Title: NUTs and Bolts: free energy via susy localization

Speakers: Chiara Toldo

Series: Quantum Fields and Strings

Date: May 10, 2019 - 2:30 PM

URL: http://pirsa.org/19050013

Abstract: The partition function of three-dimensional N=2 SCFTs on circle bundles of closed Riemann surfaces \Sigma_g was recently computed via supersymmetric localization. In this talk I will describe supergravity solutions having as conformal boundary such circle bundle. These configurations are solutions to N=2 minimal gauged supergravity in 4d and pertain to the class of AdS-Taub-NUT and AdS-Taub-Bolt preserving 1/4 of the supersymmetries. I will discuss the conditions for the uplift of these solutions to M-theory and I provide the expression for the on-shell action of the Bolt solutions, computed via holographic renormalization. I will show that, when the uplift condition is satisfied, the Bolt free energy matches with the large N limit of the partition function of the corresponding dual field theory. I will finally comment on possible subtleties that arise in our framework when a given boundary geometry admits multiple bulk fillings.

Filling the boundary: NUTs, Bolts and susy localization

Chiara Toldo
CPHT, Ecole Polytechnique

Perimeter Institute, May 10, 2019

based on 1712.08861 with B. Willett and work in progress w. B. Willett and A. Passias

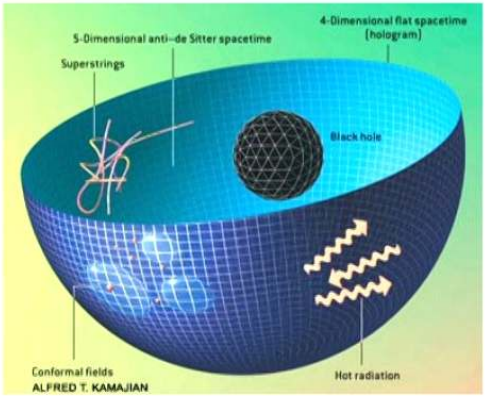
Intro and motivation

AdS/CFT correspondence:
[Maldacena '97]

Theory with gravity in AdS_{d+1}

dual to

conformal field theory in d dimensions



- Model strongly coupled field theory processes by gravity dual
- Microscopic understanding of the entropy of AdS_{d+1} black holes via the dual CFT_d

Intro and motivation

Black holes seen as thermodynamic ensembles: they emit radiation and possess entropy $S_{BH} = \kappa_B \frac{Ac^3}{4G\hbar}$

Microstate counting and entropy matching by [Strominger,Vafa'96] in some specific cases

Use information of the dual field theory to count black hole microstates

Counting usually requires supersymmetry: in AdS extremal rotating black holes can be supersymmetric

- Astrophysical ones are extremal-Kerr, rotating.

Need to construct AdS solutions with embedding in string/M-theory: exact holographic dual. Work in the low-energy limit of string theory $\rightarrow$ supergravity

i.e. AdS_4 with field theory duals in the class of ABJM [Aharony, Bergman, Jafferis, Maldacena, '08]

Intro and motivation

Recent success: microstate counting for susy AdS_4 black holes

Static 1/4 BPS black hole exist in 4d $\mathcal{N} = 2$ *gauged supergravity* [Cacciatori, Klemm '09]: scalar potential allows for susy AdS_4 vacua, field theory dual is ABJM

- extremal black holes are flows from AdS_4 to $AdS_2 \times \Sigma_g$ near horizon geometry
- magnetic gauge field cancels spin connection in the susy equations (topological twist)

Intro and motivation

ABJM partition function on $S^1 \times S^2$ with magnetic fluxes s_i on S^2 computed via susy localization, in the large N limit, [Benini, Hristov, Zaffaroni '15], [Benini, Zaffaroni '16]

$$\log Z_{S^1 \times S^2} \approx -\frac{2\pi N^{3/2}}{3}\sqrt{2m_1 m_2 m_3 m_4}\sum_{i=1}^{4}\frac{s_i}{m_i}$$

reproduces BPS black hole entropy upon extremization on m_i:

$$S_{BH} = \frac{A_{BH}}{4G_N} = \log \mathcal{Z}|_{crit}(s^i)$$

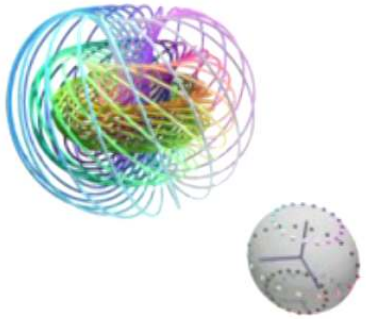
Extremization corresponds to the attractor mechanism in sugra.

