Title: A Self-consistent Model of Quantum Black Hole

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Series: Quantum Gravity

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Abstract: What is the black hole in quantum mechanics? We examine this problem in a self-consistent manner. First, we analyze time evolution of a 4D spherically symmetric collapsing matter including the back reaction of particle creation that occurs in the time-dependent spacetime. As a result, a compact high-density star with no horizon or singularity is formed and eventually evaporates. This is a quantum black hole. We can construct a self-consistent solution of the semi-classical Einstein equation showing this structure. In fact, we construct the metric, evaluate the expectation values of the energy momentum tensor, and prove the self-consistency under some assumptions. Large pressure appears in the angular direction to support this black hole, which is consistent with 4D Weyl anomaly. When the black hole is formed adiabatically in the heat bath, integrating the entropy density over the interior volume reproduces the area law.

A Self-consistent Model of Quantum Black Hole

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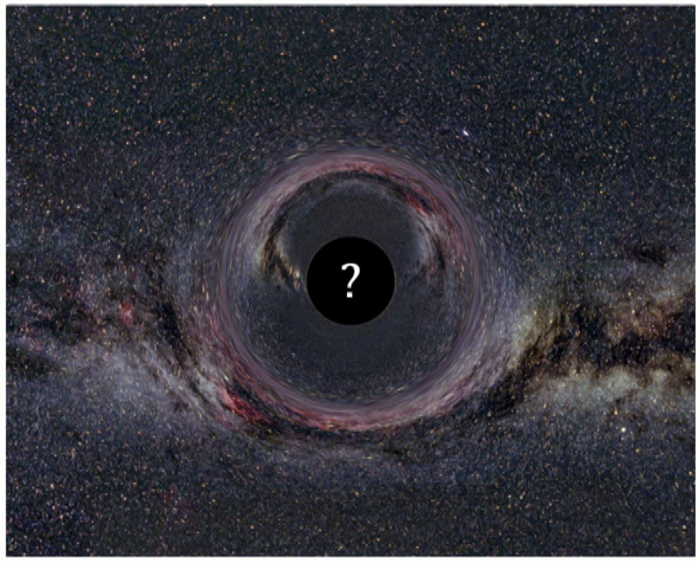
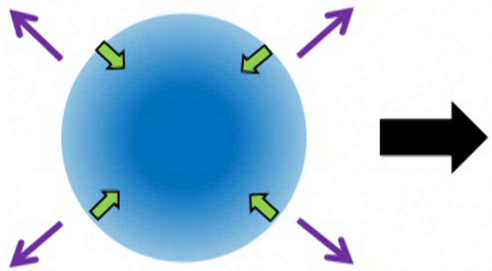


What is the "black hole" in QM?
⇒BH evaporates by nature.

Our approach

- Reconsider the time evolution of a spherical collapsing matter.



What happens?
Horizon is formed or not?

Generically, particle creation occurs in a time-dependent metric (even without horizon).
⇒ We need to include the back reaction of the matter and particle creation.

But....the back reaction is negligible?

$$\Delta t_{eva} \sim M^3 \gg M \sim \Delta\tau_{collapse}$$

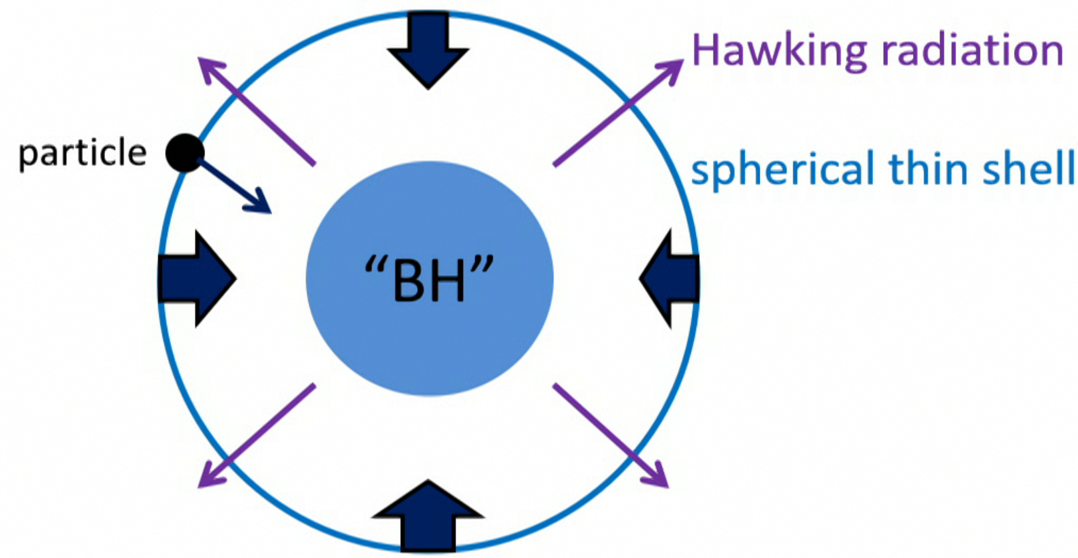
⇒No!
⇒Consider the both evolution in a single time coordinate!

Basic idea

- H. Kawai, Y. Matsuo and Y. Y, Int. J. Mod. Phys. A 28, 1350050 (2013)
- H. Kawai and Y. Y, Int. J. Mod. Phys. A 30, 1550091 (2015)
- H. Kawai and Y. Y, Phys.Rev.D.93.044011 (2016)
- H. Kawai and Y. Y, Universe 3, 51, (2017)
- H. Kawai and Y.Y, Work in Progress

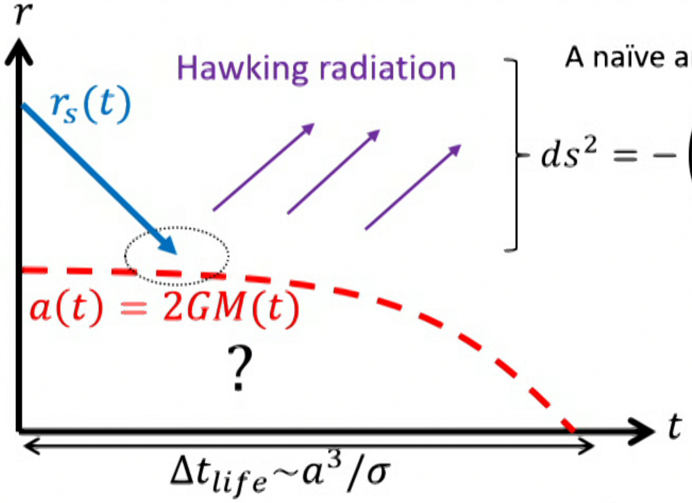
Basic idea: step1

Imagine that a spherical “BH” is evaporating.
Add a spherical thin shell (or a particle) to it.



What happens if evolution of the matter and spacetime is considered?
⇒The shell will never reach the Schwarzschild radius.

