Title: Conformal dimensions in the large charge sectors at the Wilson-Fisher fixed point using qubit formulations

Speakers: Shailesh Chandrasekharan

Series: Condensed Matter

Date: May 28, 2019 - 3:30 PM

URL: http://pirsa.org/19050022

Abstract:

Using Monte Carlo methods we explore how well does the recent proposal for computing conformal dimensions, using a large charge expansion, work. We focus on the O(2) and the O(4) Wilson-Fisher fixed points as test cases. Since the traditional Monte Carlo approach suffers from a severe signal-to-noise ratio problem in the large charge sectors, we use worldline formulations that eliminate such problems. In particular we argue that the O(4) model can be simplified drastically by studying what we refer to as a "qubit" formulation. Such simpler formulations of quantum field theories have become interesting recently from the perspective of quantum computing. Using our studies we confirm that the conformal dimensions of both conformal field theories with O(2) and O(4) symmetries obey a simple formula predicted by the large charge expansion. We also compute the two leading universal low energy constants in both cases , that play an important role in the large charge expansion.

Conformal dimensions in large charge sectors at the Wilson-Fisher fixed point using "qubit" regularizations

Shailesh Chandrasekharan
(Duke University)

Perimeter Institute

Collaborators
D.Banerjee, D.Orlando,S.Reffert
T.Bhattacharya, R.Gupta, H.Singh, R.Somma

Supported by:
US Department of Energy

Motivation

Conformal field theories are characterized by conformal dimensions D_Q of primary field operators.

Another proposal: “Q-expansion” (large charge expansion)

Hellerman, Orlando, Reffert, Watanabe JHEP 12(2015) 71.

Alvarez-Gaume, Loukas, Orlando, Reffert, JHEP 4 (2017) 59.

Idea:

Identify a conserved charge Q in the theory

Consider computing the conformal dimension D_Q associated with the primary field with large Q.

Use “radial quantization”, to argue that computing D_Q is equivalent to computing the energy of the theory on a sphere with unit radius.

When Q is large this energy can be computed in a semiclassical expansion in an powers of 1/Q starting using ideas of effective field theories with unknown constants.

Idea:
Since the only scale in the problem is R we must be able to compute E(R) as a function of R in the charge Q sector.

$$E(R) = 4\pi R^2 \times (EnergyDensity)$$

$$(Charge\ Density) \sim \left(\frac{Q}{4\pi R^2}\right)$$

Q: How well does this approach work in practice?

A: Compute D_Q using a Monte Carlo method and check!

Challenge: Computing D_Q using Monte Carlo methods suffers from severe signal to noise ratio problems with conventional methods for large Q.

New ideas for studying CFTs using Monte Carlo Methods!

New ideas for studying CFTs using Monte Carlo Methods!

Worldline Formulations

Qubit Formulations

The O(2) Model

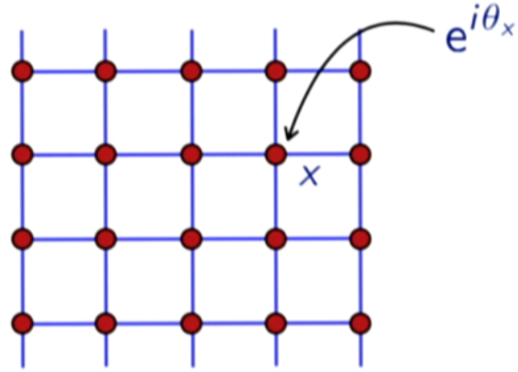
Banerjee, SC, Orlando PRL 120, (2016) 061603

The O(2) Model

Banerjee, SC, Orlando PRL 120, (2016) 061603

Traditional

$$Z = \int [d\theta]\ e^{\beta \sum_{x,\alpha} \cos(\theta_x - \theta_{x+\alpha})}$$

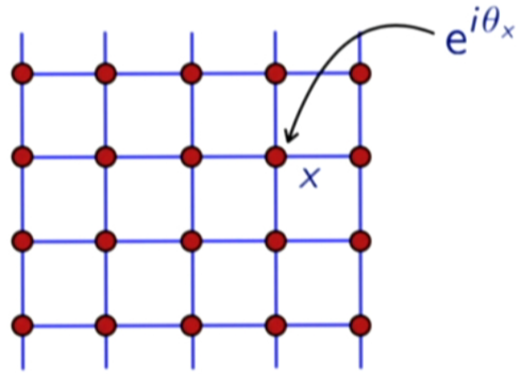


The O(2) Model

Banerjee, SC, Orlando PRL 120, (2016) 061603

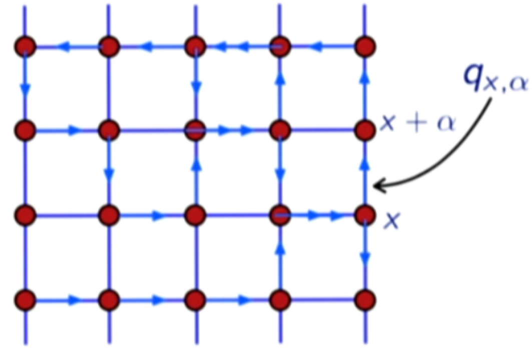
Traditional

$$Z = \int [d\theta]\ e^{\beta \sum_{x,\alpha} \cos(\theta_x - \theta_{x+\alpha})}$$



Worldline

$$Z = \sum_{[q]} \left[\prod_{x,\alpha} I_{q_{x,\alpha}}(\beta/2) \right] \left[\prod_x \delta\Big(\sum_\alpha (q_{x,\alpha} - q_{x-\alpha,\alpha}) \Big) \right]$$



Partition function with sources and sinks

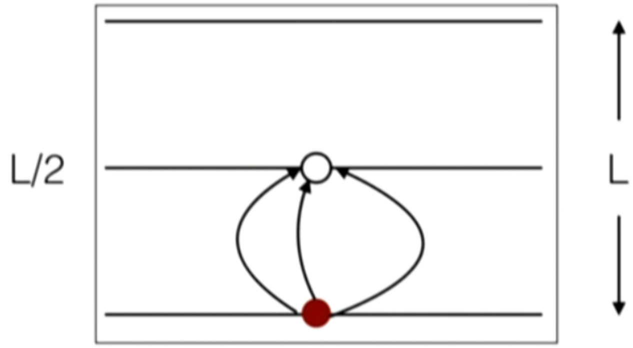
$$Z_Q = \sum_{[q]} \Big[\prod_{x,\alpha} I_{q_{x,\alpha}}(\beta/2)\Big] \Big[\prod_{x\neq x_i,x_f} \delta\Big(\sum_\alpha (q_{x,\alpha} - q_{x-\alpha,\alpha})\Big)\Big]$$

$$\delta\Big(\sum_\alpha (q_{x_i,\alpha} - q_{x_i-\alpha,\alpha} - Q\Big)\ \delta\Big(\sum_\alpha (q_{x_f,\alpha} - q_{x_f-\alpha,\alpha} + Q\Big)$$

Partition function with sources and sinks

$$Z_Q = \sum_{[q]} \left[\prod_{x,\alpha} I_{q_{x,\alpha}}(\beta/2)\right] \left[\prod_{x\neq x_i,x_f} \delta\Big(\sum_{\alpha}(q_{x,\alpha} - q_{x-\alpha,\alpha})\Big)\right]$$

$$\delta\Big(\sum_{\alpha}(q_{x_i,\alpha} - q_{x_i-\alpha,\alpha} - Q\Big)\ \delta\Big(\sum_{\alpha}(q_{x_f,\alpha} - q_{x_f-\alpha,\alpha} + Q\Big)$$

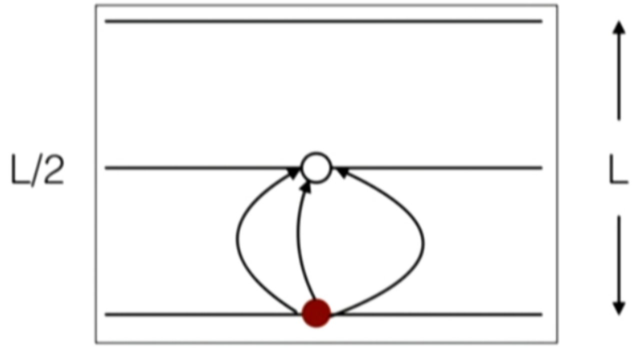


L x L box

Partition function with sources and sinks

$$Z_Q = \sum_{[q]} \Big[\prod_{x,\alpha} I_{q_{x,\alpha}}(\beta/2)\Big] \Big[\prod_{x\neq x_i,x_f} \delta\Big(\sum_{\alpha}(q_{x,\alpha} - q_{x-\alpha,\alpha})\Big)\Big]$$

$$\delta\Big(\sum_{\alpha}(q_{x_i,\alpha} - q_{x_i-\alpha,\alpha} - Q\Big)\ \delta\Big(\sum_{\alpha}(q_{x_f,\alpha} - q_{x_f-\alpha,\alpha} + Q\Big)$$



L x L box

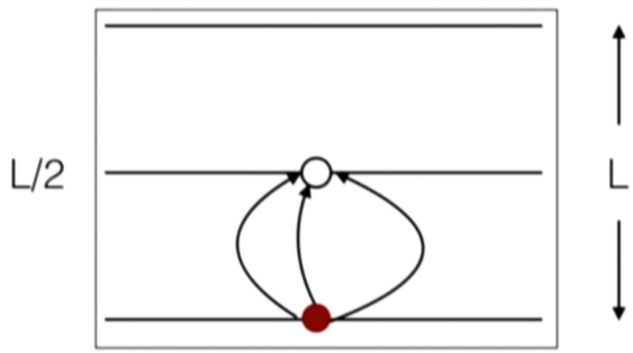
Scaling:

$Z_Q \sim 1/L^{D_Q}$

Partition function with sources and sinks

$$Z_Q = \sum_{[q]} \left[\prod_{x,\alpha} I_{q_{x,\alpha}}(\beta/2) \right] \left[\prod_{x \neq x_i, x_f} \delta\left(\sum_{\alpha} (q_{x,\alpha} - q_{x-\alpha,\alpha}) \right) \right]$$

$$\delta\left(\sum_{\alpha} (q_{x_i,\alpha} - q_{x_i-\alpha,\alpha} - Q \right) \delta\left(\sum_{\alpha} (q_{x_f,\alpha} - q_{x_f-\alpha,\alpha} + Q \right)$$