Intro and motivation

Extend these considerations to more general case of $\mathcal{M}_{g,p} = S^1$ p-bundle over 2d Riemann surfaces Σ_g [Closset, Kim, Willett '17]

$$ds^2 = (d\psi + a)^2 + d\Omega_\kappa^2 \qquad -\frac{1}{4\pi}\int_{\Sigma_g} da = p$$

- $\mathcal{M}_{g,0} \simeq S^1 \times \Sigma_g$ reduces to [Benini, Zaffaroni '16]: black hole physics
- $\mathcal{M}_{0,1} \simeq S^3$ partition function (F-theorem, entanglement entropy...)
- $\mathcal{M}_{0,p} \simeq S^3/\mathbb{Z}_p$ Lens space



Intro and motivation

Holographic check with bulk dual solutions.

ABJM partition function $Z_{ABJM}(S^1 \times \Sigma_g)$

$\rightarrow$ black holes, whose boundary is $S^1 \times \Sigma_g$

ABJM partition function $Z_{ABJM}(\mathcal{M}_{g,p})$

$\rightarrow$ free energy of configurations with boundary $\mathcal{M}_{g,p}$

The latter are 4d (Euclidean) solutions with NUT charge

Outline

Outline:

- NUTs and Bolts, gravity on-shell action
- ABJM partition function on $\mathcal{M}_{g,p}$
- Conclusions and outlook

Minimal gauged $\mathcal{N} = 2$ supergravity

Euclidean minimal 4d gauged supergravity contains only the gravity multiplet (no vector multiplets or hypermultiplets). Bosonic action is Einstein-Maxwell-Λ:

$$S = \int \mathrm{d}^4x \sqrt{g} \left[R - F_{\mu\nu} F^{\mu\nu} + \frac{6}{l^2} \right]$$

For a BPS solution the gravitino susy variation is zero

$$\delta_\epsilon \psi = \left(\partial_\mu + \frac{1}{4} \omega_{ab} \gamma^{ab} + \frac{1}{2l} \gamma_\mu + \frac{i}{l} A_\mu + \frac{i}{4} F_{\nu\rho} \gamma^{\nu\rho} \gamma_\mu \right) \epsilon = 0$$

where $\gamma_\mu \in \mathit{Cliff}(4,0)$ and $\{\gamma_\mu, \gamma_\nu\} = 2 g_{\mu\nu}$. Any solution M_4 of this theory uplifts locally to a solution of $M_4 \times Y^7$ of 11d sugra

General solution with NUT

$\mathcal{M}_{g,p}$ boundary: related to the presence of NUT charge s.

General solution to the equations of motion [Chamblin, Emparan, Johnson,Myers '98] depends on M, Q, P, κ, s

$$ds^2 = \frac{\lambda(r)}{r^2 - s^2}(d\tau + 2s\, f(\theta) d\phi)^2 + \frac{dr^2(r^2 - s^2)}{\lambda(r)} + (r^2 - s^2) d\Omega_\kappa^2$$

$$\lambda(r) = (r^2 - s^2)^2 + (\kappa - 4s^2)(r^2 + s^2) - 2Mr + P^2 - Q^2$$

and

$$f(\theta) = \begin{cases} \cos\theta \\ -\theta \\ -\cosh\theta \end{cases} \qquad d\Omega_\kappa^2 = \begin{cases} d\theta^2 + \sin^2\theta d\phi^2 & \text{for} \quad \kappa = 1 \\ d\theta^2 + d\phi^2 & \text{for} \quad \kappa = 0 \\ d\theta^2 + \sinh^2\theta d\phi^2 & \text{for} \quad \kappa = -1 \end{cases}$$

Compact Σ_g by appropriately taking the quotient of $\mathbf{R}^2$ and $\mathbf{H}^2$

General solution

The gauge field is

$$A_\tau = \frac{-2sQr + P(r^2+s^2)}{r^2-s^2}$$

$$A_\phi = \begin{cases} \cos\theta \frac{P(r^2+s^2)-2s\,Q\,r}{r^2-s^2} & \text{for} \quad \kappa = 1 \\ -\theta \frac{P(r^2+s^2)-2s\,Q\,r}{r^2-s^2} & \text{for} \quad \kappa = 0 \\ -\cosh\theta \frac{P(r^2+s^2)-2s\,Q\,r}{r^2-s^2} & \text{for} \quad \kappa = -1 \end{cases}$$

Boundary ($r \to \infty$) is a circle bundle over a constant curvature Σ_g

$$ds^2 = \frac{dr^2}{r^2} + r^2\left(4s^2(d\psi + f(\theta)d\phi)^2 + d\Omega_\kappa^2\right)$$

In particular for the choice $p = 1$, $g = 0$ and imposing periodicity $\Delta\psi = 4\pi$ the boundary is a biaxially squashed S^3.

