Title: Line of fixed points in a bosonic tensor model

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Series: Quantum Gravity

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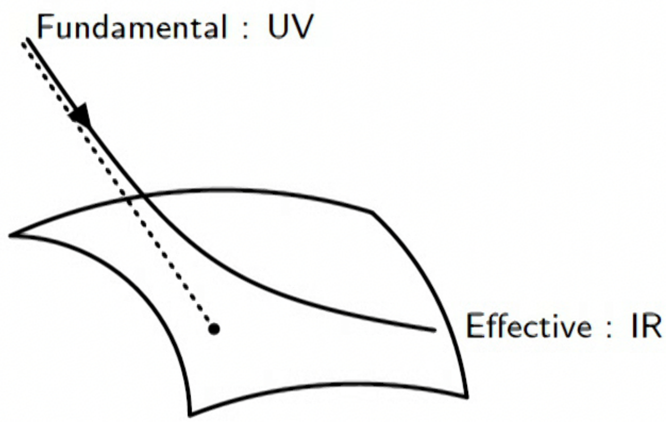
Abstract: Tensor models exhibit a melonic large N limit: this is a non trivial family of Feynman graphs that can be explicitly summed in many situations. In d dimensions, they give rise to a new family of conformal field theories and provide interesting examples of the renormalization group flow from a free theory in the UV to a melonic large N CFT in the IR.

We consider here a bosonic tensor model in rank three and $d<4$ dimensions. After giving a short introduction to tensor models, I will present the renormalization group flow of the model. At leading order in $1/N$ but non perturbatively in the coupling constants, we found a real and infrared fixed point.

General motivations: renormalization

Physics change with the energy scale: **Renormalization group**



Flow in the space of theories with respect to the energy scale [Polchinski 1984, ...]:

$\rightarrow$ fixed points and trajectories

General motivations: Quantum gravity and strong coupling

Quantum gravity frontier

- Quantum geometry ?
- RG flow near Planck length ?

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QFT: weak versus strong coupling

Weak coupling	**Strong coupling**
Perturbation theory	?
Non perturbative ?	?

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$\Updownarrow$ Holography

QFT: weak versus strong coupling

Weak coupling	Strong coupling
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Random tensors

- Generalization of matrix models
- First introduced in the context of random geometry and quantum gravity in $d > 2$ [Gurau, Bonzom, Rivasseau, ...]
- Recent applications to strongly coupled QFTs and holography [Witten, Klebanov, Tarnopolsky, ...]
- Large N limit: non perturbative study in the coupling constant

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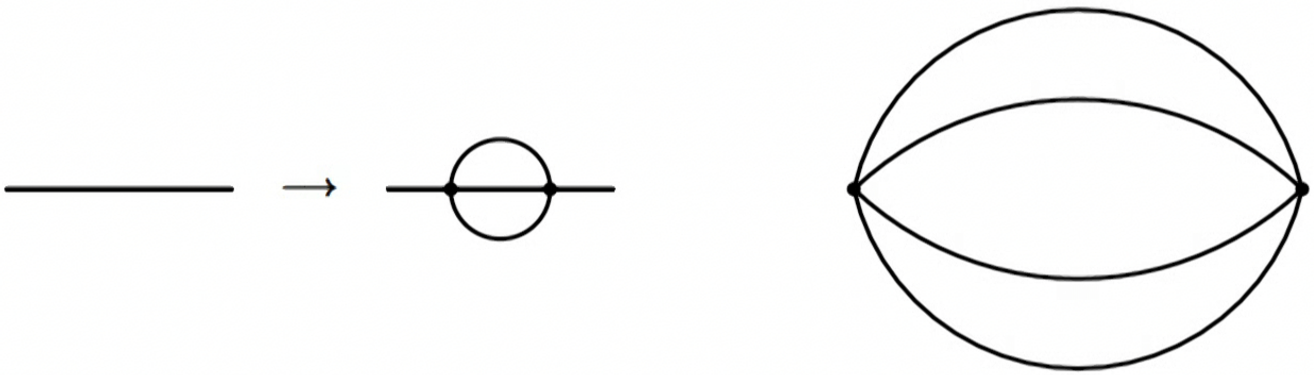
Melon diagrams



Leading order Feynman diagrams in different large N theories:

