

Title: Line of fixed points in a bosonic tensor model

Speakers: Sabine Harribey

Series: Quantum Gravity

Date: May 16, 2019 - 2:30 PM

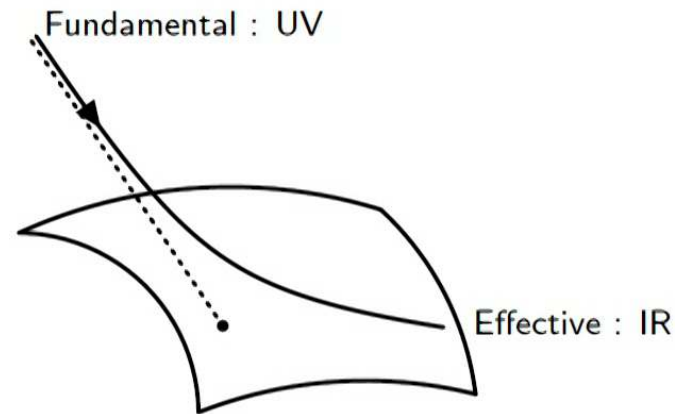
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Abstract: Tensor models exhibit a melonic large  $N$  limit: this is a non trivial family of Feynman graphs that can be explicitly summed in many situations. In  $d$  dimensions, they give rise to a new family of conformal field theories and provide interesting examples of the renormalization group flow from a free theory in the UV to a melonic large  $N$  CFT in the IR.&nbsp;

We consider here a bosonic tensor model in rank three and  $d \leq 4$  dimensions. After giving a short introduction to tensor models, I will present the renormalization group flow of the model. At leading order in  $1/N$  but non perturbatively in the coupling constants, we found a real and infrared fixed point.

## General motivations: renormalization

Physics change with the energy scale: **Renormalization group**



Flow in the space of theories with respect to the energy scale [Polchinski 1984, ...]:

→ fixed points and trajectories

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 Holography

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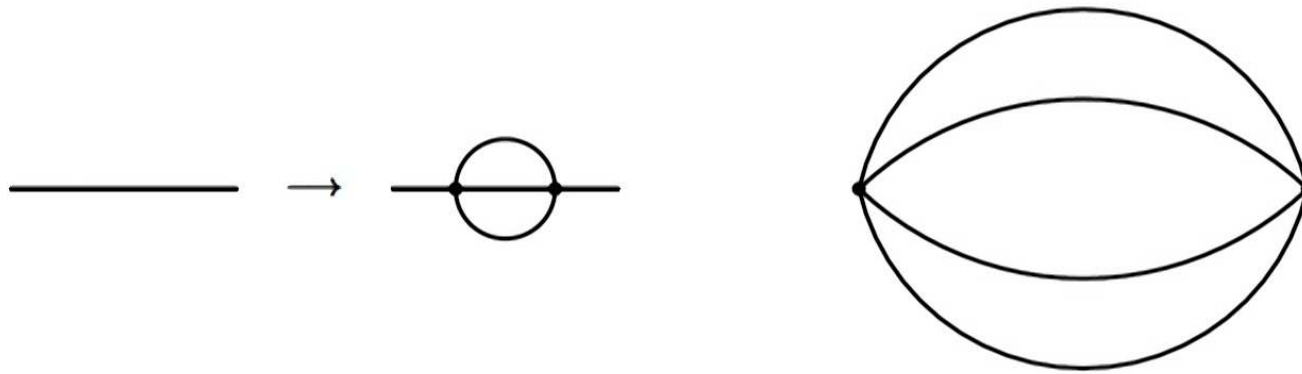
# Random tensors

- Generalization of matrix models
- First introduced in the context of random geometry and quantum gravity in  $d > 2$  [Gurau, Bonzom, Rivasseau, ...]
- Recent applications to strongly coupled QFTs and holography [Witten, Klebanov, Tarnopolsky, ...]
- Large  $N$  limit: non perturbative study in the coupling constant