NUTs and Bolts (spherical)

Impose regularity (non-singular solutions). Two classes depending on the dimension of the fixed point for the Killing vector ∂_ψ.

- "NUT" (AdS-Taub-NUT): isolated fixed point
- "BOLT" (AdS-Taub-Bolt): 2d fixed point

(spherical) NUTs and Bolts

"NUT" ($\mathbf{R}^4$ topology)

- $\lambda(r)$ has double root at $r = s$, identified with the origin of $\mathbf{R}^4$
- Regularity condition gives $\Delta\psi = 4\pi$ ("mildly singular" $\Delta\psi = 4\pi/p$)
- the boundary is a squashed S^3

"Bolt" (topology $\mathcal{O}(-p) \to S^2$)

- $\lambda(r)$ has simple root at $r_b > s$
- regularity condition at the Bolt $r = r_b$ requires: $\frac{r_b^2 - s^2}{s\lambda'(r_b)} = \frac{2}{p}$
- this gives $\Delta\psi = 4\pi/p$ so that boundary is squashed Lens space $S^3/\mathbb{Z}_p$

more general NUTs and Bolts

Spherical Bolts can be generalized into configurations where the base is a higher genus Riemann surface Σ_g:

- toroidal and higher genus Bolt solutions, with topology $\mathcal{O}(-p) \to \Sigma_g$ and $\Delta\psi = \frac{4\pi(g-1)}{p}$ for $g > 1$, $\Delta\psi = \frac{4\pi}{p}$ (g=1).

Imposing regularity (+BPS) restricts the moduli space of solutions: constraints between the parameters $Q(s), M(s), P(s)$.

Solutions might exist for a certain squashing interval $s \in [0, s_0]$.

BPS solutions: spherical NUTs and Bolts

BPS solutions: [Martelli, Passias, Sparks '12]

1/2 BPS solution

$$P = -s\sqrt{4s^2-1} \qquad M = Q\sqrt{4s^2-1}$$

1/4 BPS solution

$$P = \tfrac{1}{2}(4s^2-1) \qquad M = 2sQ$$

Focus on the 1/4 BPS solutions. End goal is to compute the on-shell action and compare with the ABJM free energy on $\mathcal{M}_{g,p}$ computed via localization.

BPS NUTs and Bolts

Generalize these BPS solutions to Σ_g from the results of
[Alonso-Alberca, Meessen, Ortin, '99], [Nozawa,Klemm '13]

1/4 BPS solution

$$P = \frac{1}{2}(4s^2 - \kappa) \qquad M = 2sQ$$

We solved the Killing spinor equation $\delta_\epsilon \psi = 0$

$$\epsilon_\pm = \begin{pmatrix} f_1(r) \\ f_2(r) \end{pmatrix} \otimes \chi \qquad f_1(r) = \sqrt{\frac{(r-r_3)(r-r_4)}{r+s}} \qquad f_2(r) = i\sqrt{\frac{(r-r_1)(r-r_2)}{r-s}}$$

with $\chi = \begin{pmatrix} 0 \\ \chi_{(0)} \end{pmatrix}$, $\chi_{(0)}$ constant.

In particular, Killing spinor has only radial dependence.

Regularity and parameter range

- NUT solutions are obtained by a double root at $r = s$, giving $Q = P = \frac{1}{2}(4s^2 - 1)$.
- For the Bolt we need to impose the regularity condition

$$\left.\frac{r^2 - s^2}{2s\lambda'(r)}\right|_{r_b} = \frac{2}{\mathbf{p}}$$

which constrains the value of Q:

$$Q_+^{\pm} = \frac{p^2 \mp (16s^2 - p)\sqrt{f_+}}{128s^2} \qquad Q_-^{\pm} = -\frac{p^2 \mp (16s^2 + p)\sqrt{f_-}}{128s^2}$$

$$f_{\pm} = (16s^2 \pm p)^2 - 128\kappa s^2$$

obtaining up to four different branches, denoted with $\text{Bolt}_{\pm}$.

