Title: The Functional Renormalization Group Equation as an Approach to  the Continuum Limit of Tensor Models for Quantum Gravity

Speakers: Tim Koslowski

Series: Quantum Gravity

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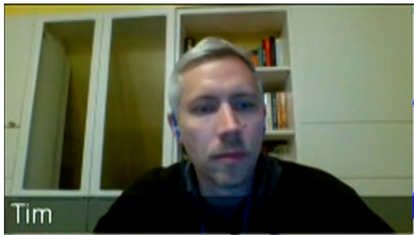
Abstract: Tensor Models provide one of the calculationally simplest approaches to defining a partition function for random discrete geometries. The continuum limit of these discrete models then provides a background-independent construction of a partition function of continuum geometry, which are candadates for quantum gravity. The blue-print for this approach is provided by the matrix model approach to two-dimensional quantum gravity. The past ten years have seen a lot of progress using (un)colored tensor models to describe state sums if discrete geometries in more than two dimensions. However, so far one has not yet been able to find a continuum limit of these models that corresponds geometries with more than two continuum dimensions. This problem can be studied systematically using exact renormalization group techniques. In this talk I will report on joint work with Astrid Eichhorn, Antonio Perreira, Joseph Ben Geloun, Daniele Oriti, Johannes Lumma, Alicia Castro and Victor Mu\~noz in this direction. In a separate part of the talk I will explain that the renormalization group is not only a tool to help investigating the continuum limit, but that it in fact also provides a stand-alone approach to quantum gravity. In particular, I will show how scaling relations follow from cylidrical consistency relations.




# …nctional Renormalization Group Equation as a …ch to the Continuum Limit of Tensor Models for Quantum Gravity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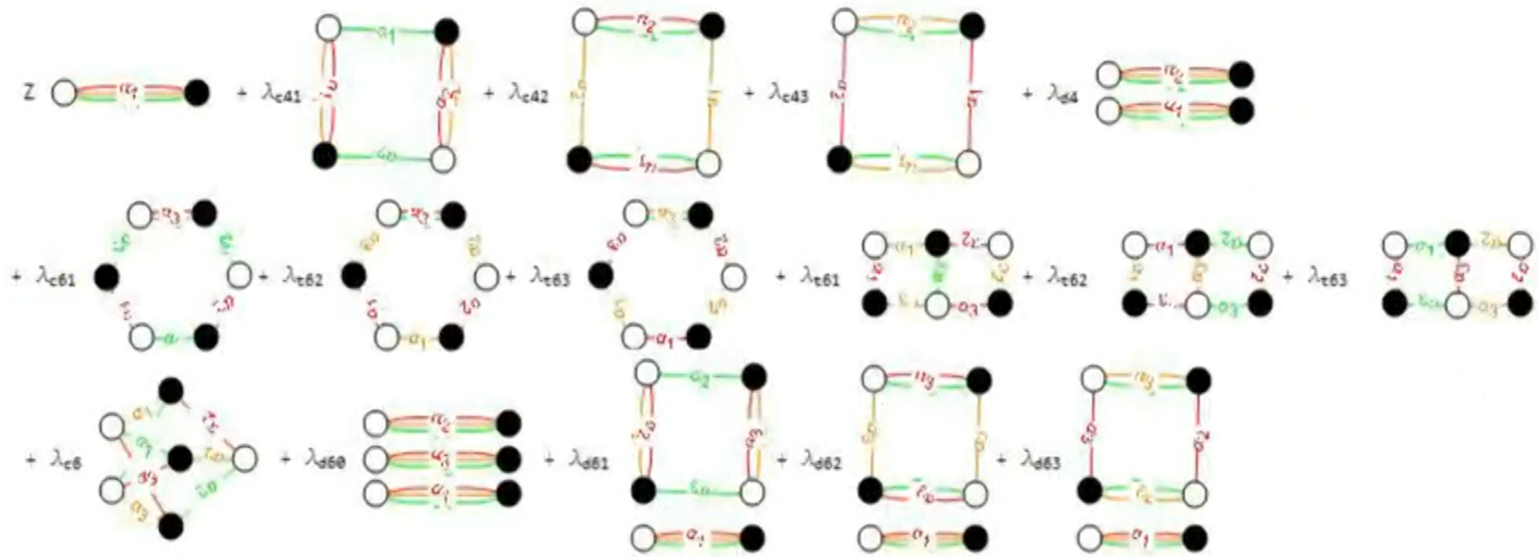


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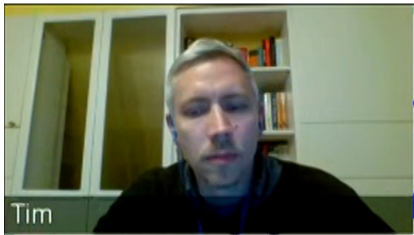


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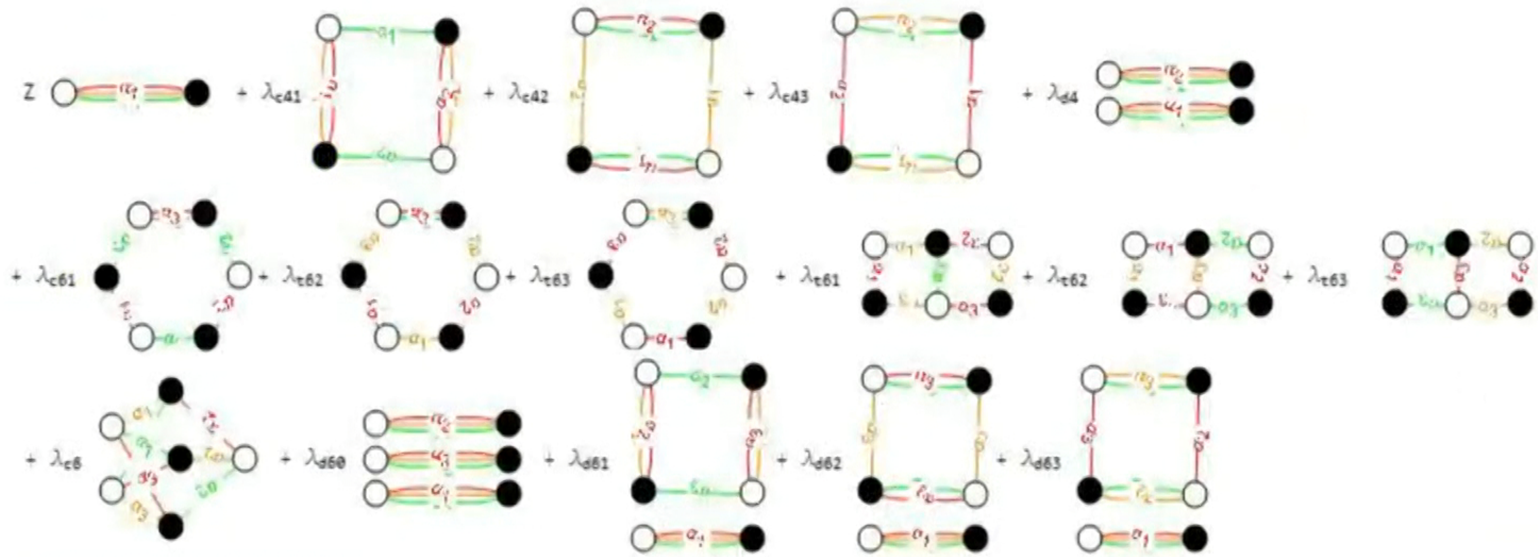


