Title: NUTs and Bolts: free energy via susy localization

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Series: Quantum Fields and Strings

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Abstract: The partition function of three-dimensional N=2 SCFTs on circle bundles of closed Riemann surfaces \Sigma_g was recently computed via supersymmetric localization. In this talk I will describe supergravity solutions having as conformal boundary such circle bundle. These configurations are solutions to N=2 minimal gauged supergravity in 4d and pertain to the class of AdS-Taub-NUT and AdS-Taub-Bolt preserving 1/4 of the supersymmetries. I will discuss the conditions for the uplift of these solutions to M-theory and I provide the expression for the on-shell action of the Bolt solutions, computed via holographic renormalization. I will show that, when the uplift condition is satisfied, the Bolt free energy matches with the large N limit of the partition function of the corresponding dual field theory. I will finally comment on possible subtleties that arise in our framework when a given boundary geometry admits multiple bulk fillings.

Filling the boundary: NUTs, Bolts and susy localization

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Perimeter Institute, May 10, 2019

based on 1712.08861 with B. Willett and work in progress w. B. Willett and A. Passias

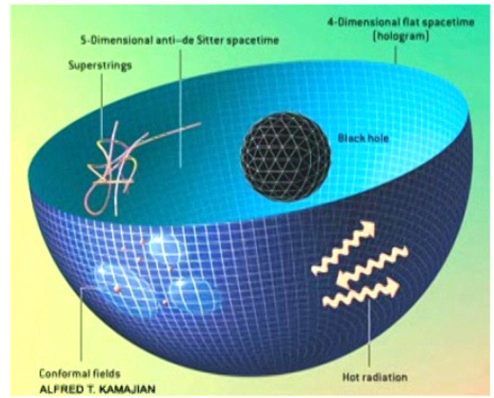
Intro and motivation

AdS/CFT correspondence:
[Maldacena '97]

Theory with gravity in AdS_{d+1}

dual to

conformal field theory in d dimensions



- Model strongly coupled field theory processes by gravity dual
- Microscopic understanding of the entropy of AdS_{d+1} black holes via the dual CFT_d

Intro and motivation

Black holes seen as thermodynamic ensembles: they emit radiation and possess entropy $S_{BH} = \kappa_B \frac{Ac^3}{4G\hbar}$

Microstate counting and entropy matching by [Strominger,Vafa'96] in some specific cases

Use information of the dual field theory to count black hole microstates

Counting usually requires supersymmetry: in AdS extremal rotating black holes can be supersymmetric

- Astrophysical ones are extremal-Kerr, rotating.

Need to construct AdS solutions with embedding in string/M-theory: exact holographic dual. Work in the low-energy limit of string theory $\rightarrow$ supergravity

i.e. AdS_4 with field theory duals in the class of ABJM [Aharony, Bergman, Jafferis, Maldacena, '08]

Intro and motivation

Recent success: microstate counting for susy AdS_4 black holes

Static 1/4 BPS black hole exist in 4d $\mathcal{N} = 2$ *gauged supergravity* [Cacciatori, Klemm '09]: scalar potential allows for susy AdS_4 vacua, field theory dual is ABJM

- extremal black holes are flows from AdS_4 to $AdS_2 \times \Sigma_g$ near horizon geometry
- magnetic gauge field cancels spin connection in the susy equations (topological twist)

Intro and motivation

ABJM partition function on $S^1 \times S^2$ with magnetic fluxes s_i on S^2 computed via susy localization, in the large N limit, [Benini, Hristov, Zaffaroni '15], [Benini, Zaffaroni '16]

$$\log Z_{S^1 \times S^2} \approx -\frac{2\pi N^{3/2}}{3} \sqrt{2 m_1 m_2 m_3 m_4} \sum_{i=1}^{4} \frac{s_i}{m_i}$$

reproduces BPS black hole entropy upon extremization on m_i:

$$S_{BH} = \frac{A_{BH}}{4 G_N} = \log \mathcal{Z}|_{crit}(s^i)$$

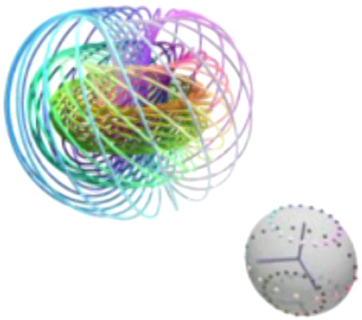
Extremization corresponds to the attractor mechanism in sugra.

