightarrow$ 1-loop Scattering

$$
\begin{aligned}
\tfrac{1}{2}\tfrac{1}{2}\mathcal{M}^{(2)}_{tot}(\vec{q}) \simeq & \left[G^2 m_a m_b \left(6(m_a+m_b)S - \frac{41}{5}L\right) - i4\pi G^2 m_a^2 m_b^2 \frac{L}{q^2}\frac{m_r}{p_0}\right]\chi_f^{a\dagger}\chi_i^a\chi_f^{b\dagger}\chi_i^b \\
& + \left[G^2\left(\frac{24m_a^3 + 56m_a^2 m_b + 45 m_a m_b^2 + 12 m_b^3}{2(m_a+m_b)}S - \frac{128 m_a + 87 m_b}{10}L\right)\right. \\
& \left. + \frac{G^2 m_a^2 m_b^2 (4m_a + 3m_b)}{(m_a+m_b)}\left(-i\frac{2\pi L}{p_0 q^2} + \frac{S}{p_0^2}\right)\right]\frac{i}{m_a}\vec{S}_a\cdot\vec{p}\times\vec{q}\,\chi_f^{b\dagger}\chi_i^b \\
& + \left[G^2\left(\frac{12m_a^3 + 45m_a^2 m_b + 56 m_a m_b^2 + 24 m_b^3}{2(m_a+m_b)}S - \frac{87 m_a + 128 m_b}{10}L\right)\right. \\
& \left. + \frac{G^2 m_a^2 m_b^2 (3m_a + 4m_b)}{(m_a+m_b)}\left(-i\frac{2\pi L}{p_0 q^2} + \frac{S}{p_0^2}\right)\right]\chi_f^{a\dagger}\chi_i^a \frac{i}{m_b}\vec{S}_b\cdot\vec{p}\times\vec{q} \\
& + G^2 m_a m_b \frac{19m_a^2 + 36 m_a m_b + 19 m_b^2}{2(m_a+m_b)} S \, \frac{\vec{S}_a\cdot\vec{q}\,\vec{S}_b\cdot\vec{q} - \vec{q}^2\vec{S}_a\cdot\vec{S}_b}{m_a m_b} \\
& - G^2 m_a m_b L \frac{11\vec{S}_a\cdot\vec{q}\,\vec{S}_b\cdot\vec{q} - 16\vec{q}^2\vec{S}_a\cdot\vec{S}_b}{15 m_a m_b} \\
& + \frac{G^2 m_a^3 m_b^3}{m_a+m_b}\frac{S}{p_0^2}\frac{\vec{S}_a\cdot\vec{q}\,\vec{S}_b\cdot\vec{q} - \vec{q}^2\vec{S}_a\cdot\vec{S}_b}{m_a m_b} \\
& + \frac{G^2 m_a^3 m_b^3}{m_a+m_b}\left(-i\frac{4\pi L}{p_0 q^2}\right)\frac{\vec{S}_a\cdot\vec{q}\,\vec{S}_b\cdot\vec{q} - \frac{1}{2}\vec{q}^2\vec{S}_a\cdot\vec{S}_b}{m_a m_b} \qquad (95)
\end{aligned}
$$

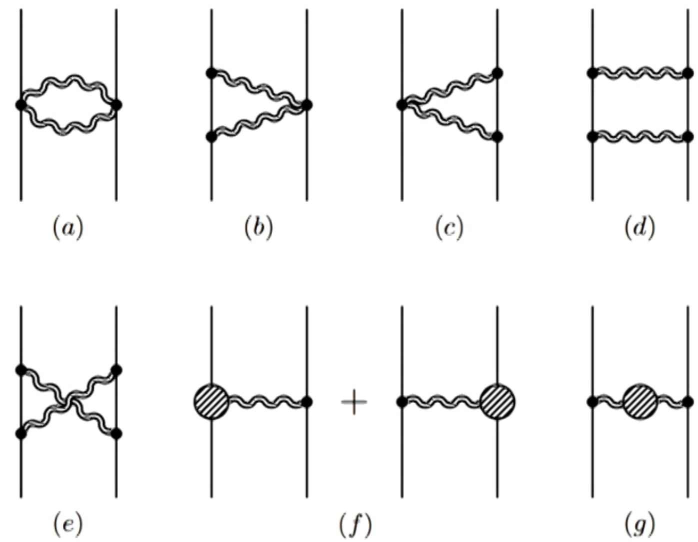
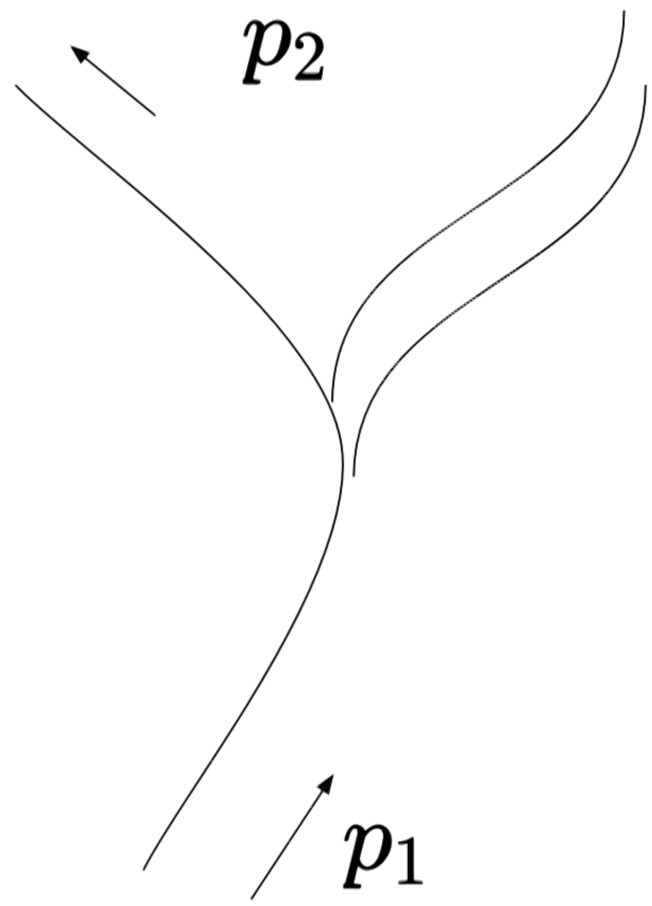