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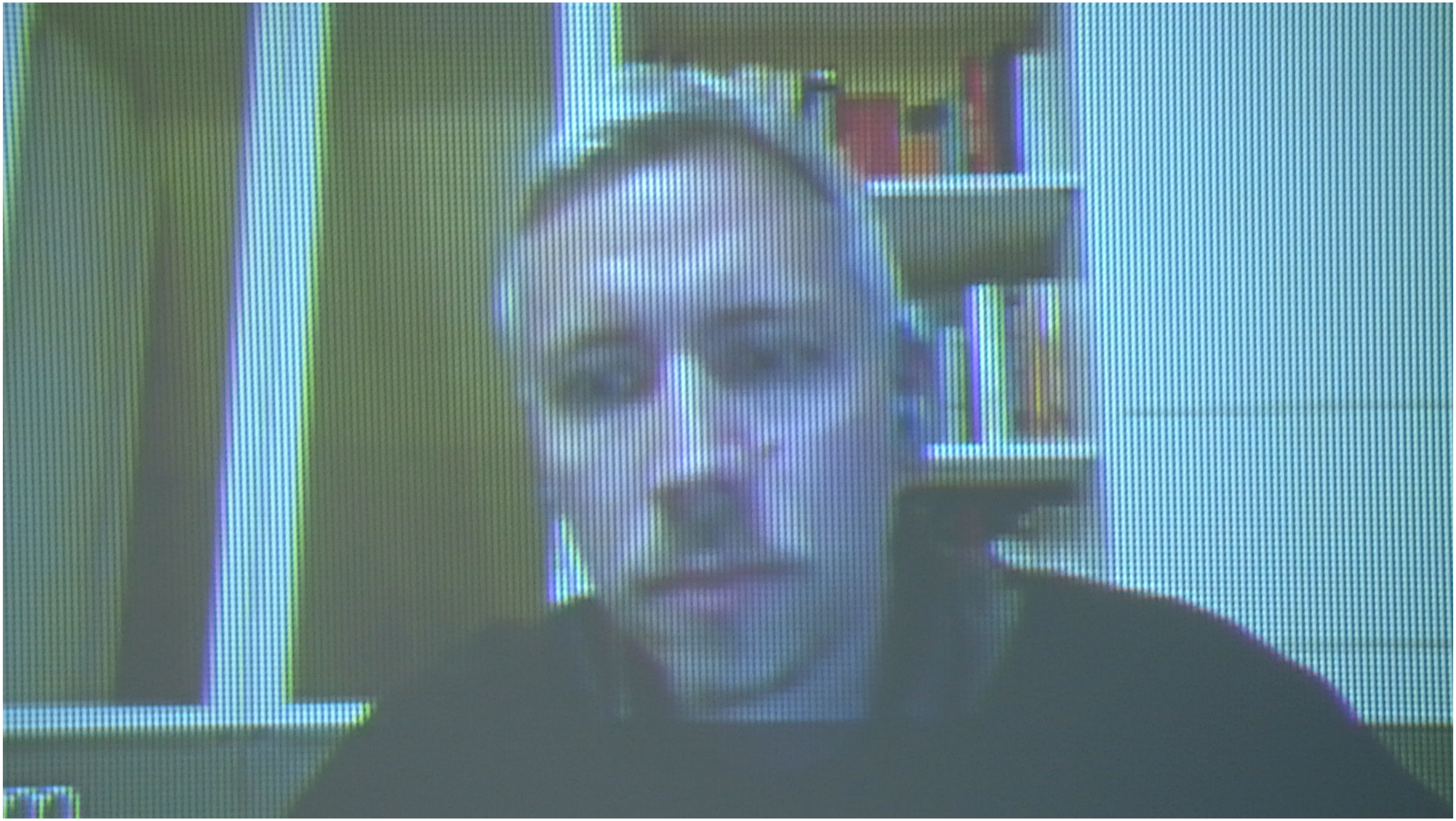

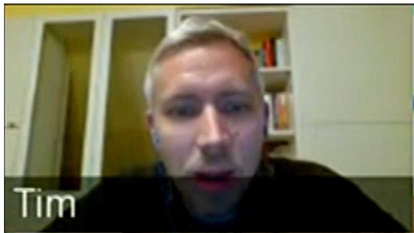


# ad to Quantum Gravity


Each approach has built-in *features* and inherent *difficulties*

$\Rightarrow$

**Combine approaches** to use built-in feature of one to solve difficulty of another approach

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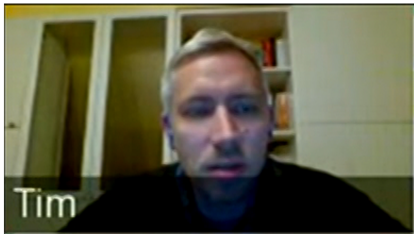


# his talk


1. **Matrix- and Colored Tensor Models** (motivation from QG)
2. **Functional Renormalization Group** (general setup and the role of symmetry)
3. **Matrix Model** (specific setup, results, numerical importance of Ward identities)
4. **Colored Tensor Models** (foundations and results)
5. **Summary**

based on work with A. Eichhorn: Phys.Rev. D88 (2013) 084016,
Phys.Rev. D90 (2014) no.10, 104039
Ann.Inst.H.Poincare Comb.Phys.Interact. 5 (2018) no.2, 173-210
Universe 5 (2019) no.2, 53 (with A. Pereira, J. Lumma) as well as arXiv:1811.00814
AND: Phys.Rev. D97 (2018) no.12, 126018 (with J. Ben Geloun, A. Pereira, D.Oriti)
As well as unpublished work with A. Pereira, A. Eichhorn, J. Lumma, A. Castro and V. Muñoz.