- Tensor models
- Sachdev-Ye-Kitaev model

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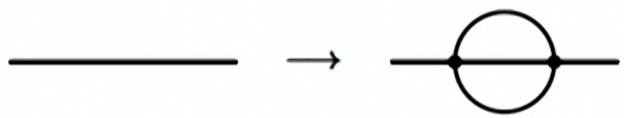

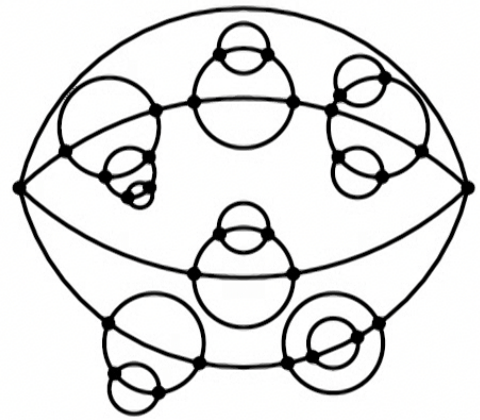

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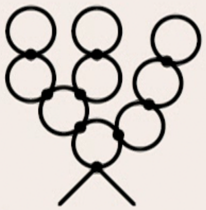
From vector to tensor models

Vector ϕ_a

$\frac{\lambda}{N}(\phi_a \phi_a)^2$

Cactus diagrams



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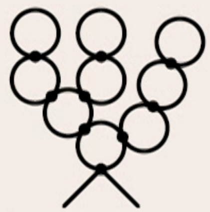
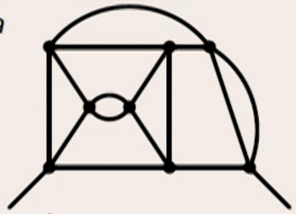


$\rightarrow$ Easy

Matrix M_{ab}

$\frac{\lambda}{N} M_{ab}M_{bc}M_{cd}M_{da}$

Planar diagrams



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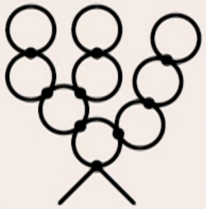

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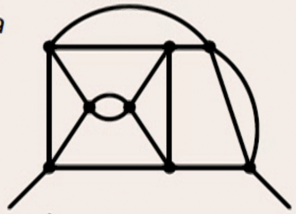
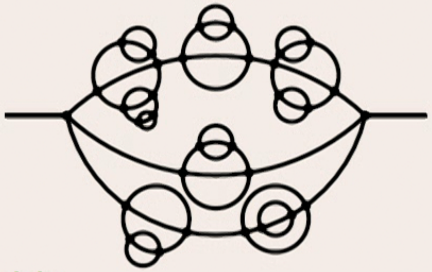


$\rightarrow$ Hard

Tensor T_{abc}

$\frac{\lambda}{N^{3/2}} T_{aeb}T_{cfb}T_{ced}T_{afd}$

Melon diagrams



$\rightarrow$ Tractable

Different types of melonic limits

- **Colored** tensor models: 4 tensor fields, $O(N)^6$ symmetry

$$\varphi^i_{abc}$$

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- **Irreducible** tensor models: 1 tensor field, $O(N)$ symmetry

- SYK model

$$J_{i_1 i_2 i_3 i_4} \psi_{i_1} \psi_{i_2} \psi_{i_3} \psi_{i_4}$$

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- Multi matrix models with a large number of matrices [Ferrari, Schaposnik Massolo, Valette ...], models in higher rank [Bonzom,Lionni, ...]

Melonic quantum field theory

- Tensor models in zero dimension: random geometry
- Tensor models in one dimension: SYK model without disorder

Melonic quantum field theory

- Tensor models in zero dimension: random geometry
- Tensor models in one dimension: SYK model without disorder
- Tensor models in higher dimension: new class of conformal field theories

- Problem: divergences in the computation of the two-point function and of the dimensions of operators
- Renormalization group: non trivial fixed points, existence of an attractive IR fixed point

Outline

1. CTKT Model
2. Two-point function
3. Four-point couplings and beta functions
4. Dimensions of operators

Action

- Real tensor field $\varphi_{a_1 a_2 a_3}$, $a_i = 1, \ldots, N$
- $O(N)^3$ symmetry

$$\varphi_{a_1 a_2 a_3}(x) \to \varphi'_{b_1 b_2 b_3}(x) = O_1{}^{a_1}_{b_1} O_2{}^{a_2}_{b_2} O_3{}^{a_3}_{b_3} \varphi_{a_1 a_2 a_3}(x)$$