Intro and motivation

Extend these considerations to more general case of $\mathcal{M}_{g,p} = S^1$ p-bundle over 2d Riemann surfaces Σ_g [Closset, Kim, Willett '17]

$$ds^2 = (d\psi + a)^2 + d\Omega_\kappa^2 \qquad -\frac{1}{4\pi}\int_{\Sigma_g} da = p$$

- $\mathcal{M}_{g,0} \simeq S^1 \times \Sigma_g$ reduces to [Benini, Zaffaroni '16]: black hole physics
- $\mathcal{M}_{0,1} \simeq S^3$ partition function (F-theorem, entanglement entropy...)
- $\mathcal{M}_{0,p} \simeq S^3/\mathbb{Z}_p$ Lens space



Intro and motivation

Holographic check with bulk dual solutions.

ABJM partition function $Z_{ABJM}(S^1 \times \Sigma_g)$

$\rightarrow$ black holes, whose boundary is $S^1 \times \Sigma_g$

ABJM partition function $Z_{ABJM}(\mathcal{M}_{g,p})$

$\rightarrow$ free energy of configurations with boundary $\mathcal{M}_{g,p}$

The latter are 4d (Euclidean) solutions with NUT charge

Outline

Outline:

- NUTs and Bolts, gravity on-shell action
- ABJM partition function on $\mathcal{M}_{g,p}$
- Conclusions and outlook

Minimal gauged $\mathcal{N} = 2$ supergravity

Euclidean minimal 4d gauged supergravity contains only the gravity multiplet (no vector multiplets or hypermultiplets). Bosonic action is Einstein-Maxwell-Λ:

$$S = \int \mathrm{d}^4x \sqrt{g}\left[R - F_{\mu\nu}F^{\mu\nu} + \frac{6}{l^2}\right]$$

For a BPS solution the gravitino susy variation is zero

$$\delta_\epsilon \psi = \left(\partial_\mu + \frac{1}{4}\omega_{ab}\gamma^{ab} + \frac{1}{2l}\gamma_\mu + \frac{i}{l}A_\mu + \frac{i}{4}F_{\nu\rho}\gamma^{\nu\rho}\gamma_\mu\right)\epsilon = 0$$

where $\gamma_\mu \in \mathit{Cliff}(4,0)$ and $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$. Any solution M_4 of this theory uplifts locally to a solution of $M_4 \times Y^7$ of 11d sugra

General solution with NUT

$\mathcal{M}_{g,p}$ boundary: related to the presence of NUT charge s.

General solution to the equations of motion [Chamblin, Emparan, Johnson,Myers '98] depends on M, Q, P, κ, s

$$ds^2 = \frac{\lambda(r)}{r^2 - s^2}(d\tau + 2s\, f(\theta)d\phi)^2 + \frac{dr^2(r^2 - s^2)}{\lambda(r)} + (r^2 - s^2)d\Omega_\kappa^2$$

$$\lambda(r) = (r^2 - s^2)^2 + (\kappa - 4s^2)(r^2 + s^2) - 2Mr + P^2 - Q^2$$

and

$$f(\theta) = \begin{cases} \cos\theta \\ -\theta \\ -\cosh\theta \end{cases} \qquad d\Omega_\kappa^2 = \begin{cases} d\theta^2 + \sin^2\theta d\phi^2 & \text{for} \quad \kappa = 1 \\ d\theta^2 + d\phi^2 & \text{for} \quad \kappa = 0 \\ d\theta^2 + \sinh^2\theta d\phi^2 & \text{for} \quad \kappa = -1 \end{cases}$$

Compact Σ_g by appropriately taking the quotient of $\mathbf{R}^2$ and $\mathbf{H}^2$