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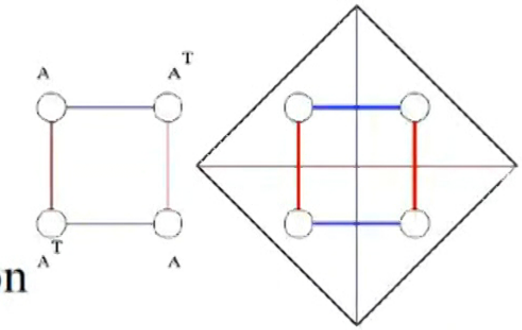
# on: Continuum Limit of Lattice QG


model action

$$S[M] = \frac{1}{2}\mathrm{Tr}(A.A^T) + \frac{g}{4\,N}\mathrm{Tr}(A.A^T.A.A^T)$$

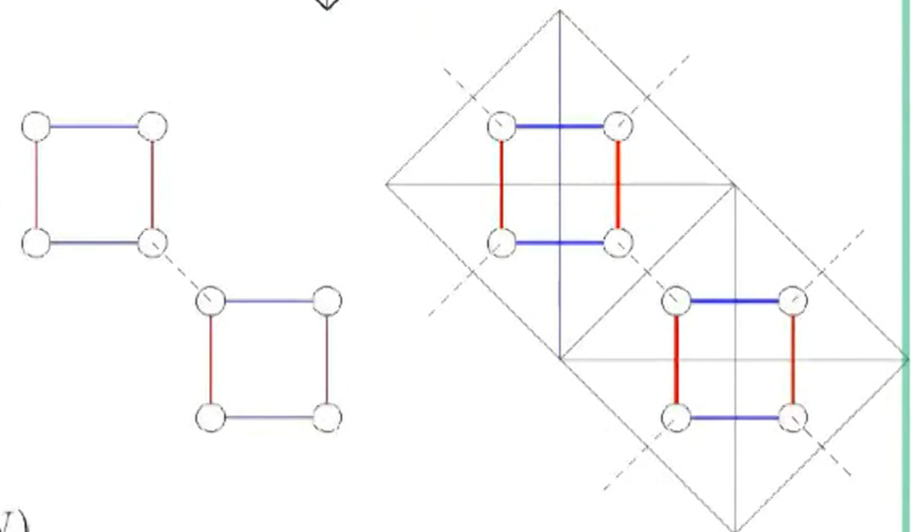



for real matrices $A$ to generate the partition function

$$Z = \int [dM]_N \, \exp(-S[M]) = \sum_\gamma A(\gamma) = \sum_{\Delta(\gamma)} e^{-(-\ln(A(\gamma)))}$$

$\Rightarrow$ can be interpreted as a partition function for random square-tesselations with weights $-\ln(A(\gamma(\Delta)))$ expressible by the Regge action:

$$S_{\mathrm{Regge}}(\Delta) = k_d \, N_d(\Delta) - k_{d-2} \, N_{d-2}(\Delta)$$



where $k_{d-2} - \alpha k_d \propto 1/G$ and $k_d \propto \Lambda/G$

with $k_{d-2} = \ln(N)$ and $k_d = \ln(g) - \frac{d(d-1)}{4}\ln(N)$

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# on: (contd.)


we want to take the continuum limit $a \to 0$ of the tesselation by squares at fixed volume $\langle V \rangle = a_o^d \langle N_d \rangle$

$\Rightarrow$ take matrix size $N$ to infinity $\Rightarrow$ $G$ *vanishes!*

$\Rightarrow$ For finite $G$ we need a critical scaling of $g(N)$ with matrix size $N$ in continuum limit.

$\Rightarrow$ to investigate the continuum limit of gravity on a random lattice we need to investigate the double scaling limit of the matrix model partition function

$$Z = \int [dM]_N \exp(-S[M]) = \sum_{\gamma} A(\gamma) = \sum_{\Delta(\gamma)} e^{-(-\ln(A(\gamma)))}$$

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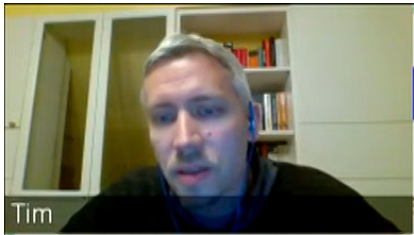


# lodels as a blueprint


te Euclidean lattice quantum gravity partition function

$$Z = \lim_{\Lambda \to \infty} \int [dg/\sim]_\Lambda e^{-\int \Sigma\left(\frac{R}{16\pi G} - \Lambda\right)\sqrt{|g|}} \quad \to \lim_{a \to 0} \sum_\Delta e^{-\frac{N_0 - \frac{1}{2}N_2}{8G} + (\ldots)a^2 N_0}$$

by evaluating the Hermitian random matrix model partition function with e.g.

$$Z = \lim_{N \to \infty} \int [dM_{ij}]_{N \times N} e^{-S_{matrix}(M)} \qquad S_{matrix} = \frac{1}{2}\mathrm{Tr}(M.M) + \frac{g}{\sqrt{N}}\mathrm{Tr}(\mathrm{M.M.M})$$

using the identification of the matrix model amplitude $N^{\chi(\Sigma)} g^{N_2} = e^{-S_{lattice}}$ with lattice Boltzmann factor

in the large N limit $\Rightarrow$ investigate critical behavior $N \to \infty$ (see e.g. Brezin, Zinn-Justin: PLB 288 (1992) 54; C. Ayala: PLB 311 (1993) 55)

**Analytic results** (benchmarks for RG methods):

1. Existence of a critical QG theory **double scaling limit** (one critical exponent: $\theta := N\partial_N \beta(g)|_{g=g_*} = \frac{4}{5}$)

2. Existence of a tower of **multicritical** points (have interpretation of gravity coupled to matter, e.g. a hard dimer)

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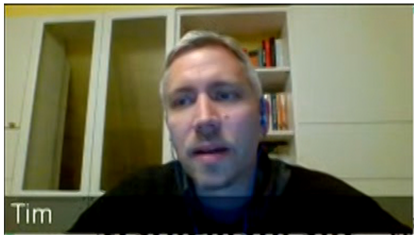


# Definition of a Measure with symmetries

**Mathematically:** We want to find non-Gaussian measures for tensor models with a desired symmetry (e.g. ON(N)^3) which exist in the large N limit

**Basic tool:** *Cylindrical consistency*. I.e. an infinite-dimensional cylindrical measure induces a measure for all functions on finite dimensional (cylindrical) subspaces.

These induced measures are not independent, but satisfy consistency relations:

$$\int_{U,V} d\mu(u,v)\, f_{U,V}(u) = \int_U d\mu(u)\, f_U(u) \quad \textit{for all } f(u) \textit{ that just depend on } U$$

Remarkably, one can define a cylindrical measure (e.g. LQG) through a set of cylindrically consistent measures.