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- CTKT action [Carrozza, Tanasa, Klebanov, Tarnopolsky]

$$S[\varphi] = \frac{1}{2} \int d^d x \, \varphi_{\mathbf{a}}(x)(-\Delta)^{\zeta} \varphi_{\mathbf{a}}(x) + S^{\text{int}}[\varphi] \, ,$$

$$S^{\text{int}}[\varphi] = \frac{m^{2\zeta}}{2} \int d^d x \, \varphi_{\mathbf{a}}(x) \delta_{\mathbf{ab}} \varphi_{\mathbf{b}}(x)$$
$$+ \int d^d x \, \varphi_{\mathbf{a}}(x) \varphi_{\mathbf{b}}(x) \varphi_{\mathbf{c}}(x) \varphi_{\mathbf{d}}(x) \left(\frac{\lambda}{4N^{3/2}} \delta^t + \frac{\lambda_p}{4N^2} \delta^p + \frac{\lambda_d}{4N^3} \delta^d \right)$$

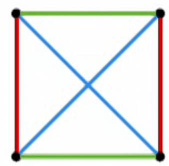
Quartic invariants

Graphic representation:

- Tensor → Vertex
- Contraction of two indices → edge with color corresponding to the position of the indices

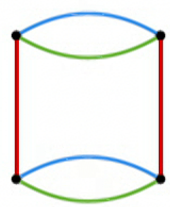
- Tetrahedron:

  $\varphi_{a_1 a_2 a_3} \varphi_{a_1 b_2 b_3} \varphi_{b_1 a_2 b_3} \varphi_{b_1 b_2 a_3}$



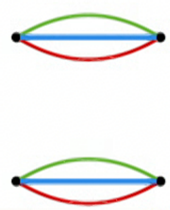
- Pillow:

  $\varphi_{a_1 a_2 a_3} \varphi_{b_1 a_2 a_3} \varphi_{a_1 b_2 b_3} \varphi_{b_1 b_2 b_3}$



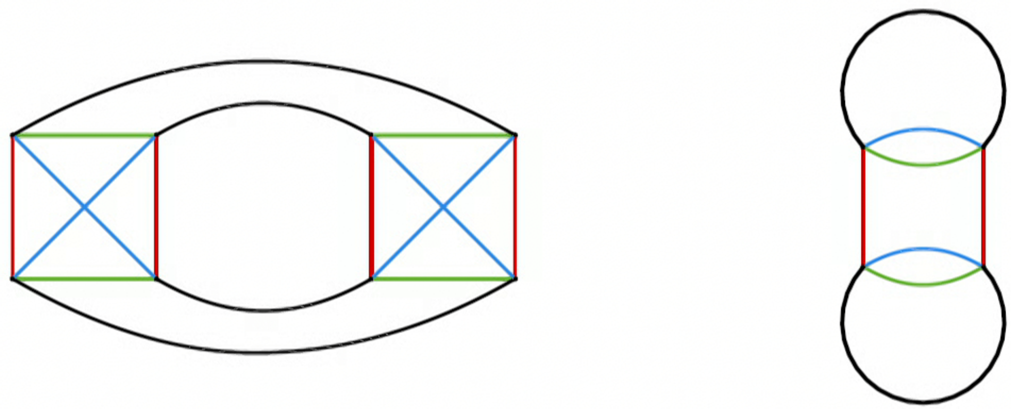
- Double trace:

  $\varphi_{a_1 a_2 a_3} \varphi_{a_1 a_2 a_3} \varphi_{b_1 b_2 b_3} \varphi_{b_1 b_2 b_3}$



Feynman graphs

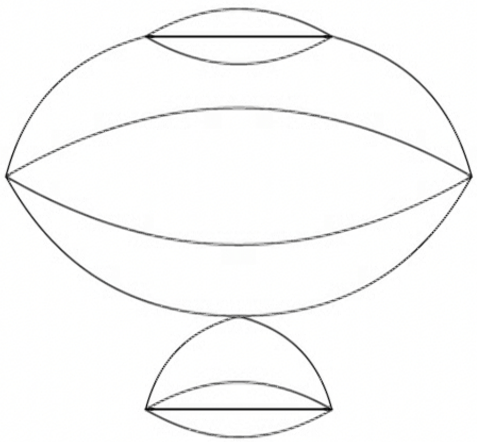
- Interaction invariant → 3-colored graph
- Pairing with propagators → edge of a new color between the vertices