L x L box

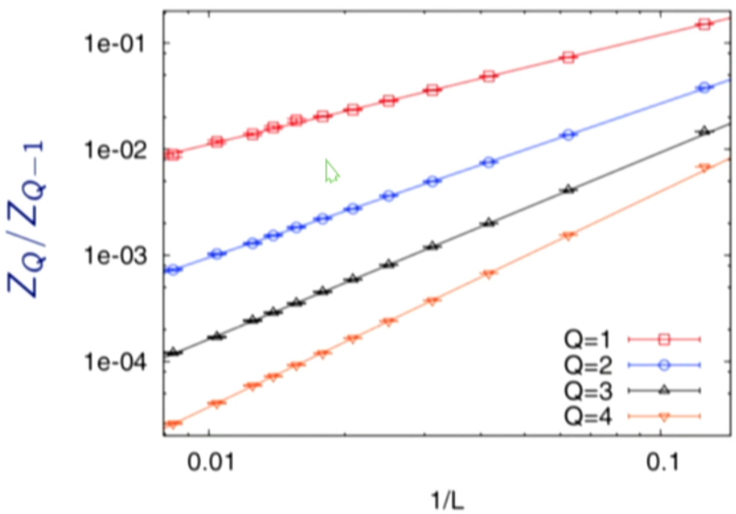
Scaling:

$$Z_Q \sim 1/L^{D_Q}$$

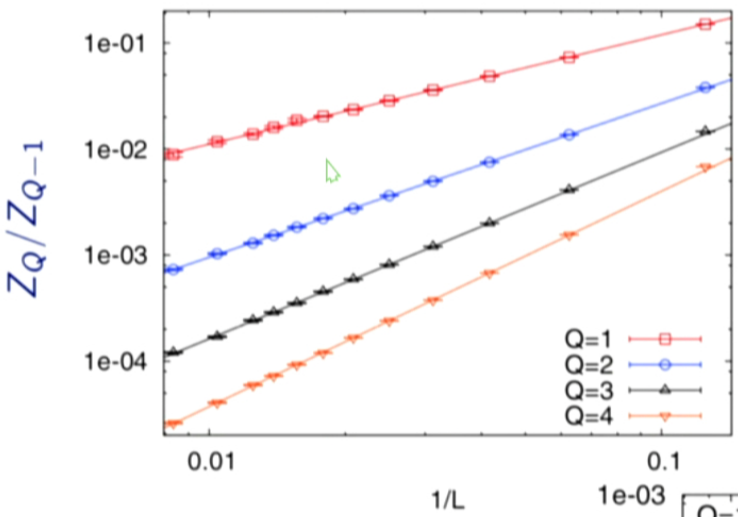
Worm algorithms can compute

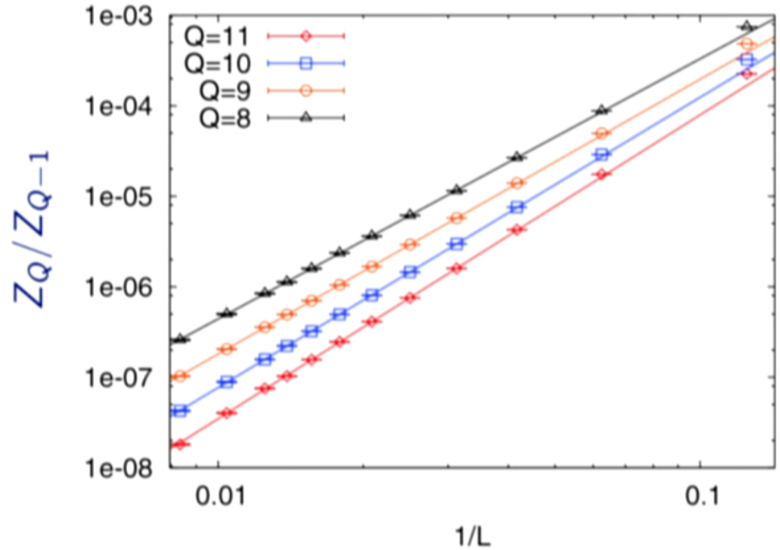
$$Z_Q/Z_{Q-1} \sim 1/L^{\Delta_Q}$$

$$\Delta_Q = D_Q - D_{Q-1}$$



Z_Q/Z_{Q-1}
1e-01
1e-02
1e-03
1e-04
0.01
0.1
1/L
Q=1
Q=2
Q=3
Q=4


Z_Q/Z_{Q-1}
1e-01
1e-02
1e-03
1e-04
0.01
0.1
1/L
Q=1
Q=2
Q=3
Q=4


Z_Q/Z_{Q-1}
1e-03
1e-04
1e-05
1e-06
1e-07
1e-08
0.01
0.1
1/L
Q=11
Q=10
Q=9
Q=8

Previous calculations for D_Q only up to Q=4

MC results are from Hasenbusch, Vicari, PRB 84 (2011) 125136

Q	ϵ^5	λ^6	MC	Bootstrap
1	0.518(1)	···	0.5190(1)	0.5190(1)
2	1.234(3)	1.23(2)	1.236(1)	1.236(3)
3	2.10(1)	2.10(1)	2.108(2)	···
4	3.114(4)	3.103(8)	3.108(6)	···

Previous calculations for D_Q only up to Q=4

MC results are from Hasenbusch, Vicari, PRB 84 (2011) 125136

Q	ϵ^5	λ^6	MC	Bootstrap
1	0.518(1)	...	0.5190(1)	0.5190(1)
2	1.234(3)	1.23(2)	1.236(1)	1.236(3)
3	2.10(1)	2.10(1)	2.108(2)	...
4	3.114(4)	3.103(8)	3.108(6)	...

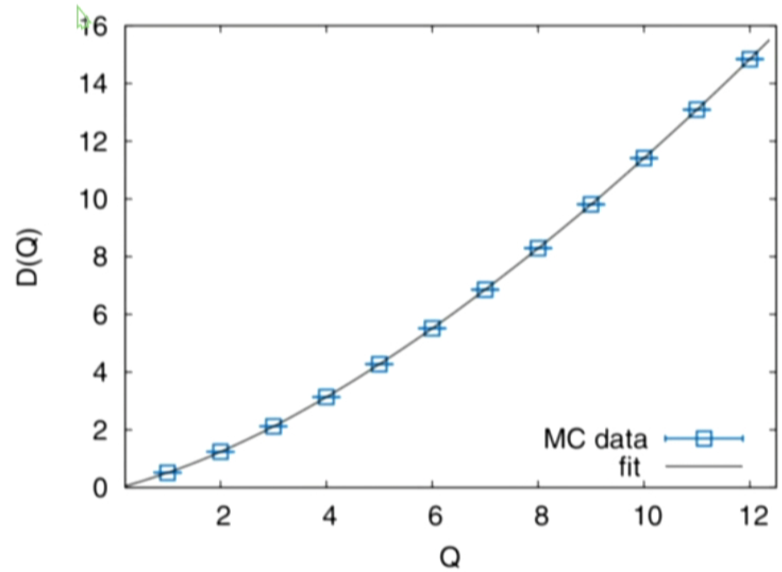
Our results: Banerjee, SC, Orlando PRL 120, (2016) 061603

Q	$\Delta(Q)$	$D(Q)$	Q	$\Delta(Q)$	$D(Q)$
1	0.516(3)	0.516(3)	7	1.332(5)	6.841(8)
2	0.722(4)	1.238(5)	8	1.437(4)	8.278(9)
3	0.878(4)	2.116(6)	9	1.518(2)	9.796(9)
4	1.012(2)	3.128(6)	10	1.603(2)	11.399(10)
5	1.137(2)	4.265(6)	11	1.678(5)	13.077(11)
6	1.243(3)	5.509(7)	12	1.748(5)	14.825(12)

Q: How well does the Q-expansion work?

Q: How well does the Q-expansion work?

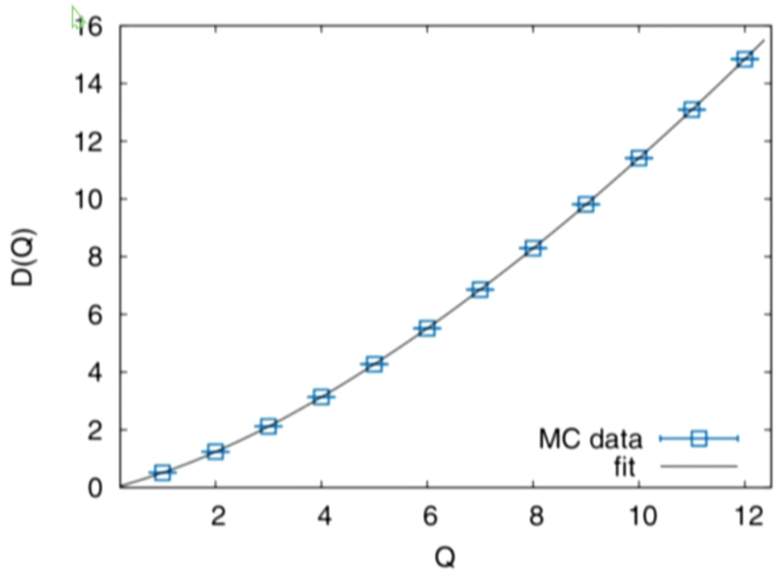
Q: How well does the Q-expansion work?



Fit Data: $D_Q \;=\; 1.195(10)\; Q^{3/2} \;+\; 0.075(10)\; Q^{1/2} \;-\; 0.094$

analytic calculation

Q: How well does the Q-expansion work?



Fit Data: $D_Q = 1.195(10)\ Q^{3/2} + 0.075(10)\ Q^{1/2} - 0.094$

analytic calculation

Q: What about the O(4) Wilson-Fisher fixed point, especially since it has two charges (j_L, j_R) that characterizes "charged sectors."

Q: What about the O(4) Wilson-Fisher fixed point, especially since it has two charges (j_L, j_R) that characterizes "charged sectors."

Since the traditional dual representations are quite complex, can we construct a simpler formulation of the theory?

Q: What about the O(4) Wilson-Fisher fixed point, especially since it has two charges (j_L, j_R) that characterizes "charged sectors."

Since the traditional dual representations are quite complex, can we construct a simpler formulation of the theory?

Q: What about the O(4) Wilson-Fisher fixed point, especially since it has two charges (j_L, j_R) that characterizes "charged sectors."

Since the traditional dual representations are quite complex, can we construct a simpler formulation of the theory?

Qubit formulation of the O(4) Wilson-Fisher fixed point!

