

Title: Supersymmetric Landau-Ginzburg Tensor Models

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Series: Quantum Fields and Strings

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Abstract: Melonic tensor model is a new type of solvable model, where the melonic Feynman diagrams dominate in the large N limit. The melonic dominance, as well as the solvability of the model, relies on a special type of interaction vertex, which generically would not be preserved under renormalization group flow. I will discuss a class of 2d $N=(2,2)$ melonic tensor models, where the non-renormalization of the superpotential protects the melonic dominance. Another important feature of our models is that they admit a novel type of deformations which gives a large IR conformal manifold. At generic point of the conformal manifold, all the flavor symmetries (including the $O(N)^{q-1}$ symmetry) are broken and all the flat directions in the potential are lifted. I will also discuss how the operator spectrum and the chaos exponent depend on the deformation parameters.

Supersymmetry Landau-Ginsberg Tensor Model
work with Mukund Rangamani, Sean Collin-Ellerine

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Motivation, backgrounds

Melonic tensor model

- solvable (large N)
- SYK, AdS_2 QG, BH
- $d \geq 2$, SUSY

SYK, AdS₂ QG, BH
 • d ≥ 2, SUSY

• New large N limit

Gurau - Witten

Carrozza - Tanasa - Klebanov

Tornopolsky

1 d QM

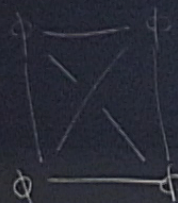
$\phi_{a_1, a_2, \dots, a_{q-1}}$

$a_r = 1, \dots, N, \quad r = 1, \dots, q-1$

$$\frac{1}{2}(\partial\phi)^2 + g[\phi^q]$$

q even, $q \geq 4$

$$[\phi^4] = \phi^{a_1 a_2 a_3} \phi^{a_1 b_2 b_3} \phi^{b_1 a_2 b_3} \phi^{b_1 b_2 a_3}$$



SYK, AdS₂ QG, BH
 • d=2, SUSY

• New large N limit

Gurau-Witten

Carrozza-Tanasa-Klebanov

Tarnopolsky

1d QM

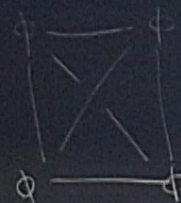
$\phi^{a_1, a_2, \dots, a_{2r}}$

$a_r = 1, \dots, N, \quad r = 1, \dots, \frac{q}{2} - 1$

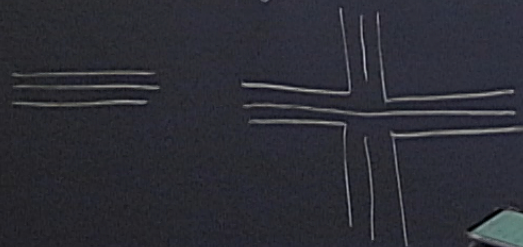
$$\frac{1}{2}(\partial\phi)^2 + g[\phi^{\frac{q}{2}}]$$

q even, $q \geq 4$

$$[\phi^4] = \phi^{a_1 a_2 a_3} \phi^{a_1 b_2 b_3} \phi^{b_1 a_2 b_3} \phi^{b_1 b_2 a_3}$$



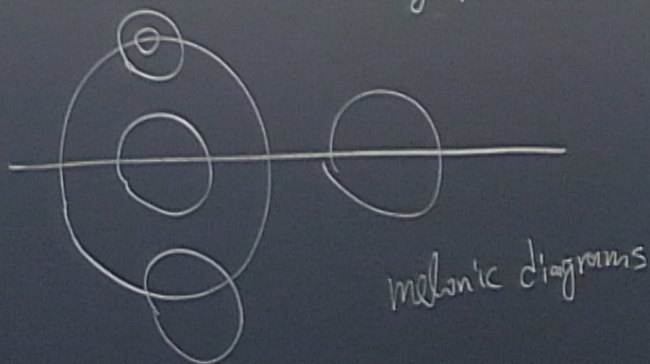
$N \rightarrow \infty$, fixing $g^2 N^3 \equiv J^2$



$$\text{Diagram 1} + \text{Diagram 2} + \dots$$

Diagram 1: Two parallel horizontal lines. ~ 1

Diagram 2: Two parallel horizontal lines with a self-energy loop on the top line. $\sim g^2 N^3 = J^2$



