Title: Quantum gravity predictions for black hole interior geometry

Speakers: Daniele Pranzetti

Series: Quantum Gravity

Date: May 30, 2019 - 2:30 PM

URL: http://pirsa.org/19050030

Abstract: We derive an effective Hamiltonian constraint for the Schwarzschild geometry starting from the full loop quantum gravity Hamiltonian constraint and computing its expectation value on coherent states sharply peaked around a spherically symmetric geometry. We use this effective Hamiltonian to study the interior region of a Schwarzschild black hole, where a homogeneous foliation is available. Descending from the full theory, our effective Hamiltonian preserves all relevant information about the graph structure of quantum space and encapsulates all dominant quantum gravity corrections to spatially homogeneous geometries at the effective level. It carries significant differences from the effective Hamiltonian postulated in the context of minisuperspace loop quantization models in the previous literature. We show how, for several geometrically and physically well motivated choices of coherent states, the classical black hole singularity is replaced by a homogeneous expanding Universe. The resultant geometries have no significant deviations from the classical Schwarzschild geometry in the pre-bounce sub-Planckian curvature regime, evidencing the fact that large quantum effects are avoided in these models. In all cases, we find no evidence of a white hole horizon formation. However, various aspects of the post-bounce effective geometry depend on the choice of quantum states. Finally, we show how a de Sitter Universe extending the classical spacetime past the singularity can be recovered by means of the simplicity constraint.

Quantum gravity predictions for black hole interior geometry

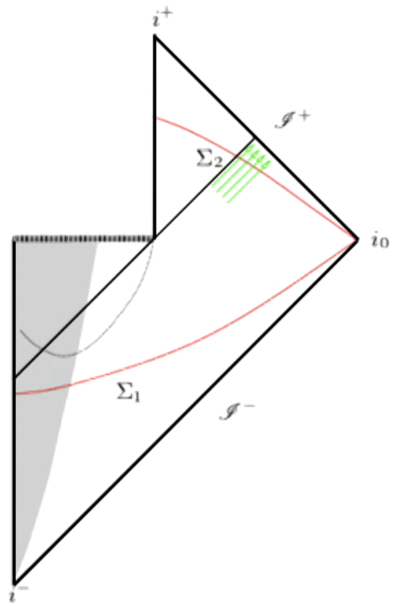
Daniele Pranzetti

Based on work in collaboration with Emanuele Alesci and Sina Bahrami

- Phys. Rev. D98, no. 4, 044052 (2018), [gr-qc/1802.06251];
- Phys. Rev. D98, 046014 (2018), [gr-qc/1807.07602];
- [gr-qc/1904.12412].

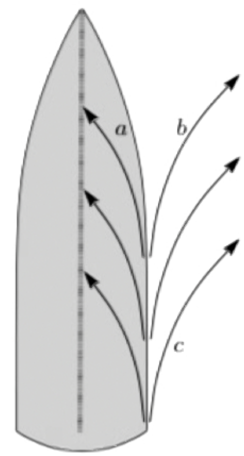
PERIMETER INSTITUTE

Black hole evaporation and information loss

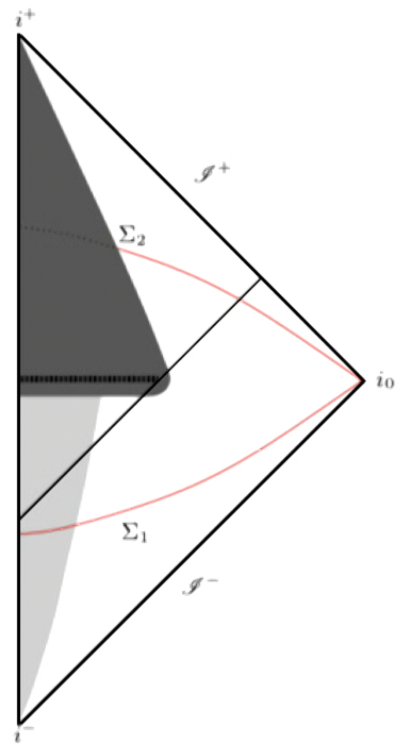


The standard semiclassical global causal structure representing black hole formation and the subsequent evaporation. The hyper surface $\Sigma 2$ representing an 'instant' of time after the complete evaporation of the initial black hole, fails to be a Cauchy surface of the whole space-time.

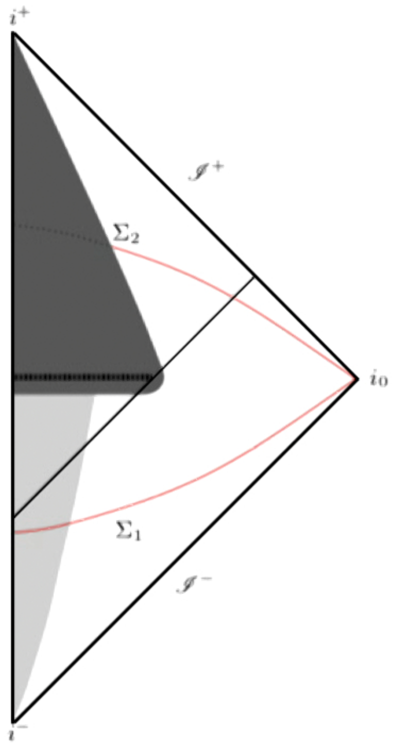
$\Rightarrow$

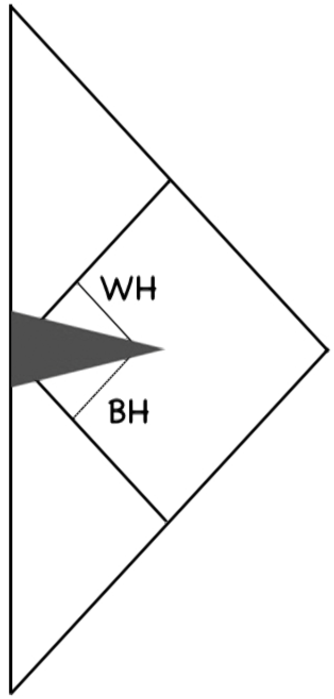


In the complementarity scenario, late particles b are expected to carry non trivial correlations with early particles c as to purify the final state of the Hawking radiation. This view is in contradiction with the equivalence principle and the validity of QFT in the mean field scenario.