General solution

The gauge field is

$$A_\tau = \frac{-2sQr + P(r^2+s^2)}{r^2-s^2}$$

$$A_\phi = \begin{cases} \cos\theta \frac{P(r^2+s^2)-2s\,Q\,r}{r^2-s^2} & \text{for} \quad \kappa = 1 \\ -\theta \frac{P(r^2+s^2)-2s\,Q\,r}{r^2-s^2} & \text{for} \quad \kappa = 0 \\ -\cosh\theta \frac{P(r^2+s^2)-2s\,Q\,r}{r^2-s^2} & \text{for} \quad \kappa = -1 \end{cases}$$

Boundary ($r \to \infty$) is a circle bundle over a constant curvature Σ_g

$$ds^2 = \frac{dr^2}{r^2} + r^2\left(4s^2(d\psi + f(\theta)d\phi)^2 + d\Omega_\kappa^2\right)$$

In particular for the choice $p = 1$, $g = 0$ and imposing periodicity $\Delta\psi = 4\pi$ the boundary is a biaxially squashed S^3.

NUTs and Bolts (spherical)

Impose regularity (non-singular solutions). Two classes depending on the dimension of the fixed point for the Killing vector ∂_ψ.

- "NUT" (AdS-Taub-NUT): isolated fixed point
- "BOLT" (AdS-Taub-Bolt): 2d fixed point

(spherical) NUTs and Bolts

"NUT" ($\mathbf{R}^4$ topology)

- $\lambda(r)$ has double root at $r = s$, identified with the origin of $\mathbf{R}^4$
- Regularity condition gives $\Delta\psi = 4\pi$ ("mildly singular" $\Delta\psi = 4\pi/p$)
- the boundary is a squashed S^3

"Bolt" (topology $\mathcal{O}(-p) \to S^2$)

- $\lambda(r)$ has simple root at $r_b > s$
- regularity condition at the Bolt $r = r_b$ requires: $\frac{r_b^2 - s^2}{s\lambda'(r_b)} = \frac{2}{p}$
- this gives $\Delta\psi = 4\pi/p$ so that boundary is squashed Lens space $S^3/\mathbb{Z}_p$

more general NUTs and Bolts

Spherical Bolts can be generalized into configurations where the base is a higher genus Riemann surface Σ_g:

- toroidal and higher genus Bolt solutions, with topology $\mathcal{O}(-p) \to \Sigma_g$ and $\Delta\psi = \frac{4\pi(g-1)}{p}$ for $g > 1$, $\Delta\psi = \frac{4\pi}{p}$ (g=1).

Imposing regularity (+BPS) restricts the moduli space of solutions: constraints between the parameters $Q(s), M(s), P(s)$.

Solutions might exist for a certain squashing interval $s \in [0, s_0]$.

BPS solutions: spherical NUTs and Bolts

BPS solutions: [Martelli, Passias, Sparks '12]

1/2 BPS solution

$$P = -s\sqrt{4s^2 - 1} \qquad M = Q\sqrt{4s^2 - 1}$$

1/4 BPS solution

$$P = \frac{1}{2}(4s^2 - 1) \qquad M = 2sQ$$

Focus on the 1/4 BPS solutions. End goal is to compute the on-shell action and compare with the ABJM free energy on $\mathcal{M}_{g,p}$ computed via localization.

BPS NUTs and Bolts

Generalize these BPS solutions to Σ_g from the results of [Alonso-Alberca, Meessen, Ortin, '99], [Nozawa,Klemm '13]

1/4 BPS solution

$$P = \frac{1}{2}(4s^2 - \kappa) \qquad M = 2sQ$$

We solved the Killing spinor equation $\delta_\epsilon \psi = 0$

$$\epsilon_\pm = \begin{pmatrix} f_1(r) \\ f_2(r) \end{pmatrix} \otimes \chi \qquad f_1(r) = \sqrt{\frac{(r - r_3)(r - r_4)}{r + s}} \qquad f_2(r) = i\sqrt{\frac{(r - r_1)(r - r_2)}{r - s}}$$

with $\chi = \begin{pmatrix} 0 \\ \chi_{(0)} \end{pmatrix}$, $\chi_{(0)}$ constant.