Non-melonic:

$$\text{Diagram 3} \quad \text{Diagram 4}$$

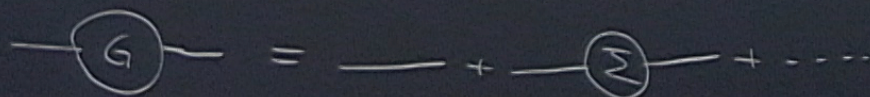
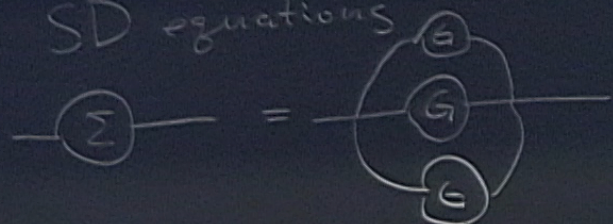
Diagram 3: A single horizontal line with a small circle on top. $\sim g N = \frac{J}{\sqrt{N}}$

Diagram 4: Two parallel horizontal lines with a self-energy loop on the top line. $\sim \frac{J^4}{N^2}$

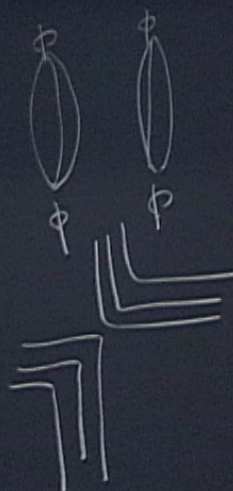
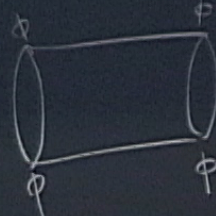


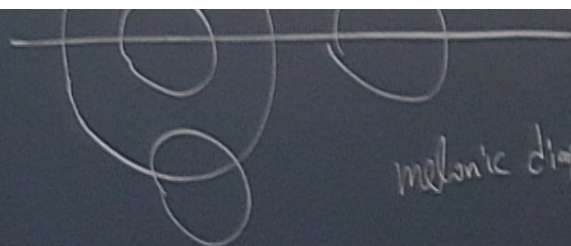
$$\sim gN = \frac{J}{\sqrt{N}}$$

SD equations



other vertices

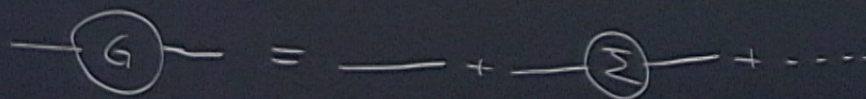
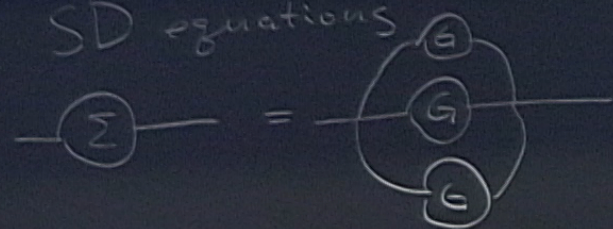




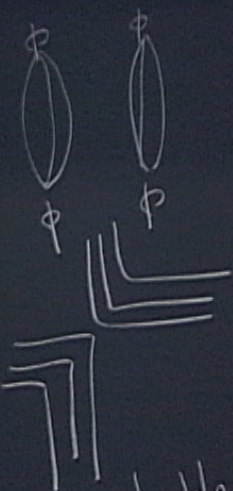
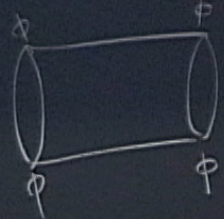
melonic diagrams

$$\sim gN = \frac{J}{N}$$

SD equations

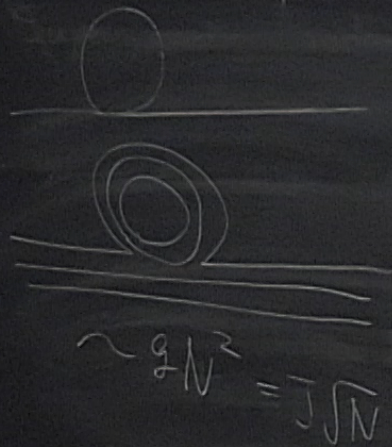
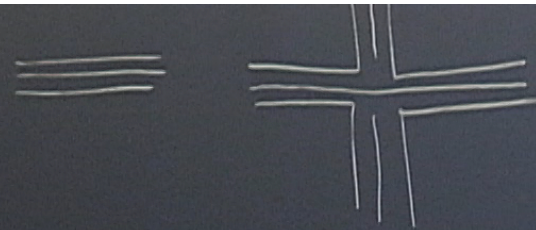