The motion of the particle near the "BH"



A naïve ansatz: ≈ time-dependent Schwarzschild metric

$$ds^2 = -\left(1 - \frac{a(t)}{r}\right)dt^2 + \left(1 - \frac{a(t)}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

$$\frac{d}{dt}a(t) = -\frac{2\sigma}{a(t)^2}$$

←Stephan-Boltzmann law of $T_H = \frac{\hbar}{4\pi a}$

$a \equiv 2GM$
intensity: $\sigma = O(1) \sim \hbar G N$
(N=d.o.f. of fields)

For $r_s \sim a$,
a particle with any (l, m) behaves ultra-relativistic:

$$\frac{dr_s(t)}{dt} = -\frac{r_s(t) - a(t)}{r_s(t)}.$$

Solve e.o.m of the particle

$$\frac{d}{dt}a(t) = -\frac{2\sigma}{a(t)^2}$$

We are now interested in $r_s(t) \sim a(t)$ and focus on

$$\Delta r(t) \equiv r_s(t) - a(t) \ll a(t).$$

Then, we have

$$\frac{dr_s(t)}{dt} = -\frac{r_s(t) - a(t)}{r_s(t)} \approx -\frac{r_s(t) - a(t)}{a(t)}$$

$$\Rightarrow \frac{d\Delta r(t)}{dt} = -\frac{\Delta r(t)}{a(t)} - \frac{da(t)}{dt}.$$

⇒The general solution is given by

$$\Delta r(t) = c_0\, e^{-\int_{t_0}^{t} \frac{dt'}{a(t')}} + \int_{t_0}^{t} dt' \{ -(\frac{da}{dt}(t'))\, e^{-\int_{t'}^{t} dt'' \frac{1}{a(t'')}} \}.$$

In the time scale $\Delta t \sim a(t) \left(\ll \frac{a^3}{\sigma} \right)$

$a(t)$ ≈const. and $\frac{da(t)}{dt}$ ≈const.

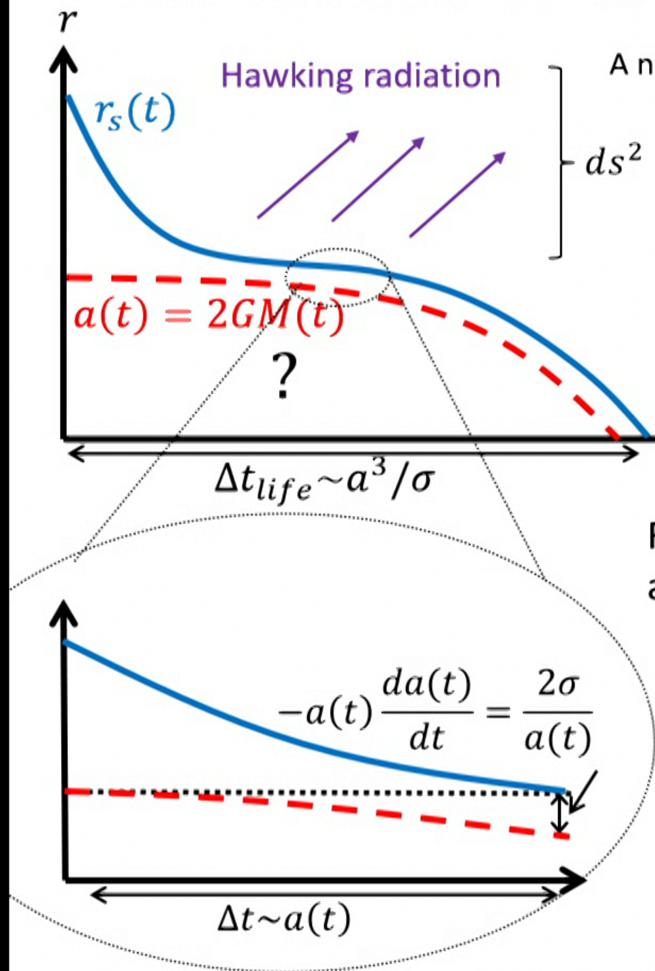
$$\approx -\frac{da(t)}{dt} \int_{t_0}^{t} dt'\, e^{-\frac{t-t'}{a(t)}}$$

$$\approx -\frac{da(t)}{dt} a(t) (1 - e^{-\frac{t-t_0}{a(t)}})$$

Thus, we obtain

$$\Delta r(t) \approx -\frac{da(t)}{dt} a(t) + C e^{-\frac{t}{a(t)}}$$

The motion of the particle near the "BH"



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$$\frac{d}{dt}a(t) = -\frac{2\sigma}{a(t)^2}$$ ←Stephan-Boltzmann law of $T_H = \frac{\hbar}{4\pi a}$

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(N=d.o.f. of fields)

For $r_s \sim a$,
a particle with any (l, m) behaves ultra-relativistic:

$$\frac{dr_s(t)}{dt} = -\frac{r_s(t) - a(t)}{r_s(t)}.$$

$$\Rightarrow r_s(t) \simeq a(t) - a(t)\frac{da(t)}{dt} + Ce^{-\frac{t}{a(t)}}$$

$$\rightarrow a(t) + \frac{2\sigma}{a(t)}$$

←Effect of back reaction

⇒Universally, particles will approach $a(t) + \frac{2\sigma}{a(t)}$.

The particle will never cross $a(t)$.

- The proper length of $\Delta r = \frac{2\sigma}{a}$ is given by

$$\Delta l = \sqrt{g_{rr}}\Delta r = \sqrt{\frac{r}{r-a(t)}}\Big|_{r=a(t)+\frac{2\sigma}{a(t)}} \frac{2\sigma}{a(t)}$$

$$\approx \sqrt{\frac{a(t)}{\frac{2\sigma}{a(t)}}} \frac{2\sigma}{a(t)}$$

$$= \sqrt{2\sigma} \sim \sqrt{N} l_p \gg l_p$$

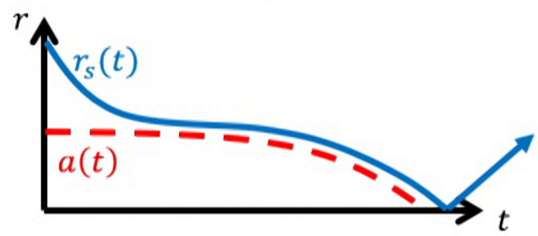
if $N \gg 1$

intensity: $\sigma = O(1) \sim \hbar G N$
(N=d.o.f. of fields)

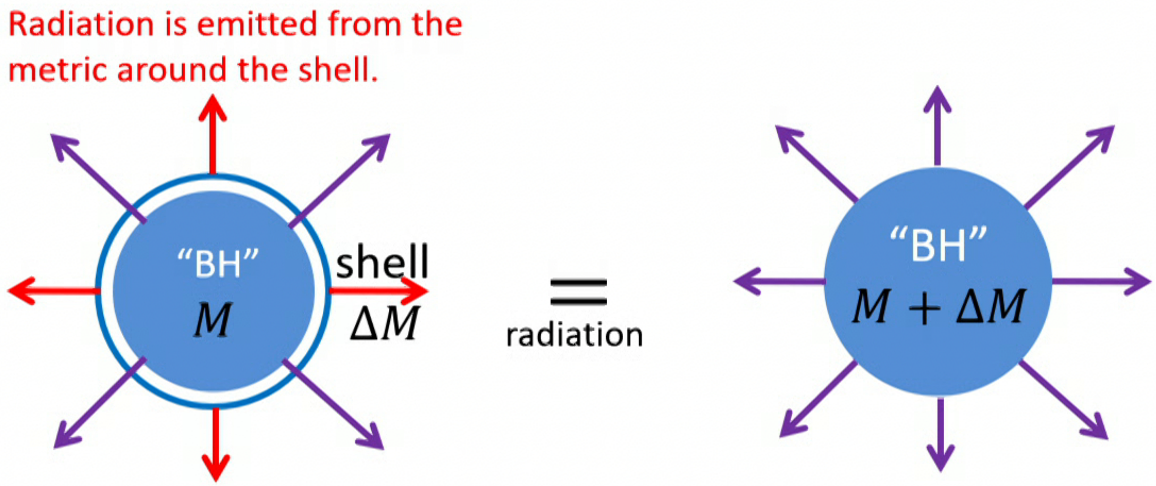
$$\boxed{l_p \equiv \sqrt{\hbar G}}$$

⇒Physically (at the semi-classical level) the shell is always outside $a(t)$.