Figure 2: One loop diagrams of gravitational scattering.

B. Holstein and A. Ross (0802.0716)

What do we learn from Soft Theorems?



What do we learn from Soft Theorems?

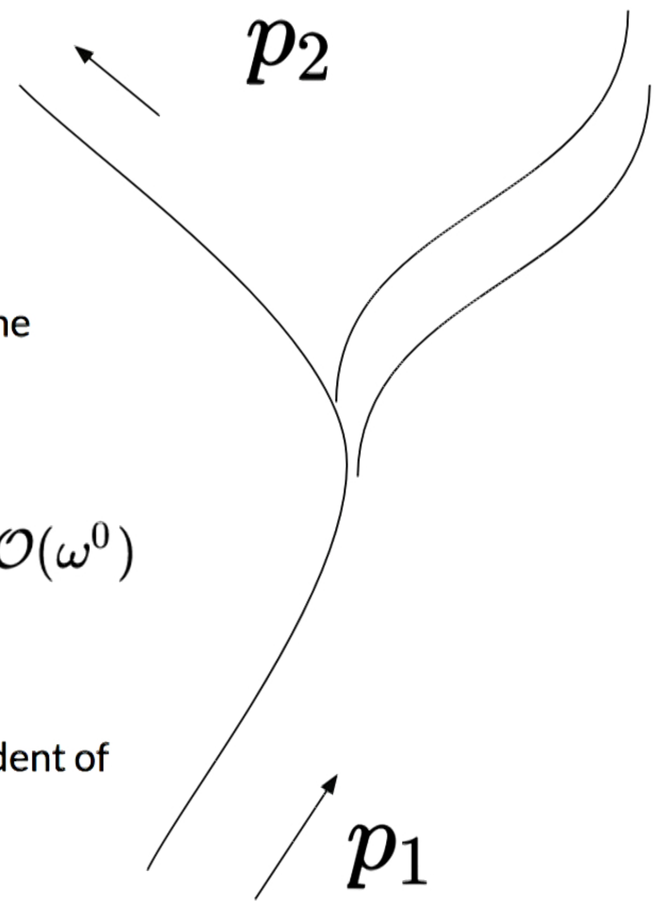
- Massive body accelerating in a finite interval of time

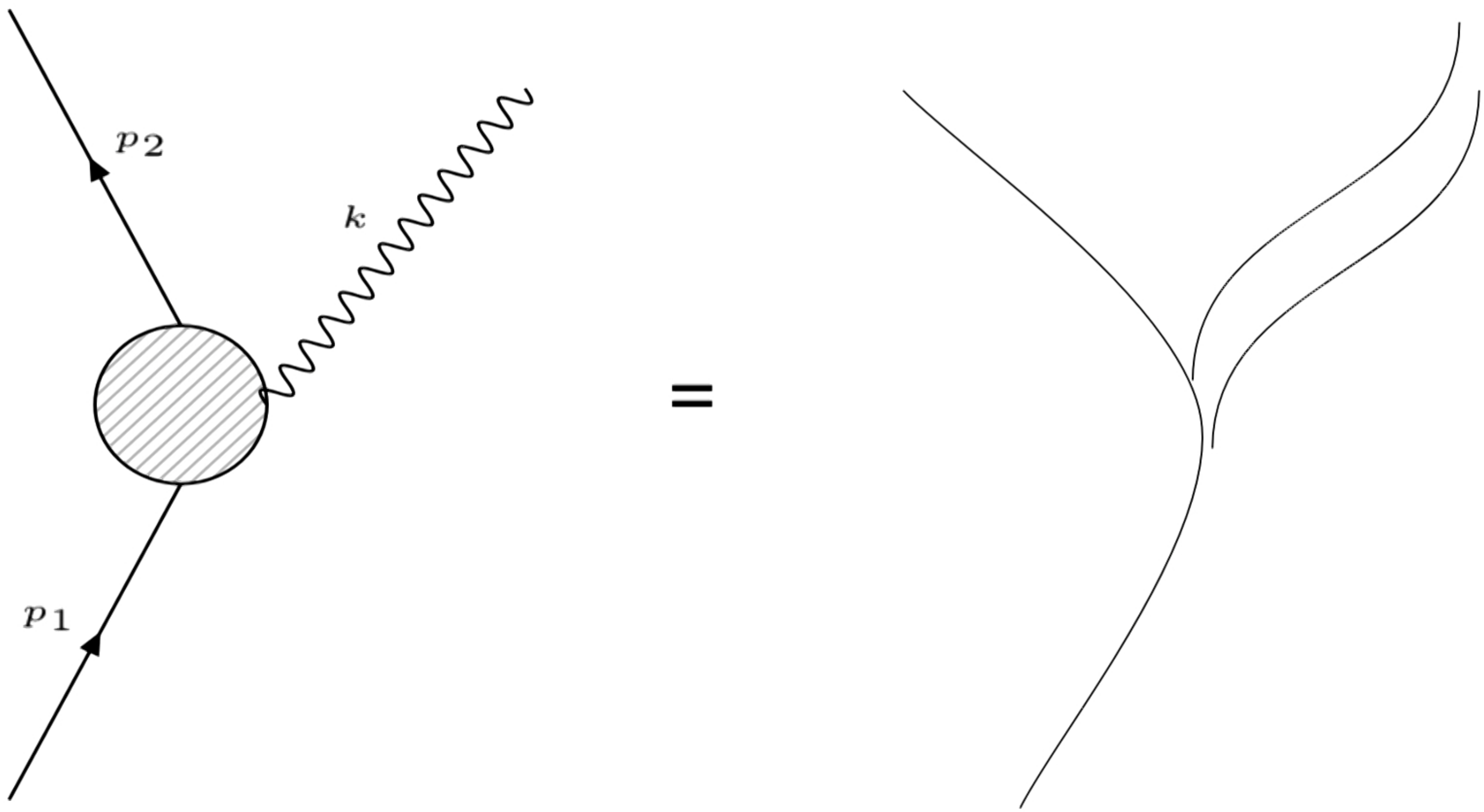
- At $r \to \infty$ we can drop Coulomb modes to get