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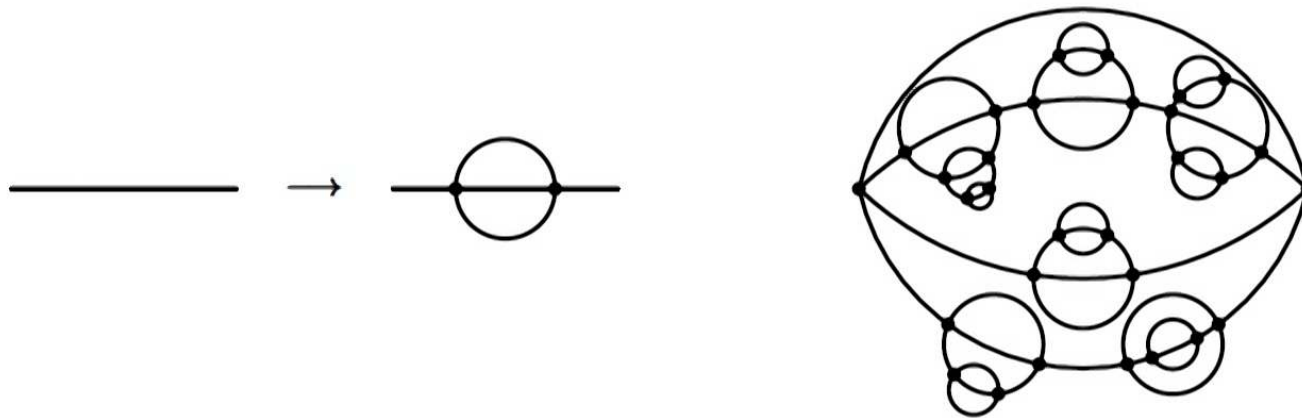
## Melon diagrams



Leading order Feynman diagrams in different large  $N$  theories:

- Tensor models
- Sachdev-Ye-Kitaev model

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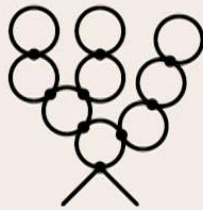
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## From vector to tensor models

### Vector $\phi_a$

$$\frac{\lambda}{N} (\phi_a \phi_a)^2$$

Cactus  
diagrams



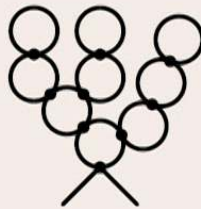
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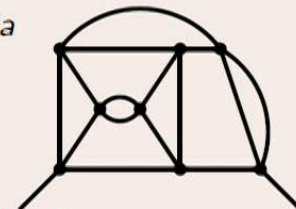


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## Matrix $M_{ab}$

$$\frac{\lambda}{N} M_{ab} M_{bc} M_{cd} M_{da}$$

Planar diagrams



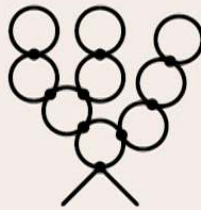
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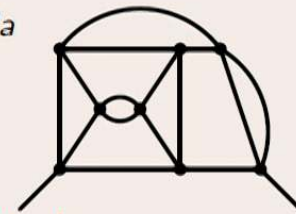


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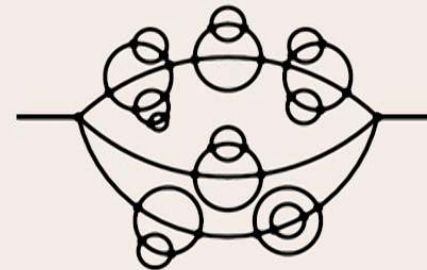


→ Hard

## Tensor $T_{abc}$

$$\frac{\lambda}{N^{3/2}} T_{aeb} T_{cfb} T_{ced} T_{afd}$$

Melon diagrams



→ Tractable

## Different types of melonic limits

- Colored tensor models: 4 tensor fields,  $O(N)^6$  symmetry

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- **Irreducible tensor models:** 1 tensor field,  $O(N)$  symmetry

- SYK model

$$J_{i_1 i_2 i_3 i_4} \psi_{i_1} \psi_{i_2} \psi_{i_3} \psi_{i_4}$$

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- Multi matrix models with a large number of matrices [Ferrari, Schaposnik Massolo, Valette ...], models in higher rank [Bonzom, Lionni, ...]