⇒The shell will never cross $a(t)$. (Note: No coordinate singularity.)



Basic idea: step2



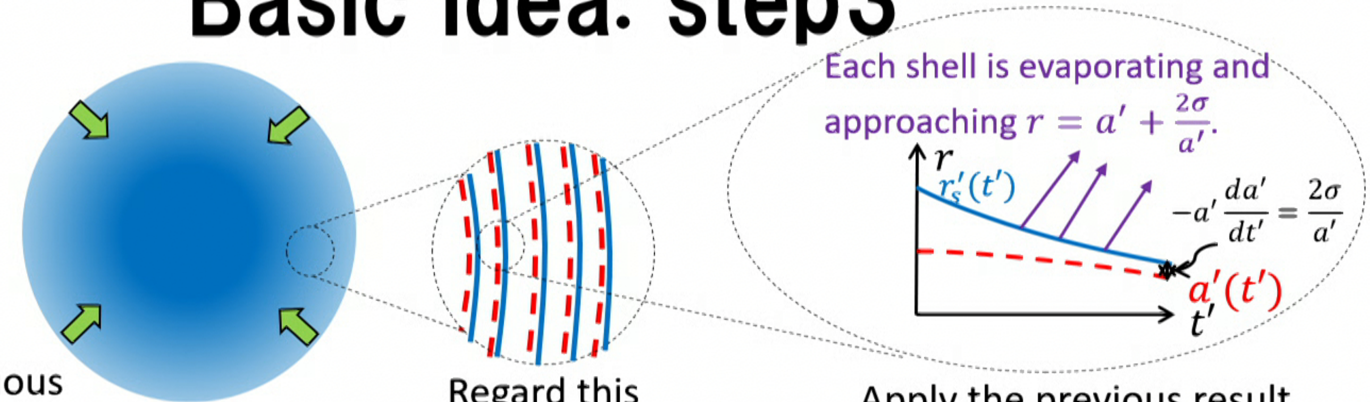
(radiation from the shell with ΔM)

+(redshift factor) (radiation from the core BH with M)

= (radiation from a BH with $M + \Delta M$)

⇒This composite system (=shell+BH) behaves like a larger BH.

Basic idea: step3

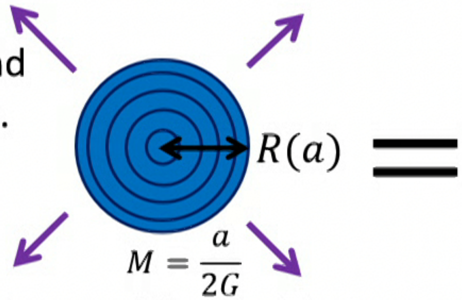


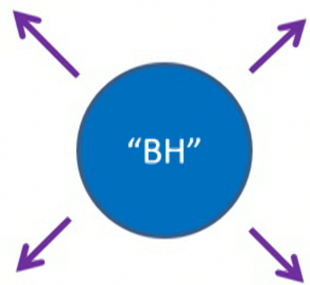
continuous
collapsing matter (not BH)

Regard this
as many shells.

Apply the previous result
to each shell recursively.

The shells pile up and
form a dense object.





The matter and radiation are filled inside

$$r = a + \frac{2\sigma}{a} \equiv R(a),$$

which is the boundary of the object (***surface***).
⇒ No horizon exists.

This looks like an ordinary BH
from the outside.
⇒This should be quantum BH!

Talk Plan

How to solve this in a self-consistent way?

Step2: Evaluate $\langle T_{\mu\nu} \rangle$ on $g_{\mu\nu}$.

$$\boxed{G_{\mu\nu} = 8\pi G \langle \hat{T}_{\mu\nu} \rangle}$$

Step1: Construct a candidate metric $g_{\mu\nu}$ by a simple model.

Step3: Put this and determine the self-consistent $g_{\mu\nu}$.

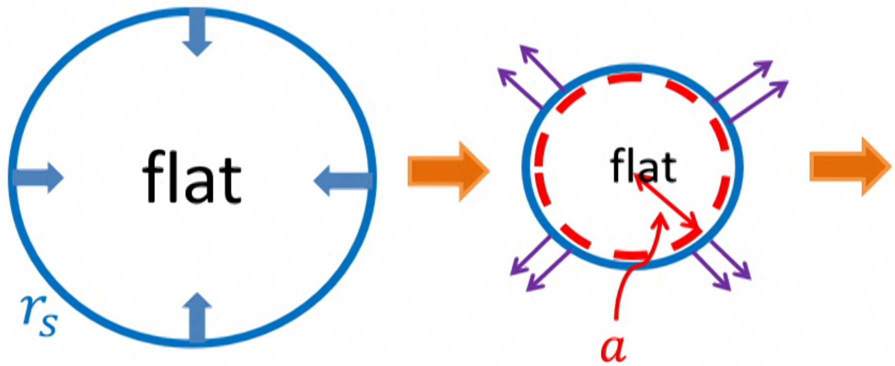
Step4: Consistency check
Curvature,
Stephan-Boltzmann law,
Hawking temperature,
energy condition,
entropy-information.

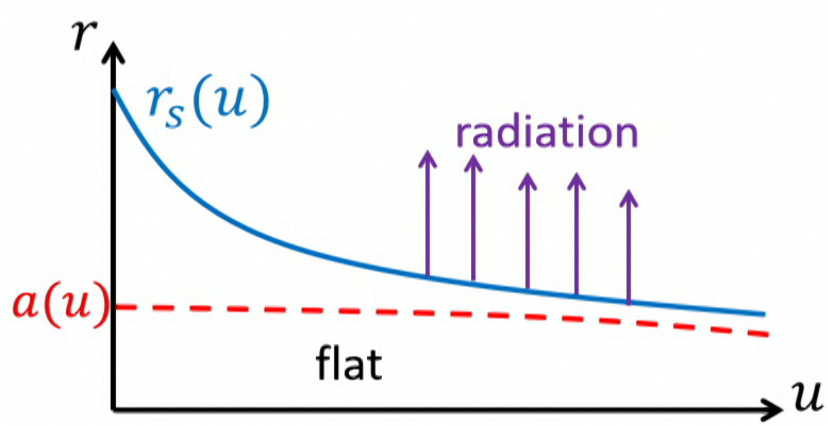
Step1. Construction of the candidate metric $g_{\mu\nu}$

Preliminary: case of single shell (1/3)

Consider time evolution of a spherical null shell.