Usual Feynman graphs: shrinking each interaction vertex to a point

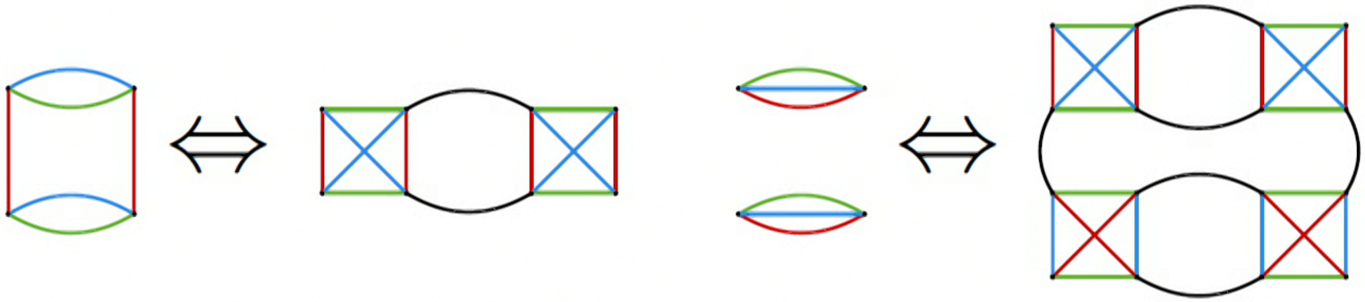
Large N expansion

- Factor N per face of color $0i$
- $\mathcal{A} \propto N^{F-\frac{3}{2}n_t-2n_p-3n_d}$
- Large N expansion:
  - Melons with tetrahedral vertices
  - Tadpoles with pillow or double trace vertices

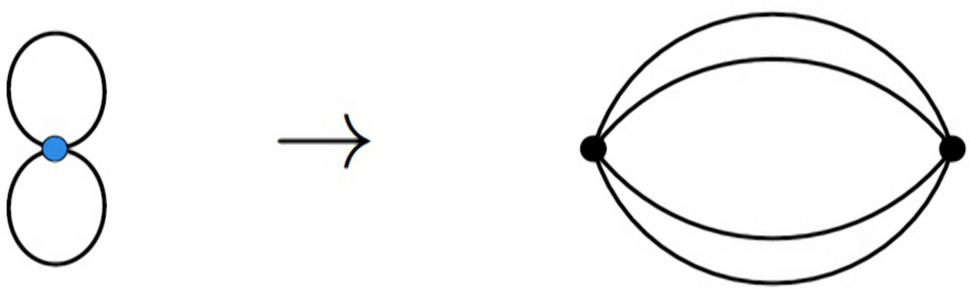


Why tadpoles ?

Radiative corrections



Leading order graphs are melonic after substitution of pillow and double trace



Motivation

- $\zeta = 1$: non trivial conformal infrared limit [Klebanov, Tarnopolsky, Giombi 2017]
- Scaling behavior in the IR: $G(p) \sim p^{-d/2}$
- Theory with interaction S^{int}: UV divergences in any d

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- Set $\zeta = \frac{d}{4}$: bare covariance reproducing the IR scaling of the two-point function
- Makes the interactions power counting marginal
- SYK model in one dimension [Gross, Rosenhaus 2017]

Two-point function

- Two-point graphs are power divergent
- Use of parametric cut-offs:

$$C_k^{\Lambda}(p) = C(p)\chi_k^{\Lambda}(p) \ , \quad \chi_k^{\Lambda}(p) = \frac{1}{\Gamma(\zeta)} \int_{p^2/\Lambda^2}^{p^2/k^2} d\alpha \ \alpha^{\zeta-1} e^{-\alpha}$$