$$T_{\mu\nu}(k) = \sqrt{8\pi G}\left(\frac{p_{1\mu}p_{1\nu}}{p_1 \cdot k + i\epsilon} - \frac{p_{2\mu}p_{2\nu}}{p_2 \cdot k - i\epsilon}\right) + \mathcal{O}(\omega^0)$$

i.e. Weinberg's Soft Factor

- Long-wavelength behaviour is universal. Independent of the acceleration or internal structure

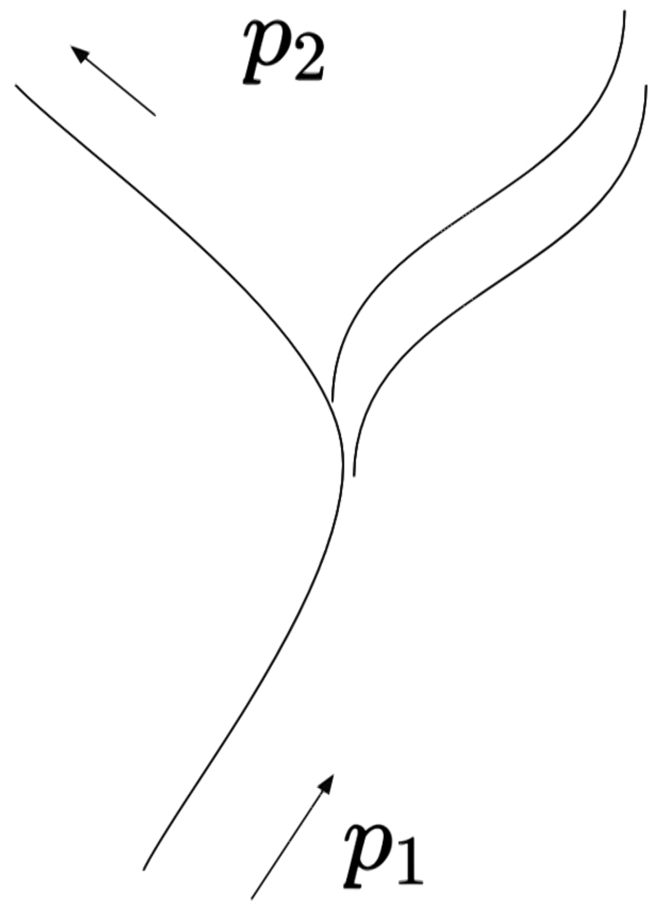




p_2
k
p_1
=

$$\left(\frac{p_{1\mu}p_{1\nu}}{p_1\cdot q+i\epsilon} - \frac{p_{2\mu}p_{2\nu}}{p_2\cdot q-i\epsilon}\right)_{p_2=p_1+q} =$$