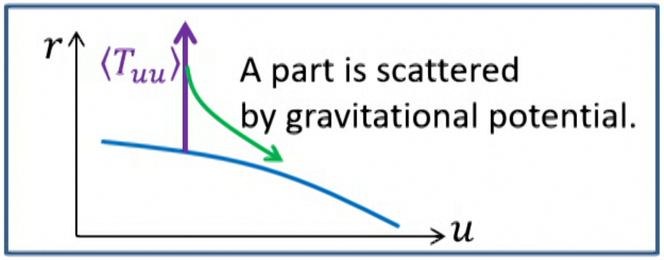
As we have seen, any matter behaves lightlike near the Schwarzschild radius.





Preliminary: case of single shell (2/3)

Approximation: Neglect reflection of radiation for simplicity.

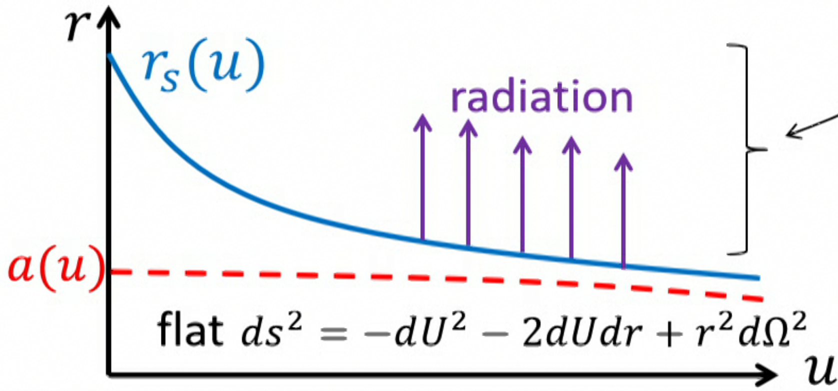


(Later we will remove this artificial assumption.)

⇒outgoing Vaidya metric:

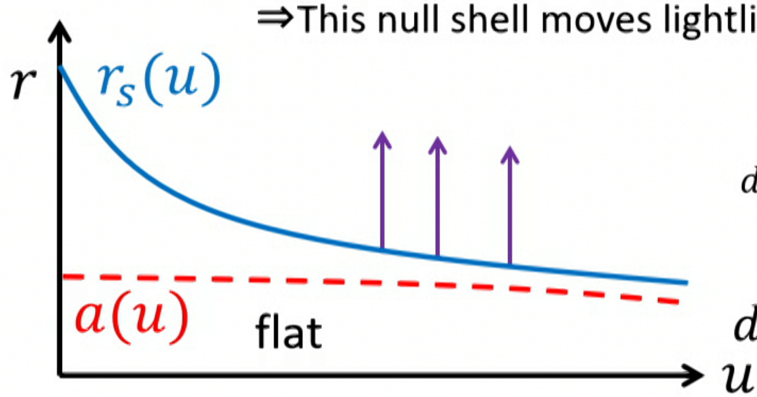
$$ds^2 = -\frac{r - a(u)}{r} du^2 - 2drdu + r^2 d\Omega^2$$

outgoing radial null energy flow~ $G_{uu} = -\frac{\dot{a}(u)}{r^2}$, $a(u)$ is not fixed yet.



Preliminary: case of single shell (3/3)

We need to connect two time coordinates U and u.
⇒This null shell moves lightlike in the both coordinates.



$$ds^2 = -\Bigg[\underbrace{\frac{r-a(u)}{r}du + 2dr}_{\simeq 0 \text{ at } r = r_s(u)}\Bigg]du + r^2\cancel{d\Omega^2}$$

$$ds^2 = -\underbrace{[dU + 2dr]}_{\simeq 0 \text{ at } r = r_s(u)}dU + r^2\cancel{d\Omega^2}$$

⇒connection condition:

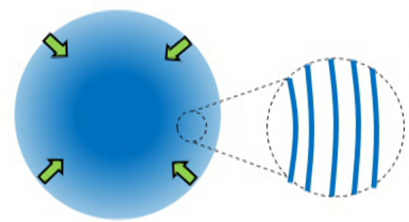
$$dU = -2dr_s = \left(1 - \frac{a(u)}{r_s(u)}\right)du$$

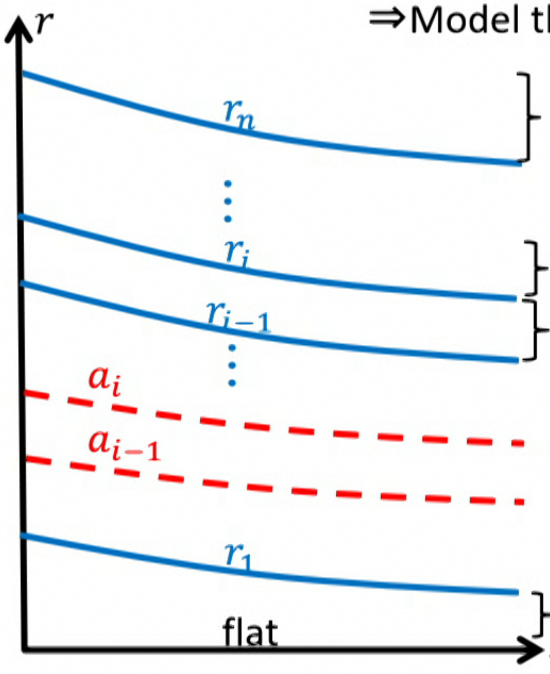
⇒If a function $a(u)$ is given,

$\boxed{U = U(u)}$ $\boxed{\dfrac{dr_s(u)}{du} = -\dfrac{r_s(u) - a(u)}{2r_s(u)}}$ are determined.

A multi-shell model

Consider a continuous spherical matter.
⇒Model this as many null shells.





$$ds_{out}^2 = -\frac{r-a}{r}du^2 - 2dudr + r^2 d\Omega^2$$

$$ds_i^2 = -\frac{r-a_i}{r}du_i^2 - 2du_i dr + r^2 d\Omega^2$$

$$ds_{i-1}^2 = -\frac{r-a_{i-1}}{r}du_{i-1}^2 - 2du_{i-1} dr + r^2 d\Omega^2$$

$$ds_0^2 = -dU^2 - 2dUdr + r^2 d\Omega^2$$

⇒"Initial condition": $a_0 = 0,\ u_0 = U.$

connection condition $\frac{r_i - a_i}{r_i}du_i = -2dr_i = \frac{r_i - a_{i-1}}{r_i}du_{i-1}$

$$\Rightarrow \frac{dr_i}{du_i} = -\frac{r_i - a_i}{2r_i}, \qquad \frac{du_i}{du_{i-1}} = \frac{r_i - a_{i-1}}{r_i - a_i}$$

⇒We want to determine $u_i(U)$.

