

Title: PSI 2018/2019 - Explorations in Cosmology - Lecture 13

Speakers: Kendrick Smith

Collection: PSI 2018/2019 - Explorations in Cosmology (Smith)

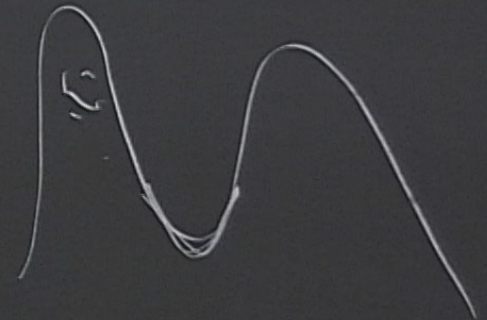
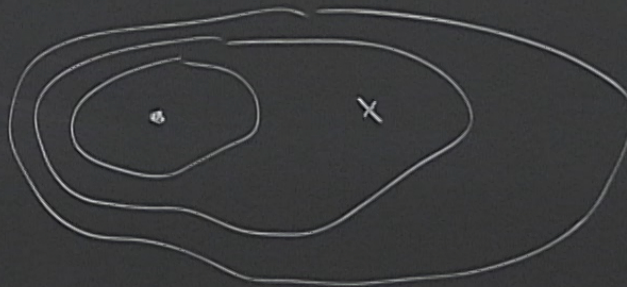
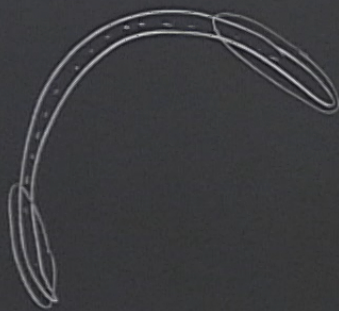
Date: May 03, 2019 - 11:30 AM

URL: <http://pirsa.org/19050007>

$$U = -\lambda \int_{-\infty}^{\infty} dt H_I(t), \quad U = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dx H_I(t, x)$$

$$L_q \sim \lambda^2 (\partial_\mu a \partial_\mu b)$$

$$\approx \lambda F^{-1}$$



CMS POLARIZATION

SCALAR FIELD: $\langle \phi(\ell)^* \phi(\ell') \rangle = C_\ell (2\pi)^2 \delta^2(\ell - \ell')$
 $\hookrightarrow$ ONLY DEPENDS ON $\ell = |\vec{\ell}|$

N SCALAR FIELDS ϕ_i : $\langle \phi_i^*(\ell) \phi_j(\ell') \rangle = C_{\ell}^{ij} (2\pi)^2 \delta^2(\ell - \ell')$
 $\hookrightarrow$ SYMMETRIC, POSITIVE SEMIDEFINITE, REAL

$\vec{e}$

$\delta(\vec{e} - \vec{e}')$

ETRIC, POSITIVE
ITE, REAL

	$\text{Re}(\phi_{e_1}) ; \text{Im}(\phi_{e_1})$	$\text{Re}(\phi_{e_2})$	$\text{Im}(\phi_{e_2})$
$\text{Re}(\psi_{e_1})$	$C_{e_1}^{is}$		
$\text{Im}(\psi_{e_1})$		$C_{e_2}^{is}$	
$\text{Re}(\psi_{e_2})$			$C_{e_2}^{is}$
$\text{Im}(\psi_{e_2})$			

CMB POLARIZATION

FLAT SKY ASSUMED:

SCALAR FIELD: $\langle \phi(\ell)^* \phi(\ell') \rangle = C_\ell (2\pi)^2 \delta^2(\ell - \ell')$
 $\hookrightarrow$ ONLY DEPENDS ON $\ell = |\vec{\ell}|$

N SCALAR FIELDS ϕ_i : $\langle \phi_i^*(\ell) \phi_j(\ell') \rangle = C_{\ell}^{ij} (2\pi)^2 \delta^2(\ell - \ell')$
 $\hookrightarrow$ SYMMETRIC, POSITIVE SEMIDEFINITE, REAL