**Logical program:**

1. Identify cylindrical consistency relations as integration over complementary subspaces
2. Turn the integration over complementary subspace into an interpolation by integrating over a Gaussian suppression factor => Polchinski-type flow equation
3. Legendre transform of Polchinski-type equation gives a Wetterich-type equation (FRGE)

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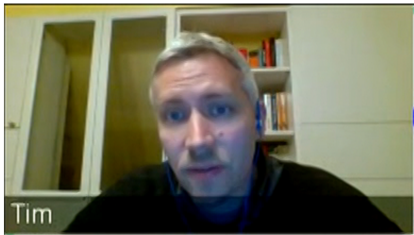


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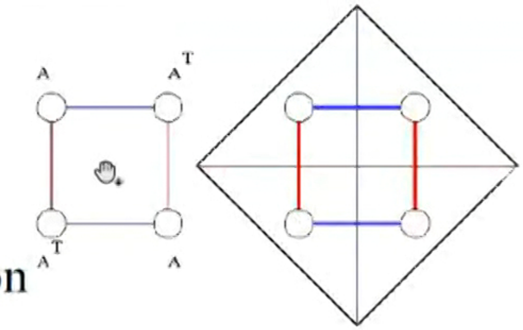


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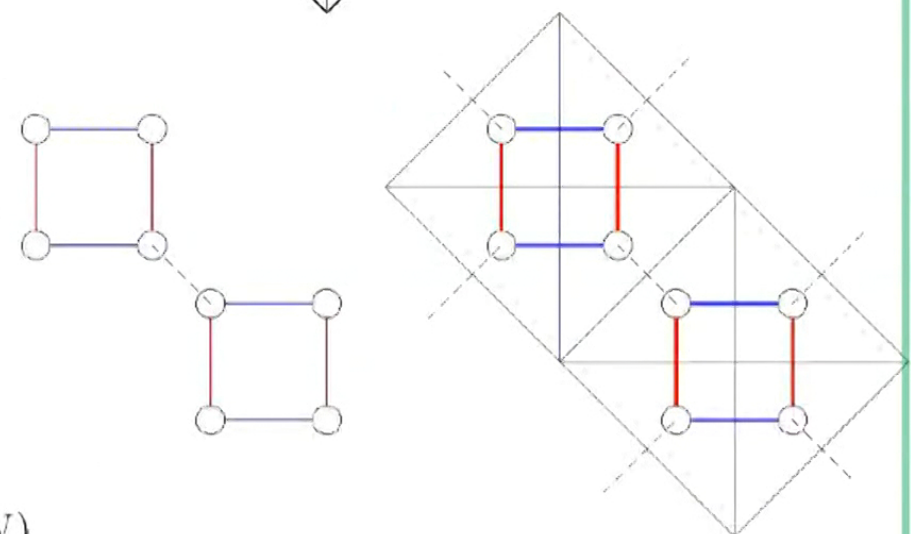

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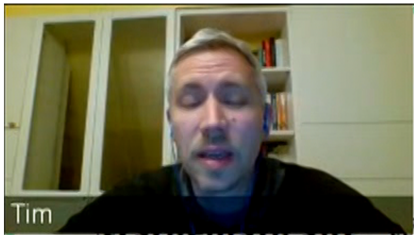


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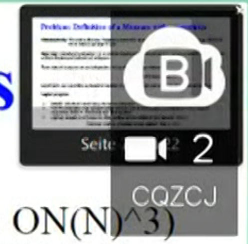


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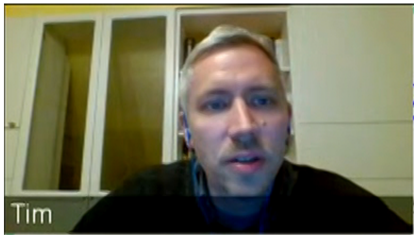



# al Renormalization Group Equation

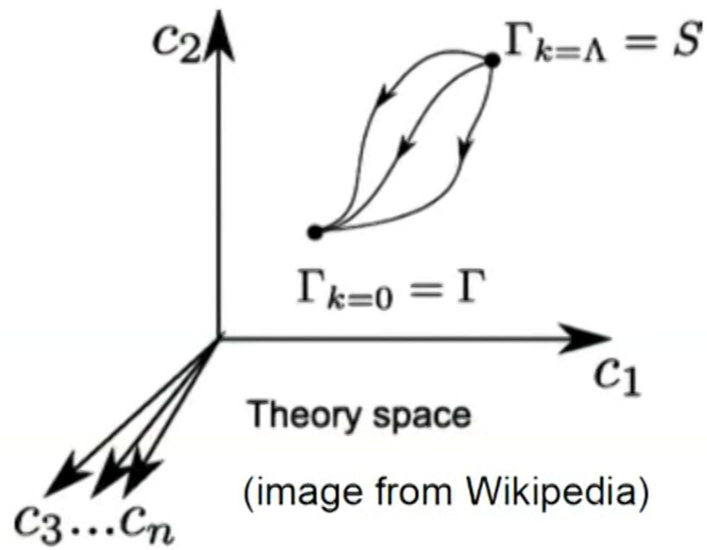
$W_k[J] = \int [d\phi]_\Lambda e^{-S_o[\phi] - \frac{1}{2}\phi - R_k.\phi + J.\phi} \Rightarrow$ field vacuum expectation value $\phi = \frac{\delta W_k}{\delta J}$

with an IR suppression term $\frac{1}{2}\phi.R_k.\phi$ (scale-dependent "mass" term of order $k$ for IR d.o.f.)

effective average action $\Gamma_k[\phi] = (\phi.J_k[\phi] - W_k[J_k[\phi]]) - \frac{1}{2}\phi.R_k.\phi$

obeys a flow equation $\partial_k \Gamma_k = \frac{1}{2}\mathrm{Tr}\left(\partial_k R_k \left(\frac{\delta^2 \Gamma_k[\phi]}{\delta\phi\,\delta\phi} + R_k\right)^{-1}\right)$

(see e.g. Wetterich *Phys. Lett. B*, **301**: 90)



(image from Wikipedia)

**Interpretation**:

1. UV limit: saddle point around $\frac{1}{2}\phi.R_k.\phi$ gives $\Gamma_{k\to\Lambda\to\infty}[\phi] \to S_o[\phi]$
2. IR limit: suppression term drops out $\Gamma_{k\to 0}[\phi] \to \Gamma[\phi]$

$\Rightarrow$ interpolation between bare action and quantum effective action

$\Rightarrow$ tool for systematic investigation of bare actions (limits $k \to \Lambda \to \infty$ )

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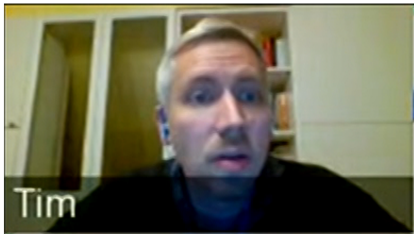


# ts: Regulator and Theory space

for the systematic investigation of:

1. possible UV actions (fundamental theories)
2. generic IR behaviour (universality)

It requires only:

1. notion of scale separation (encoded in *IR suppression* term $\frac{1}{2}\phi.R_k.\phi$)
2. "*theory space*" of admissible action functionals (field content and symmetries of bare action)
3. and in practical calculations a *truncation* ansatz and *projection* onto truncation
   (i.e. understanding which effective operators are most important
   and a how to find these operators on RHS of FRGE)

however: IR suppression term is often required to break symmetry of the bare action