In particular, Killing spinor has only radial dependence.

Regularity and parameter range

- NUT solutions are obtained by a double root at $r = s$, giving $Q = P = \frac{1}{2}(4s^2 - 1)$.
- For the Bolt we need to impose the regularity condition

$$\left.\frac{r^2 - s^2}{2s\lambda'(r)}\right|_{r_b} = \frac{2}{\mathbf{p}}$$

which constrains the value of Q:

$$Q_+^\pm = \frac{p^2 \mp (16s^2 - p)\sqrt{f_+}}{128s^2} \qquad Q_-^\pm = -\frac{p^2 \mp (16s^2 + p)\sqrt{f_-}}{128s^2}$$

$$f_\pm = (16s^2 \pm p)^2 - 128\kappa s^2$$

obtaining up to four different branches, denoted with $\text{Bolt}_\pm$.

Regularity

In general a solution exists for a range of parameter s. Different branches of solutions joining each others at special points.

- NUT solution ($g = 0$) exist for every value of squashing s. $s = 1/2$ boundary is a round sphere
- Bolt solutions exist for a finite range of s for $p = 1, 2$. For $p \geq 3$ Bolts are present for every $s > 0$.

Flux

Flux through the Bolt computed as

$$\mathcal{F}_{Bolt\pm} = \int_{\Sigma_g} \frac{F}{2\pi} = (g-1) \pm \frac{p}{2}$$

For the uplift to 11d on SE^7 [Gauntlett, Varela '07] to be well defined

$$ds^2_{11} = R^2 \left(\frac{1}{4} ds^2_4 + (d\chi + \sigma + \frac{1}{2} A)^2 + ds^2_6 \right) \qquad G = R^3 (\frac{3}{8} vol_4 - \star dA \wedge d\eta)$$

The flux through the Bolt 2-cycle needs to be quantized

$$\chi \sim \chi + 2\pi \frac{I}{4} \qquad \frac{4}{2\pi I} \int_{\Sigma_g} \frac{F}{2} \in \mathbb{Z} \qquad \text{I} = \text{"Fano number" of } Y^7$$

Flux

Inserting the flux

$$\pm p + 2(g-1) = 0 \quad \text{mod} \;\; I(Y^7)$$

In particular $I(S^7) = 4$ so we have condition

$$\pm p + 2(g-1) = 0 \quad \text{mod} \;\; 4$$

for uplift on S^7 (ABJM $k = 1$)

On-shell action

On shell action

Renormalized on-shell action

Evaluating bulk supergravity action

$$I = -\frac{1}{16\pi G_4}\int d^4x\sqrt{g}(R+6-F^2)$$

on a NUT/Bolt solution leads to divergencies: regularize with the introduction of cutoff r_{inf} and add the boundary term from holographic renormalization [Skenderis, '02]

$$I_{ct} = \frac{1}{8\pi G_4}\int_{\partial M} d^3x\sqrt{\gamma}\,(2+\frac{1}{2}R(\gamma)-K)$$

Manifold closes off at $r_0 = s$ for NUT and $r_0 = r_b$ for Bolt.

Renormalized on-shell action

Evaluating bulk terms we obtain

$$I_{grav}^{bulk} = \frac{1}{8\pi G_4}\frac{16\pi^2}{p}[2sr_{inf}^3 - 6s^3 r_{inf} - 2sr_0 + 6s^3 r_0]$$

and

$$I_{F,NUT} = \frac{2\pi}{G_4}\frac{(\kappa - 4s^2)^2}{4}$$

$$I_{F,Bolt}^{bulk} = \frac{\pi s r_b \left(r_b^2\left(\left(\kappa - 4s^2\right)^2 + 4Q^2\right) + 8\left(4s^2 - \kappa\right) sQr_b\right)}{2G_4\left(s^2 - r_b^2\right)^2 p} + \\ + \frac{\pi s r_b \left(s^2\left(\left(\kappa - 4s^2\right)^2 + 4Q^2\right)\right)}{2G_4\left(s^2 - r_b^2\right)^2 p}$$