other vertices



double-sum

$$[\phi^4] = \phi^{a_1 a_2 a_3} \phi^{a_1 b_2 b_3} \phi^{b_1 a_2 b_3} \phi^{b_1 b_2 a_3}$$



$$\sim g N^2 = J \sqrt{N}$$

RG $\rightarrow$ all interaction vertices

$$2d \quad N=(2,2), \quad \bar{c}=4$$

chiral superfield: $X^{a_1 a_2 a_3}$

$$S = \int d^2x d^4\theta K(X, \bar{X}) - \int d^2x d^2\theta W(X) + c.c.$$

$$K(X, \bar{X}) = \bar{X}^{a_1 a_2 a_3} X^{a_1 a_2 a_3}$$

$$W = \frac{1}{4} g X^{a_1 a_2 a_3} X^{a_1 b_1 b_3} X^{b_1 a_2 b_3} X^{b_1 b_2 a_3}$$

- $N \rightarrow \infty$, fixing $T^2 = g^2 N^3$

$\Rightarrow$ melonic dominance (supergraph)

- $O(N)^3$

Anisotropic deformation:

$$W(X) = \frac{1}{4} g \sum_{a_i, b_i} \alpha_{a_1 b_1, a_2 b_2, a_3 b_3} X^{\dots}$$

pillow

double-sum

$$W = \frac{1}{4} g X^{a_1 a_2 a_3} X^{a_1 b_2 b_3} X^{b_1 a_2 b_3} X^{b_1 b_2 a_3}$$

• $N \rightarrow \infty$, fixing $T^2 = g^2 N^3$
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• $O(N)^3$

Anisotropic deformation:

$$W(X) = \frac{1}{4} g \sum_{a_i, b_i} \alpha_{a_1 b_1, a_2 b_2, a_3 b_3} X^*$$

$$g \rightarrow \lambda g, \alpha \rightarrow \lambda^{-1} \alpha$$

$$\text{break } O(N)^3 \rightarrow O(1)^{3N} \simeq \mathbb{Z}_2^{3N}$$

$$N \rightarrow \infty, \text{ fixing } g \propto \alpha_{a_1 b_1, a_2 b_2, a_3 b_3} N^{\frac{3}{2}}$$

$$g = O(N^{-\frac{3}{2}}), \alpha = O(N^0)$$

$$W = \frac{1}{4} g X^{a_1 a_2 a_3} X^{a_1 b_2 b_3} X^{b_1 a_2 b_3} X^{b_1 b_2 a_3}$$

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$$\sim O(N)^3$$

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$$N \rightarrow \infty, \text{ fixing } g \alpha_{a_1 b_1, a_2 b_2, a_3 b_3} N^{\frac{3}{2}}$$

$$g = O(N^{-\frac{3}{2}}), \alpha = O(N^0)$$

vacuum supergraph n_V vertices, n_L index loops

$$\sum_{a_1, a_2, \dots, a_{n_L}=1}^N (\alpha^{n_V})_{a_1, a_2, \dots, a_{n_L}} = O(N^{n_L})$$

Anisotropic deformation:

$$W(X) = \frac{1}{4} g \sum_{a_i, b_i} \alpha_{a_1 b_1, a_2 b_2, a_3 b_3} X^4$$

$$g \rightarrow \lambda g, \alpha \rightarrow \lambda^{-1} \alpha$$

Non-renormalization thm:

1. promote g to a background superfield

2. global sym

	$U(1)_R$	$U(1)_F$
X	$\frac{1}{2}$	1
g	0	-4
W	2	0

Non-renormalization thm:

1. promote g to a background superfield
2. global sym

	$U(1)_R$	$U(1)_f$
X	$\frac{1}{2}$	1
g	0	-4
W	2	0

3. effective superpotential

$$W_{\text{eff}} = g f(X)$$

$f(X)$ homogeneous degree 4

$$f(X) = a_4 [X^4]_{\text{tetr}} + b_4 [X^4]_{\text{other}} + \frac{[X^8]}{[X^4]_{\text{tetr}}} + \dots$$