$$p_{1\mu}p_{1\nu}\,\bar{\delta}(p_1\cdot q) =$$



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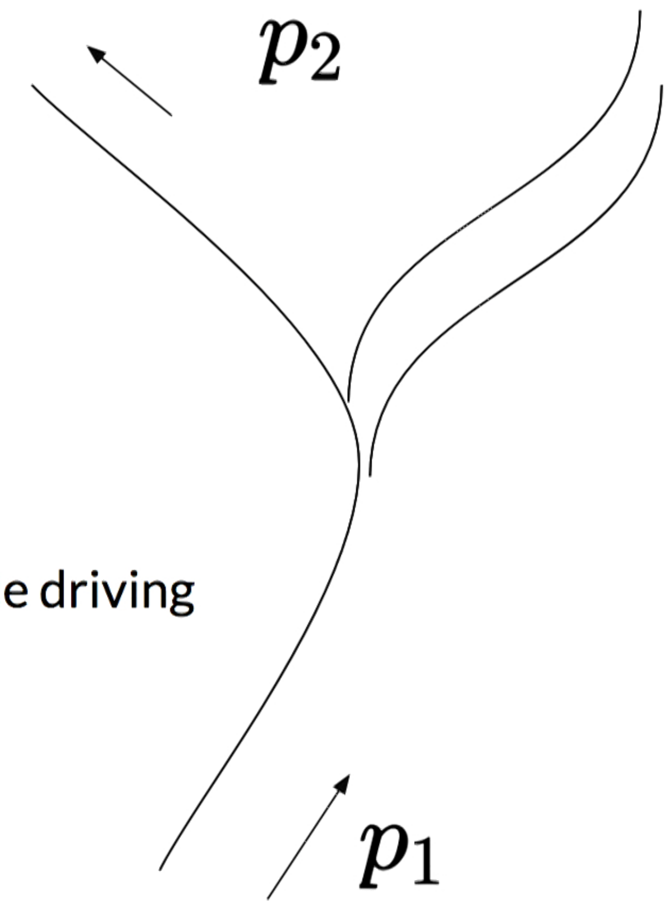
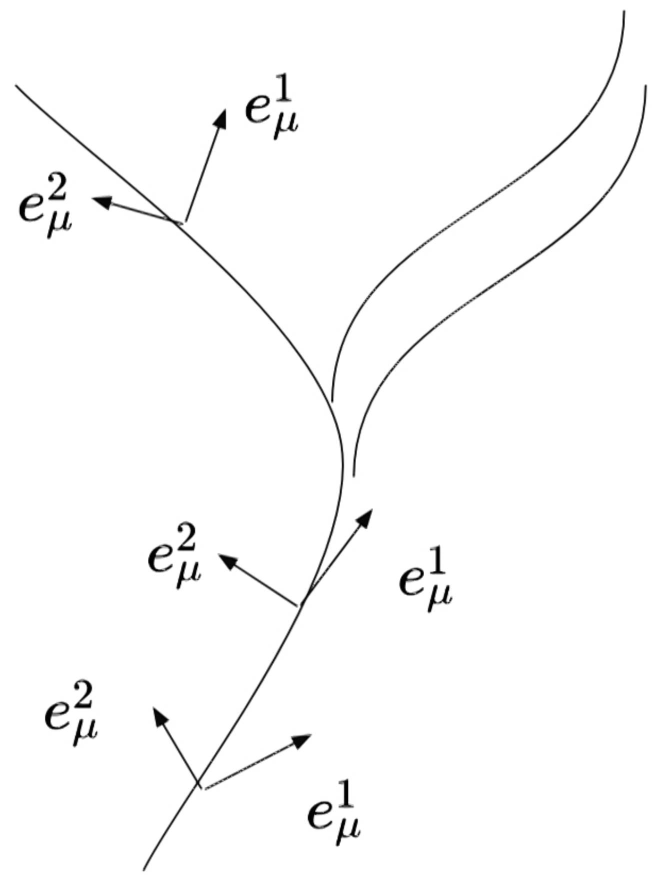


- No support for radiation! (need to include driving force)
- Still useful as building block
- Unique, fixed by little group
- Double Copy of photon vertex

What about Spin?