Ansatz in the asymptotic stage

Ansatz Each shell behaves like the ordinary evaporating BH when we look at it from the outside:

$$\frac{da_i}{du_i} = -\frac{\sigma}{a_i^2},$$

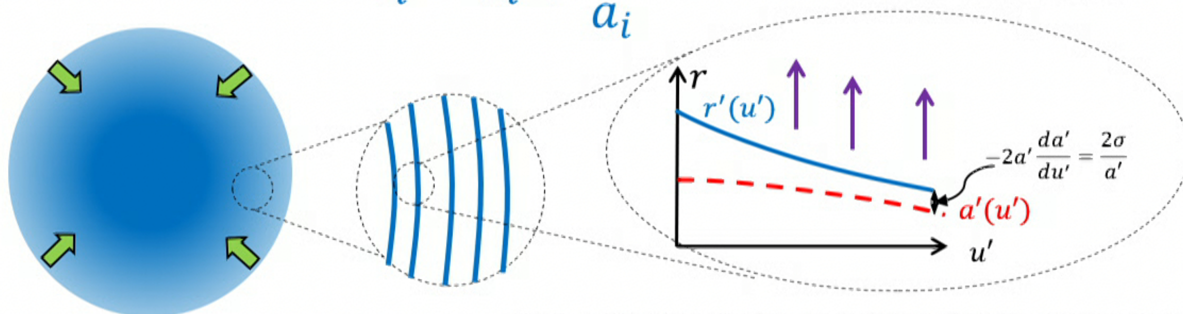
Note: At this stage, this eq is NOT derived.

and that each shell has already come close to

$$r_i = a_i + \frac{2\sigma}{a_i}$$

∼asymptotic state

Basic idea.



(We will check the validity of this ansatz later.)

⇒After taking the continuum limit $\Delta a_i \equiv a_i - a_{i-1} \ll 1$,
we can obtain $u_i(U)$.

Note: Derivation of the interior metric (1/2)

Then, $\eta_i \equiv \log \frac{dU}{du_i}$ can be evaluated as follows:

$$\boxed{\frac{du_i}{du_{i-1}} = \frac{r_i - a_{i-1}}{r_i - a_i}}$$

$$\eta_i - \eta_{i-1} = \log \frac{\frac{dU}{du_i}}{\frac{dU}{du_{i-1}}} = -\log \frac{du_{i-1}}{du_i} = -\log\left(1 + \frac{a_i - a_{i-1}}{r_i - a_i}\right)$$

$$\boxed{\Delta a \equiv a_i - a_{i-1} \ll 1} \longrightarrow \approx -\frac{a_i - a_{i-1}}{r_i - a_i}$$

$$\boxed{r_i = R(a_i) = a_i + \frac{2\sigma}{a_i}} \longrightarrow = -\frac{(a_i - a_{i-1})}{\frac{2\sigma}{a_i}}$$

$$\boxed{2a_i \approx a_i + a_{i-1}} \longrightarrow \approx -\frac{1}{4\sigma}(a_i^2 - a_{i-1}^2)$$

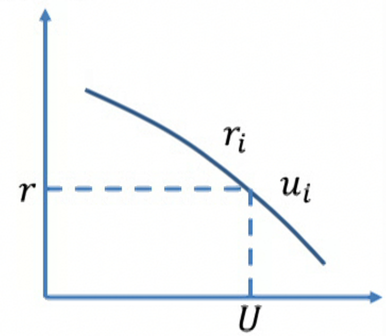
With the initial conditions $\eta_0 = a_0 = 0$, we obtain

$$\eta_i = -\frac{1}{4\sigma} a_i^2$$

Note: Derivation of the interior metric (2/2)

The metric at a point (U, r) is determined by considering the shell that passes the point:

$$ds^2 = -\left(1 - \frac{a_i}{r}\right) du_i^2 - 2du_i dr + r^2 d\Omega^2$$



with
$$\begin{cases} 1 - \frac{a_i}{r} = \frac{r_i - a_i}{r_i} = \frac{\frac{2\sigma}{a_i}}{r_i} \approx \frac{2\sigma}{r^2}, \\ \frac{du_i}{dU} = \exp(-\eta_i) = \exp\left(\frac{1}{4\sigma} a_i^2\right) \approx \exp\left(\frac{1}{4\sigma} r^2\right). \end{cases}$$

$\eta_i = -\frac{1}{4\sigma} a_i^2$

$r_i = R(a_i) = a_i + \frac{2\sigma}{a_i}$

⇒In the continuum limit, we have the interior metric:

$$ds^2 = -\left(1 - \frac{a_i}{r}\right)\left(\frac{du_i}{dU}\right)^2 dU^2 - 2\left(\frac{du_i}{dU}\right) dU dr + r^2 d\Omega^2$$

$$\approx -\frac{2\sigma}{r^2} e^{\frac{1}{2\sigma} r^2} dU^2 - 2e^{\frac{1}{4\sigma} r^2} dU dr + r^2 d\Omega^2 \, .$$ ←Static!

Note: The inside metric is static although each shell is shrinking.

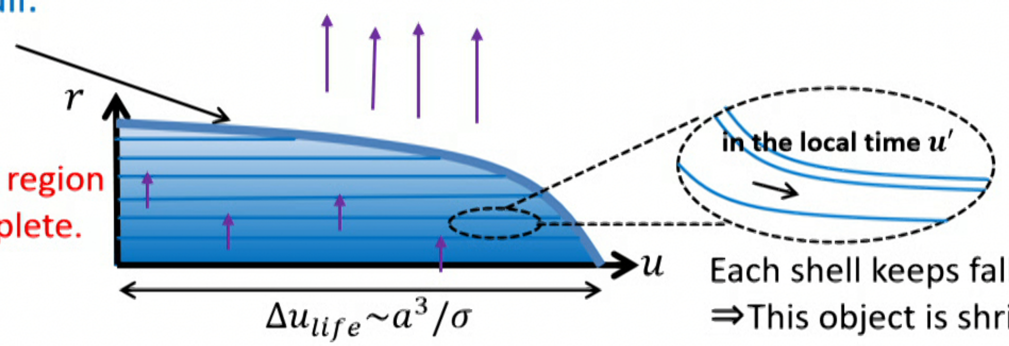
The candidate metric

$$ds^2 = \begin{cases} -\left(1-\frac{a(u)}{r}\right)du^2 - 2dudr + r^2 d\Omega^2 & \leftarrow \text{time-dep} \\ \underbrace{-\frac{2\sigma}{r^2} e^{-\frac{1}{2\sigma}\left(R(a(u))^2-r^2\right)}}_{\text{Large redshif}} du^2 - 2e^{-\frac{1}{4\sigma}\left(R(a(u))^2-r^2\right)} dudr + r^2 d\Omega^2 & \leftarrow \text{static} \end{cases}$$

→The interior is frozen.

$$\frac{da(u)}{du} = -\frac{\sigma}{a(u)^2}, \qquad R(a) \equiv a + \frac{2\sigma}{a}$$

The surface is null:
$r = R(a(u))$

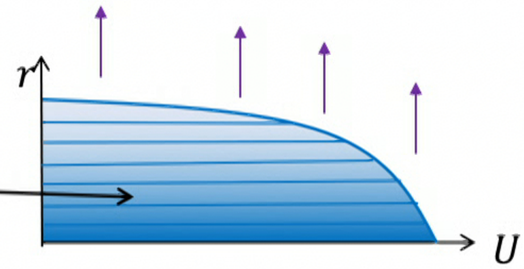


No horizon or trapped region
⇒(u, r) coord. is complete.