Regularity

In general a solution exists for a range of parameter s. Different branches of solutions joining each others at special points.

- NUT solution ($g = 0$) exist for every value of squashing s. $s = 1/2$ boundary is a round sphere
- Bolt solutions exist for a finite range of s for $p = 1, 2$. For $p \geq 3$ Bolts are present for every $s > 0$.

Flux

Flux through the Bolt computed as

$$\mathcal{F}_{Bolt\pm} = \int_{\Sigma_g} \frac{F}{2\pi} = (g-1) \pm \frac{p}{2}$$

For the uplift to 11d on SE^7 [Gauntlett, Varela '07] to be well defined

$$ds_{11}^2 = R^2 \left(\frac{1}{4} ds_4^2 + (d\chi + \sigma + \frac{1}{2}A)^2 + ds_6^2 \right) \qquad G = R^3 (\frac{3}{8} vol_4 - \star dA \wedge d\eta)$$

The flux through the Bolt 2-cycle needs to be quantized

$$\chi \sim \chi + 2\pi \frac{I}{4} \qquad \frac{4}{2\pi I} \int_{\Sigma_g} \frac{F}{2} \in \mathbb{Z} \qquad I = \text{"Fano number" of } Y^7$$

Flux

Inserting the flux

$$\pm p + 2(g-1) = 0 \mod I(Y^7)$$

In particular $I(S^7) = 4$ so we have condition

$$\pm p + 2(g-1) = 0 \mod 4$$

for uplift on S^7 (ABJM $k = 1$)

On-shell action

On shell action

Renormalized on-shell action

Evaluating bulk supergravity action

$$I = -\frac{1}{16\pi G_4}\int d^4x\sqrt{g}(R+6-F^2)$$

on a NUT/Bolt solution leads to divergencies: regularize with the introduction of cutoff r_{inf} and add the boundary term from holographic renormalization [Skenderis, '02]

$$I_{ct} = \frac{1}{8\pi G_4}\int_{\partial M} d^3x\sqrt{\gamma}\,(2+\frac{1}{2}R(\gamma)-K)$$

Manifold closes off at $r_0 = s$ for NUT and $r_0 = r_b$ for Bolt.

Renormalized on-shell action

Evaluating bulk terms we obtain

$$I_{grav}^{bulk} = \frac{1}{8\pi G_4}\frac{16\pi^2}{p}\left[2sr_{inf}^3 - 6s^3 r_{inf} - 2sr_0 + 6s^3 r_0\right]$$

and

$$I_{F,NUT} = \frac{2\pi}{G_4}\frac{(\kappa - 4s^2)^2}{4}$$

$$I_{F,Bolt}^{bulk} = \frac{\pi s r_b\left(r_b^2\left(\left(\kappa-4s^2\right)^2+4Q^2\right)+8\left(4s^2-\kappa\right)sQr_b\right)}{2G_4\left(s^2-r_b^2\right)^2 p} + \\ + \frac{\pi s r_b\left(s^2\left(\left(\kappa-4s^2\right)^2+4Q^2\right)\right)}{2G_4\left(s^2-r_b^2\right)^2 p}$$

Renormalized on-shell action

Boundary counterterms give

$$I_{ct} = \frac{1}{8\pi G_4}\frac{16\pi^2}{p}[4Qs^2 - 2sr_{inf}^3 + 6s^3 r_{inf} + O(r_{inf}^{-1})]$$

Putting all together, the renormalized on-shell action has a simple form, independent of the squashing parameter s [CT,Willett '17]