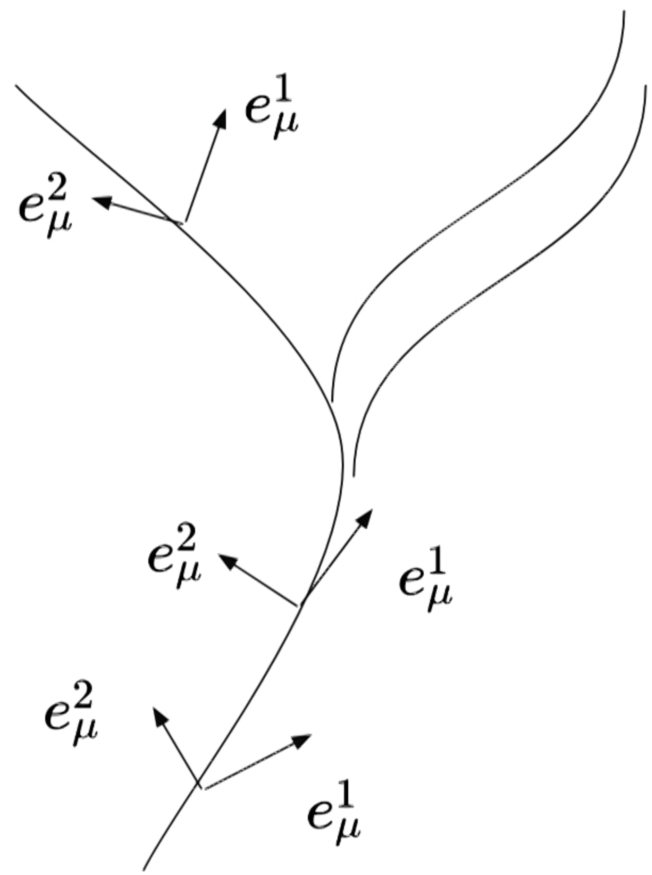


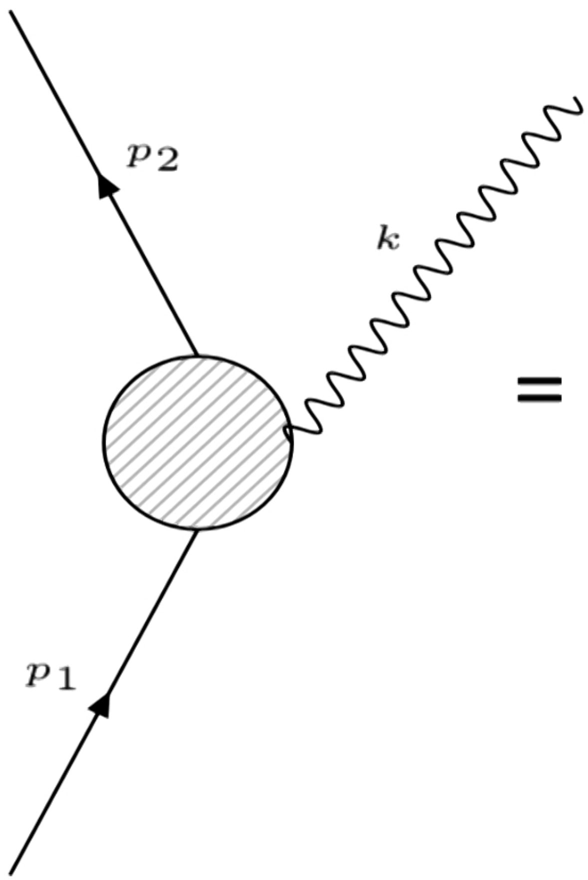
What about Spin?

$$\Omega^{IJ} \equiv \eta^{\mu\nu} e^I_\mu \frac{d}{ds} e^J_\nu$$

$$S_{pp} = \int ds \Big[-\dot{x}^\mu e^I_\mu p_I + \tfrac{1}{2} S^{IJ} \Omega_{IJ} + \tfrac{1}{2} e \left(p^I p_I - m^2 \right) + e \lambda_I S^{IJ} p_J \Big]$$

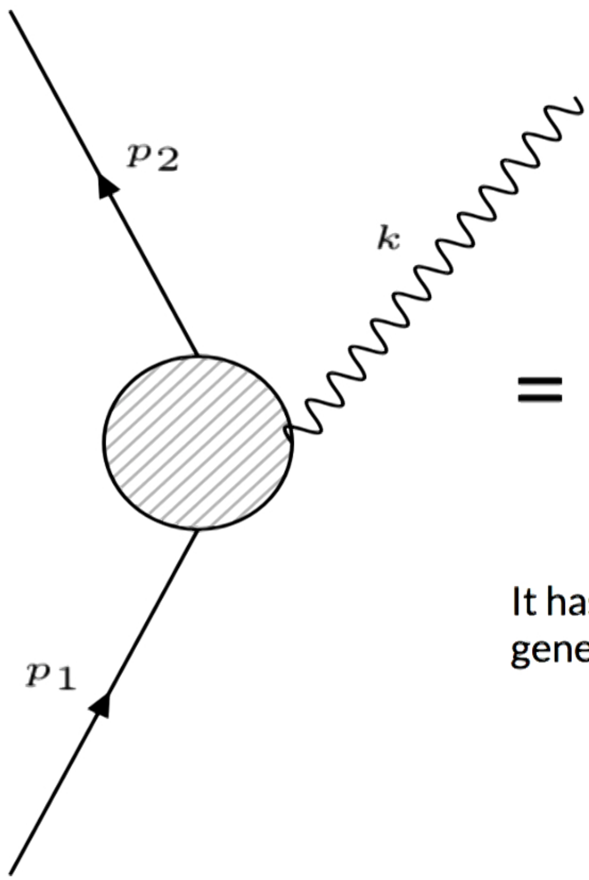
$$S_{IJ} e^I_\mu e^J_\mu = \epsilon_{\mu\nu\rho\sigma} p^\rho a^\sigma$$





$$= \quad (\epsilon \cdot p)^2 + \frac{g}{4}(\epsilon \cdot p) F_{\mu\nu} J^{\mu\nu} + \mathcal{O}(k^2 J^2)$$

$$(F_{\mu\nu} = 2k_{[\mu}\epsilon_{\nu]})$$



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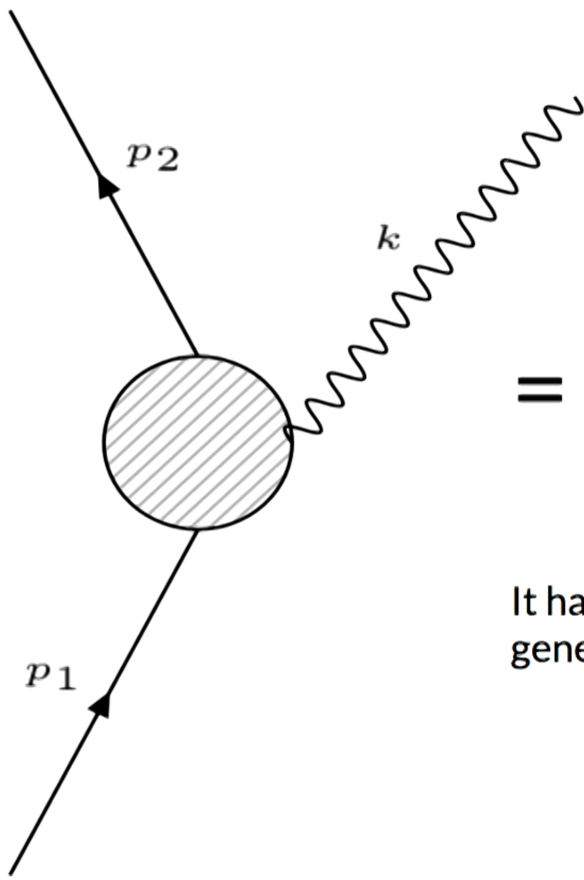