Renormalized on-shell action

Boundary counterterms give

$$I_{ct} = \frac{1}{8\pi G_4}\frac{16\pi^2}{p}[4Qs^2 - 2sr_{inf}^3 + 6s^3 r_{inf} + O(r_{inf}^{-1})]$$

Putting all together, the renormalized on-shell action has a simple form, independent of the squashing parameter s [CT,Willett '17]

$$I_{ren,NUT} = \frac{\pi}{2G_4} ,$$

$$I_{ren,Bolt_\pm} = I_{bulk} + I_{ct} = \frac{\pi\left(4(1-g) \mp p\right)}{8G_4}$$

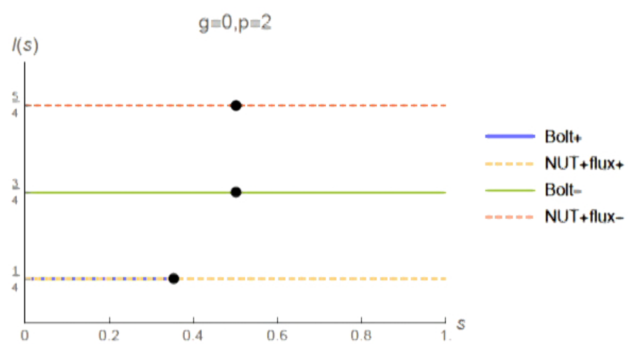
g=0 case reduces to [Martelli,Passias,Sparks]

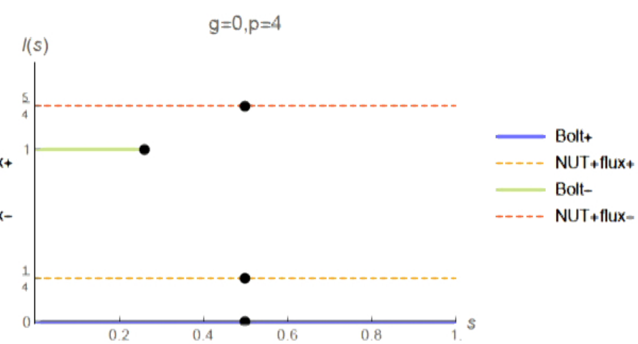
Renormalized on shell action

Also consider AdS-TN/$\mathbb{Z}_p$ with $\pm\frac{p}{2}-1$ units of magnetic flux, with same boundary data. On shell action is higher.

$$I_{NUT+flux} = \frac{\pi}{2G_4 p}\left(1+(\pm\frac{p}{2}-1)^2\right)$$

Compare vales of I_{NUT} and $I_{Bolt\pm}$ and range of existence:





Renormalized on shell action

In order to have regular Bolt solutions we need

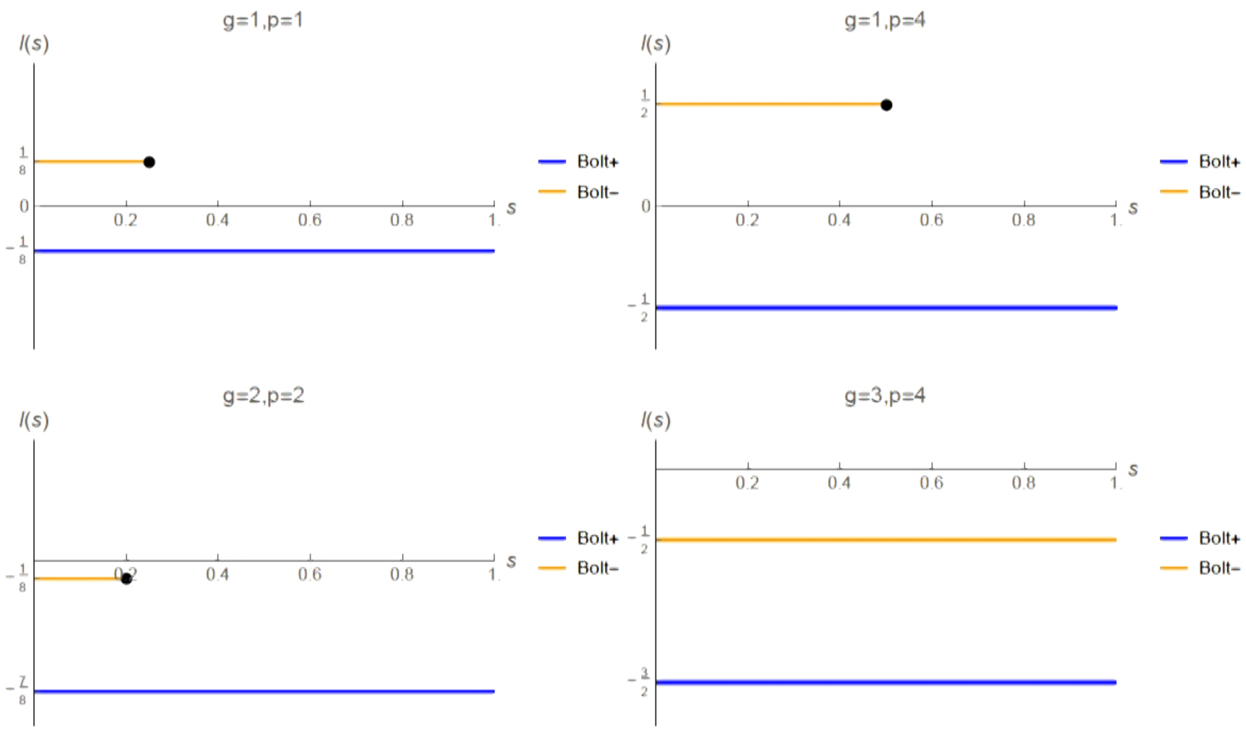
- $r_b > s$
- $f_\pm > 0$
- $r_+(Q_+) > r_-(Q_-)$ for Bolt+ and $r_-(Q_-) > r_+(Q_-)$ for Bolt-

A given solution branch, if it's not defined everywhere, either joins other another branch (i.e. NUT$/\mathbb{Z}_p$ solution), or ends "annihilating" another branch

For $p = 2$: NUT$/\mathbb{Z}_2$ and Bolt+ free energy coincide, solutions are continuously connected

Renormalized on shell action

More on moduli space..



Recap

Free energy of SUSY solution with uplift to 11d on S^7 with ABJM dual. Using $1/G_4 = 2\sqrt{2}N^{3/2}/3$ we obtain

NUT with squashed S^3 boundary

$$I_{NUT} = \frac{\sqrt{2}\pi N^{3/2}}{3} \qquad p = 1, g = 0$$

Bolt solutions with boundary $\mathcal{M}_{g,p}$

$$I_{Bolt_\pm} = \frac{\pi N^{3/2}}{6\sqrt{2}} (4(1-g) \mp p)$$

which exist for $\pm p + 2(g-1) = 0 \mod 4$.

Comparison with field theory

Free energy result for S^3

$\mathcal{N} = 2$ CS theories on the squashed S^3 studied in [Hama, Hosomichi, Lee '11]: one family found to be independent of the squashing s.
Computation of the ABJM partition function on S^3 gives [Jafferis, Klebanov, Pufu, Safdi, '11]

$$\log Z_{S^3} = -\frac{4\pi N^{3/2}}{3}\sqrt{2m_1 m_2 m_3 m_4}$$

At the conformal point $m_i = 1/2$, we have

$$F_{S^3} = -Log(Z_{S^3}) = \frac{\sqrt{2}\pi N^{3/2}}{3} = I_{NUT}$$

Indeed the NUT free energy does not depend on the squashing, already noticed in [Martelli, Passias, Sparks '12].

ABJM free energy on $\mathcal{M}_{g,p}$

We want to reproduce the Bolt free energy.