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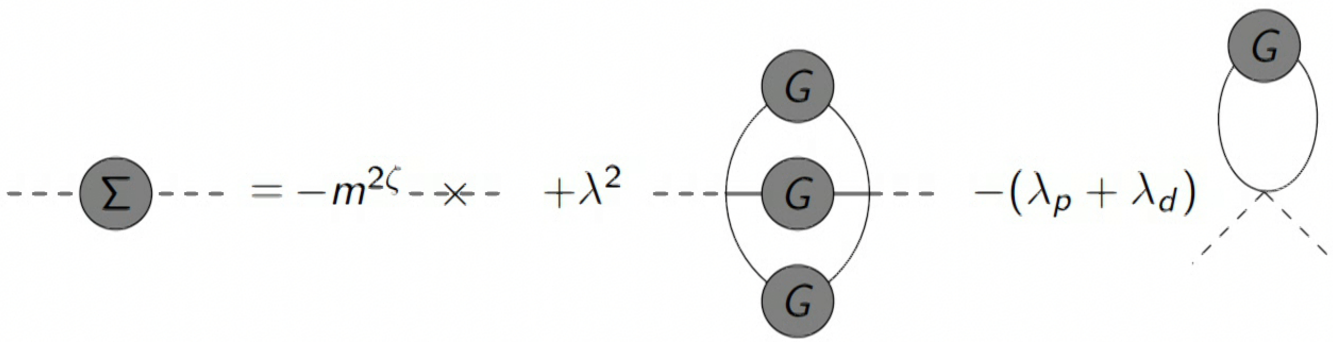
- Regularization of the Schwinger-Dyson equation

$$G(p)^{-1} = C(p)^{-1} - \Sigma(p)$$

- Closed equation for G: characteristic feature of tensor models at large N



Two-point function

- Subtraction of the divergence in the bare mass $\to$ tune the renormalized mass to zero
- Finite wave function renormalization
- Scaling of the propagator with a non integer power of the momentum: match the conformal scaling

$$Z^4 - Z^3 = \lambda^2 \frac{1}{(4\pi)^d} \frac{\Gamma\left(1 - \frac{d}{4}\right)}{\frac{d}{4}\Gamma\left(3\frac{d}{4}\right)} \, .$$

Four-point function

- Renormalization of the mass and the wave function
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- Four point couplings ?
- Power counting: logarithmically divergent
- Three different couplings: λ, λ_p and λ_d

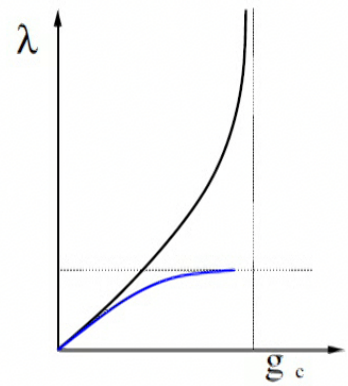
Four-point couplings: tetrahedral coupling

No radiative correction, finite $Z \to$ no running:

$$g = k^{d-4\zeta}\frac{\lambda}{Z^2} = \frac{\lambda}{Z^2}$$

$$Z = \frac{1}{1-\frac{g^2}{g_c^2}} \qquad \lambda = gZ^2$$

- λ **real**: $\lambda(g)$ invertible for $g < g_c$ and Z diverges when $\lambda \to \infty$ or $g \to g_c$.
- λ **purely imaginary**: $\lambda(g)$ invertible for $\mid g \mid < 3^{-1/2} g_c$ and Z is bounded.



Four-point couplings: pillow and double-trace couplings

- Mixing: the trace part of the pillow interaction contributes to the double-trace interaction
- $\rightarrow$ parametrize in terms of two independent couplings

$$\lambda_1 = \frac{\lambda_p}{3}, \qquad \lambda_2 = \lambda_d + \lambda_p$$

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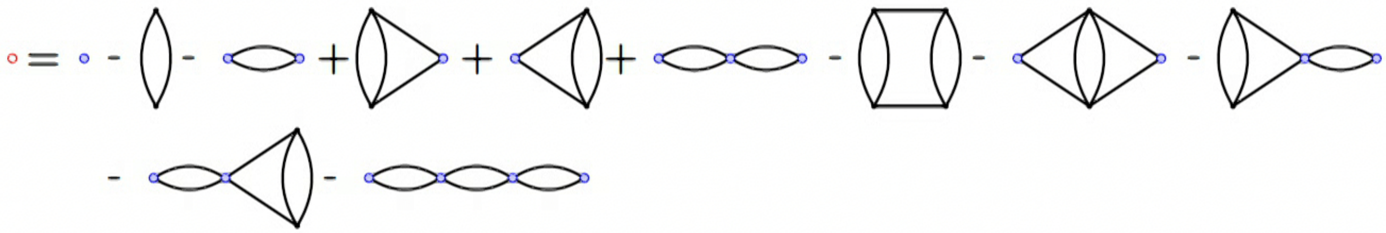
- Leading four-point graphs: series of ladders and bubbles