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Weinberg's Soft Factor (Equivalence principle)

Cachazo-Strominger Soft Factor (g=2)

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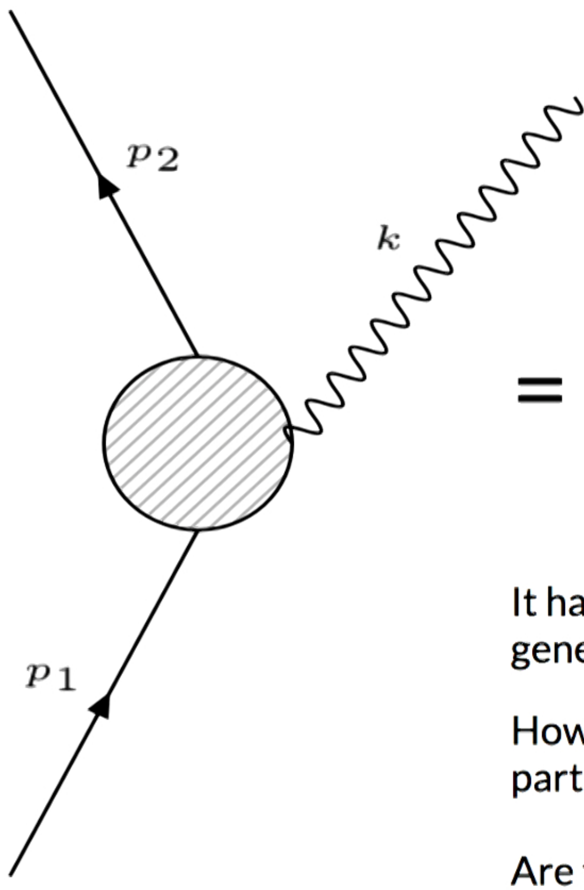

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How do we fix higher multipoles? We need to consider spin-s particles to get spin^{2s} terms

Are these universal in any sense?

How to fix all orders in spin? We can...

1. Consider only minimal coupling to gravity (Guevara 17/ Guevara, Ochirov, Vines 18/ Chung 18)

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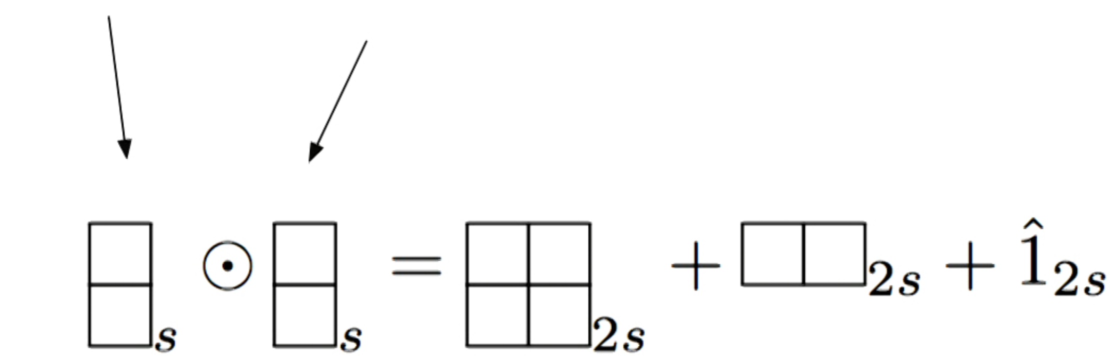

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$$A_3^{\mathrm{gr,s}}=\tfrac{\kappa}{2}(\epsilon\cdot p)^2\times\exp\left(\frac{F_{\mu\nu}J^{\mu\nu}}{2\epsilon\cdot p}\right)$$

$$\Longrightarrow \quad =\tfrac{\kappa}{2}(\epsilon\cdot p)^2\left(1+\frac{F_{\mu\nu}J^{\mu\nu}}{2\epsilon\cdot p}+\left[\frac{F_{\mu\nu}J^{\mu\nu}}{2\epsilon\cdot p}\right]^2+\dots\right)$$

(truncates for finite reps.)

Agrees with Kerr Stress Energy tensor in momentum space! First order in G, all orders in spin.