4. $g \rightarrow 0$, $a_4 = 1$, $b_4 = 0$

$$W = \frac{1}{4} g X^{a_1 a_2 a_3} X^{a_1 b_1 b_3} X^{b_1 a_2 b_3} X^{b_1 b_2 a_3}$$

$N \rightarrow \infty$, fixing $T^2 = g^2 N^3$
(supergraph)

break $O(N)^3 \rightarrow O(1)^{3N} \cong \mathbb{Z}_2^{3N}$

$N \rightarrow \infty$, fixing $g \propto a_1 b_1 a_2 b_2 a_3 b_3 N^{\frac{3}{2}}$
 $g = O(N^{-\frac{3}{2}}) = O(N^0)$

$$2N = J\sqrt{N}$$

$$K(X, \bar{X}) = \overline{X^{a_1 a_2 a_3}} X^{a_1 a_2 a_3}$$

1. derivatives $\Rightarrow$ irrelevant

$$2. K_{\text{eff}}(X, \bar{X}) = Z_2 X \bar{X} + Z_4 (X \bar{X})^2 + \dots$$

3. assumption: no moduli

$\Rightarrow X$ finite anomalous dim

$$4. K_{\text{eff}} = \sum_{a_i=1}^{\infty} Z_{a_1 a_2 a_3} \overline{X^{a_1 a_2 a_3}} X^{a_1 a_2 a_3}$$

$$2N = J\sqrt{N}$$

$$K(X, \bar{X}) = \bar{X}^{a_1 a_2 a_3} X^{a_1 a_2 a_3}$$

Kähler potential

1. derivatives $\Rightarrow$ irrelevant

2. $K_{\text{eff}}(X, \bar{X}) = Z_2 X \bar{X} + Z_4 (X \bar{X})^2 + \dots$

3. assumption: no moduli

$\Rightarrow X$ finite anomalous dim

4. $K_{\text{eff}} = \sum_{a_i=1}^{\infty} Z_{a_1 a_2 a_3} \bar{X}^{a_1 a_2 a_3} X^{a_1 a_2 a_3}$

$$\tilde{X} = \sqrt{Z} X$$

$$W_4 = \frac{1}{4} \tilde{\alpha} \tilde{X}^4 = \frac{1}{4} g \alpha X^4$$

$$\tilde{\alpha} = g \alpha \sqrt{Z Z Z Z}$$

Anisotropic deformation:

$$W(X) = \frac{1}{4} g \sum_{a_i, b_i} \alpha_{a_i b_i} a_1 b_1 a_2 b_2 a_3 b_3 X^4$$

$$g \rightarrow \lambda g, \alpha \rightarrow \lambda^{-1} \alpha$$

$$a_1, a_2, \dots, a_{n_L} = 1$$

UV $N=(2,2)$ LG model $\longrightarrow$ IR $N=(2,2)$ SCFT
solvable in large N

• Large IR conformal manifold

$$\Delta(X) = \frac{1}{4} \Leftarrow \Delta(W) = 1$$

• R-sym anomaly matching

$$c = 3 \sum_i (1 - 2\Delta_i) = \frac{3}{2} N^3$$

• elliptic genus, S^2 partition function

effective superpotential

$$W_{\text{eff}} = g f(X)$$

homogeneous degree 4

$$[X^4]_{\text{tetra}} + b_4 [X^4]_{\text{other}}$$

$$b_4 = 1, b_4 = 0$$

Anisotropic deformation:

$$W(X) = \frac{1}{4} g \sum_{a_i, b_i} \alpha_{a_i b_i} X^4$$

$$g \rightarrow \lambda g, \alpha \rightarrow \lambda^{-1} \alpha$$

$$a_1, a_2, \dots, a_{n_L} = 1$$

UV $N=(2,2)$ LG model $\longrightarrow$ IR $N=(2,2)$ SCFT
solvable in large N

no L arc IR conformal manifold

$$\frac{1}{4} \Leftarrow \Delta(W) = 1$$

anomaly matching

$$(1 - 2\Delta_c) = \frac{3}{2} N^2$$

S^2 partition function

Result from large N

• Isotropic model

same large N correlation fn
as 2d susy SYK

Two point function:

$$X = X + \theta \psi + \theta^2 F$$

$$\langle X(z_1) X(z_2) \rangle = \frac{b}{|z_1 - z_2|^{2\Delta}}$$

$$\text{SD eq} \Rightarrow \Delta = \frac{1}{4}$$

$$b = \frac{1}{\sqrt{2\pi J}}$$

assumption: no moduli
 $\Rightarrow X$ finite anomalous dim
 4. $k_{\text{eff}} = \sum_{a_i=1}^{\infty} \sum_{a_1, a_2, a_3} X^{a_1, a_2, a_3} X^{a_1, a_2, a_3}$