$\Rightarrow$ flowing Ward identities

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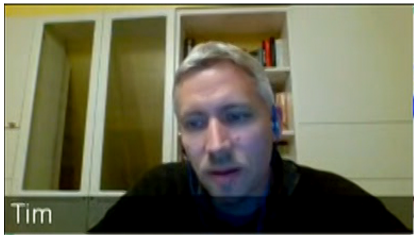



# Ward Identities

ce of the bare action under a symmetry generated by $\mathcal{G}_\epsilon\, S[\phi] = \epsilon^B \frac{\delta S[\phi]}{\delta \phi^A} f_B^A[\phi]$

(for simplicity assume invariance of the measure $[d\phi]_\Lambda$ under this symmetry)

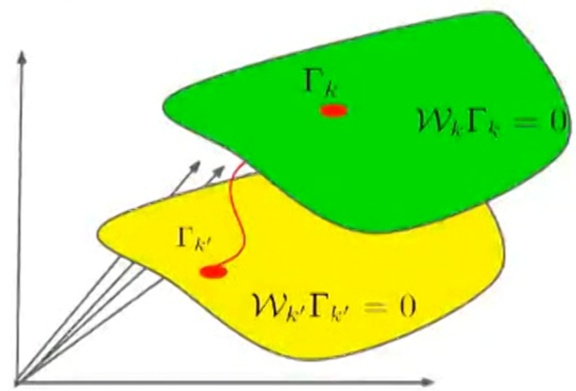
$\Rightarrow$ Legendre transform yields $\mathcal{W}_k = \mathcal{G}_\epsilon \Gamma_k - \frac{1}{2}\mathcal{G}_\epsilon \left( \langle \phi.R_k.\phi \rangle - \phi.R_k.\phi \right)$

Which ensures that the effective action satisfies the correct Ward identity $\lim_{k\to 0} \mathcal{W}_k \Gamma_k = \mathcal{W}\Gamma = 0$

*Moreover*: analogous to derivation of FRGE one finds

$$\partial_k \mathcal{W}_k \Gamma_k = -\frac{1}{2}\mathrm{Tr}\left( (\Gamma_k^{(2)} + R_k)^{-1}.\partial_k R_k.(\Gamma_k^{(2)} + R_k).(\mathcal{W}_k\Gamma_k)^{(2)} \right)$$

$\Rightarrow$ if initial effective average action satisfies initial WTI then the effective action satisfies the normal WTI




$\Rightarrow$ symmetry improved flow by solving mWTI and using symmetric couplings as coordinates on $\mathcal{W}_k\Gamma_k = 0$

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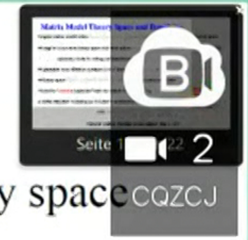
# odel Theory Space and Regulator




del action $S_{matrix} = \frac{1}{2}\mathrm{Tr}(M.M) + \frac{g}{\sqrt{N}}\mathrm{Tr}(\mathrm{M.M.M})$ generates complicated odd theory space

$\Rightarrow$ simpler to use even theory space with even action $S_{matrix} = \frac{1}{2}\mathrm{Tr}(M.M^T) + \frac{g}{N}\mathrm{Tr}(M.M^T.M.M^T)$ with real $M$

symmetry under bi-orthogonal transformations $M \to O_1.M.O_2^T$

$\Rightarrow$ generates even effective operators (i.e. of form $\mathrm{Tr}(M^{2n_1})\ldots\mathrm{Tr}(M^{2n_k})$ )

$\Rightarrow$ theory space $\Gamma_k[M] = f_k\left(\mathrm{Tr}(M^2), \mathrm{Tr}(M^4), \mathrm{Tr}(M^6), \ldots\right)$ has no occurrence of scale

$\Rightarrow$ need to "*invent* a Laplacian" that says which d.o.f. are IR, e.g. $\Delta M_{ab} := (a+b)\, M_{ab}$

This invention breaks *U(N)* resp. *O(N)*$^2$ symmetry !

A useful regulator is (analogous to Litim's optimized profile):

$$\Delta_N S[M] = M_{ab} R_N(a,b) M_{ab} \quad \text{with} \quad R_N(a,b) = Z\left(\frac{2N}{a+b} - 1\right)\theta\left(1 - \frac{2N}{a+b}\right)$$

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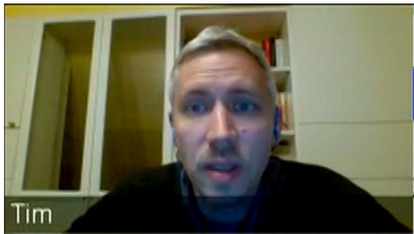


# …lodel: Dimension from 1/N expandability

…cal dimension" of operators in this theory space, but since we are interested in the large $N$ limit,

we have to impose that the beta functions admit a *1/N* expansion, i.e. $\beta_{g_i} = b_i^1(g_1, ...) + 1/N\, b_i^2(g_1, ...) + O(1/N^2)$

this fixes the scaling of the operators, by generating upper and lower bounds that admit only one solution at the end.

E.g. tadpole of one $g_4 Tr(M^4)^{(2)}$ flows into $Z$

and two-vertex diagram with two $g_4 Tr(M^4)^{(2)}$ flows into $g_4$

$\Rightarrow$ dimension of $g_4$ is fixed; analogously all other operators.

$\Rightarrow$ for couplings defined as $\Gamma_N[M] = \sum g^i_{n_1 \dots n_i} \mathrm{Tr}(M^{2n_1}) \dots \mathrm{Tr}(M^{2n_i})$

one obtains the canonical dimension from *1/N* expandability as

$$\dim(g^i_{n_1 \dots n_i}) = N^{i-1+\sum_{k=1}^{i} n_k}$$

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# lodel: projection on truncation

Evaluation of FRGE already orders terms by trace structure (single-, multitrace operators)

$\Rightarrow$ simplest case: insert "constant" matrix in each trace summand $M_{ab} = \phi\, \delta_{ab}\, \theta(N-a)$

This rule distinguishes between all index-independent (i.e. *U(N)* symmetric) operators

(it can be regarded as the first term for a projection rule with

index -dependent terms, e.g. by inserting orthogonal polynomials

$$M_{ab} = \phi_{ij}\, u_{ij}(a,b)\theta(N-a)\theta(N-b)$$

*however: spectral sums quickly become very complicated*)

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# odel: FRGE results (w/o symmetry)


truncation: $\Gamma_N[M] = \frac{Z}{2}\mathrm{Tr}(M^2) + \sum_{n\geq 2} \frac{\bar{g}_{2n}}{2n}\mathrm{Tr}(M^{2n})$ with dimensionless couplings $\bar{g}_i = Z^{\frac{i}{2}} N^{\frac{i}{2}-1} g_i$