Qubit Regularization of QFTs

Qubit Regularization of QFTs

Canonical commutation relation of QFTs requires an infinite dimensional Hilbert space per lattice site.

$$[\phi(x), \pi(y)] = i\delta_{x,y}$$

Qubit Regularization of QFTs

Canonical commutation relation of QFTs requires an infinite dimensional Hilbert space per lattice site.

$$[\phi(x), \pi(y)] = i\delta_{x,y}$$

Traditional formulations of scalar and gauge field theories begin with this commutation relation and hence require an infinite dimensional Hilbert space per spatial site.

Definition: Qubit Regularization of a QFT reproduces the QFT of interest with a finite dimensional Hilbert space per lattice site.

Qubit Regularization of QFTs

Canonical commutation relation of QFTs requires an infinite dimensional Hilbert space per lattice site.

$$[\phi(x), \pi(y)] = i\delta_{x,y}$$

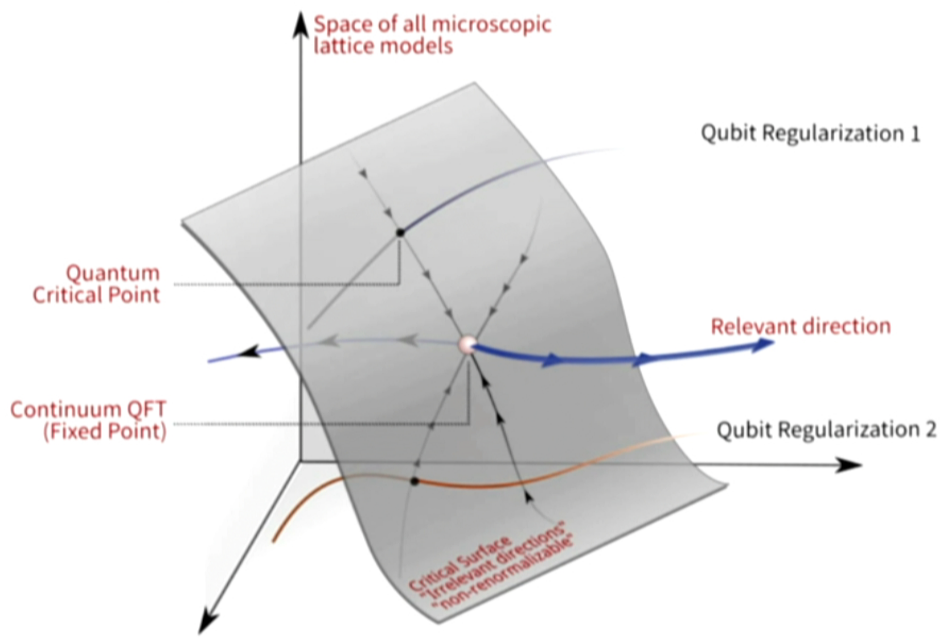
Traditional formulations of scalar and gauge field theories begin with this commutation relation and hence require an infinite dimensional Hilbert space per spatial site.

Definition: Qubit Regularization of a QFT reproduces the QFT of interest with a finite dimensional Hilbert space per lattice site.

Fermions are already qubits, but with anti-commutation relations.

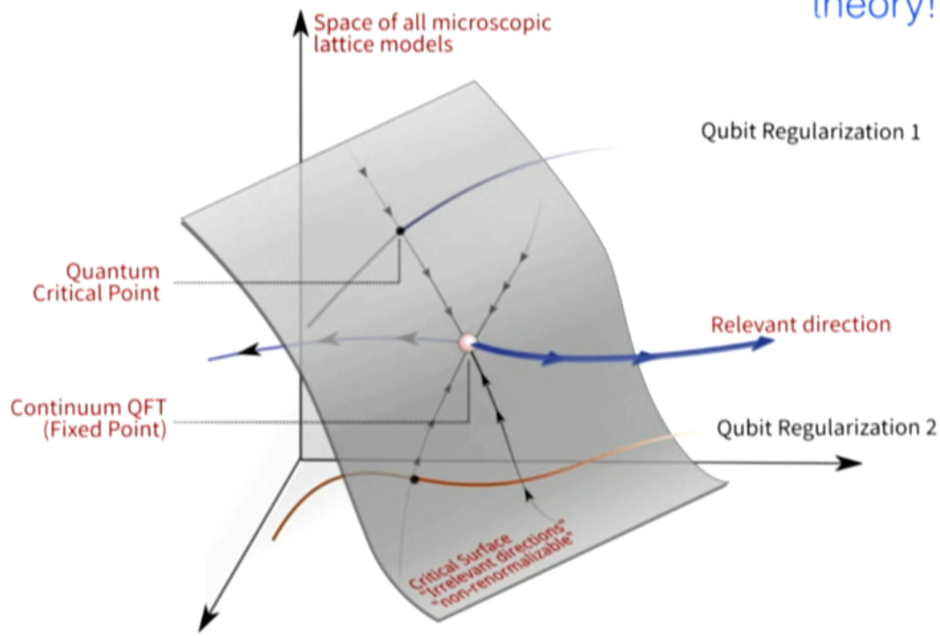
Insight from non-perturbative Wilson's RG

Insight from non-perturbative Wilson's RG



Insight from non-perturbative Wilson's RG

Identifying QCPs usually requires tools beyond perturbation theory!



It is important to identify the Quantum Critical Points that lead to the QFT of interest.

This approach to quantum field theories is well known since 1980s, but was not explored essentially due to lack of computational tools.

This approach to quantum field theories is well known since 1980s, but was not explored essentially due to lack of computational tools.

They are often standard in condensed matter literature, but the focus is usually on finding exotic phenomena.

This approach to quantum field theories is well known since 1980s, but was not explored essentially due to lack of computational tools.

They are often standard in condensed matter literature, but the focus is usually on finding exotic phenomena.

New algorithms become available and they are often simpler than traditional QFT but still reproduce the physics of interest.

D-theory approach, Wiese (2006)

This approach to quantum field theories is well known since 1980s, but was not explored essentially due to lack of computational tools.

They are often standard in condensed matter literature, but the focus is usually on finding exotic phenomena.

New algorithms become available and they are often simpler than traditional QFT but still reproduce the physics of interest.

D-theory approach, Wiese (2006)

Perhaps some day we can also design a quantum computer and develop algorithms to study them!

Jordan, Lee, Preskill (2012) + many more in the past two years!

This approach to quantum field theories is well known since 1980s, but was not explored essentially due to lack of computational tools.

They are often standard in condensed matter literature, but the focus is usually on finding exotic phenomena.

New algorithms become available and they are often simpler than traditional QFT but still reproduce the physics of interest.

D-theory approach, Wiese (2006)

Perhaps some day we can also design a quantum computer and develop algorithms to study them!

Jordan, Lee, Preskill (2012) + many more in the past two years!

This talk: They helped us to explore the large Q-expansion in the O(4) model!

Qubit Regularization of the O(4) scalar QFT

Banerjee,SC,Orlando,Reffert, 1902.09542

Qubit Regularization of the O(4) scalar QFT

Banerjee,SC,Orlando,Reffert, 1902.09542

Model with four flavors of hardcore bosons

Qubit Regularization of the O(4) scalar QFT

Banerjee,SC,Orlando,Reffert, 1902.09542

Model with four flavors of hardcore bosons

Hamiltonian

Qubit Regularization of the O(4) scalar QFT

Banerjee,SC,Orlando,Reffert, 1902.09542

Model with four flavors of hardcore bosons

Hamiltonian

$$H = -t \sum_{\langle xy \rangle, \alpha} \left(a^\dagger_{x,\alpha} a_{y,\alpha} + a^\dagger_{y,\alpha} a_{x,\alpha} \right)$$

(hopping term)

$$-t \sum_{\langle xy \rangle, \alpha} \left(a^\dagger_{x,\alpha} a^\dagger_{y,\alpha} + a_{y,\alpha} a_{x,\alpha} \right) \qquad + \mu \sum_{x,\alpha} a^\dagger_{x,\alpha} a_{x,\alpha}$$

(pair creation-annihilation term) (chemical potential term)

Local Hilbert Space is five dimensional (requires 3-qubits)

$|s, \mathbf{r}\rangle$

Fock vacuum

$|q^z_L, q^z_R, \mathbf{r}\rangle, \quad q^z_L, q^z_R = 1/2, -1/2$

O(4) vector (1/2,1/2) sector

Local Hilbert Space is five dimensional (requires 3-qubits)

$|s, \mathbf{r}\rangle$

Fock vacuum

$|q_L^z, q_R^z, \mathbf{r}\rangle, \quad q_L^z, q_R^z = 1/2, -1/2$

O(4) vector (1/2,1/2) sector

We denote (q_L^z, q_R^z)
as the z-components in the (j_L, j_R) sector

Local Hilbert Space is five dimensional (requires 3-qubits)

$|s, \mathbf{r}\rangle$

Fock vacuum

$|q_L^z, q_R^z, \mathbf{r}\rangle, \quad q_L^z, q_R^z = 1/2, -1/2$

O(4) vector (1/2,1/2) sector

We denote (q_L^z, q_R^z)
as the z-components in the (j_L, j_R) sector

Local Hilbert Space is five dimensional (requires 3-qubits)

$|s, \mathbf{r}\rangle$

Fock vacuum

$|q^z_L, q^z_R, \mathbf{r}\rangle, \quad q^z_L, q^z_R = 1/2, -1/2$

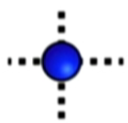
O(4) vector (1/2,1/2) sector

We denote (q^z_L, q^z_R)

as the z-components in the (j_L, j_R) sector

Pictorially:

(0,0) sector



Fock Vacuum
(Monomers)

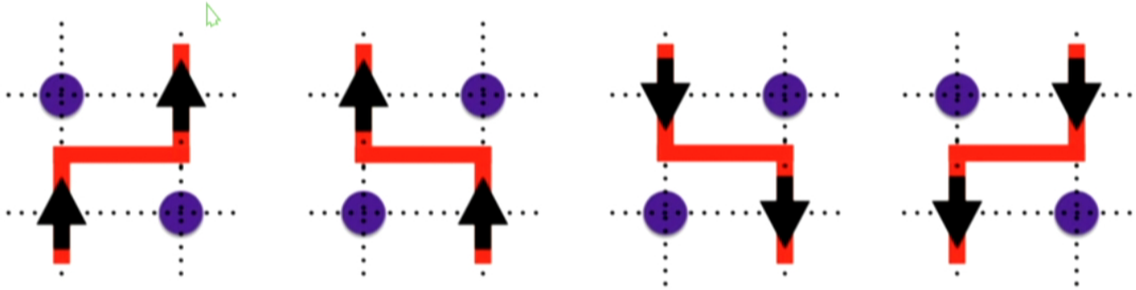
(1/2,1/2) sector

(1/2,1/2) (-1/2,1/2) (1/2,-1/2) (-1/2,-1/2)