VECTOR FIELD $V_i(\vec{x})$

"OBVIOUS" APPROACH:

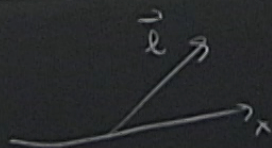
$$V_i(\vec{l}) = \begin{pmatrix} x(\vec{l}) \\ y(\vec{l}) \end{pmatrix}$$

$$\langle V_i(\vec{l})^* V_j(\vec{l}') \rangle = \begin{pmatrix} C_{\vec{l}}^{xx} & C_{\vec{l}}^{xy} \\ C_{\vec{l}}^{xy} & C_{\vec{l}}^{yy} \end{pmatrix} (2\pi)^2 \delta^2(\vec{l} - \vec{l}')$$

NOT TRUE THAT $C_{\vec{l}}^{xx}$ ONLY DEPENDS ON $l = |\vec{l}|$

BETTER APPROACH

$$\hat{l}_i = \frac{l_i}{|l|}$$



$$\vec{l} = l(\cos \phi_l \hat{x} + \sin \phi_l \hat{y})$$

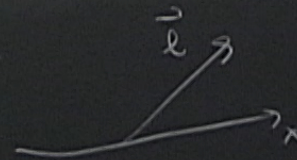
$$V_i(\vec{l}) = G(\vec{l}) \hat{l}_i + C(\vec{l}) \epsilon_{ij} \hat{l}_j$$

$$\begin{pmatrix} X(\vec{l}) \\ Y(\vec{l}) \end{pmatrix} = \begin{pmatrix} \cos(\phi_l) & -\sin(\phi_l) \\ \sin(\phi_l) & \cos(\phi_l) \end{pmatrix} \begin{pmatrix} G(\vec{l}) \\ C(\vec{l}) \end{pmatrix}$$

THEN G AND C DO NOT "MIX" UNDER ROTATIONS

BETTER APPROACH

$$\hat{e}_i = \frac{e_i}{|e|}$$



$$\vec{l} = l(\cos \phi_l \hat{e}_1 + \sin \phi_l \hat{e}_2)$$

$$V: \vec{r} \mapsto$$

$$V_i(\vec{l}) = G(\vec{l}) \hat{l}_i + C(\vec{l}) \epsilon_{ij} \hat{l}_j$$

$$\begin{pmatrix} X(\vec{l}) \\ Y(\vec{l}) \end{pmatrix} = \begin{pmatrix} \cos(\phi_l) & -\sin(\phi_l) \\ \sin(\phi_l) & \cos(\phi_l) \end{pmatrix} \begin{pmatrix} G(\vec{l}) \\ C(\vec{l}) \end{pmatrix}$$

THEN G AND C DO NOT "MIX" UNDER ROTATIONS

$$\langle G(\vec{l})^* G(\vec{l}') \rangle = C_l^{GG} (2\pi)^2 \delta^2(\vec{l} - \vec{l}')$$

$$\langle G(\vec{l})^* C(\vec{l}') \rangle = C_l^{GC} (2\pi)^2 \delta^2(\vec{l} - \vec{l}')$$

$$\langle C(\vec{l})^* C(\vec{l}') \rangle = C_l^{CC} (2\pi)^2 \delta^2(\vec{l} - \vec{l}')$$