Global causal structure in the remnant paradigm. The darken region represents the region where quantum gravity effects are expected to be large. The black hole evaporates following the semiclassical (mean field) expectation until it becomes Planckian in size. Most of the initial mass is radiated as Hawking radiation and one ends up with a Planckian mass remnant moving along a time-like world-tube of Planck size in (essentially) flat Minkowski space-time with a very long lifetime. Here we assume the remnant is stable so it goes up to i^+.

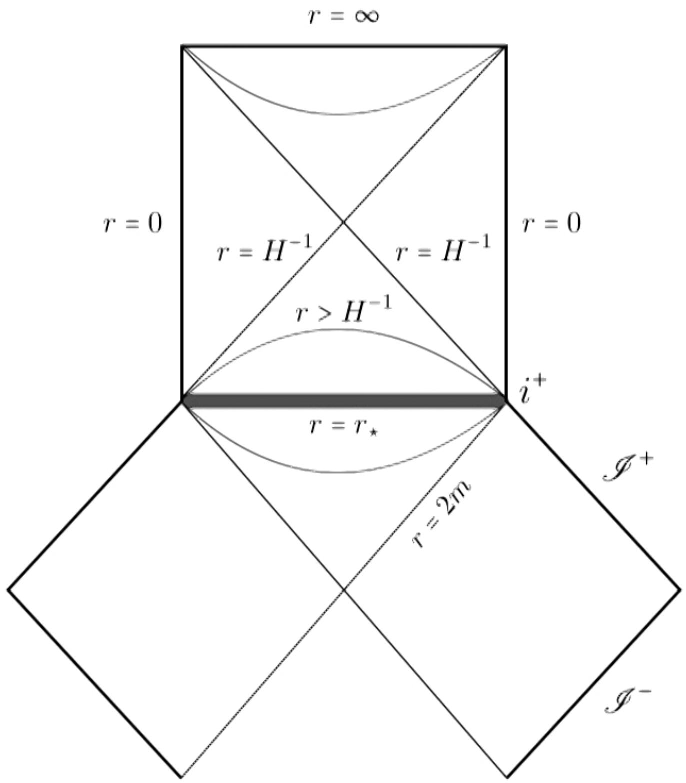




[Bianchi, Christodoulou, D'Ambrosio, Haggard, Rovelli, CQG 2018]

Global causal structure in the remnant paradigm. The darken region represents the region where quantum gravity effects are expected to be large. The black hole evaporates following the semiclassical (mean field) expectation until it becomes Planckian in size. Most of the initial mass is radiated as Hawking radiation and one ends up with a Planckian mass remnant moving along a time-like world-tube of Planck size in (essentially) flat Minkowski space-time with a very long lifetime. Here we assume the remnant is stable so it goes up to i^+.

Through a black hole into a new universe?



[Frolov, Markov, Mukhanov, PLB 1988]: Under the assumption that some limiting curvature exists, the Schwarzschild metric inside a black hole can be attached to a closed de Sitter one at some space-like junction surface which may represent a short transition layer.

In general, conservative solutions to restore unitary evolution rely on elimination of singularities [Hossenfelder, Smolin, PRD 2010].

At the end of the day, only a full quantum gravity calculation can discriminate between different scenarios.

Loop Quantum Gravity (LQG) provides a non-perturbative framework to investigate BH singularity resolution.

However, the analysis is strongly affected by an important choice:

Reduction or Quantization first?

The two in general do NOT commute!

General Relativity in Ashtekar variables

$$A_a^i(x), E_i^a(x)$$

Symmetry reduction

Mini-superspace

$$A_a^i(t), E_i^a(t)$$

'LQG derived' quantization

Polymer BH

[Ashtekar, Boehmer, Bojowald, Brahma, Campiglia, Corichi, Gambini, Kastrup, Modesto, Olmedo, Pullin, Singh, Swiderski, Vandersloot]

Use of point holonomies:
Graph DOF is lost

Quantum Reduced Loop Gravity program

- Symmetry reduced models have "smart" frames: systems of coordinates adapted to the symmetries
- In these coordinate systems the imposition of the symmetries allows to further simplify the form of the metric and Einstein Equations

Symmetry reduction in two steps:

1) Partial Gauge fixing of the metric (without symmetry reduction)
   Study the second class constraint system: Reduced Phase Space

   A. Solve the second class constraints
   B. Dirac Brackets
   C. Gauge Unfixing [Mitra, Rajaraman, Anishetty, Vytheeswaran]:
      - Ordinary Poisson Brackets for the non gauge fixed variables
      - Modified Constraints to preserve the gauge fixing during the evolution

2) Implement the symmetry reduction in the reduced phase space

Quantization

A. Quantize the classically Reduced Phase Space (with or without symmetry)
B. Quantize Dirac Brackets
C. QRLG: 4 steps

1. Impose the second class constraints weakly in the Full Hilbert Space:
   Selects the reduced states i.e. the quantum reduced phase space

2. Project the constraints defined in the full theory to represent the classical gauge unfixed constraints (preserving the gauge fixing)

3. Impose the symmetry reduction on the reduced states using coherent states

4. Define the effective constraints by taking the expectation value of the quantum reduced constraints on the symmetry reduced states