The new 3pt. Amplitude

- It corresponds to a Lorentz transformation, e.g. “Spin holonomy” or parallel transport

- It can be glued into higher point amplitudes

- In particular this builds the **Compton amplitude** => Radiation and 2PM Scattering Angle. At least up to S^4 (hexadecapole) order

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$$\exp\left(\frac{F_{\mu\nu}J^{\mu\nu}}{2\epsilon\cdot p}\right)p_1 = p_2 \qquad \exp\left(\frac{F_{\mu\nu}J^{\mu\nu}}{2\epsilon\cdot p}\right)|\epsilon_1^s\rangle = |\epsilon_2^s\rangle$$

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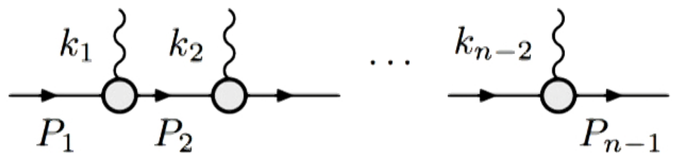

$$= \prod_i (P_i\cdot\epsilon_i)^h \langle\varepsilon_2|e^{J_{n-2}}\cdots e^{J_1}|\varepsilon_1\rangle = \prod_i (P_i\cdot\epsilon_i)^h \langle\varepsilon_2|\tilde{\varepsilon}_2\rangle$$

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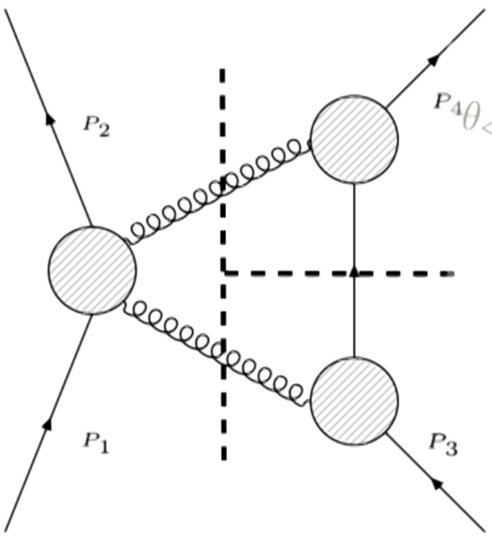

$$\theta_{\triangleleft} = \pi G^2 E \frac{m_b}{2v^4} \frac{\partial}{\partial b} \int_{\Gamma_{\text{LS}}} \frac{dz}{2\pi i} \frac{(1-vz)^4}{(z^2-1)^{3/2}} \int \frac{d^2\boldsymbol{k}}{2\pi|\boldsymbol{k}|} \exp\left(i\boldsymbol{k}\cdot\left[\boldsymbol{b} - z\hat{\boldsymbol{p}}\times\boldsymbol{a}_b - \frac{z-v}{1-vz}\hat{\boldsymbol{p}}\times\boldsymbol{a}_a\right]\right)$$

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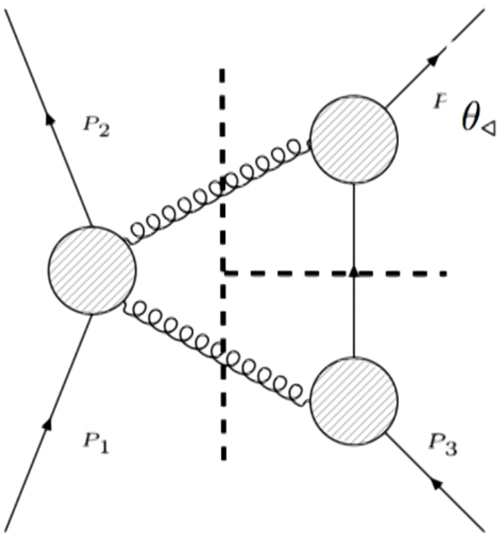

$$\begin{aligned}\theta_\triangleleft &= \pi G^2 E \frac{m_b}{2v^4}\frac{\partial}{\partial b}\int_{\Gamma_{\rm LS}}\frac{dz}{2\pi i}\frac{(1-vz)^4}{(z^2-1)^{3/2}}\int\frac{d^2\boldsymbol{k}}{2\pi|\boldsymbol{k}|}\exp\Big(i\boldsymbol{k}\cdot\Big[\boldsymbol{b}-z\hat{\boldsymbol{p}}\times\boldsymbol{a}_b-\frac{z-v}{1-vz}\hat{\boldsymbol{p}}\times\boldsymbol{a}_a\Big]\Big)\\ &= \pi G^2 E \frac{m_b}{2v^4}\frac{\partial}{\partial b}\int_{\Gamma_{\rm LS}}\frac{dz}{2\pi i}\frac{(1-vz)^4}{(z^2-1)^{3/2}}\left|b-za_b-\frac{z-v}{1-vz}a_a\right|^{-1}, \qquad (3.\end{aligned}$$

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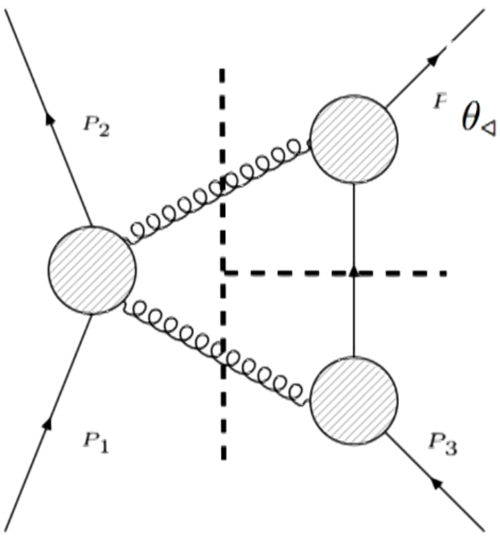

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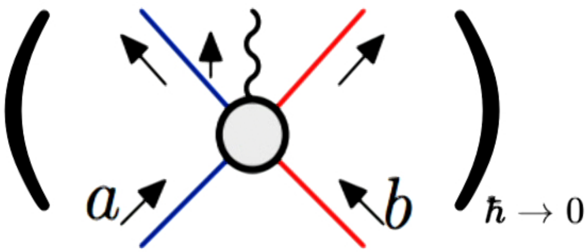
(from Guevara, Ochirov, Vines)

- All orders in a_b. Agreement up to a_a^3, **all orders in v.**
- Can be matched to effective (bounded) Hamiltonian (Siemonsen and Vines)

What about radiation?