Each shell keeps falling.
⇒This object is shrinking.

Step2. Evaluation of $\langle T_{\mu\nu} \rangle$ on the metric $g_{\mu\nu}$

Setup



Consider the interior region.

The background metric is static:

$$ds^2 = -\frac{2\sigma}{r^2} e^{\frac{r^2}{2\sigma}} dU^2 - 2e^{\frac{r^2}{4\sigma}} dU dr + r^2 d\Omega^2$$
$$= -e^{\varphi(r(U,V))} dU dV + r(U,V)^2 d\Omega^2,$$

$\Rightarrow \langle T_{\mu\nu} \rangle$ also should be static:

$$\langle T_{\mu\nu} \rangle = \langle T_{\mu\nu}(r) \rangle, \qquad \langle T_{UU} \rangle = \langle T_{VV} \rangle$$

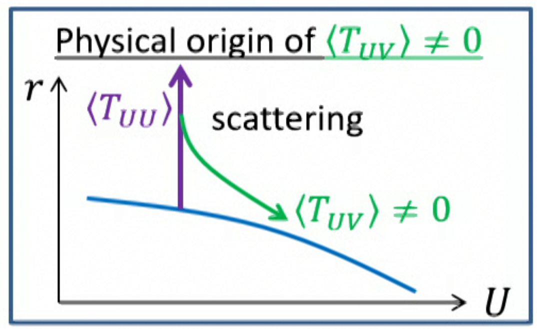
$\Rightarrow$Neglecting the scattering means

$$\langle T_{UV} \rangle = 0,$$

$\Rightarrow$We need to determine only

$$\langle T_{UU} \rangle, \qquad \langle T^{\theta}_{\theta} \rangle.$$

($\Rightarrow$Later $\langle T_{UV} \rangle \neq 0$.)



The relations of $\langle T_{\mu\nu} \rangle$

・1st eq.

$\langle T^{\mu}_{\mu} \rangle = 2g^{UV} \underbrace{\langle T_{UV} \rangle}_{=0} + 2\langle T^{\theta}_{\theta} \rangle$ leads to

$$\langle T^{\theta}_{\theta} \rangle = \frac{1}{2} \langle T^{\mu}_{\mu} \rangle$$

4D Weyl anomaly

For simplicity, consider conformal matters. ←NOT necessary

⇒ $\langle T^{\mu}_{\mu} \rangle$ is determined by the 4D Weyl anomaly:

$$\langle T^{\mu}_{\mu} \rangle = \hbar c_w \mathcal{F} - \hbar a_w \mathcal{G}$$ ←state-independent id.

where

$$\mathcal{F} \equiv C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta}, \qquad \mathcal{G} \equiv R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$$

⇒The metric determines

$$\langle T^{\mu}_{\mu} \rangle = \frac{\hbar c_w}{3\sigma^2}$$

step3: Determine the self-consistent $g_{\mu\nu}$

The $g_{\mu\nu}$ is the self-consistent solution.

Thus , we have obtained

$$\langle T_\theta^\theta \rangle = \frac{\hbar c_W}{6\sigma^2}, \qquad \langle T_{UU} \rangle = \langle T_{VV} \rangle = \frac{\hbar c_W}{3r^4} e^{\frac{1}{2\sigma} r^2}, \qquad \langle T_{UV} \rangle = 0$$

On the other hand, the metric gives

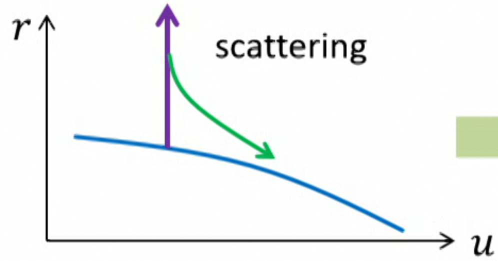
$$G_\theta^\theta = \frac{1}{2\sigma}, \qquad G_{UU} = G_{VV} = \frac{\sigma}{r^4} e^{\frac{1}{2\sigma} r^2}, \qquad G_{UV} = 0$$

$\Rightarrow G_{\mu\nu} = 8\pi G \langle T_{\mu\nu} \rangle$ is satisfied if we identify

$$\sigma = \frac{8\pi G\hbar}{3} c_W.$$

Removing the assumption $\langle T_{UV} \rangle = 0$

Origin of $\langle T_{UV} \rangle \neq 0$



phenomenological eq

$\langle T_{UV} \rangle = f \langle T_{UU} \rangle$

$f(t, r) = O(1)$

Apply to this case

$\boxed{\text{conformal matters} \Rightarrow f = \text{const.} = O(1)}$

General ansatz: $ds^2 = -\frac{1}{B(r)} e^{A(r)} dT^2 + B(r) dr^2 + r^2 d\Omega^2$

$\Rightarrow A(r), B(r)$ can be determined by

$$\begin{cases} G^\mu_\mu = 8\pi G(\hbar c_w \mathcal{F} - \hbar a_w \mathcal{G}) \\ G_{UV} = f G_{UU} \end{cases}$$

$$ds^2 = -\frac{2\sigma}{r^2} e^{-\frac{1}{2\sigma(1+f)}\left(R(a(u))^2 - r^2\right)} du^2 - 2e^{-\frac{1}{4\sigma(1+f)}\left(R(a(u))^2 - r^2\right)} du dr + r^2 d\Omega^2$$

$$\boxed{\sigma \equiv \frac{8\pi G \hbar c_W}{3(1+f)^2}}$$

Self-consistent solution

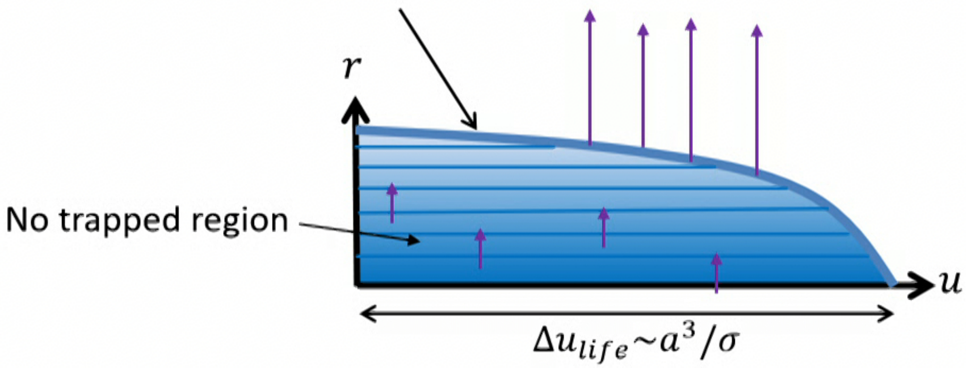
$$ds^2 = \begin{cases} -\left(1-\dfrac{a(u)}{r}\right)du^2 - 2dudr + r^2 d\Omega^2 \\ -\dfrac{2\sigma}{r^2} e^{-\frac{1}{2\sigma(1+f)}\left(R(a(u))^2 - r^2\right)} du^2 - 2e^{-\frac{1}{4\sigma(1+f)}\left(R(a(u))^2 - r^2\right)} dudr + r^2 d\Omega^2 \end{cases}$$