Four-point function: $\langle XX \bar{X} \bar{X} \rangle$

$$\overline{\overline{\quad}} + \overline{\overline{\quad}} + \overline{\overline{\quad}} + \dots$$

$\bar{F}_0$

$$\bar{F}_1 = k \bar{F}_0$$

$$\bar{F}_2 = k^2 \bar{F}_0$$

$$k = \text{diagram}$$

$$\bar{F} = \frac{1}{1-k} \bar{F}_0$$

$\Rightarrow$ no moduli
 $\Rightarrow$ finite anomalous dim
 $4, k_{eff} = \sum_{a_i=1}^3 \sum_{a_1, a_2, a_3} X^{a_1, a_2, a_3} X^{a_1, a_2, a_3}$

Four-point function: $\langle XX \bar{X} \bar{X} \rangle$
 $\bar{X} \bar{X} \sim \# T$

$$\overline{\overline{\quad}} + \overline{\overline{\quad}} + \overline{\overline{\quad}} + \dots$$

$$\tilde{F}_0, \quad \tilde{F}_1 = K \tilde{F}_0, \quad \tilde{F}_2 = K^2 \tilde{F}_0$$

$$K = \overline{\overline{\quad}}$$

$$\tilde{F} = \frac{1}{1-K} \tilde{F}_0$$

$$\tilde{F} \supset \frac{g^2}{2c} \bar{z}^2, \quad c = \frac{3}{2} N^3$$

assumption: no moduli
 $\Rightarrow X$ finite anomalous dim
 4. $k_{\text{eff}} = \sum_{a_i=1}^{\infty} \sum_{a_1, a_2, a_3} X^{a_1, a_2, a_3} X^{a_1, a_2, a_3}$

Four-point function: $\langle XX XX \rangle$
 $XX \sim \# T$

$$\overline{\overline{\quad}} + \overline{\overline{\quad}} + \overline{\overline{\quad}} + \dots$$

$$\tilde{F}_0, \quad \tilde{F}_1 = K \tilde{F}_0, \quad \tilde{F}_2 = K^2 \tilde{F}_0$$

$$K = \overline{\overline{\quad}}$$

$$\tilde{F} = \frac{1}{1-K} \tilde{F}_0$$

$$\tilde{F} \supset \frac{S^2}{2c} z^2, \quad c = \frac{3}{2} N^3$$

spectrum: $\frac{1}{1 - k(h, \bar{h})}$
 $k(h, \bar{h})$ eigenvalue of K

$$\Delta - J = 2h + 2\bar{h} + \Sigma(n, J)$$

$$\Sigma(n, J) \sim O(1)$$

$$X(\partial \bar{\partial})^n \partial^J X$$

$$\Sigma \rightarrow 0$$

 $n \rightarrow \infty$
 or $J \rightarrow \infty$

- R-sym anomaly matching
- $C = 3 \sum_i (1 - 2\Delta_i) = \frac{3}{2} N^3$
- elliptic genus, S^2 partition function

$$\langle \chi(z_1) \chi(z_2) \rangle = \frac{1}{|z_1 - z_2|^{2\Delta}}$$

SD eq $\Rightarrow \Delta = \frac{1}{4}$

$$b = \frac{1}{\sqrt{2\pi J}}$$

$$\langle \chi_{a_1 a_2 a_3} \chi_{a_1 a_2 a_3} \rangle = \frac{b \beta_{a_1 a_2 a_3}}{|z_1 - z_2|^{2\Delta}}$$

$$\beta_{a_1 a_2 a_3}^{-1} = \frac{1}{N^3} \sum_{b_i=1}^N |\alpha_{a_1 b_1, a_2 b_2, a_3 b_3}|^2 \beta_{a_1 b_2 b_3} \beta_{b_1 a_2 b_3} \beta_{b_1 b_2 a_3}$$