$$I_{ren,NUT} = \frac{\pi}{2G_4}\,,$$

$$I_{ren,Bolt_\pm} = I_{bulk} + I_{ct} = \frac{\pi\,(4(1-g) \mp p)}{8G_4}$$

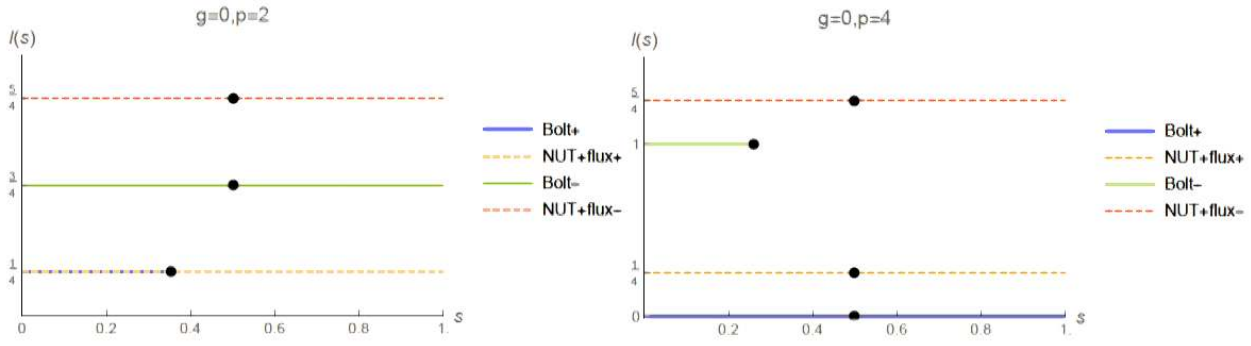
g=0 case reduces to [Martelli,Passias,Sparks]

Renormalized on shell action

Also consider AdS-TN/$\mathbb{Z}_p$ with $\pm\frac{p}{2}-1$ units of magnetic flux, with same boundary data. On shell action is higher.

$$I_{NUT+flux} = \frac{\pi}{2G_4 p}\left(1+(\pm\frac{p}{2}-1)^2\right)$$

Compare vales of I_{NUT} and $I_{Bolt\pm}$ and range of existence:



Renormalized on shell action

In order to have regular Bolt solutions we need

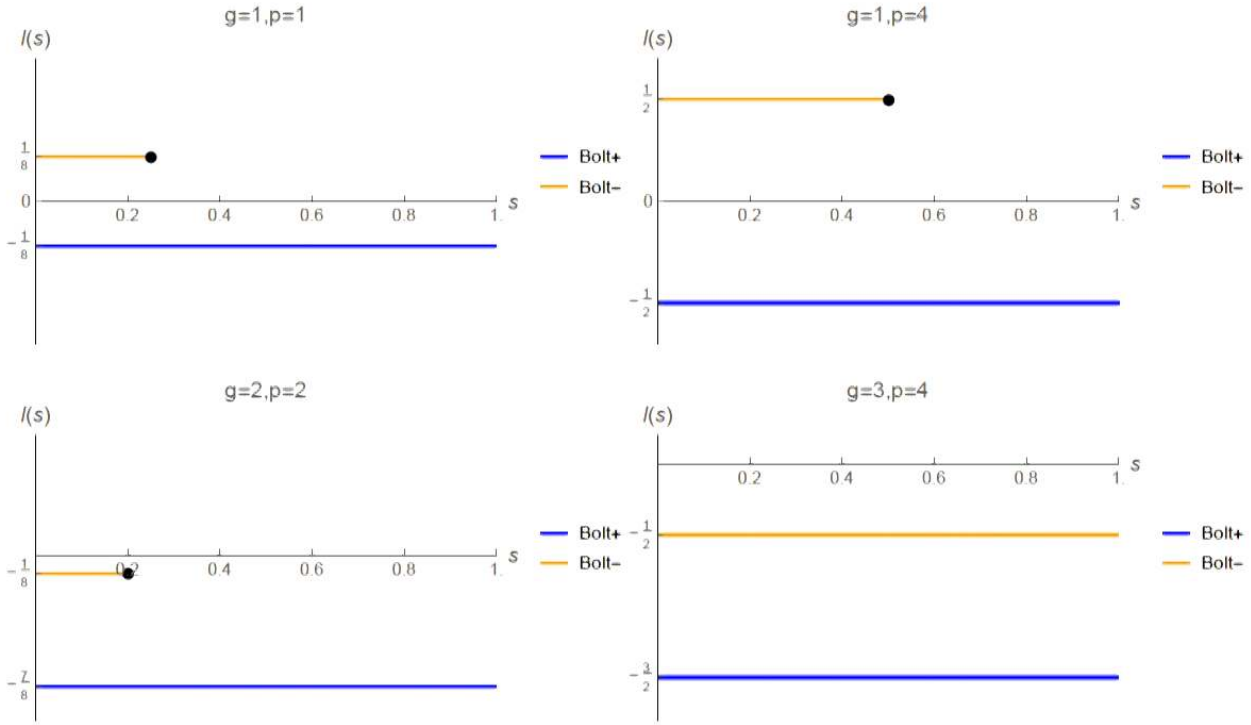
- $r_b > s$
- $f_\pm > 0$
- $r_+(Q_+) > r_-(Q_-)$ for Bolt+ and $r_-(Q_-) > r_+(Q_-)$ for Bolt-

A given solution branch, if it's not defined everywhere, either joins other another branch (i.e. NUT/$\mathbb{Z}_p$ solution), or ends "annihilating" another branch

For $p = 2$: NUT/$\mathbb{Z}_2$ and Bolt+ free energy coincide, solutions are continuously connected

Renormalized on shell action

More on moduli space..