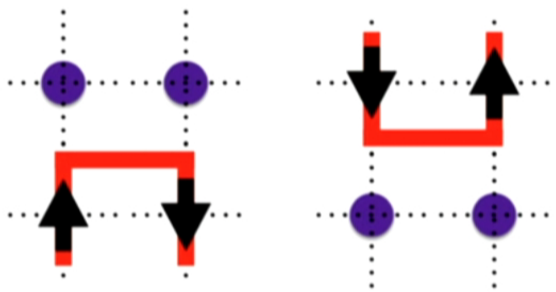
(q^z_L, q^z_R)

particles (worldlines)

Hopping term



Pair Creation/Annihilation term



Euclidean Qubit O(4) Model

Euclidean Qubit O(4) Model

$$Z = \sum_k \int [dt_k ... dt_1] \mathrm{Tr}\left(e^{-(\beta - t_k)H_1} (-H_2)\ e^{-(t_k - t_{k-1})H_1} \cdots (-H_2) e^{-(t_1)H_1} \right)$$

$$Z = \sum_{[s,m]} \prod_{\langle ij \rangle} W_{\langle ij \rangle}$$

Euclidean Qubit O(4) Model

$$Z = \sum_k \int [dt_k ... dt_1] \mathrm{Tr}\Big(e^{-(\beta - t_k)H_1} (-H_2)\ e^{-(t_k - t_{k-1})H_1} \cdots (-H_2) e^{-(t_1)H_1} \Big)$$

$$Z = \sum_{[s,m]} \prod_{\langle ij \rangle} W_{\langle ij \rangle}$$

Relativistic Limit

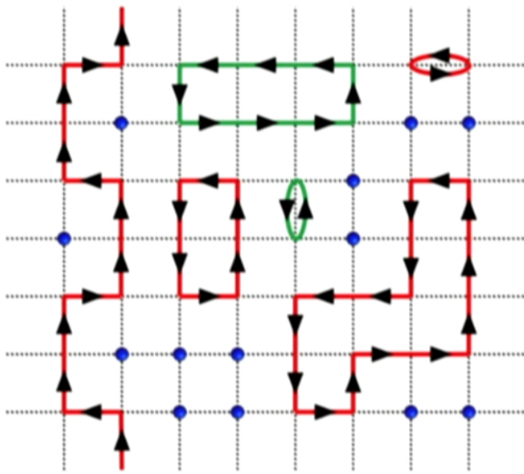
$$\varepsilon = 1$$

$$W_t = W_s$$

Euclidean Qubit O(4) Model

$$Z = \sum_k \int [dt_k ... dt_1] \mathrm{Tr}\Big(e^{-(\beta - t_k)H_1} (-H_2)\ e^{-(t_k - t_{k-1})H_1} \cdots (-H_2) e^{-(t_1)H_1} \Big)$$

$$Z = \sum_{[s,m]} \prod_{\langle ij \rangle} W_{\langle ij \rangle}$$



Relativistic Limit $\varepsilon = 1$, $W_t = W_s$

Hamiltonian limit $\varepsilon \to 0$

Can study using classical QMC
(directed loop/worm algorithms)

$W_s = \varepsilon t \qquad W_t = \exp(-\varepsilon \mu)$

Observables

Order Parameter Suceptibility

$$\chi = \frac{1}{ZL^d} \sum_{\mathbf{r},\mathbf{r}'} \int_0^\beta dt \; \mathrm{Tr}\Big(e^{-(\beta-t)H} a_{\mathbf{r},\mathbf{m}} e^{-tH} a^\dagger_{\mathbf{r}',\mathbf{m}} \Big)$$

Winding Number Susceptibility

$$\rho_s = \frac{1}{L^{d-2}\beta} \langle \, (Q_w)^2 \, \rangle$$

Observables

Order Parameter Suceptibility

$$\chi = \frac{1}{ZL^d} \sum_{\mathbf{r},\mathbf{r}'} \int_0^\beta dt\ \mathrm{Tr}\left(e^{-(\beta-t)H} a_{\mathbf{r},\mathbf{m}} e^{-tH} a^\dagger_{\mathbf{r}',\mathbf{m}}\right)$$

Winding Number Susceptibility

$$\rho_s = \frac{1}{L^{d-2}\beta} \langle\ (Q_w)^2\ \rangle$$

Near the critical point we expect

$$\chi / L^{2-\eta} = f((U - U_c) L^{1/nu})$$

$$\rho_s L = g((U - U_c) L^{1/nu})$$

Observables

Order Parameter Suceptibility

$$\chi = \frac{1}{ZL^d}\sum_{\mathbf{r},\mathbf{r}'}\int_0^\beta dt\ \mathrm{Tr}\left(e^{-(\beta-t)H}a_{\mathbf{r},\mathbf{m}}e^{-tH}a^\dagger_{\mathbf{r}',\mathbf{m}}\right)$$

Winding Number Susceptibility

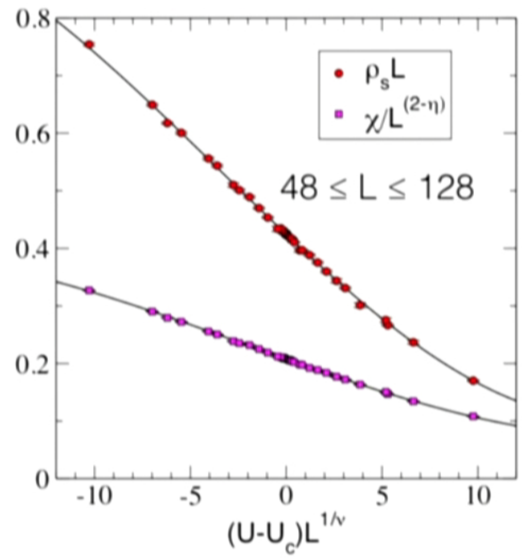
$$\rho_s = \frac{1}{L^{d-2}\beta}\langle\ (Q_w)^2\ \rangle$$

Near the critical point we expect

$$\chi/L^{2-\eta} = f((U-U_c)L^{1/nu})$$

$$\rho_s L = g((U-U_c)L^{1/nu})$$

Qubit Model Results



$$U_c = 1.655394(3)$$

$$\nu = 0.746(3), \eta = 0.0353(10)$$

Observables

Order Parameter Suceptibility

$$\chi = \frac{1}{ZL^d} \sum_{\mathbf{r},\mathbf{r}'} \int_0^\beta dt \; \mathrm{Tr}\left(e^{-(\beta-t)H} a_{\mathbf{r},\mathbf{m}} e^{-tH} a^\dagger_{\mathbf{r}',\mathbf{m}} \right)$$

Winding Number Susceptibility

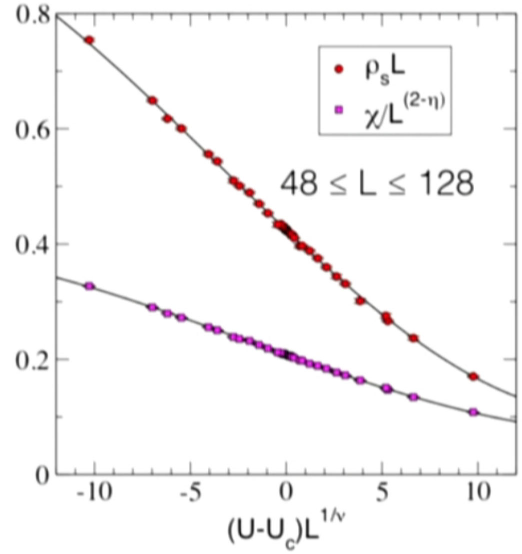
$$\rho_s = \frac{1}{L^{d-2}\beta} \langle \, (Q_w)^2 \, \rangle$$

Near the critical point we expect

$$\chi/L^{2-\eta} = f((U - U_c)L^{1/nu})$$

$$\rho_s L = g((U - U_c)L^{1/nu})$$

Qubit Model Results



$$U_c = 1.655394(3)$$

$$\nu = 0.746(3), \eta = 0.0353(10)$$

Pelisetto, Vicari Phys. Repts. (2002)

$$\nu = 0.749(2), \eta = 0.0365(10)$$

Large Charge Sectors "Q"

Now sectors are labeled with Q $\equiv$ $(j_L\ ,\ j_R\ )$

Large Charge Sectors "Q"

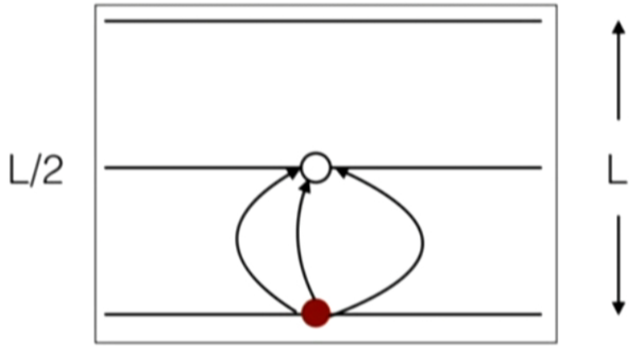
Now sectors are labeled with Q $\equiv$ $(j_L \ , \ j_R \ )$

Can choose any subsector (j_L, q_L^z, j_R, q_R^z) D_Q will be the same!

Large Charge Sectors "Q"

Now sectors are labeled with Q ≡ $(j_L \, , \, j_R)$

Can choose any subsector (j_L, q_L^z, j_R, q_R^z) D_Q will be the same!

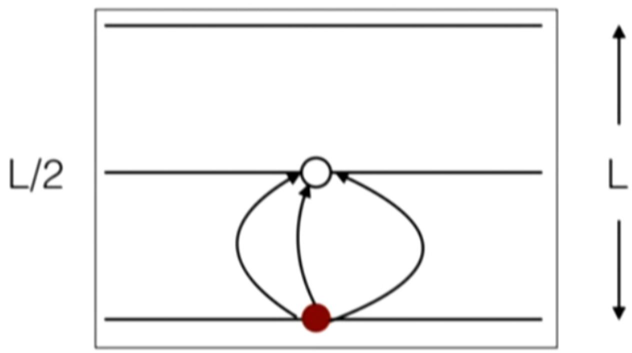


L x L box

Large Charge Sectors "Q"

Now sectors are labeled with Q ≡ $(j_L\,,\ j_R)$

Can choose any subsector (j_L, q^z_L, j_R, q^z_R) D_Q will be the same!