$$Z_{a_1 a_2 a_3} = b^{-\frac{1}{2}} \beta_{a_1 a_2 a_3}^{-\frac{1}{2}} \quad \underline{\alpha = 1 + \epsilon}$$

$$\tilde{F}_z = \sum_{n=0}^{\infty} r_n(\alpha) K^n \tilde{F}_0 \equiv A(\alpha, k) \tilde{F}_0$$

$$r_0 = N^{-3} \sum_{a_i=1}^N \beta_{a_1 a_2 a_3}^2, r_1, r_2, \dots$$

spectrum:
poles of $A(\alpha, k(h, \bar{h}))$

$$A(\alpha, k) \neq \frac{r_0(\alpha)}{1-k}$$

• isotropic

$$\langle X^{a_1 a_2 a_3} X^{a_1 a_2 a_3} \rangle = \frac{b \beta_{a_1 a_2 a_3}}{|z_1 - z_2|^{2\Delta}}$$

$$\beta_{a_1 a_2 a_3}^{-1} = \frac{1}{N^3} \sum_{b_i=1}^N |\alpha_{a_1 b_1, a_2 b_2, a_3 b_3}|^2 \beta_{a_1 b_2 b_3} \beta_{b_1 a_2 b_3} \beta_{b_1 b_2 a_3}$$

$$Z_{a_1 a_2 a_3} = b^{-\frac{1}{2}} \beta_{a_1 a_2 a_3}^{-\frac{1}{2}} \quad \underline{\alpha = 1 + \epsilon}$$

$$\vec{r} = \sum_{n=0}^{\infty} r_n(\alpha) k^n \vec{r}_0 \equiv A(\alpha, k) \vec{r}_0$$

$$r_0 = N^{-3} \sum_{a_i=1}^N \beta_{a_1 a_2 a_3}^2, r_1, r_2, \dots$$

spectrum:

poles of $A(\alpha, k(h, \bar{h}))$

$$A(\alpha, k) \neq \frac{r_0(\alpha)}{1-k}$$

• isotropic

• first direction got lifted by anisotropic deformation

UV (2,2) solvable mod

$$\bar{F} = \sum_{n=0}^{\infty} r_n(\alpha) K^n \bar{F}_0 \equiv A(\alpha, K) \bar{F}_0$$

$$r_0 = N^{-3} \sum_{q_1=1}^V \beta_{q_1 q_2 q_3}^2, r_1, r_2, \dots$$

UV (2,2) solvable model
 melnic, anisotropic deformation



IR SCFT large conformality

Future directions

1.

$$\tilde{F}_2 = \sum_{n=0}^{\infty} r_n(\alpha) K^n \tilde{F}_0 \equiv A(\alpha, k) \tilde{F}_0$$

$$r_0 = N^{-3} \sum_{q_1=1}^V \beta_{q_1 q_2 q_3}^2, r_1, r_2, \dots$$

UV (2,2) solvable model
metric, anisotropic deformation

↓
IR SCFT large conformality

Future directions

1. solve $A(\alpha, k)$

⇒ How spectrum, chaos changes

2. lift stringy modes?
pure supergravity dual

3. 1d tensor model
No $O(N)^{f-1}$ sym

$$\alpha = \beta \sqrt{2222}$$

$$k_{eff} = \sum_{i=1}^n \lambda_i^2$$

4. $k_{eff} = \sum_{a_i=1}^{\infty} \sum_{a_1, a_2, a_3} \text{anomalies dim}$

 $\bar{x} \times \sim \# T$

spectrum: $\frac{1}{1 - k(h, h)}$

 $k(h, \bar{h})$ eigenvalue of K

$$\tilde{F} = \frac{1}{1-K} F$$

1

$$\xi(n, T) \sim O(1)$$

$$\bar{X}(\partial\bar{J})^n\partial\bar{J} \times$$

$$\begin{aligned} \Sigma &\rightarrow 0 \\ n &\rightarrow \infty \\ \text{or } j &\rightarrow \infty \end{aligned}$$

$$\alpha_{a_1 b_1} \alpha_{a_2 b_2} \alpha_{a_3 b_3} = \alpha_{a_2 b_2} \alpha_{a_3 b_3}$$