←Non-perturbative solution w.r.t. $\hbar$

$$\boxed{\sigma \equiv \frac{8\pi G\hbar c_W}{3(1+f)^2}}$$

The surface exists at

$$R(a(u)) \equiv a(u) + \frac{2\sigma}{a(u)}$$



$$\langle T^\mu_\mu \rangle = \hbar c_w C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta} - \hbar a_W \mathcal{G}$$

Validity of the classical gravity

In the macroscopic region ($r > l_P$),

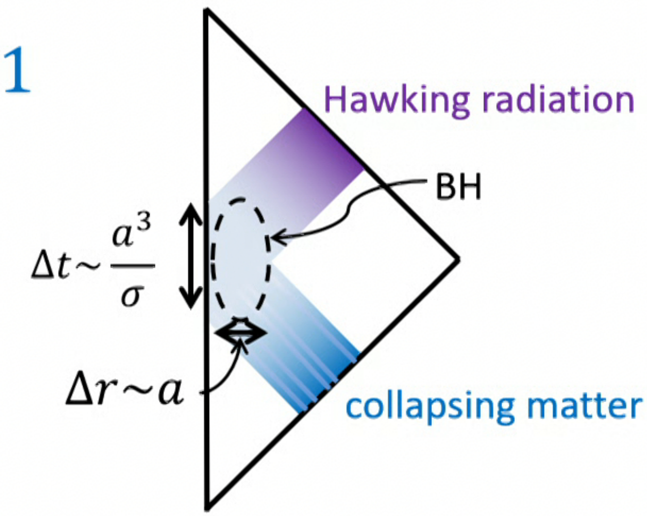
$$R, \sqrt{R_{\alpha\beta}R^{\alpha\beta}}, \sqrt{R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}} \sim \frac{1}{c_w l_p^2} \ll \frac{1}{l_p^2}$$

(c_w plays a role of N in the introduction.)

if $c_w \gg 1$

⇒No singularity:

$G_{\mu\nu} = 8\pi G \langle \hat{T}_{\mu\nu} \rangle$ is valid as long as $a > l_p$ and $L_{gravity} = R + O(1)R^2$.



Note:

1) Cannot describe the final stage of the evaporation.

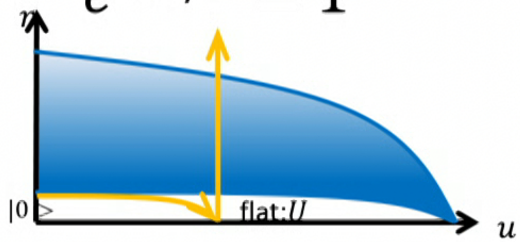
2) Consistent with the singularity theorem.

Hawking radiation

- Hawking radiation appears self-consistently:

By a similar manner to Hawking's derivation, we can show

$$\langle 0 | \hat{N}_\omega | 0 \rangle = \frac{1}{e^{\hbar\omega/T} - 1}, \qquad T = \frac{\hbar}{4\pi a(u)}$$



- Stefan-Boltzmann law is obtained:

$$\frac{da}{du} = -\frac{\sigma}{a^2}, \qquad \sigma \equiv \frac{8\pi G\hbar c_W}{3(1+f)^2}$$

Energy condition of $\langle T_{\mu\nu} \rangle$

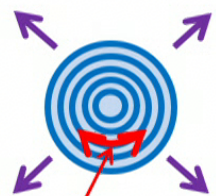
$$\boxed{4\pi \int_{\sim\sqrt{c_W} l_p}^{R(a)} dr\, r^2 (-\langle T^t_t \rangle) \approx \frac{a}{2G} = M}$$

$$0 < -\langle T^t_t \rangle = \frac{1}{8\pi G} \frac{1}{r^2}, \langle T^r_r \rangle = \frac{1}{8\pi G} \frac{1-f}{1+f} \frac{1}{r^2}$$

The dominant energy condition ($\rho \geq p_i > 0$) is broken. $\hat{\wedge}$

$\neq$ anisotropic ⇒ Not a fluid

$$\langle T^\theta_\theta \rangle = \frac{1}{8\pi G} \frac{3}{16\pi c_W l_p^2}$$



This large angular pressure supports the object.

TOV eq: $0 = \partial_r p_r + \partial_r \log \sqrt{-g_{tt}} (\rho + p_r) + \frac{2}{r}(p_r - p_\theta)$

balance!

Importance of 4D Weyl anomaly

- Our key eq is

$$G^{\mu}_{\mu} = 8\pi G\langle T^{\mu}_{\mu}\rangle$$
$$\Leftrightarrow \quad \mathcal{R}\sim N l_p^2 \mathcal{R}^2$$

$$\langle T^{\mu}_{\mu}\rangle = \hbar c_w \mathcal{F} - \hbar a_w \mathcal{G}$$

$\Rightarrow$The solutions are categorized into three cases:

$$\mathcal{R}\sim \frac{1}{L^2} \ll 1$$ $\leftarrow$ perturbative solution w.r.t. $\hbar$

$$\mathcal{R} = \infty$$ $\leftarrow$nonsense

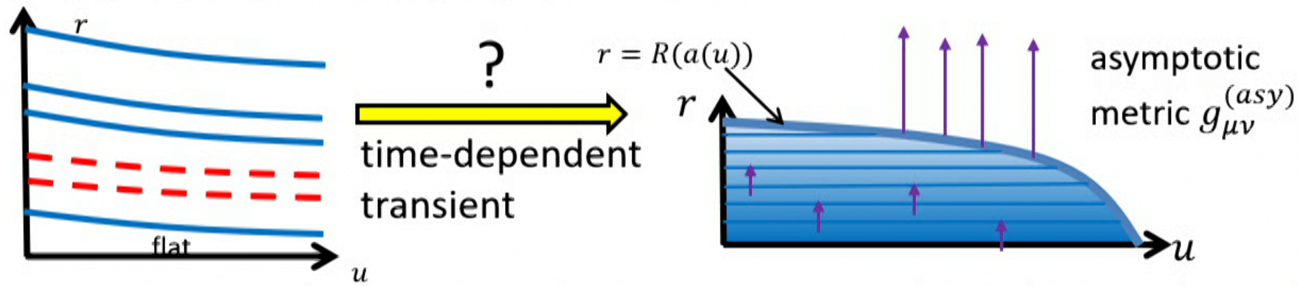
$$\mathcal{R}\sim \frac{1}{N l_p^2}$$ $\leftarrow$non-perturbative solution w.r.t. $\hbar$

- Summing up vacuum fluctuation modes with large angular momentum leads to 4D anomaly and the large pressure:

$$\langle T^{\theta}_{\theta}\rangle \approx \frac{1}{2}\langle T^{\mu}_{\mu}\rangle = \frac{1}{8\pi G}\frac{3}{16\pi c_W l_p^2}$$

Validity of the ansatz $\frac{da_i}{du_i} = -\frac{\sigma}{a_i^2}, r_i = a_i + \frac{2\sigma}{a_i}$

- It is difficult to study the time-depend transient process in which the matter approaches the asymptotic metric.