$\Rightarrow$ beta functions: $\eta = g_4\,[\dot{R}\,P^2]$

$$\beta(g_{2n}) = ((1+\eta)n - 1)\,g_{2n} + 2n \sum_{i;\vec{m}:\sum m_k = n} (-1)^{\sum_i m_i}[\dot{R}P^{1+\sum_i m_i}] \begin{pmatrix} \sum_i m_i \\ m_1 m_2 \dots \end{pmatrix} \prod_i g_{2(i+1)}^{m_i}$$

Finding fixed points with one relevant direction, but $\theta \approx 1.0, \dots, 1.1$ instead of analytic $\theta = 0.8$ *(in all truncations)*
(and all other crit. exponents near negative integers and aligned with $g_{2n}\,:\,n > 4$)

2. Multitrace truncation: only $\mathrm{Tr}(M^2)\mathrm{Tr}(M^{2n})$ flow into single-trace operators at large $N$

$\Rightarrow$ include $g_{2,2}$ and $g_{2,4}$ in truncation, but critical exponents actually get *worse*:

$$\theta_1 = 1.21,\ \theta_2 = -0.69,\ \theta_3 = -1.01,\ \theta_4 = -1.88$$

(inclusion of further multitrace operators does not improve result)

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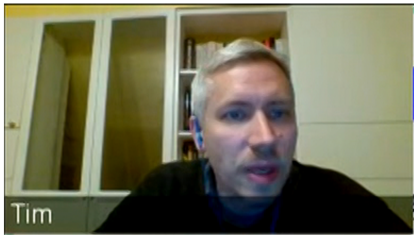



# odel: FRGE results (with symmetry)

generated by $M \to O^T.M.O = \phi + \epsilon\,[M, A] + O(\epsilon^2)$

and leads to Ward-identity $\mathcal{W}_N\Gamma_N[M] = \mathcal{G}_\epsilon\Gamma_N[M] - \mathrm{tr}_{op}\left(\frac{[A, R_N]}{\Gamma_N^{[2]}[M] + R_N}\right) = 0$

**Observation:** Tadpole approximation of flowing WTI vanishes
(i.e. no index dependence of tadpoles of index-independent operators!)

1. Tadpole approximation of single trace truncation

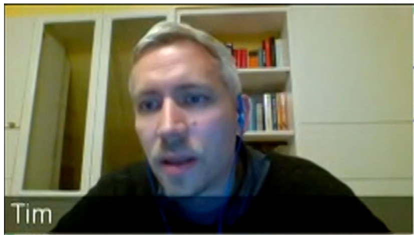
$$\eta = 2g_4\,x \qquad\qquad \beta(g_{2n}) = ((n-1) + n\eta)g_{2n} - 2\,n\,x\,g_{2(n+1)}$$

$\Rightarrow$ find $\theta_1 = 1$ (is *20%* off, but all further multicritical exponents with good accuracy )

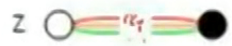
2. Tadpole approximation with multirace operators

including multitrace operators $g_{2n}$, $g_{2,2n}$, $g_{4,2n}$, $g_{2,2,2n}$ in truncation gives arbitrarily close values to $\theta_1 = \frac{4}{5}$

and multicritical exponents also in *O(1%)* precision




# odel: Setup analogous to Matrix Model

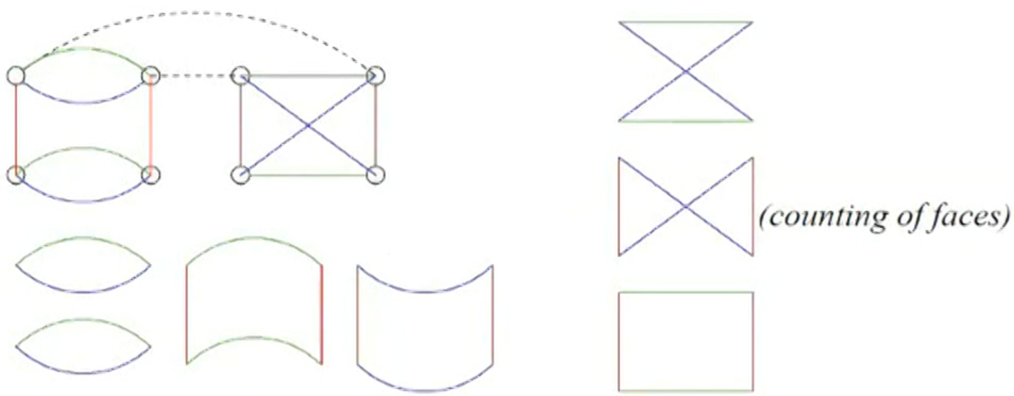
Theory space: $U(N)^3$ symmetry ⇒ colored bipartite graphs 

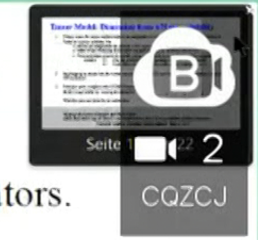
Invented Laplacian: $\Delta T_{abc} = (a+b+c)\,T_{abc}$

Regulator: $R_N(a,b,c) = Z\left(\frac{3N}{a+b+c} - 1\right)\theta\left(1 - \frac{3N}{a+b+c}\right)$

**Now:** Spectral sums become significantly more complicated!

Interactions:
*(dual to gluing of triangulations)*



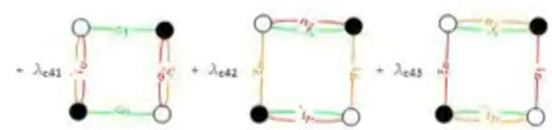

# odel: Dimension from 1/N expandability

e for tensor models requires an assignment of scaling dimension for all possible operators.
These are a priori arbitrary, but:

a. Almost all assignments do prevent a 1/N expansion for all beta functions
b. Many of the remaining assignments collapse to Gaussian model in large N-limit
c. One can always remove an overall scaling of the fields by a redefinition of the integration variable => One can always choose a Gaussian term to be dimensionless
=> Seed of the construction is the scaling of an interaction term

2. By plugging an ansatz into the vertex expansion of the FRGE, one can show directly whether it admits a 1/N expansion

3. Example: pure, complex rank 3 U(N)^3-symmetric model:
Model is non-trivial for scaling s(4-melon)=2



With this seen one finds for all melons that

$$s(\gamma) = 3 - \frac{1}{2}(3p(\gamma) - F(\gamma))$$

Where p=# of pairs of tensors and F=# of faces
**AND:** this choice can be shown to be consistent with the 1/N expandability of all beta functions

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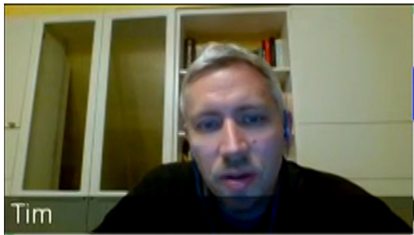