L x L box

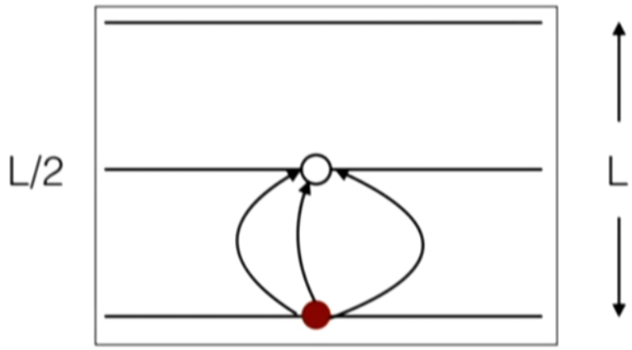
Scaling:

$$Z(j_J, j_R) \sim \frac{1}{L^{D(j_L, j_R)}}$$

Large Charge Sectors “Q”

Now sectors are labeled with Q ≡ (j_L, j_R)

Can choose any subsector (j_L, q_L^z, j_R, q_R^z) D_Q will be the same!



L x L box

Scaling:

$$Z_{(j_J, j_R)} \sim \frac{1}{L^{D(j_L, j_R)}}$$

Worm algorithms can compute

$$Z_{j_L j_R} / Z_{j_L' j_R'} \sim \frac{1}{L^{\Delta}}$$

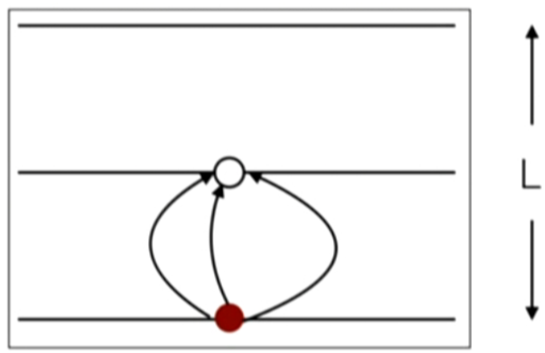
$$\Delta = D(j_L, j_R) - D(j_L', j_R')$$

Large Charge Sectors "Q"

Now sectors are labeled with Q ≡ $(j_L \ , \ j_R \ )$

Can choose any subsector (j_L, q_L^z, j_R, q_R^z) D_Q will be the same!

L/2



L x L box

Scaling:

$$Z_{(j_J, j_R)} \sim \frac{1}{L^{D(j_L, j_R)}}$$

Worm algorithms can compute

$$Z_{j_L j_R} / Z_{j_L' j_R'} \sim \frac{1}{L^{\Delta}}$$

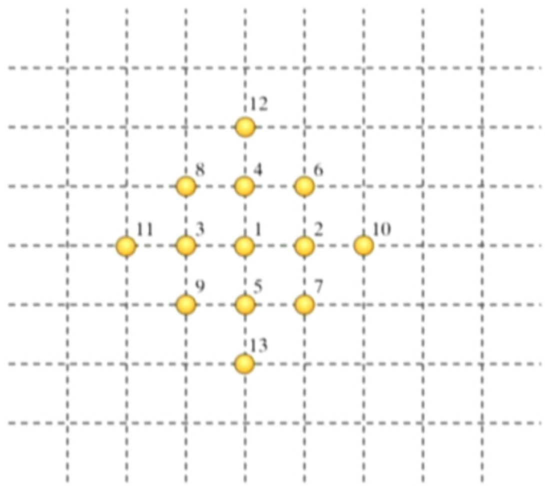
$$\Delta = D(j_L, j_R) - D(j_L', j_R')$$

Q: How do we construct an operator in a given (j_L ,$q_L{}^z$, j_R, $q_R{}^z$) sector?

We need to spread out the charges due to the hard core constraint!

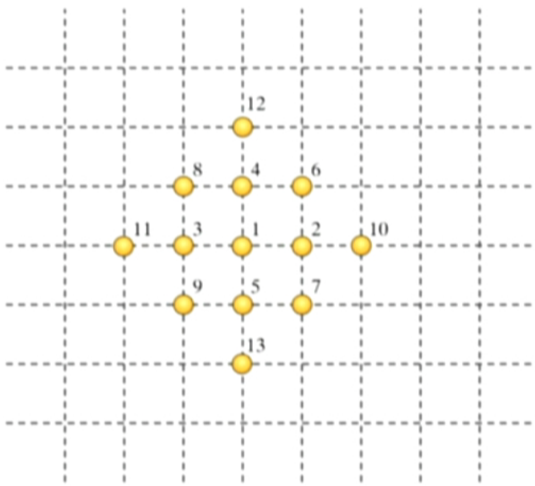
We need to spread out the charges due to the hard core constraint!

Location of charges
at t=0 and t=L/2

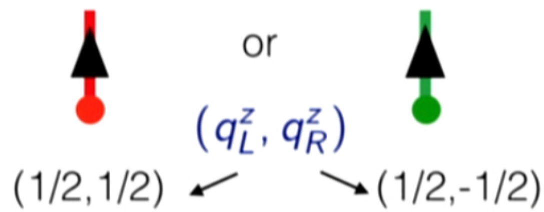


We need to spread out the charges due to the hard core constraint!

Location of charges
at t=0 and t=L/2



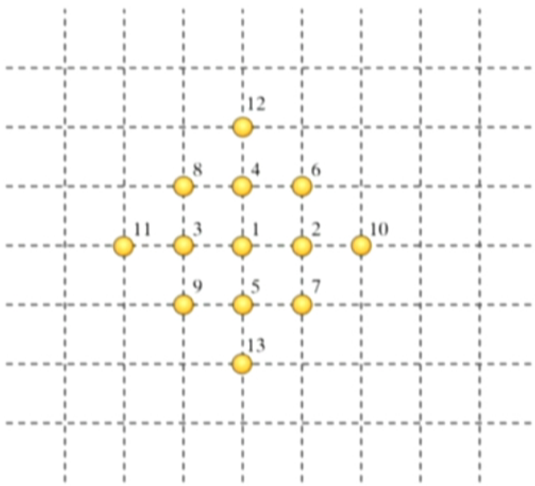
At t=0 on each site we can
create sources of two types
of particles in (1/2,1/2) sector



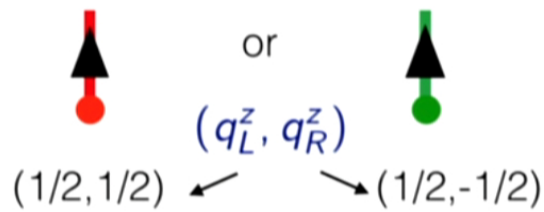
We can annihilate them at t = L/2

We need to spread out the charges due to the hard core constraint!

Location of charges
at t=0 and t=L/2



At t=0 on each site we can
create sources of two types
of particles in (1/2,1/2) sector

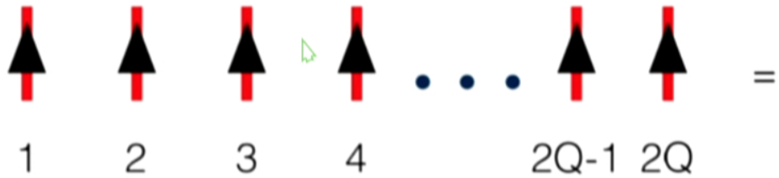


We can annihilate them at t = L/2

We then need to project on to one
of the (j_L,j_R) sector.

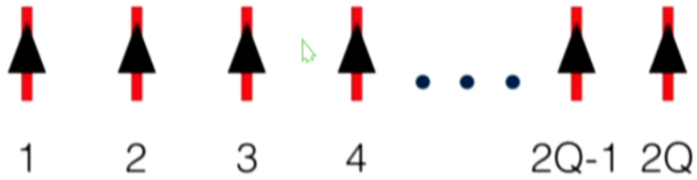
The (Q,Q) sector is easy. We simply create sources from the same particle.

The (Q,Q) sector is easy. We simply create sources from the same particle.



$$|j_L = Q, q_L^z = Q;\ j_R = Q, q_R^z = Q\rangle$$

The (Q,Q) sector is easy. We simply create sources from the same particle.



$=$

$$|j_L = Q, q_L^z = Q;\ j_R = Q, q_R^z = Q\rangle$$

Other sectors need some work.

For example: (Q,Q-1) can be obtained from

$=$

$$\alpha\ |j_L = Q, q_L^z = Q;\ j_R = Q, q_R^z = Q-1\rangle$$

$$+\ \beta\ |j_L = Q, q_L^z = Q;\ j_R = Q-1, q_R^z = Q-1\rangle$$

Need superpositions to construct (Q,Q-1)!

Example: Q=1

$$|j_L = 1, q_L^z = 1;\ j_R = 1, q_R^z = 0\rangle = \frac{1}{\sqrt{2}}$$

Need superpositions to construct (Q,Q-1)!

Example: Q=1

$$|j_L = 1, q_L^z = 1;\ j_R = 1, q_R^z = 0\rangle = \frac{1}{\sqrt{2}}\left(\quad + \quad \right)$$

Need superpositions to construct (Q,Q-1)!

Example: Q=1

$$|j_L = 1, q_L^z = 1;\ j_R = 1, q_R^z = 0\rangle = \frac{1}{\sqrt{2}}$$

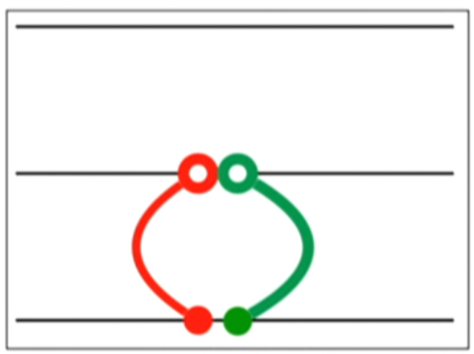
$$|j_L = 1, q_L^z = 1;\ j_R = 0, q_R^z = 0\rangle = \frac{1}{\sqrt{2}}$$

Need superpositions to construct (Q,Q-1)!

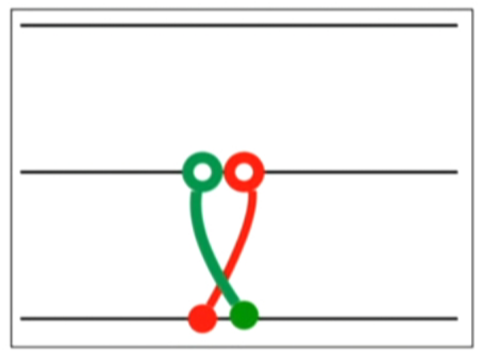
Example: Q=1

$$|j_L = 1, q_L^z = 1;\ j_R = 1, q_R^z = 0\rangle = \frac{1}{\sqrt{2}}$$

$$|j_L = 1, q_L^z = 1;\ j_R = 0, q_R^z = 0\rangle = \frac{1}{\sqrt{2}}$$



(1,1)
+
−
(1,0)



Note for j_L not equal to j_R sectors have cancellations

Predictions from EFT

Alvarez-Gaume, Loukas, Orlando, Reffert, JHEP 4 (2017) 59.