☞ Find quantum symmetry reduction compatible with given metrics
- Black Holes: Orthogonal gauge fixing

Quantum Reduced Loop Gravity: Black holes

The intrinsic metric on the spacelike hypersurfaces is:

$$d\sigma^2 = \Lambda^2 dr^2 + R^2\left(d\theta^2 + \sin^2\theta\ d\varphi^2\right)$$

$\Lambda(t,r), R(t,r)$ ADM phase space configuration variables

➥

$$E = E^r(t,r)\sin\theta\tau_3\partial_r + \left[E^1(t,r)\tau_1 + E^2(t,r)\tau_2\right]\sin\theta\partial_\theta + \left[E^1(t,r)\tau_2 - E^2(t,r)\tau_1\right]\partial_\varphi\,,$$

$$A = A_r(t,r)\tau_3 dr + \left[A_1(t,r)\tau_1 + A_2(t,r)\tau_2\right]d\theta + \sin\theta\left[A_1(t,r)\tau_2 - A_2(t,r)\tau_1\right]d\varphi + \cos\theta\tau_3 d\varphi$$

with Poisson brackets

$$\left\{A_r(t,r), E^r(t,r')\right\} = 2G\gamma\,\delta(r-r')\,,$$
$$\left\{A_1(t,r), E^1(t,r')\right\} = G\gamma\,\delta(r-r')\,,$$
$$\left\{A_2(t,r), E^2(t,r')\right\} = G\gamma\,\delta(r-r')$$

Orthogonal partial gauge fixing conditions:

$$E^r_I = 0\,,\quad I = 1,2\,,$$
$$E^A_3 = 0\,,\quad A = \theta,\phi$$

- **Step 1:** $P\mathcal{H}^K = \mathcal{H}^R$

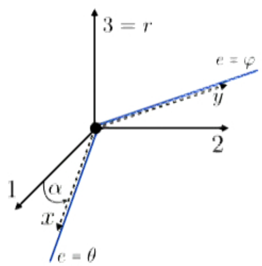
$$\mathcal{H}^R = \oplus_\Gamma \mathcal{H}^R_\Gamma$$

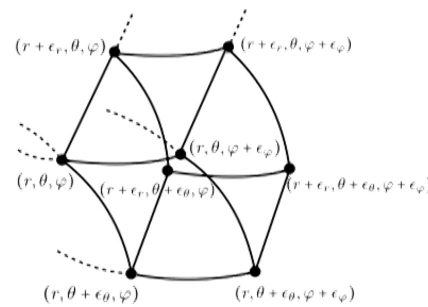
$\Gamma =$ cubulation

Carries a representation of the quantum holonomy-flux algebra:

$$E_i(S^a) = \int E_i^a n_a d\sigma_1 d\sigma_2$$

$$g_e = \mathcal{P}e^{\int_{\ell_e} \tau_i A_a^i \dot{\ell}_e^a(s)ds}$$





Assign to each link in a given tangent direction the following basis elements

$${}^xD^{j_x}_{\bar{m}_x\bar{n}_x}(g_\theta) = \langle \bar{m}_x, \vec{u}_x | D^{j_x}(g_\theta) | \bar{n}_x, \vec{u}_x \rangle \,,$$
$${}^yD^{j_y}_{\bar{m}_y\bar{n}_y}(g_\varphi) = \langle \bar{m}_y, \vec{u}_y | D^{j_y}(g_\varphi) | \bar{n}_y, \vec{u}_y \rangle \,,$$
$$D^{j_z}_{\bar{m}_z\bar{n}_z}(g_r) = \langle \bar{m}_z, j_z | D^{j_z}(g_r) | j_z, \bar{n}_z \rangle \,,$$

where $\bar{m}, \bar{n} = \pm j$

$|\bar{n}_I, \vec{u}_I\rangle =$ SU(2) coherent state having maximum or minimum magnetic number along $\vec{u}_I$

2 orthogonal unit vectors in the arbitrary internal directions $I \in \{x, y\}$

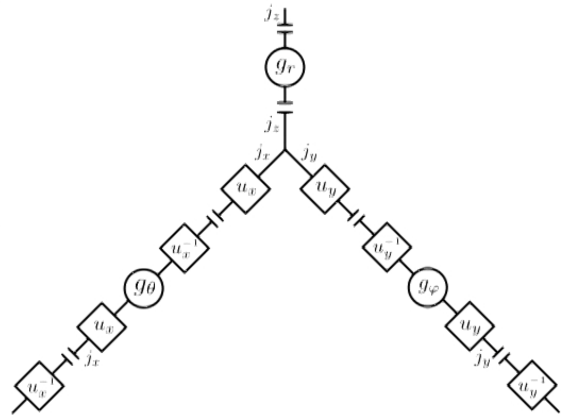
$S^a =$ Orthogonal faces of the cube dual to a 6-valent node of the reduced graph (regularization of the reduced fluxes)

On $\mathcal{H}^R$ the gauge fixing conditions are weakly satisfied:

$$\langle \hat{E}_I(S^r) \rangle = 0 \,, \quad I = 1, 2 \,,$$
$$\langle \hat{E}_3(S^A) \rangle = 0 \,, \quad A = \theta, \phi$$

Reduced 3-valent vertex state:

$$|v_3^R(j)\rangle = \int_0^{2\pi} d\alpha$$



Reduced flux operators:

$$
\begin{aligned}
{}^R\hat{E}_i(S^r) &= \hat{P}^z \hat{E}_i(S^r) \hat{P}^z\,, \\
{}^R\hat{E}_i(S^\theta) &= \hat{P}^x \hat{E}_i(S^\theta) \hat{P}^x\,, \\
{}^R\hat{E}_i(S^\varphi) &= \hat{P}^y \hat{E}_i(S^\varphi) \hat{P}^y
\end{aligned}
\qquad \text{where} \qquad
\begin{aligned}
\hat{P}^z &= \sum_{\bar{m}_z = \pm j_z} |j_z, \bar{m}_z\rangle\langle \bar{m}_z, j_z|\,, \\
\hat{P}^I &= \sum_{\bar{m}_I = \pm j_I} |\vec{u}_I, \bar{m}_I\rangle\langle \bar{m}_I, \vec{u}_I|
\end{aligned}
$$

✦ This procedure allows one to work with the complete structure of the full theory, consisting of quantum states of polymeric nature labelled by graphs and SU(2) representations

✦ At the same time, the reduced flux operators are diagonal on the reduced quantum states!