- Here, we consider the s-wave of massless scalar fields in the eikonal approximation and study the stability of the perturbation from the asymptotic metric.

$$G_{\mu\nu} = 8\pi G\langle \hat{T}_{\mu\nu}\rangle \Rightarrow \begin{cases} \dfrac{da_i}{du_i} = -\dfrac{l_p{}^2}{8\pi}\{u_i, U\} \\ \dfrac{dr_i}{du_i} = -\dfrac{r_i(u_i) - a_i(u_i)}{2r_s(u_i)} \\ \dfrac{du_i}{du_{i-1}} = \dfrac{r_i - a_{i-1}}{r_i - a_i} \end{cases}$$

Result

$$g_{\mu\nu} = g_{\mu\nu}^{(asy)} + h_{\mu\nu}$$

$$h_{\mu\nu} \to 0$$

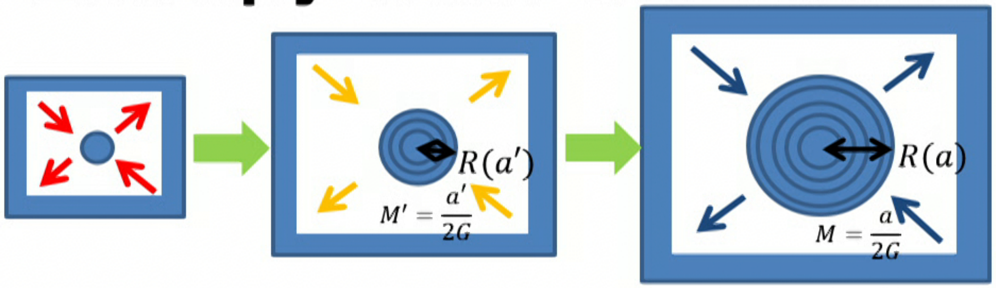
if the initial condition is fine-tuned.

⇒The ansatz is stable.

Cf. ・Schwarzian derivative $\{u, U\} \equiv \frac{\ddot{U}(u)^2}{\dot{U}(u)^2} - \frac{2\dddot{U}(u)}{3\dot{U}(u)}$

・back reaction of radiating electron $F_{self} \sim \frac{d^3x}{dt^3}$

Entropy from the interior



$$\sigma = \frac{8\pi l_p^2 c_W}{3(1+f)^2}$$

- The interior metric of the stationary BH is given by

$$ds^2 = -\frac{2\sigma}{r^2} e^{-\frac{1}{2(1+f)\sigma}(R(a)^2 - r^2)} dt^2 + \frac{r^2}{2\sigma} dr^2 + r^2 d\Omega^2.$$

- We can evaluate the entropy density as

$$s = \frac{1}{4\pi r^2} \frac{2\pi\sqrt{2\sigma}}{l_p^2}. \qquad \Rightarrow 4\pi r^2 s \sim \sqrt{N}/l_p$$

- Summing up it over the interior volume, we obtain

$$S = \int dV\, s = \int_0^{R(a)} dr \sqrt{g_{rr}}\, 4\pi r^2 s \approx \int_0^{a} dr \frac{r}{\sqrt{2\sigma}} \frac{2\pi\sqrt{2\sigma}}{l_p^2} = \frac{\pi a^2}{l_p^2} = \frac{A}{4 l_p^2}$$

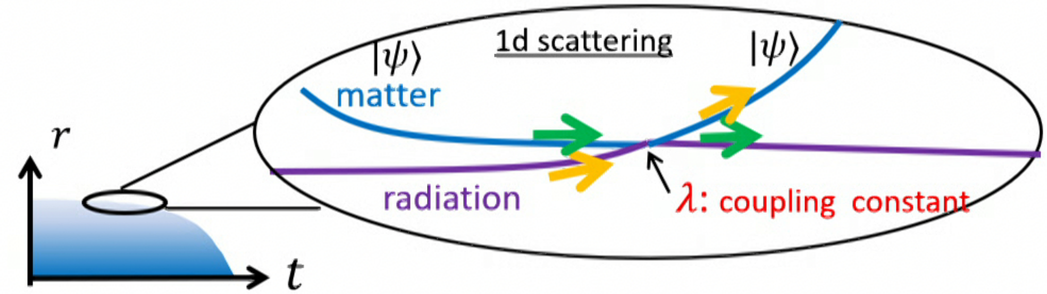
⇒The information should be stored inside the BH.

Information recovery by interaction

- Hawking radiation is created inside the collapsing matter.

⇒The collapsing matter and Hawking radiation interact.

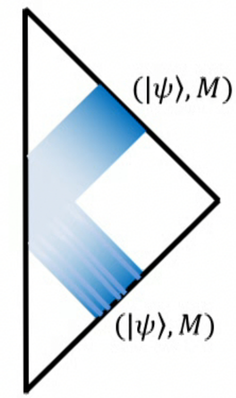
Cf: Thermodynamics
small interaction
⇒equilibrium state



⇒We can estimate the scattering time scale

$$\Delta t_{scat} \sim a \log \frac{a}{\lambda N l_p}$$

⇒information recovery? ~scrambling time



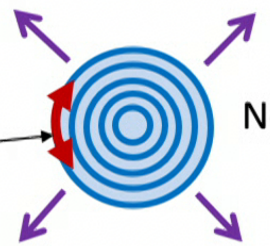
Conclusions

Quantum BH

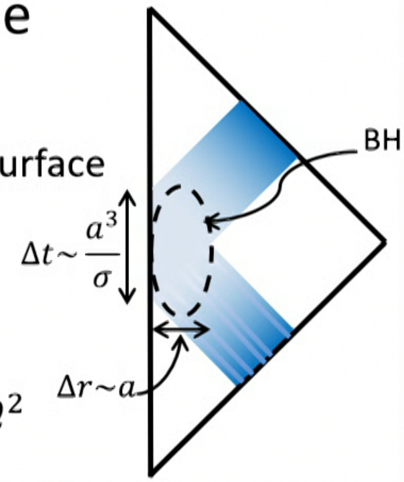
= a dense star without horizon or singularity that looks like the classical BH from the outside

Large tangential pressure $\langle \hat{T}^{\theta}_{\theta} \rangle$

⇒Stabilize it but breaks dominant energy condition.



No horizon but surface



$$ds^2 = \begin{cases} -\left(1-\frac{a(u)}{r}\right)du^2 - 2dudr + r^2 d\Omega^2 \\ -\frac{2\sigma}{r^2} e^{-\frac{1}{2\sigma(1+f)}\left(R(a(u))^2 - r^2\right)} du^2 - 2e^{-\frac{1}{4\sigma(1+f)}\left(R(a(u))^2-r^2\right)} dudr + r^2 d\Omega^2 \end{cases}$$

- This satisfies $G_{\mu\nu} = 8\pi G \langle \hat{T}_{\mu\nu} \rangle$ if there are many matter fields.
- Entropy comes from the interior: $S = \int dV\, s = \frac{A}{4 l_p^2}$

Future directions

- What happens to a fallen matter?

⇒Interaction b.w. matter and radiation?

- Can we derive the entropy density by counting microstates?

⇒ Work in progress.

- Can we predict some observable effects from this quantum BH?

⇒Something should bounce at the surface?

- Can we generalize this picture to rotating BHs?

⇒There is the universal surface of Kerr BHs [Kawai-Y, PRD2016].

No horizon but surface.
This is the BH.

Thank you very much!