Banerjee,SC,Orlando,Reffert, 1902.09542

Predictions from EFT

Alvarez-Gaume, Loukas, Orlando, Reffert, JHEP 4 (2017) 59.

Banerjee,SC,Orlando,Reffert, 1902.09542

O(4) fields: $g(\mathbf{r}, t) \in SU(2)$ $\quad \mathbf{r} \in \Sigma$ (spatial manifold (sphere))

Predictions from EFT

Alvarez-Gaume, Loukas, Orlando, Reffert, JHEP 4 (2017) 59.

Banerjee,SC,Orlando,Reffert, 1902.09542

O(4) fields: $g(\mathbf{r}, t) \in SU(2)$ $\mathbf{r} \in \Sigma$ (spatial manifold (sphere))

Effective Action at the conformal point:

$$S = \int_{\mathbb{R}\times\Sigma} \mathrm{d}t\, \mathrm{d}\Sigma \left[\frac{\sqrt{2}}{27 c_{3/2}^2} \|\mathrm{d}g\|^3 - \frac{c_{1/2}}{3\sqrt{2} c_{3/2}} R \|\mathrm{d}g\| + \dots \right],$$

$$\mathrm{Tr}\left(\partial_\mu g^\dagger \partial^\mu g\right),$$

Input: DVI - 1920x1080p@60Hz
Output: SDI - 1920x1080i@60Hz

Predictions from EFT

Alvarez-Gaume, Loukas, Orlando, Reffert, JHEP 4 (2017) 59.
Banerjee,SC,Orlando,Reffert, 1902.09542

O(4) fields: $g(\mathbf{r}, t) \in SU(2)$ $\mathbf{r} \in \Sigma$ (spatial manifold (sphere))

Effective Action at the conformal point:

$$S = \int_{\mathbb{R}\times\Sigma} \mathrm{d}t\, \mathrm{d}\Sigma \left[\frac{\sqrt{2}}{27 c_{3/2}^2} \|\mathrm{d}g\|^3 - \frac{c_{1/2}}{3\sqrt{2} c_{3/2}} R \|\mathrm{d}g\| + \dots \right],$$

$$\mathrm{Tr}\left(\partial_\mu g^\dagger \partial^\mu g\right),$$

Conserved Noether charges:

$$Q_{\mathrm{L}} = i \int \mathrm{d}\Sigma\, c_J\, \partial_0 g g^\dagger, \quad Q_{\mathrm{R}} = i \int \mathrm{d}\Sigma\, c_J\, \partial_0 g^\dagger g,$$

Predictions from EFT

Alvarez-Gaume, Loukas, Orlando, Reffert, JHEP 4 (2017) 59.
Banerjee,SC,Orlando,Reffert, 1902.09542

O(4) fields: $g(\mathbf{r}, t) \in SU(2)$ $\quad \mathbf{r} \in \Sigma$ (spatial manifold (sphere))

Effective Action at the conformal point:

$$S = \int_{\mathbb{R}\times\Sigma} \mathrm{d}t\,\mathrm{d}\Sigma \left[\frac{\sqrt{2}}{27c_{3/2}^2}\|\mathrm{d}g\|^3 - \frac{c_{1/2}}{3\sqrt{2}c_{3/2}}R\|\mathrm{d}g\| + \dots\right],$$

$$\mathrm{Tr}\left(\partial_\mu g^\dagger \partial^\mu g\right),$$

Conserved Noether charges:

$$Q_{\mathrm{L}} = i\int \mathrm{d}\Sigma\, c_J\, \partial_0\, g g^\dagger, \quad Q_{\mathrm{R}} = i\int \mathrm{d}\Sigma\, c_J\, \partial_0\, g^\dagger g,$$

Solve the theory in a semiclassical expansion in a given charged sector.

Predictions:

$j_L = j_R = j$ Large j

$$D(j,j) = \sqrt{\frac{2j^3}{\pi}}\left(c_{3/2} + c_{1/2}\frac{2\pi}{j} + \mathcal{O}\left(\frac{1}{j^2}\right)\right) + c_0, \quad c_0 = -0.094...$$

Predictions:

$j_L = j_R = j$ Large j

$$D(j,j) = \sqrt{\frac{2j^3}{\pi}}\left(c_{3/2} + c_{1/2}\frac{2\pi}{j} + \mathcal{O}\left(\frac{1}{j^2}\right)\right) + c_0, \quad c_0 = -0.094\ldots$$

$j_L \neq j_R$ Large $j_m = \max(j_L, j_R)$ but small $|j_L - j_R|/j_m$.

$$D(j_L, j_R) = \sqrt{\frac{2j_m^3}{\pi}}\left[c_{3/2} + c_{1/2}\frac{2\pi}{j_m} + \frac{1}{3c_{3/2}}\left(\frac{|j_L - j_R|}{j_m} + \lambda_2\frac{(j_L - j_R)^2}{j_m^2} + \ldots\right)\frac{2\pi}{j_m} + \ldots\right] - \frac{1}{12\sqrt{2}}.$$

$$\lambda_2 \approx 0.2455.$$

Predictions:

$j_L = j_R = j$ Large j

$$D(j,j) = \sqrt{\frac{2j^3}{\pi}}\left(c_{3/2} + c_{1/2}\frac{2\pi}{j} + \mathcal{O}\left(\frac{1}{j^2}\right)\right) + c_0, \quad c_0 = -0.094\ldots$$

$j_L \neq j_R$ Large $j_m = \max(j_L, j_R)$ but small $|j_L - j_R|/j_m$.

$$D(j_{\rm L}, j_{\rm R}) = \sqrt{\frac{2j_m^3}{\pi}}\left[c_{3/2} + c_{1/2}\frac{2\pi}{j_m} + \frac{1}{3c_{3/2}}\left(\frac{|j_{\rm L} - j_{\rm R}|}{j_m} + \lambda_2\frac{(j_{\rm L} - j_{\rm R})^2}{j_m^2} + \ldots\right)\frac{2\pi}{j_m} + \ldots\right] - \frac{1}{12\sqrt{2}}.$$

$$\lambda_2 \approx 0.2455.$$

Note: the leading corrections depend only on the coefficient $c_{3/2}$

Predictions:

$j_L = j_R = j$ Large j

$$D(j,j) = \sqrt{\frac{2j^3}{\pi}}\left(c_{3/2} + c_{1/2}\frac{2\pi}{j} + \mathcal{O}\left(\frac{1}{j^2}\right)\right) + c_0, \quad c_0 = -0.094\ldots$$

$j_L \neq j_R$ Large $j_m = \max(j_L, j_R)$ but small $|j_L - j_R|/j_m$.

$$D(j_{\rm L}, j_{\rm R}) = \sqrt{\frac{2j_m^3}{\pi}}\left[c_{3/2} + c_{1/2}\frac{2\pi}{j_m} + \frac{1}{3c_{3/2}}\left(\frac{|j_{\rm L} - j_{\rm R}|}{j_m} + \lambda_2\frac{(j_{\rm L} - j_{\rm R})^2}{j_m^2} + \ldots\right)\frac{2\pi}{j_m} + \ldots\right] - \frac{1}{12\sqrt{2}}.$$

$$\lambda_2 \approx 0.2455.$$

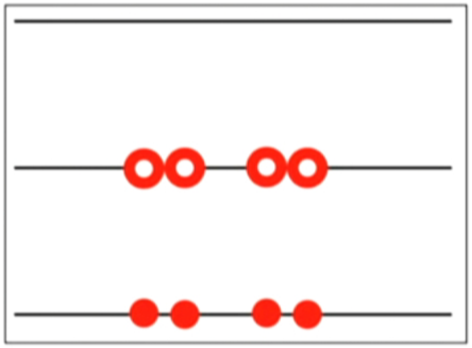
Note: the leading corrections depend only on the coefficient $c_{3/2}$

Once we know $c_{3/2}$ and $c_{1/2}$ we have a prediction for all $D(j_L,j_R)$ in the large charge expansion!