- **Step 2:**

Extended Hamiltonian constraint
(preserving the gauge fixing)

$$\tilde{H}_E[N] = \frac{1}{\kappa}\int d^3x \, \frac{N}{\sqrt{\det(E)}}\left[{}^R H_E + {}^{ext} H_E\right],$$

$$H_L[N] = -2\frac{(1+\gamma^2)}{\kappa}\int d^3x \, \frac{N\, {}^R H_L}{\sqrt{\det(E)}}$$

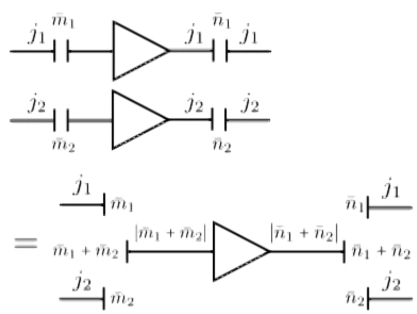
Let us focus on the reduced Euclidean Hamiltonian

$${}^R H_E = E_3^r E_I^A \epsilon^I{}_J \partial_r A_A^J + E_I^A E_J^B A_{[A}^I A_{B]}^J + E_3^r E_I^A A_r^3 A_A^I$$

Using Thiemann's techniques ➥

$${}^R\hat{H}_\square^E[N] = -\frac{2}{\kappa^2\gamma} N(v)\epsilon^{ijk}\mathrm{tr}\left[\left({}^R\hat{g}_{\alpha_{ij}} - {}^R\hat{g}_{\alpha_{ij}}^{-1}\right){}^R\hat{g}_{s_k}^{-1}\left[{}^R\hat{g}_{s_k}, {}^R\hat{V}(v)\right]\right]$$

By means of the "reduced" recoupling rule:



its action can be computed in a straightforward way

• Step 3:

Coherent semiclassical states

[Hall, Thiemann, Winkler, Sahlmann, Bahr]

$$\psi_G^\lambda(g_\ell) = \sum_{j_\ell}(2j_\ell+1)e^{-\frac{\lambda}{2}j_\ell(j_\ell+1)}\chi_{j_\ell}(g_\ell^{-1}G)$$

$\lambda =$ positive real number controlling the fluctuations of the state

$\chi_{j_\ell} =$ SU(2) character in the irreducible representation j_ℓ

$$G = g\exp\left(i\frac{\lambda}{\kappa\gamma}E_i(S^\ell)\tau^i\right)$$

SL(2,C) group element encoding the classical geometry around which we want to peak

Codes the extrinsic curvature (g)

Codes the intrinsic geometry ($E_i(S^\ell)$)

for $j_x, j_y, j_z \gg 1$ the coherent states become Gaussian weights for the fluxes peaked around the semiclassical values $\tilde{j}_\ell = \delta_\ell^2 j_\ell^0$ with

$$j_x^0 = \frac{1}{\kappa\gamma}\left(E_1^\theta\cos\tilde{\alpha} + E_2^\theta\sin\tilde{\alpha}\right),$$
$$j_y^0 = \frac{1}{\kappa\gamma}\left(-E_1^\varphi\sin\tilde{\alpha} + E_2^\varphi\cos\tilde{\alpha}\right),$$
$$j_z^0 = \frac{E_3^r}{\kappa\gamma},$$

$$\delta_x^2 = \epsilon_r\epsilon_\varphi\,,$$
$$\delta_y^2 = \epsilon_r\epsilon_\theta\,,$$
$$\delta_z^2 = \epsilon_\theta\epsilon_\varphi$$

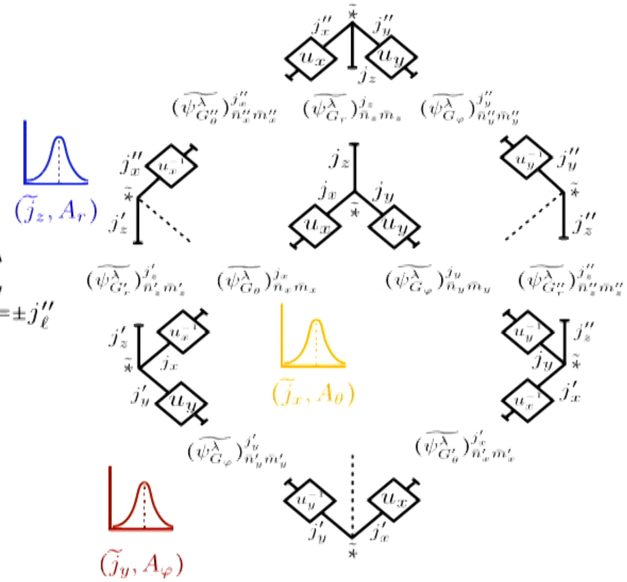
If we open up the character, we can write the quantum reduced coherent states in the compact notation:

$$\psi^{\lambda}_{G}(g_\ell) = \sum_{j_\ell=0}^{\infty} \sum_{\bar{m}_\ell, \bar{n}_\ell = \pm j_\ell} (2j_\ell + 1)(\psi^{\lambda}_{G})^{j_\ell}_{\bar{n}_\ell \bar{m}_\ell}\, {}^{\ell}D^{j_\ell}_{\bar{m}_\ell \bar{n}_\ell}(g_\ell^{-1})$$