# Melonic quantum field theory

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- Tensor models in zero dimension: random geometry
  - Tensor models in one dimension: SYK model without disorder
  - Tensor models in higher dimension: new class of conformal field theories
- Problem: divergences in the computation of the two-point function and of the dimensions of operators
  - Renormalization group: non trivial fixed points, existence of an attractive IR fixed point

# Outline

- 1 CTKT Model
- 2 Two-point function
- 3 Four-point couplings and beta functions
- 4 Dimensions of operators

## Action

- Real tensor field  $\varphi_{a_1 a_2 a_3}$ ,  $a_i = 1, \dots, N$
- $O(N)^3$  symmetry

$$\varphi_{a_1 a_2 a_3}(x) \rightarrow \varphi'_{b_1 b_2 b_3}(x) = O_1^{a_1}_{b_1} O_2^{a_2}_{b_2} O_3^{a_3}_{b_3} \varphi_{a_1 a_2 a_3}(x)$$

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- CTKT action [Carrozza, Tanasa, Klebanov, Tarnopolsky]

$$S[\varphi] = \frac{1}{2} \int d^d x \varphi_{\mathbf{a}}(x) (-\Delta)^\zeta \varphi_{\mathbf{a}}(x) + S^{\text{int}}[\varphi],$$

$$S^{\text{int}}[\varphi] = \frac{m^{2\zeta}}{2} \int d^d x \varphi_{\mathbf{a}}(x) \delta_{\mathbf{ab}} \varphi_{\mathbf{b}}(x) \\ + \int d^d x \varphi_{\mathbf{a}}(x) \varphi_{\mathbf{b}}(x) \varphi_{\mathbf{c}}(x) \varphi_{\mathbf{d}}(x) \left( \frac{\lambda}{4N^{3/2}} \delta^t + \frac{\lambda_p}{4N^2} \delta^p + \frac{\lambda_d}{4N^3} \delta^d \right)$$

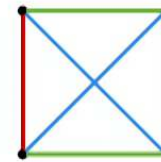
# Quartic invariants

Graphic representation:

- Tensor  $\rightarrow$  Vertex
- Contraction of two indices  $\rightarrow$  edge with color corresponding to the position of the indices

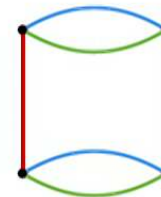
- Tetrahedron:

$$\varphi_{a_1 a_2 a_3} \varphi_{a_1 b_2 b_3} \varphi_{b_1 a_2 b_3} \varphi_{b_1 b_2 a_3}$$



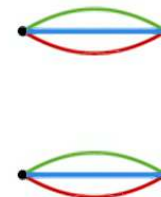
- Pillow:

$$\varphi_{a_1 a_2 a_3} \varphi_{b_1 a_2 a_3} \varphi_{a_1 b_2 b_3} \varphi_{b_1 b_2 b_3}$$



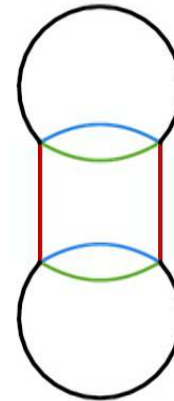
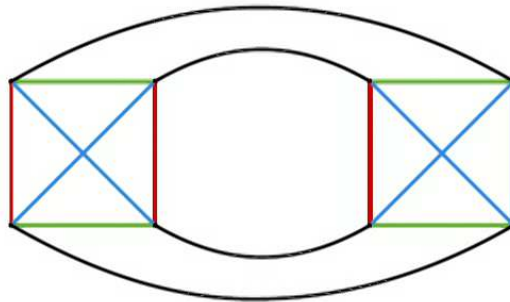
- Double trace:

$$\varphi_{a_1 a_2 a_3} \varphi_{a_1 a_2 a_3} \varphi_{b_1 b_2 b_3} \varphi_{b_1 b_2 b_3}$$



# Feynman graphs

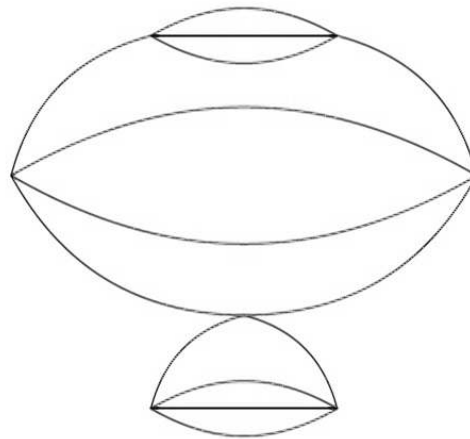
- Interaction invariant  $\rightarrow$  3-colored graph
- Pairing with propagators  $\rightarrow$  edge of a new color between the vertices