Recently the partition function of superconformal $\mathcal{N}=2$ theories on $\mathcal{M}_{g,p}$ computed in [Closset, Kim, Willett '17]. It has the form of sum over vacua

$$Z_{\mathcal{M}_{g,p}} = \sum_{u_a \in S_{BE}} \mathcal{F}_I(u_a, m_i)^p \, \mathcal{H}_I^{g-1}(u_a, m_i) \, \Pi_I^i(u_a, m_i)^{s_i}$$

where

- $\mathcal{F}_I(u_a, m_i) = exp\left(2\pi i \left(\mathcal{W}^I - \sum_i m_i \frac{\partial \mathcal{W}^I}{\partial m_i}\right)\right)$ "fibering operator"
- $\mathcal{H}_I = exp\left(2i\pi\Omega^I det_{ab} \frac{\partial^2 \mathcal{W}}{\partial u_a \partial u_b}\right)$ "handle gluing operator"
- $\Pi_I^i = exp\left(2\pi i \frac{\partial \mathcal{W}}{\partial m_i}\right)$ "flux operator"

Large N ABJM free energy on $\mathcal{M}_{g,p}$

Large N limit of the partition function is

$$\log Z_{\mathcal{M}_{g,p}}(m_i, n_i) = pW_{ext} - \sum_i (pm_i - s_i - (g-1)r_i)\partial_i W_{ext}$$

and inserting the extremal superpotential W_{ext}

$$\log Z_{\mathcal{M}_{g,p}} = \frac{2\pi N^{3/2}}{3}\sqrt{2[m_1][m_2][m_3][m_4]}\left(2p - \sum_i \frac{-pn_i + s_i + (g-1)r_i}{[m_i]}\right)$$

with the following constraints

$$\sum_{j=0}^{3} m_j = \sum_j s_j = 0\,, \qquad \sum_j [m_j] = 1\,, \qquad \sum_j (r_j - 1) + 2 = 0$$

Matching for minimal solutions

Minimal sugra: condition of constant scalars and equal fluxes gives

$$[m_i] \quad \text{and} \quad -pn_i + s_i + (g-1)r_i \quad \text{independent of} \quad i$$

hence $p + 2(g-1) = 0 \ \ mod\ 4$ and $[m_i] = [m] = 1/4$ which gives

$$-\log Z^{ABJM}_{\mathcal{M}_{g,p}} = \frac{\pi N^{3/2}}{6\sqrt{2}}\left(4(1-g) - p\right) = I_{Bolt+}$$

which matches the supergravity result!

For $p = 0$ reduces to [Azzurli, Bobev, Crichigno, Min, Zaffaroni '17] which gives the entropy of minimal sugra black holes with Σ_g horizons.

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Recap: results for ABJM

- For $k = 1$ ABJM theory Bolt free energy in sugra matches [CT, Willett '17] the large N localization result for $\mathcal{M}_{gp}$ with

$$\pm p + 2(g-1) = 0 \mod 4$$

- S^3 boundary treated separately, computed [Jafferis, Klebanov, Pufu, Safdi,'11] (also $S^3/\mathbb{Z}_p$ (no flux) in [Alday,Fluder,Sparks]):

$$I_{NUT} = I_{EAdS4} = -Z_{ABJM}(S^3)$$

Everything good so far... How about the $p = 1$ Bolt?

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Everything good so far... How about the $p = 1$ Bolt?

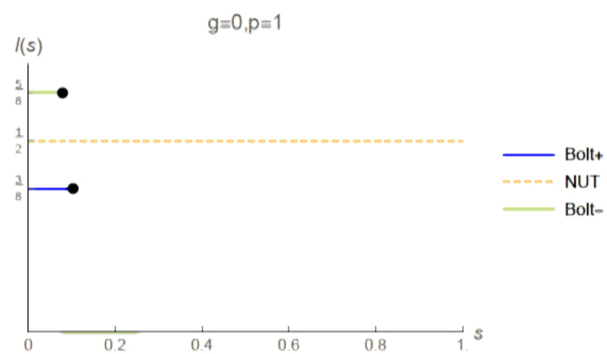
Curious case of $g = 0, p = 1$ Bolt

There are cases in which the S^3 boundary $(g = 0, p = 1)$ admits $p = 1$ Bolt fillings for certain parameter ranges $s \in [s_-, s_+]$:

Bolt free energy is

$$I_{Bolt+} = \pi \frac{4-p}{8G_4} = \frac{3\pi}{8G_4}$$

$$I_{Bolt-} = \pi \frac{4+p}{8G_4} = \frac{5\pi}{8G_4}$$



Notice that

- $I_{Bolt_+} < \frac{\pi}{2G_4} = I_{NUT} < I_{Bolt-}$

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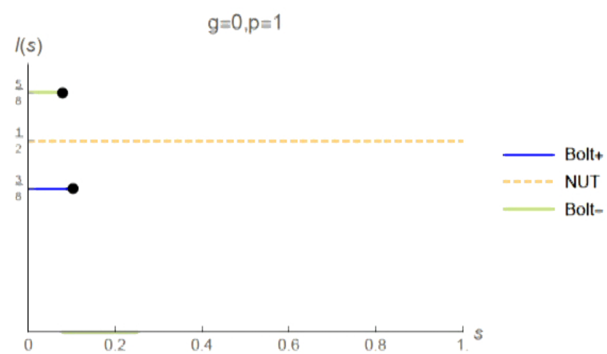

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$V^{5,2}$ and $g = 0, p = 1$ Bolt

- M-theory reduction on $M^{3,2}$: both Bolt$\pm$ with $p = 1$ uplift. Dual computation not under control.
- M-theory on $Y^7 = V^{5,2}$

$$\pm p + 2(g-1) = 0 \mod 3$$

hence $p = 1$ Bolt- uplifts! Its free energy is $I_{Bolt-} = \frac{5}{4} I_{NUT}$.

Free energy for theory dual to $V^{5,2}$ on S^3 computed by [Martelli, Sparks '08] coincides with I_{NUT}.

However... Our result for the partition function on $\mathcal{M}_{g,p}$ reproduces this *subleading* saddle point: $I_{Bolt-} > I_{NUT}$.

Curious case of $g = 0, p = 1$ Bolt

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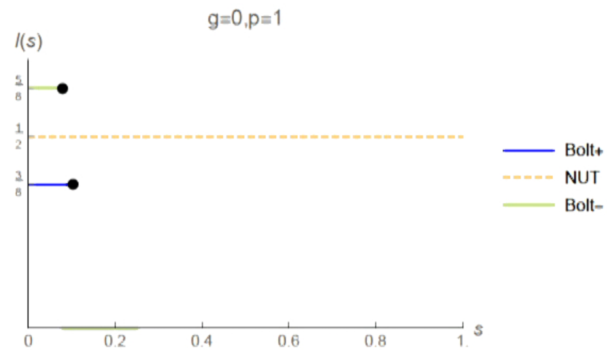

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- M-theory on $Y^7 = V^{5,2}$

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hence $p = 1$ Bolt- uplifts! Its free energy is $I_{Bolt-} = \frac{5}{4} I_{NUT}$.

Free energy for theory dual to $V^{5,2}$ on S^3 computed by [Martelli, Sparks '08] coincides with I_{NUT}.

However... Our result for the partition function on $\mathcal{M}_{g,p}$ reproduces this *subleading* saddle point: $I_{Bolt-} > I_{NUT}$.

$V^{5,2}$ and $g = 0, p = 1$ Bolt

- M-theory reduction on $M^{3,2}$: both Bolt$\pm$ with $p = 1$ uplift. Dual computation not under control.
- M-theory on $Y^7 = V^{5,2}$

$$\pm p + 2(g-1) = 0 \quad \text{mod } 3$$

hence $p = 1$ Bolt- uplifts! Its free energy is $I_{Bolt-} = \frac{5}{4} I_{NUT}$.

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Summary and Outlook

Free energy of Bolts with $\mathcal{M}_{g,p}$ boundary, when uplift is possible, is reproduced by the 3d partition function via susy localization.

Goals:

- Understand subtleties when different $p = 1$ Bolt fillings are allowed
  - stability of the Bolt configurations?
- NUT/Bolt solutions with matter multiplets [Colleoni, Klemm '11; Erbin, Halmagyi '15]
  - holographic renormalization: susy boundary conds in progress
- Reproduce on-shell action of 1/2 BPS NUTs and Bolts from field theory

Thanks for attention!