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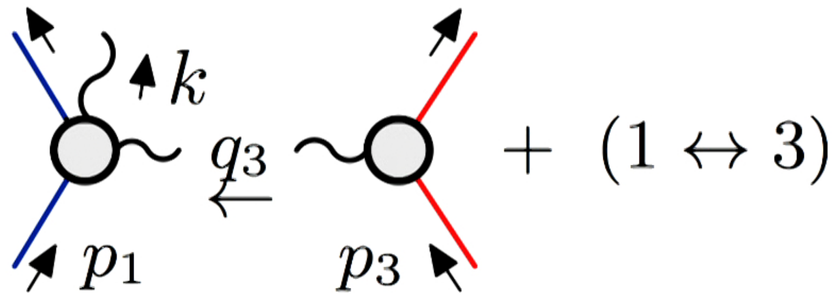
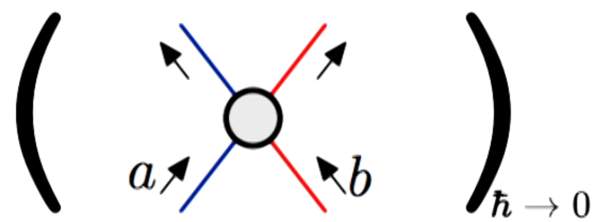


$$= \sum_{i=1,3} \mathcal{S}_i \, e^{\eta_i \left(F_p \frac{p_i \cdot k}{F_{iq}} \frac{\partial}{\partial (p_1 \cdot p_3)} + q \cdot k \frac{\partial}{\partial q^2} \right)}$$



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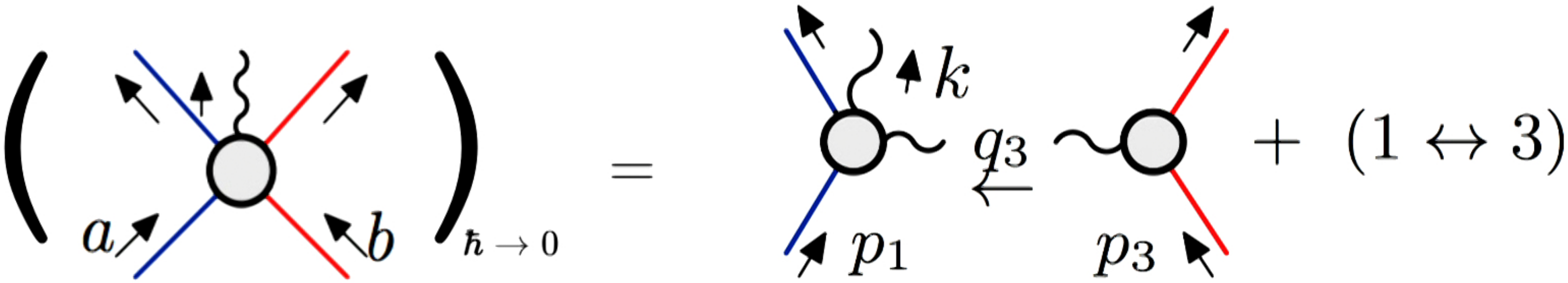

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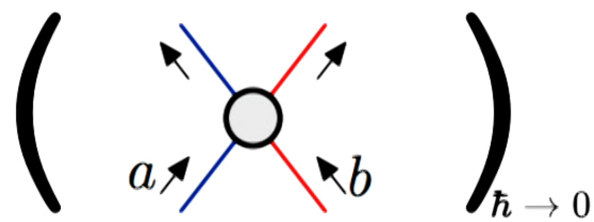

The radiation field exponentiates even for scalars! The soft theorem extends to all orders when the classical limit is taken.

It should be possible to derive from this the full multipole expansion of the radiative field.

Exempli Gratia: Einstein's Quadrupole

Consider just the LO and assume bounded NR orbits

In a spatial gauge $\epsilon^{0i} = 0$ we get

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$$p_i = m_i v_i \qquad v_i^\mu \to (1, \vec{v}_i)$$

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$$k^\mu \to \omega(1, \hat{n}) \quad \text{(radiation)}$$

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$$h^{ij} = m_a^2 m_b^2 \int dt e^{i\omega t} \frac{d^3 q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{x}_{a,b}} \frac{1}{2\vec{q}^4} \left[2q^i q^j - \frac{\vec{q}^2}{\omega} \left(v_1^i q^j + v_1^j q^i - v_3^i q^j - v_3^j q^i \right) \right]$$

$$\Rightarrow \epsilon_{ij}^* h^{ij} = \frac{\omega^2}{4m_{Pl}} \int dt e^{i\omega t} \epsilon_{ij}^*(q) Q^{ij}(t)\,, \qquad \text{with } Q^{ij} = \sum_a m_a \left(x_a^i x_a^j - \frac{1}{3}\delta^{ij}\vec{x}_a^2 \right)$$

Thank you!