Recap

Free energy of SUSY solution with uplift to 11d on S^7 with ABJM dual. Using $1/G_4 = 2\sqrt{2}N^{3/2}/3$ we obtain

NUT with squashed S^3 boundary

$$I_{NUT} = \frac{\sqrt{2}\pi N^{3/2}}{3} \qquad p = 1, g = 0$$

Bolt solutions with boundary $\mathcal{M}_{g,p}$

$$I_{Bolt_\pm} = \frac{\pi N^{3/2}}{6\sqrt{2}}(4(1-g) \mp p)$$

which exist for $\pm p + 2(g-1) = 0 \mod 4$.

Comparison with field theory

Free energy result for S^3

$\mathcal{N} = 2$ CS theories on the squashed S^3 studied in [Hama, Hosomichi, Lee '11]: one family found to be independent of the squashing s. Computation of the ABJM partition function on S^3 gives [Jafferis, Klebanov, Pufu, Safdi, '11]

$$\log Z_{S^3} = -\frac{4\pi N^{3/2}}{3}\sqrt{2m_1 m_2 m_3 m_4}$$

At the conformal point $m_i = 1/2$, we have

$$F_{S^3} = -Log(Z_{S^3}) = \frac{\sqrt{2}\pi N^{3/2}}{3} = I_{NUT}$$

Indeed the NUT free energy does not depend on the squashing, already noticed in [Martelli, Passias, Sparks '12].

ABJM free energy on $\mathcal{M}_{g,p}$

We want to reproduce the Bolt free energy.

Recently the partition function of superconformal $\mathcal{N}=2$ theories on $\mathcal{M}_{g,p}$ computed in [Closset, Kim, Willett '17]. It has the form of sum over vacua

$$Z_{\mathcal{M}_{g,p}} = \sum_{u_a \in S_{BE}} \mathcal{F}_I(u_a, m_i)^p \, \mathcal{H}_I^{g-1}(u_a, m_i) \, \Pi_I^i(u_a, m_i)^{s_i}$$

where

- $\mathcal{F}_I(u_a, m_i) = exp\left(2\pi i \left(\mathcal{W}^I - \sum_i m_i \frac{\partial \mathcal{W}^I}{\partial m_i}\right)\right)$ "fibering operator"
- $\mathcal{H}_I = exp\left(2i\pi\Omega^I det_{ab} \frac{\partial^2 \mathcal{W}}{\partial u_a \partial u_b}\right)$ "handle gluing operator"
- $\Pi_I^i = exp\left(2\pi i \frac{\partial \mathcal{W}}{\partial m_i}\right)$ "flux operator"

Large N ABJM free energy on $\mathcal{M}_{g,p}$

Large N limit of the partition function is

$$\log Z_{\mathcal{M}_{g,p}}(m_i, n_i) = pW_{ext} - \sum_i (pm_i - s_i - (g-1)r_i)\partial_i W_{ext}$$

and inserting the extremal superpotential W_{ext}

$$\log Z_{\mathcal{M}_{g,p}} = \frac{2\pi N^{3/2}}{3}\sqrt{2[m_1][m_2][m_3][m_4]}\left(2p - \sum_i \frac{-pn_i + s_i + (g-1)r_i}{[m_i]}\right)$$

with the following constraints

$$\sum_{j=0}^{3} m_j = \sum_j s_j = 0\,, \qquad \sum_j [m_j] = 1\,, \qquad \sum_j (r_j - 1) + 2 = 0$$

Matching for minimal solutions

Minimal sugra: condition of constant scalars and equal fluxes gives

$$[m_i] \text{ and } -pn_i + s_i + (g-1)r_i \text{ independent of } i$$

hence $p + 2(g-1) = 0 \ mod\ 4$ and $[m_i] = [m] = 1/4$ which gives

$$-\log Z^{ABJM}_{\mathcal{M}_{g,p}} = \frac{\pi N^{3/2}}{6\sqrt{2}}(4(1-g)-p) = I_{Bolt+}$$

which matches the supergravity result!

For $p = 0$ reduces to [Azzurli, Bobev, Crichigno, Min, Zaffaroni '17] which gives the entropy of minimal sugra black holes with Σ_g horizons.