Usual Feynman graphs: shrinking each interaction vertex to a point

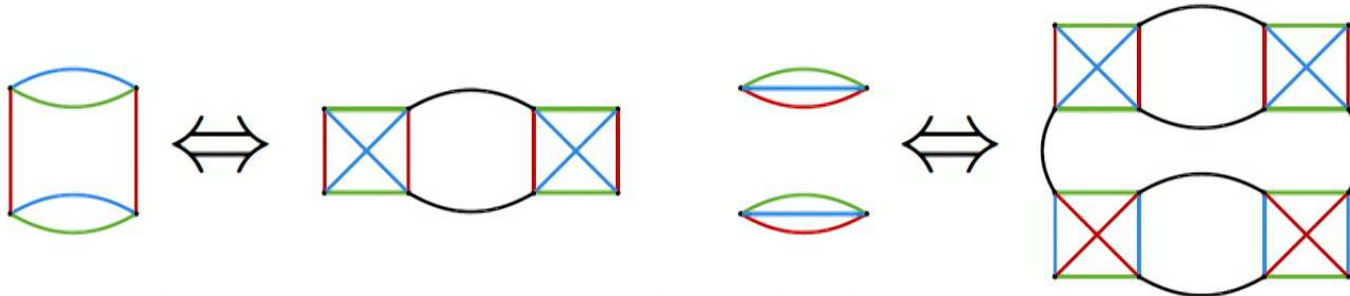
## Large $N$ expansion

- Factor  $N$  per face of color  $0i$
- $\mathcal{A} \propto N^{F - \frac{3}{2}n_t - 2n_p - 3n_d}$
- Large  $N$  expansion:
  - Melons with tetrahedral vertices
  - Tadpoles with pillow or double trace vertices

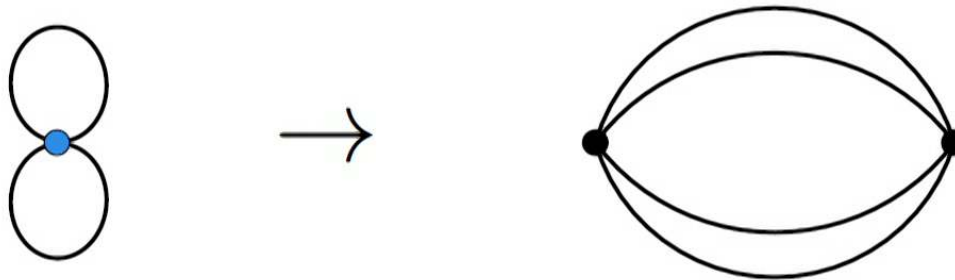


# Why tadpoles ?

Radiative corrections



Leading order graphs are melonic after substitution of pillow and double trace



## Motivation

- $\zeta = 1$ : non trivial conformal infrared limit [Klebanov, Tarnopolsky, Giombi 2017]
- Scaling behavior in the IR:  $G(p) \sim p^{-d/2}$
- Theory with interaction  $S^{int}$ : UV divergences in any d

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- Set  $\zeta = \frac{d}{4}$ : bare covariance reproducing the IR scaling of the two-point function
  - Makes the interactions power counting marginal
  - SYK model in one dimension [Gross, Rosenhaus 2017]

## Two-point function

- Two-point graphs are power divergent
- Use of parametric cut-offs:

$$C_k^\Lambda(p) = C(p)\chi_k^\Lambda(p), \quad \chi_k^\Lambda(p) = \frac{1}{\Gamma(\zeta)} \int_{p^2/\Lambda^2}^{p^2/k^2} d\alpha \alpha^{\zeta-1} e^{-\alpha}$$