# lodel: FRGE results (w/o symmetry)

truncation: $\Gamma_N[M] = \frac{Z}{2}\mathrm{Tr}(M^2) + \sum_{n\geq 2} \frac{\bar{g}_{2n}}{2n}\mathrm{Tr}(M^{2n})$ with dimensionless couplings $\bar{g}_i = Z^{\frac{i}{2}} N^{\frac{i}{2}-1} g$

$\Rightarrow$ beta functions: $\eta = g_4\,[\dot{R}\,P^2]$

$$\beta(g_{2n}) = ((1+\eta)n - 1)\, g_{2n} + 2n \sum_{i;\vec{m}:\sum m_k = n} (-1)^{\sum_i m_i} [\dot{R} P^{1+\sum_i m_i}] \begin{pmatrix} \sum_i m_i \\ m_1 m_2 \ldots \end{pmatrix} \prod_i g_{2(i+1)}^{m_i}$$

Finding fixed points with one relevant direction, but $\theta \approx 1.0, \ldots, 1.1$ instead of analytic $\theta = 0.8$ *(in all truncations)*
(and all other crit. exponents near negative integers and aligned with $g_{2n} : n > 4$)

2. Multitrace truncation: only $\mathrm{Tr}(M^2)\mathrm{Tr}(M^{2n})$ flow into single-trace operators at large $N$

$\Rightarrow$ include $g_{2,2}$ and $g_{2,4}$ in truncation, but critical exponents actually get *worse*:

$$\theta_1 = 1.21,\ \theta_2 = -0.69,\ \theta_3 = -1.01,\ \theta_4 = -1.88$$

(inclusion of further multitrace operators does not improve result)

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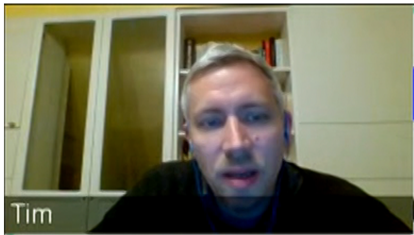


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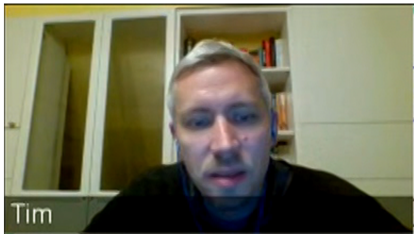
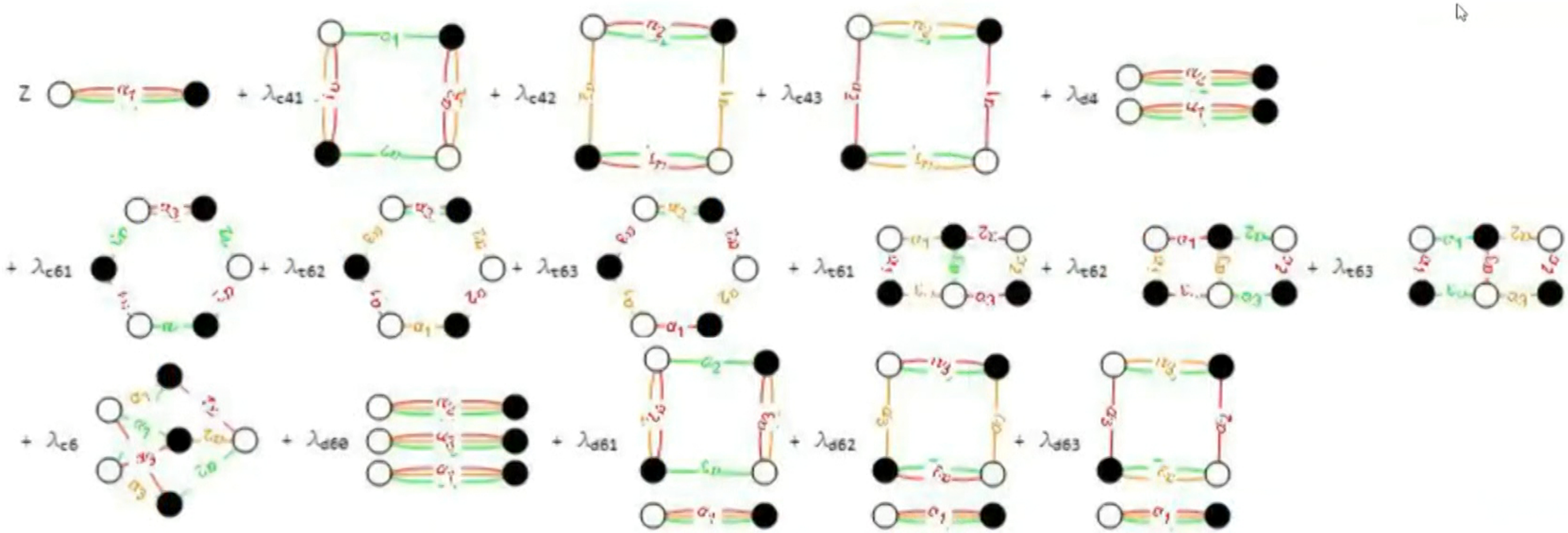



# odel: Color ⇒ 1/N expansion

There exists a 1/N expansion of amplitude in colored tensor models (analogous to matrix models)

⇒ very active research in past 9 years on colored tensor ensembles (pure models as well as tensor field theory)

Symmetric $T^6$ truncation of rank 3 pure tensor model:

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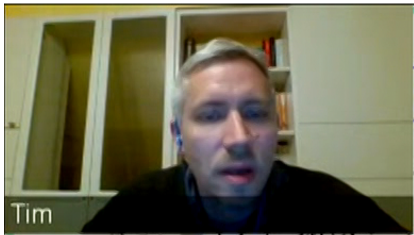
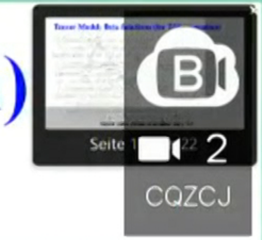