`Gaussian' wave-function

By contracting with reduced intertwiners and introducing proper normalizations, we obtain the normalized quantum reduced coherent state

$$|\widetilde{\psi^{\lambda}_{\square}}\rangle = \prod_{\ell=x,y,z} \sum_{j_\ell, j'_\ell, j''_\ell=0}^{\infty} \sum_{\bar{m}_\ell, \bar{n}_\ell=\pm j_\ell} \sum_{\bar{m}'_\ell, \bar{n}'_\ell=\pm j'_\ell} \sum_{\bar{m}''_\ell, \bar{n}''_\ell=\pm j''_\ell}$$



$$|\widetilde{v^{R}_{\square}}\rangle$$

$$
\begin{aligned}
&-2\kappa\gamma^2\langle\widetilde{\psi_\square^\lambda}|\,{}^R\hat{H}^E_\square + {}^R\hat{H}^{ext}_\square + {}^R\hat{H}^L_\square\,|\widetilde{\psi_\square^\lambda}\rangle = 4\sqrt{E^r}\Bigg[\epsilon_\theta\Bigg(\cos\left[\frac{\sqrt{A_1^2(r)+A_2^2(r)}}{2}\sin\theta\epsilon_\varphi\right]\sin\left[\frac{\sqrt{A_1^2(r+\epsilon_r)+A_2^2(r+\epsilon_r)}}{2}\sin\theta\epsilon_\varphi\right]\\
&\times\frac{\left(\sin\left[\frac{A_r(r)+A_r(r+\epsilon_r)}{2}\epsilon_r\right]A_1(r+\epsilon_r)+\cos\left[\frac{A_r(r)+A_r(r+\epsilon_r)}{2}\epsilon_r\right]A_2(r+\epsilon_r)\right)}{\sqrt{A_1^2(r+\epsilon_r)+A_2^2(r+\epsilon_r)}}\\
&-\sin\left[\frac{\sqrt{A_1^2(r)+A_2^2(r)}}{2}\sin\theta\epsilon_\varphi\right]\cos\left[\frac{\sqrt{A_1^2(r+\epsilon_r)+A_2^2(r+\epsilon_r)}}{2}\sin\theta\epsilon_\varphi\right]\\
&\times\frac{\left(\sin\left[\frac{A_r(r+\epsilon_r)-A_r(r)}{2}\epsilon_r\right]A_1(r)+\cos\left[\frac{A_r(r+\epsilon_r)-A_r(r)}{2}\epsilon_r\right]A_2(r)\right)}{\sqrt{A_1^2(r)+A_2^2(r)}}\Bigg)\\
&+\epsilon_\varphi\sin\theta\Bigg(\cos\left[\frac{\sqrt{A_1^2(r)+A_2^2(r)}}{2}\epsilon_\theta\right]\sin\left[\frac{\sqrt{A_1^2(r+\epsilon_r)+A_2^2(r+\epsilon_r)}}{2}\epsilon_\theta\right]\\
&\times\frac{\left(\sin\left[\frac{A_r(r)+A_r(r+\epsilon_r)}{2}\epsilon_r\right]A_1(r+\epsilon_r)+\cos\left[\frac{A_r(r)+A_r(r+\epsilon_r)}{2}\epsilon_r\right]A_2(r+\epsilon_r)\right)}{\sqrt{A_1^2(r+\epsilon_r)+A_2^2(r+\epsilon_r)}}\\
&-\sin\left[\frac{\sqrt{A_1^2(r)+A_2^2(r)}}{2}\epsilon_\theta\right]\cos\left[\frac{\sqrt{A_1^2(r+\epsilon_r)+A_2^2(r+\epsilon_r)}}{2}\epsilon_\theta\right]\\
&\times\frac{\left(\sin\left[\frac{A_r(r+\epsilon_r)-A_r(r)}{2}\epsilon_r\right]A_1(r)+\cos\left[\frac{A_r(r+\epsilon_r)-A_r(r)}{2}\epsilon_r\right]A_2(r)\right)}{\sqrt{A_1^2(r)+A_2^2(r)}}\Bigg)\Bigg]\\
&+2\epsilon_r\frac{E^1}{\sqrt{E^r}}\sin\left[\sqrt{A_1^2(r)+A_2^2(r)}\epsilon_\theta\right]\sin\left[\frac{\sqrt{A_1^2(r)+A_2^2(r)}}{2}(\sin\theta+\sin(\theta+\epsilon_\theta))\epsilon_\varphi\right]\\
&-2\gamma^2\frac{\epsilon_r\epsilon_\varphi}{\epsilon_\theta}\frac{E^1(r)}{\sqrt{E^r(r)}}(\sin(\theta+2\epsilon_\theta)-2\sin(\theta+\epsilon_\theta)+\sin\theta)\\
&-(1+\gamma^2)\frac{\epsilon_\theta\epsilon_\varphi}{\epsilon_r}\frac{\sin\theta}{2\sqrt{E^r(r)}(E^1(r))^2}\\
&\times\Big[E^1(r)\Big((E^r(r+\epsilon_r)-E^r(r))^2+4E^r(r)(E^r(r+2\epsilon_r)-2E^r(r+\epsilon_r)+E^r(r))\Big)\\
&-4E^r(r)(E^r(r+\epsilon_r)-E^r(r))\left(E^1(r+\epsilon_r)-E^1(r)\right)\Big]
\end{aligned}
$$