Worm algorithms can be used to compute

$$R_j = \frac{Z_{j,j}(L)}{Z_{j-1/2,j-1/2}(L)} = \frac{C}{L^{2\Delta(j)}}$$

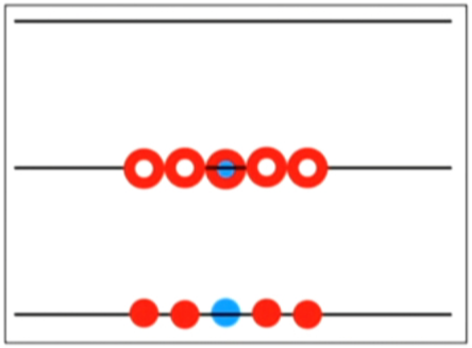
$$\Delta = D(j,j) - D(j-1/2, j-1/2)$$



Worm algorithms can be used to compute

$$R_j = \frac{Z_{j,j}(L)}{Z_{j-1/2,j-1/2}(L)} = \frac{C}{L^{2\Delta(j)}}$$

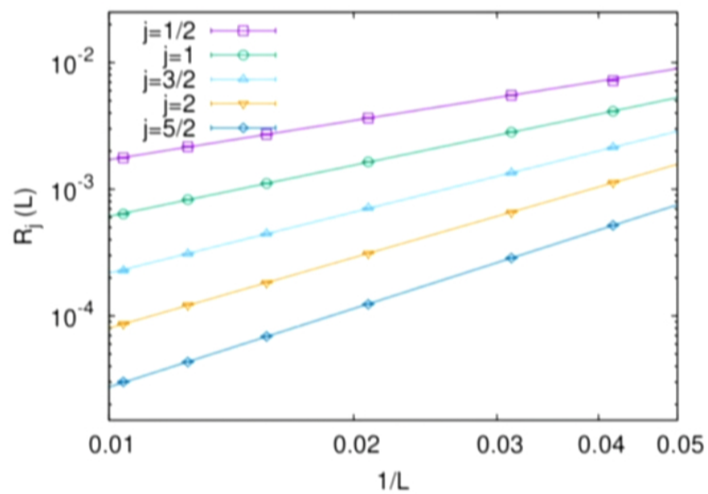
$$\Delta = D(j,j) - D(j-1/2, j-1/2)$$

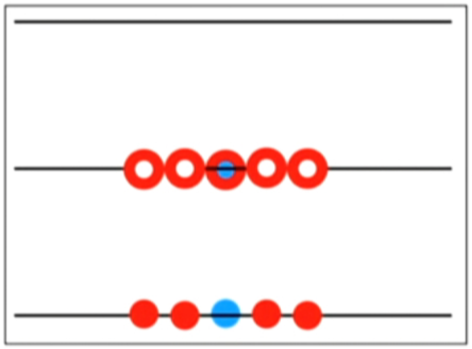


Worm algorithms can be used to compute

$$R_j = \frac{Z_{j,j}(L)}{Z_{j-1/2,j-1/2}(L)} = \frac{C}{L^{2\Delta(j)}}$$

$$\Delta = D(j,j) - D(j-1/2, j-1/2)$$

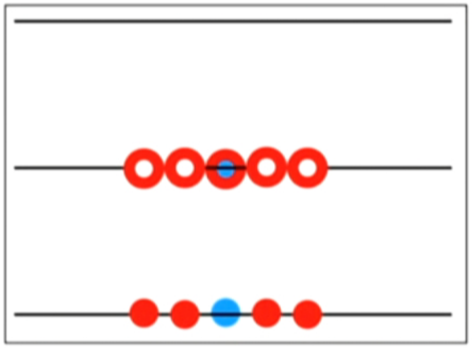


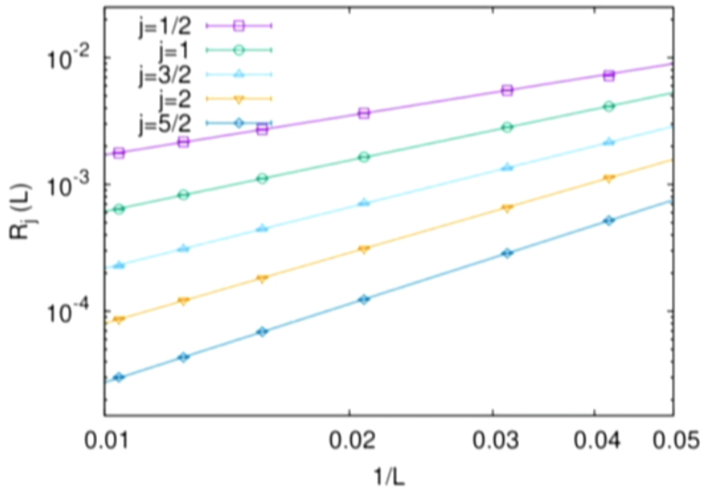


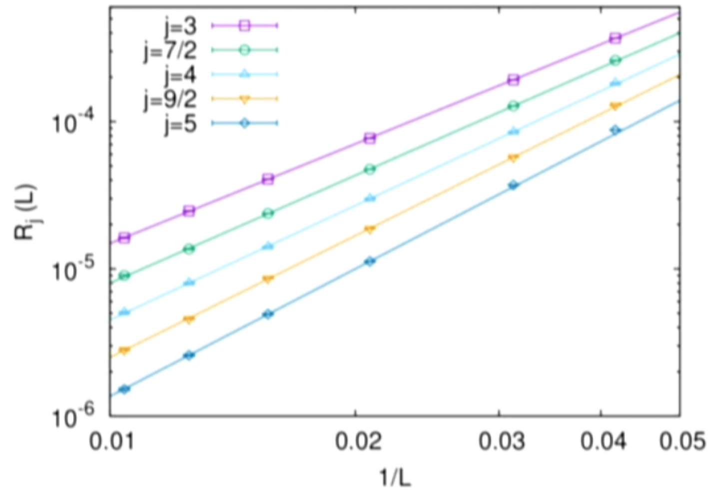
Worm algorithms can be used to compute

$$R_j = \frac{Z_{j,j}(L)}{Z_{j-1/2,j-1/2}(L)} = \frac{C}{L^{2\Delta(j)}}$$

$$\Delta = D(j,j) - D(j-1/2, j-1/2)$$





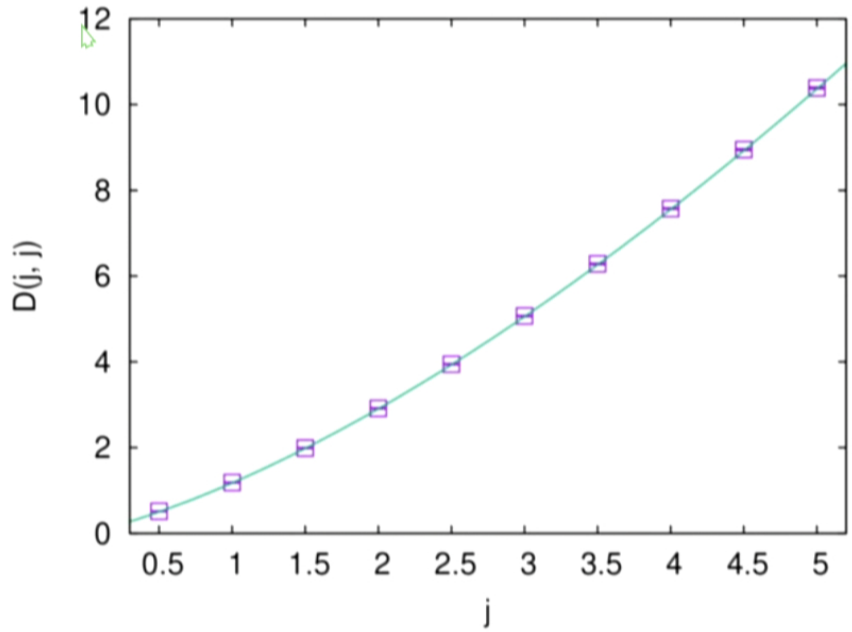


Large charge results at the O(4) Wilson-Fisher fixed point

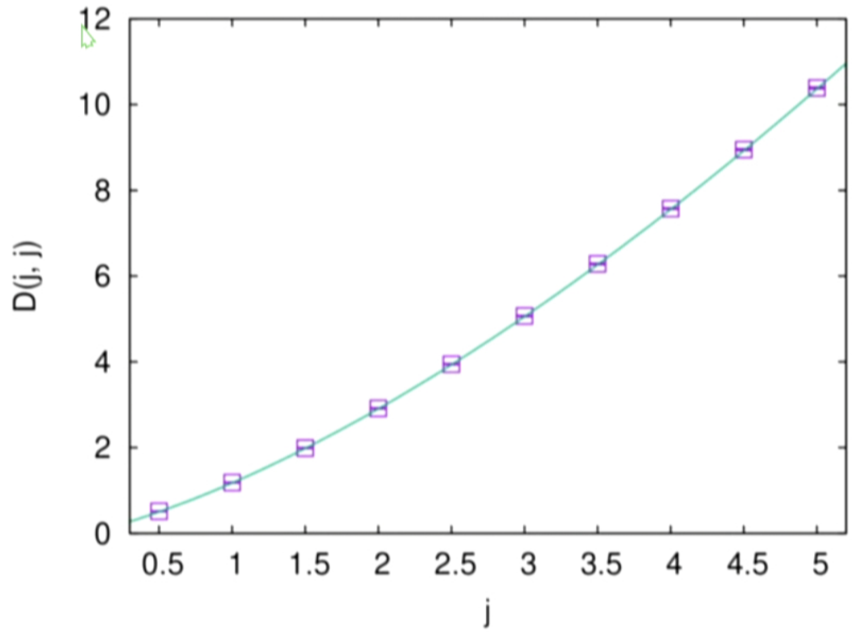
j	$D(j,j)$ (this work)	$D(j,j)$ (from [26])	j	$D(j,j)$ (this work)	$D(j,j)$ (from [26])
1/2	0.515(3)	0.5180(3)	1	1.185(4)	1.1855(5)
3/2	1.989(5)	1.9768(10)	2	2.915(6)	2.875(5)
5/2	3.945(6)	-	3	5.069(7)	-
7/2	6.284(8)	-	4	7.575(9)	-
9/2	8.949(10)	-	5	10.386(11)	-

[26] Hasenbusch, Vicari, PRB 84 (2011) 125136

Q: How well does the Q-expansion work in the O(4) case?



Q: How well does the Q-expansion work in the O(4) case?



$$D(j,j) \;=\; 1.068\, j^{3/2} \;+\; 0.083\, j^{1/2} \;-\; 0.094$$

Qubit Regularization of O(3) scalar QFT

T. Bhattacharya, SC, R. Gupta, H.Singh and R. Somma

Use two qubits per site:

$\lvert s, \mathbf{r}\rangle$	$\lvert m, \mathbf{r}\rangle,\ m = 0, +1, -1$
singlet	triplet

Qubit Regularization of O(3) scalar QFT

T. Bhattacharya, SC, R. Gupta, H.Singh and R. Somma

Use two qubits per site:

$|s, \mathbf{r}\rangle$ singlet

$|m, \mathbf{r}\rangle$, $m = 0, +1, -1$ triplet

Fock Vacuum

Spin-1 particle

Qubit Regularization of O(3) scalar QFT

T. Bhattacharya, SC, R. Gupta, H.Singh and R. Somma

Use two qubits per site:

$|s, \mathbf{r}\rangle$ singlet

$|m, \mathbf{r}\rangle,\ m = 0, +1, -1$ triplet

Fock Vacuum

Spin-1 particle

Hamiltonian is the same as the O(4) model but with three flavors of hardcore bosons!