## Two-point function

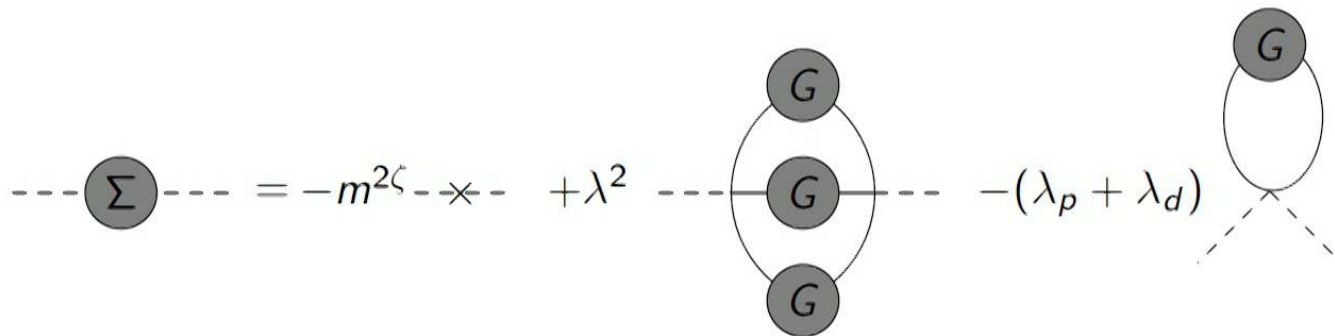
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- Regularization of the Schwinger-Dyson equation

$$G(p)^{-1} = C(p)^{-1} - \Sigma(p)$$

- Closed equation for  $G$ : characteristic feature of tensor models at large  $N$



## Two-point function

- Subtraction of the divergence in the bare mass  $\rightarrow$  tune the renormalized mass to zero
- Finite wave function renormalization
- Scaling of the propagator with a non integer power of the momentum: match the conformal scaling

$$Z^4 - Z^3 = \lambda^2 \frac{1}{(4\pi)^d} \frac{\Gamma(1 - \frac{d}{4})}{\frac{d}{4} \Gamma(3\frac{d}{4})}.$$

## Four-point function

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- Renormalization of the mass and the wave function
- Four point couplings ?
- Power counting: logarithmically divergent
- Three different couplings:  $\lambda$ ,  $\lambda_p$  and  $\lambda_d$

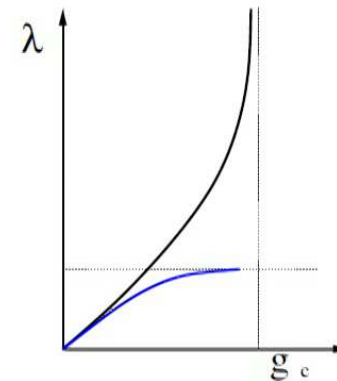
## Four-point couplings: tetrahedral coupling

No radiative correction, finite  $Z \rightarrow$  no running:

$$g = k^{d-4\zeta} \frac{\lambda}{Z^2} = \frac{\lambda}{Z^2}$$

$$Z = \frac{1}{1 - \frac{g^2}{g_c^2}} \quad \lambda = gZ^2$$

- $\lambda$  **real**:  $\lambda(g)$  invertible for  $g < g_c$  and  $Z$  diverges when  $\lambda \rightarrow \infty$  or  $g \rightarrow g_c$ .
- $\lambda$  **purely imaginary**:  $\lambda(g)$  invertible for  $|g| < 3^{-1/2}g_c$  and  $Z$  is bounded.



## Four-point couplings: pillow and double-trace couplings

- Mixing: the trace part of the pillow interaction contributes to the double-trace interaction
- → parametrize in terms of two independent couplings

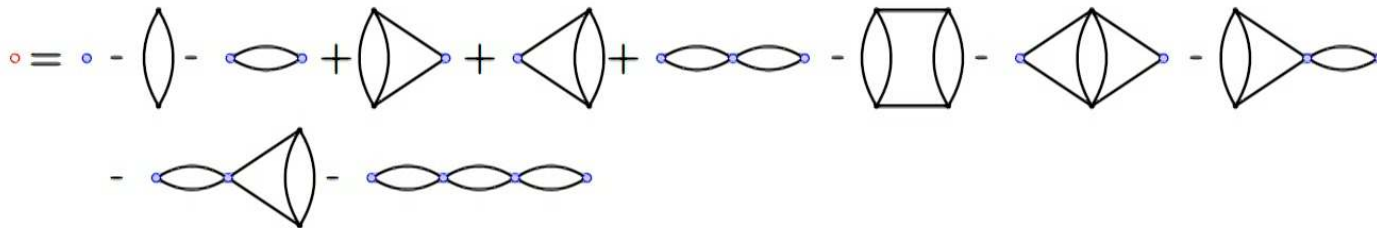
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- Leading four-point graphs: series of ladders and bubbles