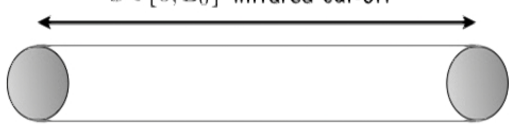
- **Step 4:**

Solving the effective dynamics

We are interested in the effective description for the interior geometry of a spherically symmetric black hole, namely we restrict our search for quantum geometries to metrics in the minisuperspace of the form

$$ds^2 = -N(\tau)^2 d\tau^2 + \Lambda(\tau)^2 dx^2 + R(\tau)^2 d\Omega^2$$

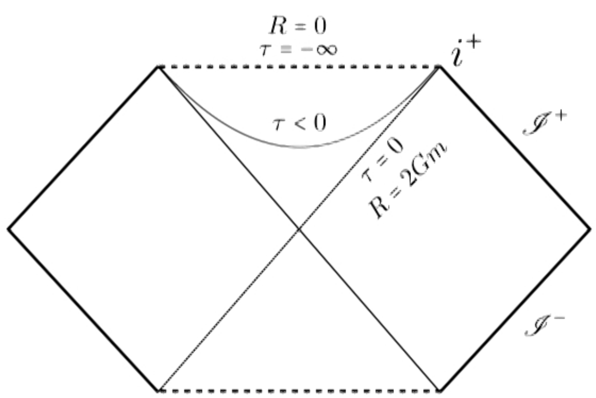
Homogeneous Cauchy slices with topology $\mathbb{R} \times S^2$



$$N_c = -\frac{R^2}{2G^2 m P_\Lambda}\,, \quad -\infty < \tau < 0$$

$\tau = 0 \rightarrow$ BH horizon

$\tau = -\infty \rightarrow$ classical singularity



ADM phase space:

$$E^r = R^2\,, \quad E^1 = R\Lambda\,,$$

$$A_r = -\frac{\gamma G}{R}\left(P_R - \frac{\Lambda P_\Lambda}{R}\right), \quad A_1 = -\frac{\gamma G}{R} P_\Lambda\,,$$

$$\rightarrow \quad \{R, P_R\} = \{\Lambda, P_\Lambda\} = 1/L_0$$

⭐ Effective Hamiltonian:

$$H_{\rm eff} = -\frac{L_0}{4\gamma^2 G\epsilon_x\epsilon^2}\Bigg[\epsilon R\sin\left(\frac{\gamma G\epsilon_x[P_R R - P_\Lambda\Lambda]}{R^2}\right)\left\{2\sin\left(\frac{\gamma G\epsilon P_\Lambda}{R}\right) + \pi H_0\left(\frac{\gamma G\epsilon P_\Lambda}{R}\right)\right\}$$
$$+\epsilon_x\Lambda\left\{8\gamma^2\cos(\epsilon)\sin\left(\frac{\epsilon}{2}\right)^2 + \pi\sin\left(\frac{\gamma G\epsilon P_\Lambda}{R}\right)H_0\left(\frac{\gamma G\epsilon P_\Lambda}{R}\right)\right\}\Bigg]$$

Struve function of order 0:

$$H_0[z] = \sum_{k=0}^{\infty}\frac{(-1)^k}{\left(\Gamma\left[k+\frac{3}{2}\right]\right)^2}\left(\frac{z}{2}\right)^{2k+1}$$

main departure from the minisuperspace quantization models: encodes the DOF associated with the 2-sphere graph structure which are frozen in all the previous treatments relying on the use of point holonomies.

Effective Hamiltonian:

$$H_{\rm eff} = -\frac{L_0}{4\gamma^2 G\epsilon_x\epsilon^2}\Bigg[\epsilon R\sin\left(\frac{\gamma G\epsilon_x[P_R R - P_\Lambda\Lambda]}{R^2}\right)\left\{2\sin\left(\frac{\gamma G\epsilon P_\Lambda}{R}\right) + \pi H_0\left(\frac{\gamma G\epsilon P_\Lambda}{R}\right)\right\}$$
$$+ \epsilon_x\Lambda\left\{8\gamma^2\cos(\epsilon)\sin\left(\frac{\epsilon}{2}\right)^2 + \pi\sin\left(\frac{\gamma G\epsilon P_\Lambda}{R}\right)H_0\left(\frac{\gamma G\epsilon P_\Lambda}{R}\right)\right\}\Bigg]$$

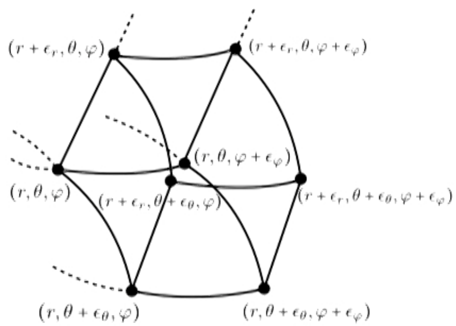
Struve function of order 0:

$$H_0[z] = \sum_{k=0}^{\infty}\frac{(-1)^k}{\left(\Gamma\left[k+\frac{3}{2}\right]\right)^2}\left(\frac{z}{2}\right)^{2k+1}$$

main departure from the minisuperspace quantization models: encodes the DOF associated with the 2-sphere graph structure which are frozen in all the previous treatments relying on the use of point holonomies.