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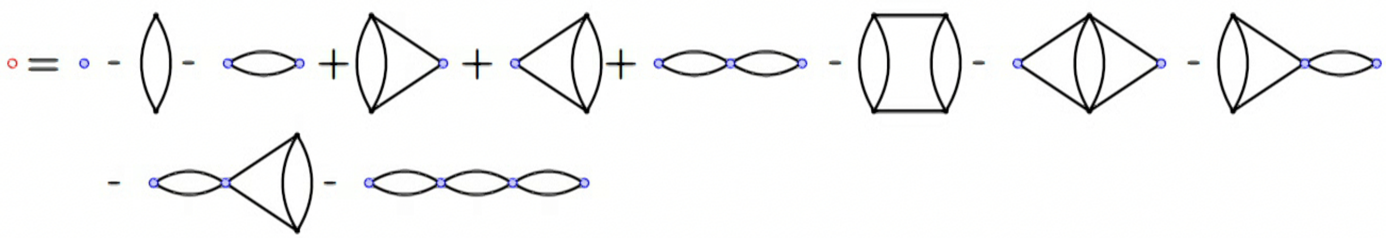

- Precise structure for the bare expansion of the couplings and the beta functions

Beta functions

- Beta function $\beta_{g_1} = k\partial_k g_1$: characterizes the flow of the theory.

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- Quadratic:

$$\beta_{g_1} = k\frac{\partial g_1}{\partial k}\Big|_{\lambda,\lambda_1} = \beta_0^g - 2\beta_1^g\, g_1 + \beta_2^g\, g_1^2 \ ,$$
$$\beta_{g_2} = k\frac{\partial g_2}{\partial k}\Big|_{\lambda,\lambda_2} = \beta_0^{\sqrt{3}g} - 2\beta_1^{\sqrt{3}g}\, g_2 + \beta_2^{\sqrt{3}g}\, g_2^2$$

At first orders:

$$\beta_0^g = \left(2\frac{\Gamma(\frac{d}{4})^2}{\Gamma(\frac{d}{2})}\right) g^2 + \mathcal{O}(g^4)\ , \qquad \beta_1^g = \mathcal{O}(g^2)\ , \qquad \beta_2^g = \left(2\frac{\Gamma(\frac{d}{4})^2}{\Gamma(\frac{d}{2})}\right) + \mathcal{O}(g^2)$$

Fixed points

β_{g_1} admits two fixed points:

$$g_{1\pm} = \frac{\beta_1^g \pm \sqrt{(\beta_1^g)^2 - \beta_0^g \beta_2^g}}{\beta_2^g} = \pm\sqrt{-g^2} + \mathcal{O}(g^2)\,.$$

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Critical exponents:

$$\beta'_{g_1}(g_{1\pm}) = \pm 2\sqrt{(\beta_1^g)^2 - \beta_0^g \beta_2^g} = \pm\sqrt{-g^2}\left(4\frac{\Gamma(\frac{d}{4})^2}{\Gamma(\frac{d}{2})}\right) + \mathcal{O}(g^3)\,.$$

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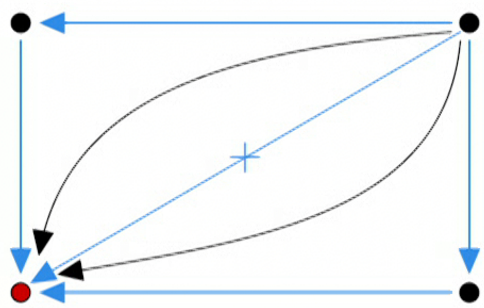
β_{g_2}: $g \to \sqrt{3}g$

- $g \to 0$: trivial fixed point
- $g = \pm i \mid g \mid$: **real** fixed points for small g