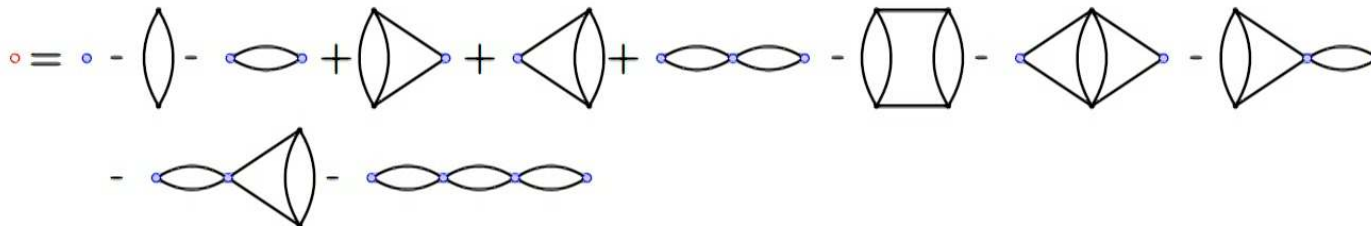


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- Precise structure for the bare expansion of the couplings and the beta functions

## Beta functions

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- Quadratic:

$$\beta_{g_1} = k \frac{\partial g_1}{\partial k} \Big|_{\lambda, \lambda_1} = \beta_0^g - 2\beta_1^g g_1 + \beta_2^g g_1^2 ,$$

$$\beta_{g_2} = k \frac{\partial g_2}{\partial k} \Big|_{\lambda, \lambda_2} = \beta_0^{\sqrt{3}g} - 2\beta_1^{\sqrt{3}g} g_2 + \beta_2^{\sqrt{3}g} g_2^2$$

At first orders:

$$\beta_0^g = \left( 2 \frac{\Gamma(\frac{d}{4})^2}{\Gamma(\frac{d}{2})} \right) g^2 + \mathcal{O}(g^4) , \quad \beta_1^g = \mathcal{O}(g^2) , \quad \beta_2^g = \left( 2 \frac{\Gamma(\frac{d}{4})^2}{\Gamma(\frac{d}{2})} \right) + \mathcal{O}(g^2)$$

## Fixed points

$\beta_{g_1}$  admits two fixed points:

$$g_{1\pm} = \frac{\beta_1^g \pm \sqrt{(\beta_1^g)^2 - \beta_0^g \beta_2^g}}{\beta_2^g} = \pm \sqrt{-g^2} + \mathcal{O}(g^2).$$

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Critical exponents:

$$\beta'_{g_1}(g_{1\pm}) = \pm 2 \sqrt{(\beta_1^g)^2 - \beta_0^g \beta_2^g} = \pm \sqrt{-g^2} \left( 4 \frac{\Gamma(\frac{d}{4})^2}{\Gamma(\frac{d}{2})} \right) + \mathcal{O}(g^3).$$

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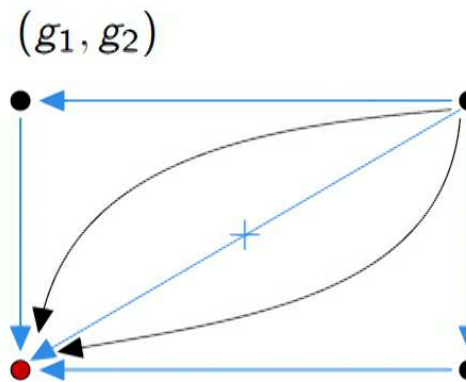
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$\beta_{g_2}: g \rightarrow \sqrt{3}g$

- $g \rightarrow 0$ : trivial fixed point
- $g = \pm i |g|$ : **real** fixed points for small  $g$