⇨ No White Hole horizon in the effective interior geometry
like, e.g., [Ashtekar, Olmedo, Singh, PRL 2018]

Quantum parameters:



$$\epsilon_\theta = \epsilon_\varphi := \epsilon = \frac{2\pi}{N}\,, \quad \epsilon_x = \frac{L_0}{N_x}\,, \quad N, N_x \gg 1$$

$$A(R) = 4\pi R^2 \simeq 4\pi\gamma\ell_P^2 j_0 N^2$$

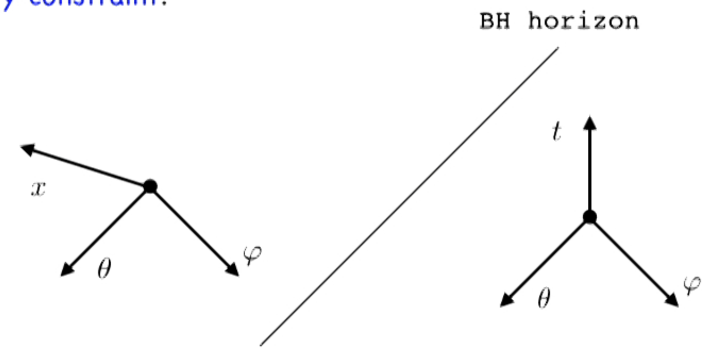
$$V(\Sigma) = 4\pi L_0\Lambda R^2 = 2(8\pi\gamma\ell_P^2)^{3/2} j\sqrt{j_0}N_x N^2$$

➥

$$\epsilon = \frac{\alpha}{R}\,, \quad \alpha := 2\pi\sqrt{\gamma j_0}\,\ell_p\,,$$
$$\epsilon_x = \frac{\beta}{\Lambda}\,, \quad \beta := \frac{4\sqrt{8\pi\gamma}\,j\,\ell_p}{\sqrt{j_0}}$$

✦ Including the simplicity constraint:

Homogenous foliation inside



Homogenous foliation outside

j = Expectation value of generator of rotations in the planes $(x,\theta)\,,(x,\varphi)$

j = Expectation value of generator of boosts in the planes $(t,\theta)\,,(t,\varphi)$

j_0 = Expectation value of generator of rotations in the plane $(\theta\,,\varphi)$

j_0 = Expectation value of generator of rotations in the plane $(\theta\,,\varphi)$

Internal gauge group $SU(2)$

Internal gauge group $SU(1,1)$

$$\langle \hat{K}_i \rangle = \gamma \langle \hat{L}_i \rangle \quad \Rightarrow \quad j = \gamma j_0$$

Interplay between canonical and covariant formulations already exploited to understand thermality properties [DP, PRD 2014]

- Effective Hamilton evolution eq.s:

$$\dot{R} = \frac{1}{L_0}\frac{\partial H_{\text{eff}}[N]}{\partial P_R} = \frac{R}{2m}\cos\left[\gamma\beta\left(\frac{P_R}{R\Lambda} - \frac{P_\Lambda}{R^2}\right)\right],$$

$$\dot{\Lambda} = \frac{1}{L_0}\frac{\partial H_{\text{eff}}[N]}{\partial P_\Lambda} = \frac{\Lambda}{2m}\left\{-\cos\left[\gamma\beta\left(\frac{P_R}{R\Lambda} - \frac{P_\Lambda}{R^2}\right)\right]\right.$$

$$\left. + \frac{\pi H_{-1}\left[\frac{\gamma\alpha P_\Lambda}{R^2}\right]\left(2\sin^2\left[\frac{\gamma\alpha P_\Lambda}{R^2}\right] - 8\gamma^2\cos\left[\frac{\alpha}{R}\right]\sin^2\left[\frac{\alpha}{2R}\right]\right) + \cos\left[\frac{\gamma\alpha P_\Lambda}{R^2}\right]\left(\pi^2 H_0^2\left[\frac{\gamma\alpha P_\Lambda}{R^2}\right] - 16\gamma^2\cos\left[\frac{\alpha}{R}\right]\sin^2\left[\frac{\alpha}{2R}\right]\right)}{\left(2\sin\left[\frac{\gamma\alpha P_\Lambda}{R^2}\right] + \pi H_0\left[\frac{\gamma\alpha P_\Lambda}{R^2}\right]\right)^2}\right\}$$

We look for solutions in the asymptotic region $\tau \to -\infty$ s.t.: $\frac{\dot{R}}{R} = \frac{\dot{\Lambda}}{\Lambda} = \frac{C}{2m}, \quad C = const < 0$

We make the ansatz $\frac{P_R}{R\Lambda} = 2\frac{P_\Lambda}{R^2} = -\frac{2\pi\sigma_\beta}{\gamma\beta}, \quad \frac{P_\Lambda}{R^2} = -\frac{\pi\sigma_\alpha}{\gamma\alpha} \quad \to \quad \frac{\alpha}{\beta} = \frac{\sigma_\alpha}{\sigma_\beta} > 1$

➥

$$\cos[\pi\sigma_\beta] = \frac{\pi}{2}\frac{\left(2H_{-1}[\pi\sigma_\alpha]\sin[\pi\sigma_\alpha]^2 + \pi H_0[\pi\sigma_\alpha]^2\cos[\pi\sigma_\alpha]\right)}{\left(2\sin[\pi\sigma_\alpha] + \pi H_0[\pi\sigma_\alpha]\right)^2}$$

$$\frac{\sigma_\alpha}{\sigma_\beta} = -\pi\frac{\sin[\pi\sigma_\alpha]H_0[\pi\sigma_\alpha]}{\sin[\pi\sigma_\beta]\left(2\sin[\pi\sigma_\alpha] + \pi H_0[\pi\sigma_\alpha]\right)}$$