Matching for minimal solutions

Minimal sugra: condition of constant scalars and equal fluxes gives

$$[m_i] \quad \text{and} \quad -pn_i + s_i + (g-1)r_i \quad \text{independent of} \quad i$$

hence $p + 2(g-1) = 0 \ \ mod\ 4$ and $[m_i] = [m] = 1/4$ which gives

$$-\log Z^{ABJM}_{\mathcal{M}_{g,p}} = \frac{\pi N^{3/2}}{6\sqrt{2}}\,(4(1-g)-p) = I_{Bolt+}$$

which matches the supergravity result!

For $p = 0$ reduces to [Azzurli, Bobev, Crichigno, Min, Zaffaroni '17] which gives the entropy of minimal sugra black holes with Σ_g horizons.

Recap: results for ABJM

- For $k = 1$ ABJM theory Bolt free energy in sugra matches [CT, Willett '17] the large N localization result for $\mathcal{M}_{gp}$ with

$$\pm p + 2(g-1) = 0 \mod 4$$

- S^3 boundary treated separately, computed [Jafferis, Klebanov, Pufu, Safdi,'11] (also $S^3/\mathbb{Z}_p$ (no flux) in [Alday,Fluder,Sparks]):

$$I_{NUT} = I_{EAdS4} = -Z_{ABJM}(S^3)$$

Everything good so far... How about the $p = 1$ Bolt?

Recap: results for ABJM

- For $k = 1$ ABJM theory Bolt free energy in sugra matches [CT, Willett '17] the large N localization result for $\mathcal{M}_{gp}$ with

$$\pm p + 2(g-1) = 0 \mod 4$$

- S^3 boundary treated separately, computed [Jafferis, Klebanov, Pufu, Safdi,'11] (also $S^3/\mathbb{Z}_p$ (no flux) in [Alday,Fluder,Sparks]):

$$I_{NUT} = I_{EAdS4} = -Z_{ABJM}(S^3)$$

Everything good so far... How about the $p = 1$ Bolt?

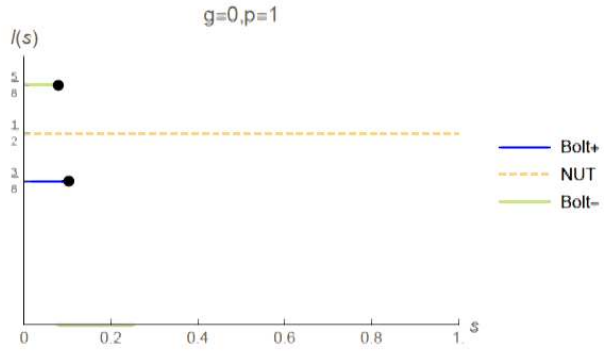
Curious case of $g = 0, p = 1$ Bolt

There are cases in which the S^3 boundary $(g = 0, p = 1)$ admits $p = 1$ Bolt fillings for certain parameter ranges $s \in [s_-, s_+]$:

Bolt free energy is

$$I_{Bolt+} = \pi \frac{4 - p}{8G_4} = \frac{3\pi}{8G_4}$$

$$I_{Bolt-} = \pi \frac{4 + p}{8G_4} = \frac{5\pi}{8G_4}$$



Notice that

- $I_{Bolt_+} < \frac{\pi}{2G_4} = I_{NUT} < I_{Bolt-}$

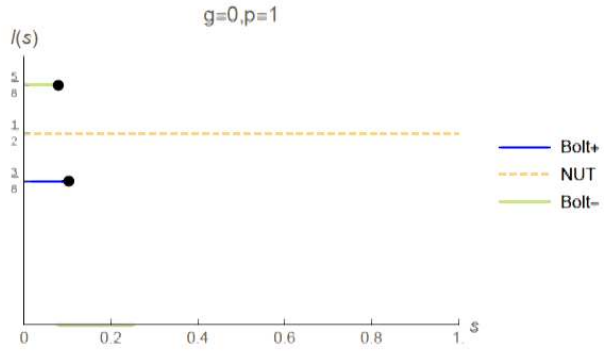
Curious case of $g = 0, p = 1$ Bolt

There are cases in which the S^3 boundary $(g = 0, p = 1)$ admits $p = 1$ Bolt fillings for certain parameter ranges $s \in [s_-, s_+]$:

Bolt free energy is

$$I_{Bolt+} = \pi \frac{4 - p}{8 G_4} = \frac{3\pi}{8 G_4}$$

$$I_{Bolt-} = \pi \frac{4 + p}{8 G_4} = \frac{5\pi}{8 G_4}$$



Notice that

- $I_{Bolt_+} < \frac{\pi}{2 G_4} = I_{EAdS_4} < I_{Bolt-}$

$V^{5,2}$ and $g = 0, p = 1$ Bolt

- M-theory reduction on $M^{3,2}$: both Bolt$\pm$ with $p = 1$ uplift. Dual computation not under control.
- M-theory on $Y^7 = V^{5,2}$

$$\pm p + 2(g-1) = 0 \mod 3$$

hence $p = 1$ Bolt- uplifts! Its free energy is $I_{Bolt-} = \frac{5}{4} I_{NUT}$.

Free energy for theory dual to $V^{5,2}$ on S^3 computed by [Martelli, Sparks '08] coincides with I_{NUT}.

However... Our result for the partition function on $\mathcal{M}_{g,p}$ reproduces this *subleading* saddle point: $I_{Bolt-} > I_{NUT}$.

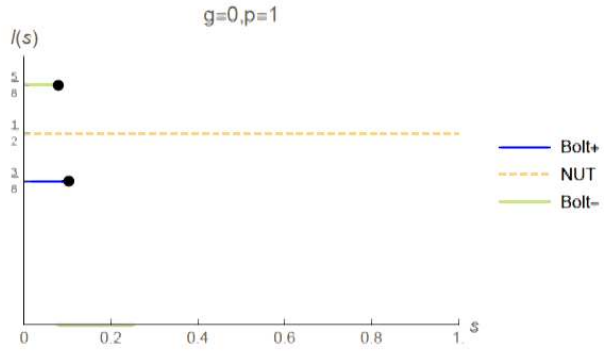
Curious case of $g = 0, p = 1$ Bolt

There are cases in which the S^3 boundary $(g = 0, p = 1)$ admits $p = 1$ Bolt fillings for certain parameter ranges $s \in [s_-, s_+]$:

Bolt free energy is

$$I_{Bolt+} = \pi \frac{4 - p}{8 G_4} = \frac{3\pi}{8 G_4}$$

$$I_{Bolt-} = \pi \frac{4 + p}{8 G_4} = \frac{5\pi}{8 G_4}$$



Notice that

- $I_{Bolt_+} < \frac{\pi}{2 G_4} = I_{EAdS_4} < I_{Bolt-}$

$V^{5,2}$ and $g = 0, p = 1$ Bolt

- M-theory reduction on $M^{3,2}$: both Bolt$\pm$ with $p = 1$ uplift. Dual computation not under control.
- M-theory on $Y^7 = V^{5,2}$

$$\pm p + 2(g-1) = 0 \mod 3$$

hence $p = 1$ Bolt- uplifts! Its free energy is $I_{Bolt-} = \frac{5}{4} I_{NUT}$.

Free energy for theory dual to $V^{5,2}$ on S^3 computed by [Martelli, Sparks '08] coincides with I_{NUT}.

However... Our result for the partition function on $\mathcal{M}_{g,p}$ reproduces this *subleading* saddle point: $I_{Bolt-} > I_{NUT}$.

$V^{5,2}$ and $g = 0, p = 1$ Bolt

- M-theory reduction on $M^{3,2}$: both Bolt$\pm$ with $p = 1$ uplift. Dual computation not under control.
- M-theory on $Y^7 = V^{5,2}$

$$\pm p + 2(g-1) = 0 \mod 3$$

hence $p = 1$ Bolt- uplifts! Its free energy is $I_{Bolt-} = \frac{5}{4} I_{NUT}$.

Free energy for theory dual to $V^{5,2}$ on S^3 computed by [Martelli, Sparks '08] coincides with I_{NUT}.

However... Our result for the partition function on $\mathcal{M}_{g,p}$ reproduces this *subleading* saddle point: $I_{Bolt-} > I_{NUT}$.

Summary and Outlook

Free energy of Bolts with $\mathcal{M}_{g,p}$ boundary, when uplift is possible, is reproduced by the 3d partition function via susy localization.

Goals:

- Understand subtleties when different $p = 1$ Bolt fillings are allowed
  - stability of the Bolt configurations?
- NUT/Bolt solutions with matter multiplets [Colleoni, Klemm '11; Erbin, Halmagyi '15]
  - holographic renormalization: susy boundary conds in progress
- Reproduce on-shell action of 1/2 BPS NUTs and Bolts from field theory

Thanks for attention!