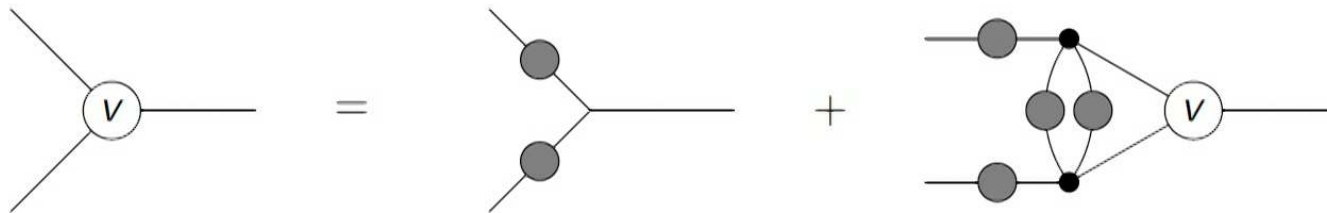
## RG trajectories

- $(g_{1+}, g_{2+})$ : infrared attractive fixed point, stable and strongly interacting
- $(g_{1-}, g_{2-})$ : ultraviolet attractive fixed point, strongly interacting
- Explicit renormalization group trajectory from  $(g_{1-}, g_{2-})$  to  $(g_{1+}, g_{2+})$



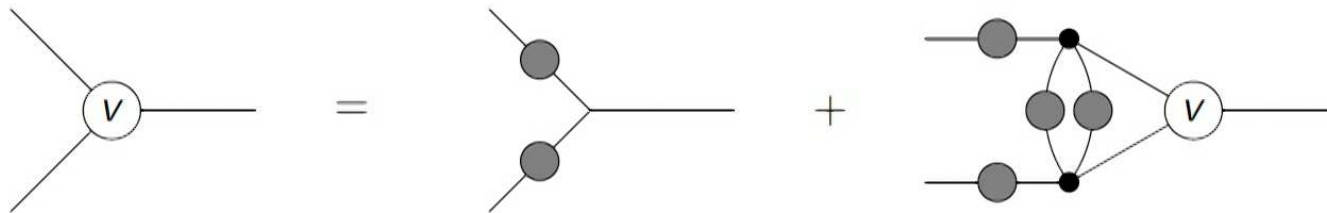
# Bethe Salpeter equation

Bilinears:  $\langle \mathcal{O}_h(x_0) \varphi_{abc}(x_1) \varphi_{abc}(x_2) \rangle \equiv v(x_0, x_1, x_2)$



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- Bilinears  $\leftrightarrow$  eigenfunctions of eigenvalue **1** of the kernel
- Conformal dimensions solutions of  $f(h) = 1$

$$f(h) = 3g^2 \Gamma(d/4)^4 \frac{\Gamma(h/2 - d/4) \Gamma(d/4 - h/2)}{\Gamma(3d/4 - h/2) \Gamma(d/4 + h/2)} .$$

## Dimension of bilinear operators

Two types of solutions:

$$h_{0\pm} = \frac{d}{2} \pm \alpha_0 \sqrt{-g^2} + \mathcal{O}(g^3)$$

$$h_n = \frac{d}{2} + 2n + \alpha_n g^2 + \mathcal{O}(g^3), \quad n \in \mathbb{N}^+$$

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- Degeneration of the solutions in  $d = 2$
- Real  $g$ :  $h_{0\pm}$  imaginary and instability
- Purely imaginary  $g$ : **real dimensions**
- $h_{0\pm}$  dimension of  $\varphi_{abc}\varphi_{abc}$  at the fixed point

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- Purely imaginary  $g$ : **real dimensions**
- $h_{0\pm}$  dimension of  $\varphi_{abc}\varphi_{abc}$  at the fixed point
- For  $|g| > |g_0| \approx 3^{-1/2}g_c$ , complex dimensions reappear

## Conclusion and perspectives

- Large  $N$  limit: four lines of fixed points parametrized by  $\lambda$
- Purely imaginary  $\lambda$ : real fixed points and **one IR attractive fixed point**
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- **Real** dimensions of bilinear operators
  
- Conjecture: appearance of complex scaling dimensions  $\Rightarrow$  spontaneous breaking of conformal invariance [Kim, Klebanov, Tarnopolsky, Zhao 2019]
- Rigorous computation of the dimensions of operators: RG techniques and complete kernel
- Detailed study of this new type of CFT: *Melonic CFT*