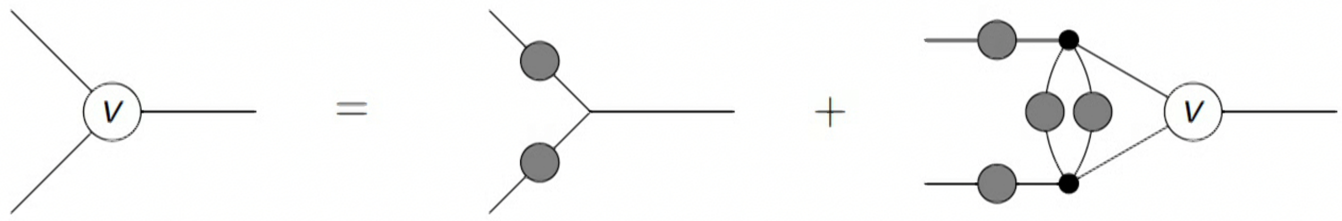
RG trajectories

- (g_{1+}, g_{2+}): infrared attractive fixed point, stable and strongly interacting
- (g_{1-}, g_{2-}): ultraviolet attractive fixed point, strongly interacting
- Explicit renormalization group trajectory from (g_{1-}, g_{2-}) to (g_{1+}, g_{2+})



Bethe Salpeter equation

Bilinears: $\langle \mathcal{O}_h(x_0)\varphi_{abc}(x_1)\varphi_{abc}(x_2)\rangle \equiv v(x_0, x_1, x_2)$



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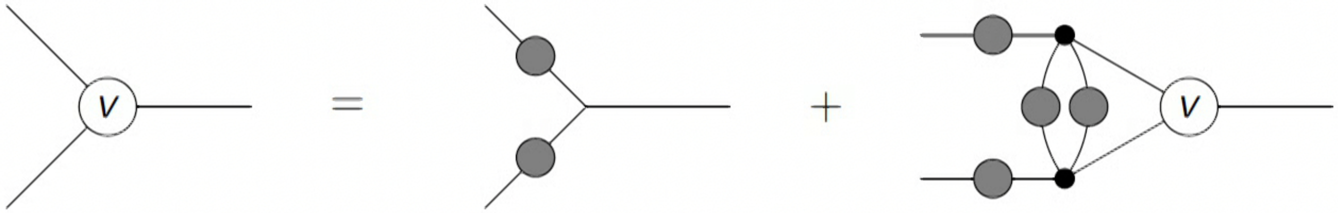

- Bilinears $\leftrightarrow$ eigenfunctions of eigenvalue 1 of the kernel
- Conformal dimensions solutions of $f(h) = 1$

$$f(h) = 3g^2\Gamma(d/4)^4 \frac{\Gamma(h/2 - d/4)\Gamma(d/4 - h/2)}{\Gamma(3d/4 - h/2)\Gamma(d/4 + h/2)} \ .$$

Dimension of bilinear operators

Two types of solutions:

$$h_{0\pm} = \frac{d}{2} \pm \alpha_0 \sqrt{-g^2} + \mathcal{O}(g^3)$$

$$h_n = \frac{d}{2} + 2n + \alpha_n g^2 + \mathcal{O}(g^3) \ , \ \ n \in \mathbb{N}^+$$

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- Degeneration of the solutions in $d = 2$
- Real g: $h_{0\pm}$ imaginary and instability

- Purely imaginary g: **real** dimensions
- $h_{0\pm}$ dimension of $\varphi_{abc}\varphi_{abc}$ at the fixed point

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- Purely imaginary g: **real** dimensions
- $h_{0\pm}$ dimension of $\varphi_{abc}\varphi_{abc}$ at the fixed point
- For $\mid g \mid > \mid g_0 \mid \approx 3^{-1/2} g_c$, complex dimensions reappear

Conclusion and perspectives

- Large N limit: four lines of fixed points parametrized by λ
- Purely imaginary λ: real fixed points and one IR attractive fixed point
- Real dimensions of bilinear operators

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- Conjecture: appearance of complex scaling dimensions $\Rightarrow$ spontaneous breaking of conformal invariance [Kim, Klebanov, Tarnopolsky, Zhao 2019]
- Rigorous computation of the dimensions of operators: RG techniques and complete kernel
- Detailed study of this new type of CFT: *Melonic CFT*