# odel: Beta functions (for T^6-truncation)




$$+ g_{4,1}^{2,3} + g_{4,2}^{2}\right)(5-\eta)$$

$$\beta_{g_{4,1}^{2,i}} = (2+2\eta) g_{4,1}^{2,i} + (g_{4,1}^{2,i})^2 \frac{13}{630}(21-4\eta) - g_{6,1}^{3,1}\frac{5-\eta}{15} - g_{6,2}^{3,1}\frac{5-\eta}{40}$$

$$\beta_{g_{4,2}^{2}} = (3+2\eta) g_{4,2}^{2} + \frac{6-\eta}{15}\Big( \left(g_{4,2}^{2}\right)^2 + 2 g_{4,2}^{2}\left(g_{4,1}^{2,1} + g_{4,1}^{2,2} + g_{4,1}^{2,3}\right) +$$

$$2\, g_{4,1}^{2,1}\, g_{4,1}^{2,2} + 2\, g_{4,1}^{2,1}\, g_{4,1}^{2,3} + 2\, g_{4,1}^{2,2}\, g_{4,1}^{2,3}\Big) - \frac{5-\eta}{20}\left(g_{6,2}^{3,1} + g_{6,2}^{3,2} + g_{6,2}^{3,3}\right)$$

$$\beta_{g_{6,1}^{3,1}} = (4+3\eta)\, g_{6,1}^{3,1} + \frac{13}{210}(21-4\eta) g_{4,1}^{2,1}\, g_{6,1}^{3,1} - 8\left(g_{4,1}^{2,1}\right)^3 \frac{5769-1049\eta}{60480}$$

$$\beta_{g_{6,1}^{2,1}} = (5{+}3\eta)\, g_{6,1}^{2,1} - g_{4,1}^{2,1}\, g_{4,1}^{2,2}\, g_{4,1}^{2,3}\, 16\frac{93869-15729\eta}{362880} - \left(g_{4,1}^{2,2}\left(g_{4,1}^{2,3}\right)^2 + g_{4,1}^{2,3}\left(g_{4,1}^{2,2}\right)^2\right) 8\frac{46500-8887\eta}{151200} +$$

$$\left(g_{4,1}^{2,2} g_{6,1}^{3,3} + g_{4,1}^{2,3} g_{6,1}^{3,2}\right) 13\frac{21-4\eta}{210} + \left(g_{4,1}^{2,2} + g_{4,1}^{2,3}\right) g_{6,1}^{2,1} 13\frac{21-4\eta}{630}$$

$$\beta_{g_{6,1}^{0}} = \left(-d_{g_{6,1}^{0}} + 3\eta\right) g_{6,1}^{0} - g_{4,1}^{2,1}\, g_{4,1}^{2,2}\, g_{4,1}^{2,3}\, 16\frac{73160-13889\eta}{604800}\frac{N^{-d^{g_{6,1}^{0}}}}{N^6}$$

$$\beta_{g_{6,2}^{3,1}} = (5{+}3\eta)\, g_{6,2}^{3,1} + \left(g_{4,1}^{2,2} + g_{4,1}^{2,3}\right) g_{6,1}^{3,1}\frac{6-\eta}{5} - \left(g_{4,1}^{2,1}\right)^2\left(g_{4,1}^{2,2} + g_{4,1}^{2,3}\right) 8\frac{2764-467\eta}{10080} +$$

$$\left(g_{4,1}^{2,2} + g_{4,1}^{2,3}\right) g_{6,2}^{3,1}\frac{6-\eta}{15} + g_{6,1}^{3,1} g_{4,2}^{2}\frac{6-\eta}{5} + g_{6,2}^{3,1} g_{4,2}^{2}\frac{6-\eta}{15} + g_{6,2}^{3,1} g_{4,1}^{2,1}\frac{399-73\eta}{315}$$

$$\beta_{g_{6,3}^{3}} = (6+3\eta)\, g_{6,3}^{3} + \Big( g_{4,1}^{2,2}\left[g_{6,2}^{3,1} + g_{6,2}^{3,3}\right] + g_{4,1}^{2,1}\left[g_{6,2}^{3,2} + g_{6,2}^{3,3}\right]$$

$$+ g_{4,1}^{2,3}\left[g_{6,2}^{3,1} + g_{6,2}^{3,2}\right]\Big)\frac{6-\eta}{15} 2 + g_{6,3}^{3}\left(g_{4,1}^{2,1} + g_{4,1}^{2,2} + g_{4,1}^{2,3}\right)\frac{6-\eta}{5} - g_{4,1}^{2,1}\, g_{4,1}^{2,2}\, g_{4,1}^{2,3}\frac{7-\eta}{84} 16 +$$

$$g_{4,2}^{2} g_{6,3}^{3}\frac{6-\eta}{5} - 8\left(g_{4,2}^{2}\right)^3\frac{7-\eta}{84} - \left(g_{4,2}^{2}\right)^2\left(g_{4,1}^{2,1} + g_{4,1}^{2,2} + g_{4,1}^{2,3}\right) 2\frac{7-\eta}{7} - \left(g_{4,1}^{2,1}\left[g_{4,1}^{2,2} + g_{4,1}^{2,3}\right] + g_{4,1}^{2,2} g_{4,1}^{2,3}\right) g_{4,2}^{2} 2\frac{7-\eta}{7}$$

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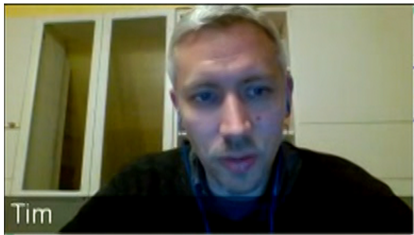


# odel: Overview of fixed points

One finds a ton of fixed points in various models and truncations:

a. Types of models investigated:
   i. pure tensor models (real O(N)^3, complex U(N)^3) up to T^8 truncation
   ii. Tensor field theory
   iii. 1+1 foliation (Benedetti-Henson model)

b. Types of fixed points:
   i. with enhanced symmetry (O(N)^3 becomes e.g. O(N^2)O(N))
   ii. truncation artifacts
   iii. candidates for QG (no enhanced symmetry, no indication for truncation artifacts)

c. To be done: investigation of tensor models which implement nontrivial propagator (i.e. a modified Ward-Identity) to investigate dually weighted tensor models, as is necessary to explore CDT and EDT theory spaces

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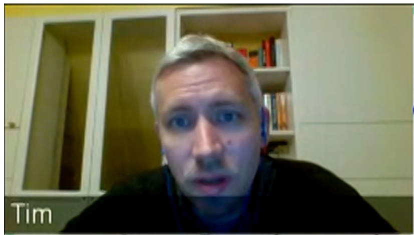


1. Starting point: desire to explore continuum limit in LQG related models
   $\Rightarrow$ use FRGE to explore tensor models for QG

2. Matrix model results:
   a. FRGE is a tool to find asymptotic safety in GW model (previous work with A. Sfondrini: IJMP A 26 (2011) 4009)
   b. FRGE finds double scaling limit and multicritical points in pure matrix models
   c. FRGE achieves numerical accuracy

3. Importance of *U(N)* - Ward-identity for numerical accuracy of critical exponents

4. Tensor model results:
   a. Setup can be applied to tensor models
   b. FRGE finds various symmetry enhanced continuum limits as well as candidates without symmetry enhancement
   c. Future work will probably need to implement *U(N)* - Ward-identity to obtain accurate critical exponents and broken Ward-identities (e.g. CDT and EDT models)

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# Thank you !