☞

$$C = -0.974474$$

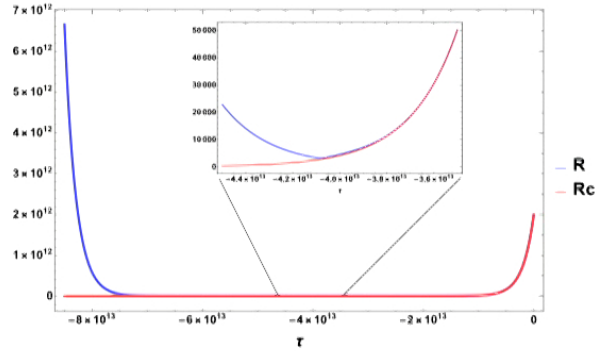
$$\frac{\alpha}{\beta} = 1.1421$$

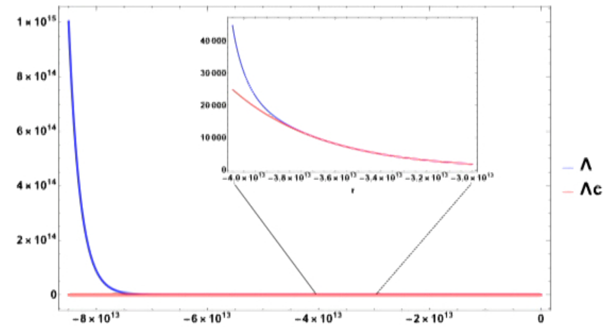
Kretschmann scalar

$$\mathcal{K}_c := R_{abcd}R^{abcd} = \frac{3\, e^{-\frac{3\tau}{Gm}}}{4(Gm)^4}$$

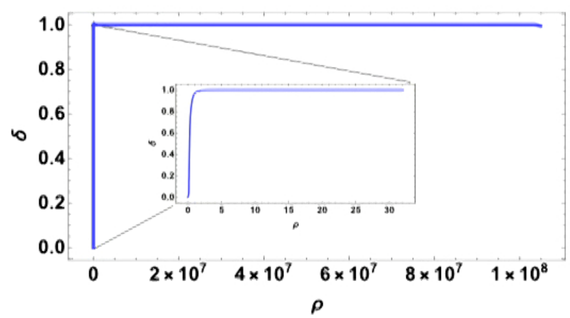
QG regime: $\mathcal{K}_c \sim 1/\ell_p^4\,,$ for $\tau_\star = \frac{Gm}{3}\log\left[\frac{3\ell_p^4}{(4G^4m^4)}\right]$

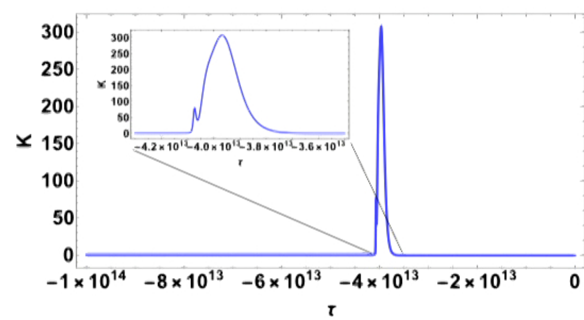
Numerical Solution: $m = 10^{12} m_p\,, \quad \tau_\star \approx -3.7\times 10^{13}$





$\delta := (\mathcal{K}/\mathcal{K}_c)^{1/4}\,, \quad \rho := R_c(\tau)/R_c(\tau_\star)$





No large quantum effects

All curvature invariants have a mass-independent bound

We predict an asymptotic Poincaré patch of de Sitter space-time inside for

$$\frac{j}{j_0} = \gamma \approx 0.2743$$

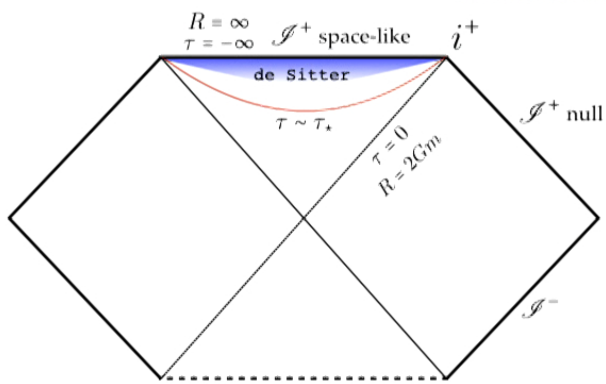
Hold on a second...

Same numerical value as from the SU(2) black hole entropy calculation!

[Agullo, Barbero, Borja, Diaz-Polo, Villasenor, PRD 2009]; [Engle, Noui, Perez, DP, PRD 2010]

☞ $$ds^2_{asym} = -\frac{\ell_p\sqrt{j}}{Gm}d\tau^2 + j^{2\delta'}\left(\frac{\ell_p}{Gm}\right)^{2\delta} e^{-\frac{\tau C}{Gm}}\left(dx^2 + \ell_p^2 d\Omega^2\right), \quad \delta, \delta' = const \sim o(1)$$

$$\tau \to -\infty\,, \quad R_{\mu\nu} = \lambda\, g_{\mu\nu}\,, \quad \text{with} \quad \boxed{\lambda = \frac{3C^2}{4\sqrt{j}}\frac{1}{Gm\,\ell_p}}$$



Discussion & Outlook

- ☑ By performing the symmetry reduction at the quantum level all relevant DOF are encoded in the effective dynamics

  -> Crucial modifications w.r.t. minisuperspace quantization models (no white holes)

- ☑ Geometric considerations + simplicity constraint

  -> Asymptotically de Sitter effective metric for the same Immirzi parameter value as in SU(2) BH entropy calculation

- ☑ Emerging cosmological constant due to quantum gravity effects

- ☐ Inclusion of matter: Does the gravitational collapse encode the history of the Universe??

- ☐ Horizon penetrating foliation: exterior and interior dynamics together for the first time.

  - Algebra of effective constraints [WIP with Camilletti, Dekhil]

  - Development of numerical techniques to solve non-local ODE's [WIP with Luzi]

⋮