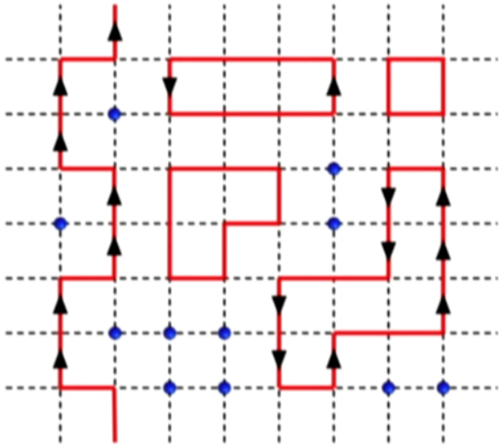
Euclidean Qubit O(3) Model

$$Z = \sum_k \int [dt_k ... dt_1] \mathrm{Tr}\left(e^{-(\beta - t_k)H_1}(-H_2)\ e^{-(t_k - t_{k-1})H_1} \cdots (-H_2) e^{-(t_1)H_1} \right)$$

$$Z = \sum_{[s,m]} \prod_{\langle ij \rangle} W_{\langle ij \rangle}$$

Relativistic Limit $\quad \varepsilon = 1$

$$W_t = W_s$$



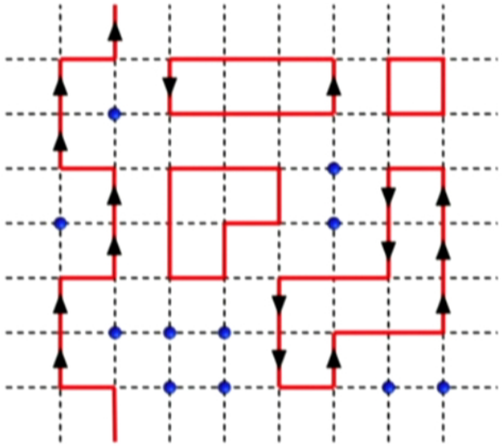
$$W_s = \varepsilon J \qquad W_t = \exp(-\varepsilon J_t)$$

$$J_h = J_p = J$$

Euclidean Qubit O(3) Model

$$Z = \sum_k \int [dt_k ... dt_1] \mathrm{Tr}\Big(e^{-(\beta - t_k)H_1}(-H_2)\; e^{-(t_k - t_{k-1})H_1} \cdots (-H_2) e^{-(t_1)H_1}\Big)$$

$$Z = \sum_{[s,m]} \prod_{\langle ij \rangle} W_{\langle ij \rangle}$$



Relativistic Limit $\quad \varepsilon = 1 \quad W_t = W_s$

Hamiltonian limit $\quad \varepsilon \to 0$

Can study using classical QMC
(directed loop/worm algorithms)

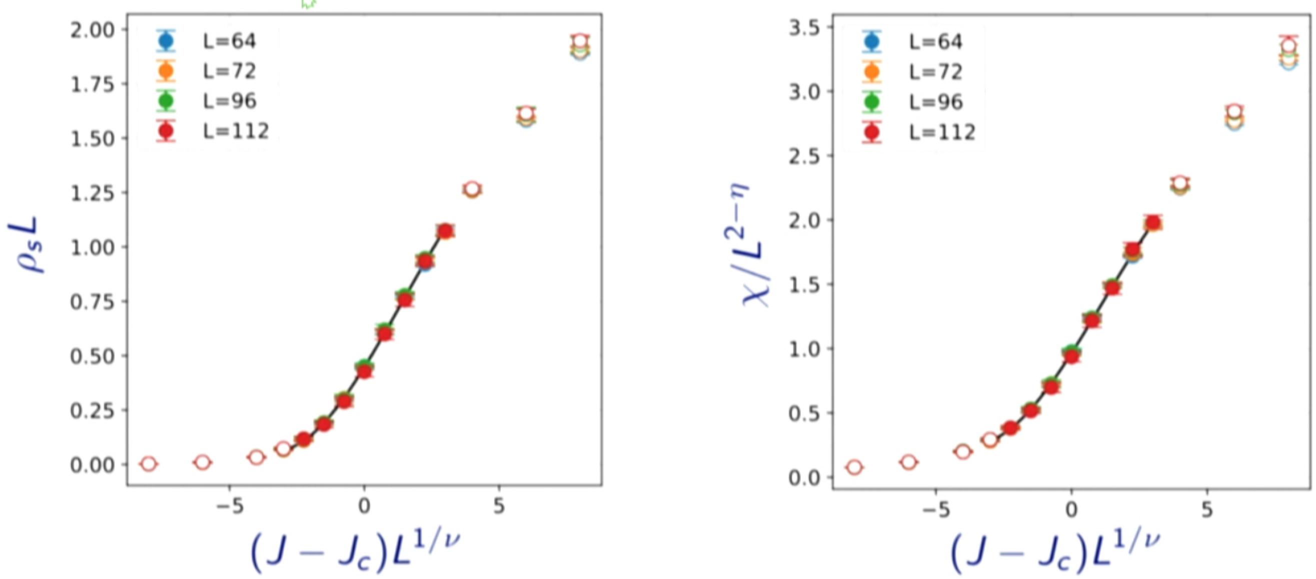
$$W_s = \varepsilon J \qquad W_t = \exp(-\varepsilon J_t)$$

$$J_h = J_p = J$$

Wilson-Fisher fixed point

$\nu = 0.7113(11), \quad \eta = 0.0378(6)$

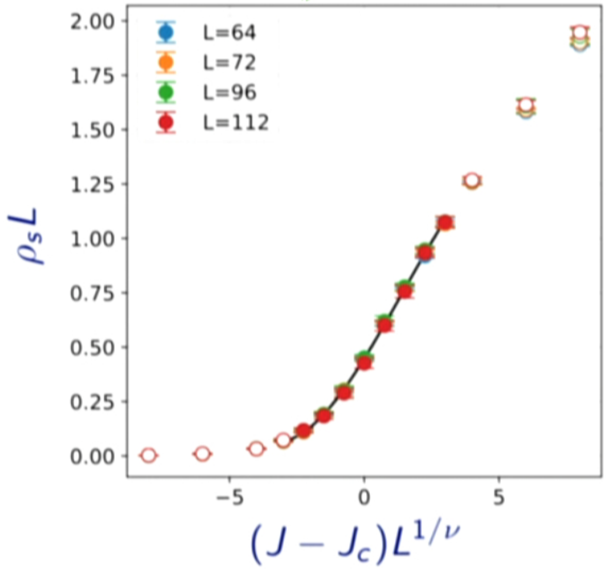
Pelisetto and Vicari Phys. Repts. (2002)

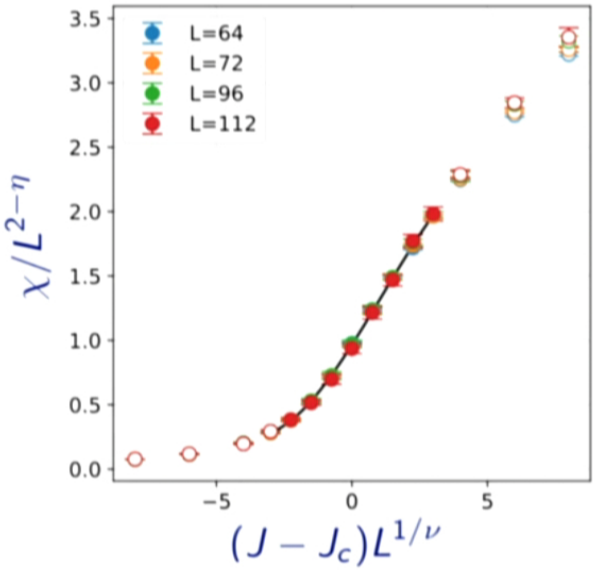


Wilson-Fisher fixed point

$\nu = 0.7113(11), \quad \eta = 0.0378(6)$

Pelisetto and Vicari Phys. Repts. (2002)



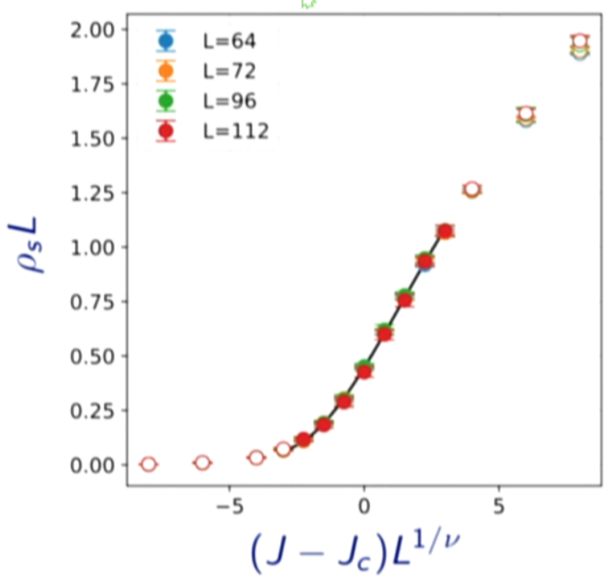


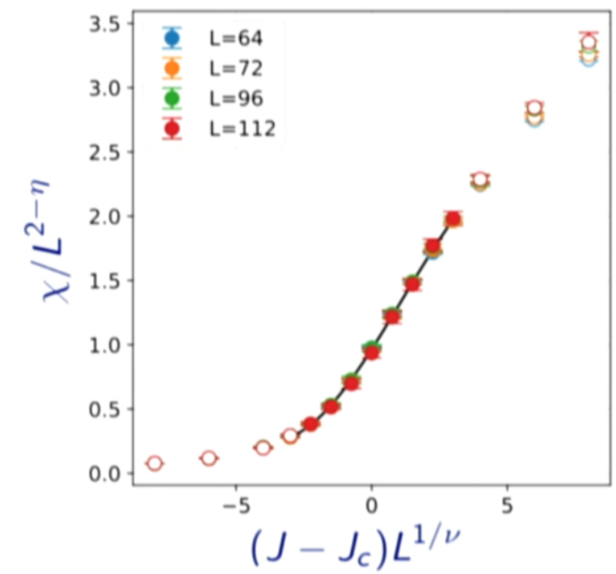
$\chi^2 = 0.53, J_c = 0.244329(11)$
$\nu = 0.7113(0), \eta = 0.038(0)$

Wilson-Fisher fixed point

$\nu = 0.7113(11), \quad \eta = 0.0378(6)$

Pelisetto and Vicari Phys. Repts. (2002)





$$\chi^2 = 0.53, J_c = 0.244329(11)$$
$$\nu = 0.7113(0), \eta = 0.038(0)$$

We see the Gaussian fixed point in d=3+1. We also see asymptotic freedom in d=1+1 but with caveats!

Conclusions

The recent proposal of Q-expansion for CFTs seems like a promising approach. It would be interesting to explore other theories with it.

Conclusions

The recent proposal of Q-expansion for CFTs seems like a promising approach. It would be interesting to explore other theories with it.

It may be possible to construct qubit Hamiltonians to study the large charge sectors more easily than traditional lattice models.

Conclusions

The recent proposal of Q-expansion for CFTs seems like a promising approach. It would be interesting to explore other theories with it.

It may be possible to construct qubit Hamiltonians to study the large charge sectors more easily than traditional lattice models.

From a quantum computational point of view, an interesting question is to ask is can we construct qubit Hamiltonians to study quantum field theories in general?

Conclusions

The recent proposal of Q-expansion for CFTs seems like a promising approach. It would be interesting to explore other theories with it.

It may be possible to construct qubit Hamiltonians to study the large charge sectors more easily than traditional lattice models.

From a quantum computational point of view, an interesting question is to ask is can we construct qubit Hamiltonians to study quantum field theories in general?

If true, perhaps this is yet another way to regularize quantum field theories! We can call it "qubit regularizations!"