→ ONLY DEPEND ON $\vec{l} = |\vec{l}|$

$$\langle \chi(\vec{l})^* \chi(\vec{l}') \rangle = \cos^2(\varphi_l) C_l^{GG} + 2 \sin(\varphi_l) \cos(\varphi_l) C_l^{GC} + \sin^2(\varphi_l) C_l^{CC}$$

$$(2\pi)^2 \delta^2(\ell - \ell')$$

$$(2\pi)^2 \delta^2(\ell - \ell')$$

$$(2\pi)^2 \delta^2(\ell - \ell')$$

→ ONLY DEPEND ON $\ell = |\vec{\ell}|$

$$G = 2 \sin(\varphi_\ell) \cos(\varphi_\ell) C_\ell^{GC} + \sin^2(\varphi_\ell) C_\ell^{CC}$$

$$\begin{pmatrix} X(\vec{x}) \\ Y(\vec{y}) \end{pmatrix} \longleftrightarrow \begin{pmatrix} X(\vec{\ell}) \\ Y(\vec{\ell}) \end{pmatrix} \longleftrightarrow \begin{pmatrix} G(\vec{\ell}) \\ C(\vec{\ell}) \end{pmatrix}$$

PARITY INVARIANCE $\Rightarrow$ $\boxed{C_l^{GC} = 0}$

$$V_i(\vec{l}) = G(\vec{l}) \hat{l}_i + C(\vec{l}) \varepsilon_{ij} \hat{l}_j$$

$\nearrow$
 $\begin{pmatrix} G(l) \\ C(l) \end{pmatrix}$

$$\begin{aligned} G(l) &\rightarrow G(l) \\ C(l) &\rightarrow -C(l) \end{aligned}$$

$$\begin{aligned} C_l^{GG} &\rightarrow C_l^{GG} \\ C_l^{CC} &\rightarrow C_l^{CC} \\ C_l^{GC} &\rightarrow -C_l^{GC} \end{aligned}$$

$$(2\pi)^2 \delta^2(l-l')$$

$$\begin{pmatrix} C_l^{cc} & 0 \\ 0 & C_l^{cc} \end{pmatrix}$$

$$(2\pi)^2 \delta^2(l-l')$$

$$(2\pi)^2 \delta^2(l-l')$$

→ ONLY DEPEND ON $l = |\vec{l}|$

$$G = 2 \sin(\varphi_l) \cos(\varphi_l) C_l^{cc} + \sin^2(\varphi_l) C_l^{cc}$$

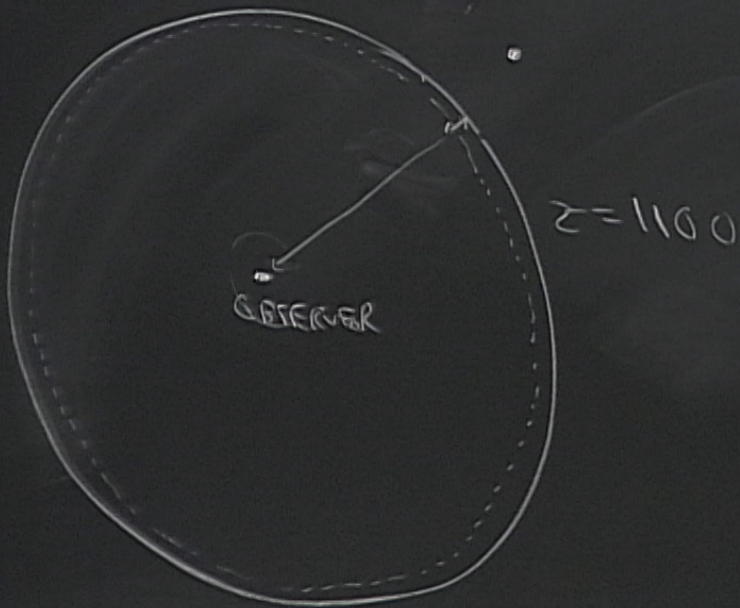
$$\begin{pmatrix} X(\vec{x}) \\ Y(\vec{y}) \end{pmatrix} \longleftrightarrow \begin{pmatrix} X(\vec{l}) \\ Y(\vec{l}) \end{pmatrix} \longleftrightarrow$$

IN REAL SPACE, THIS IS THE GRADIENT/CURL DECOMPOSITION:

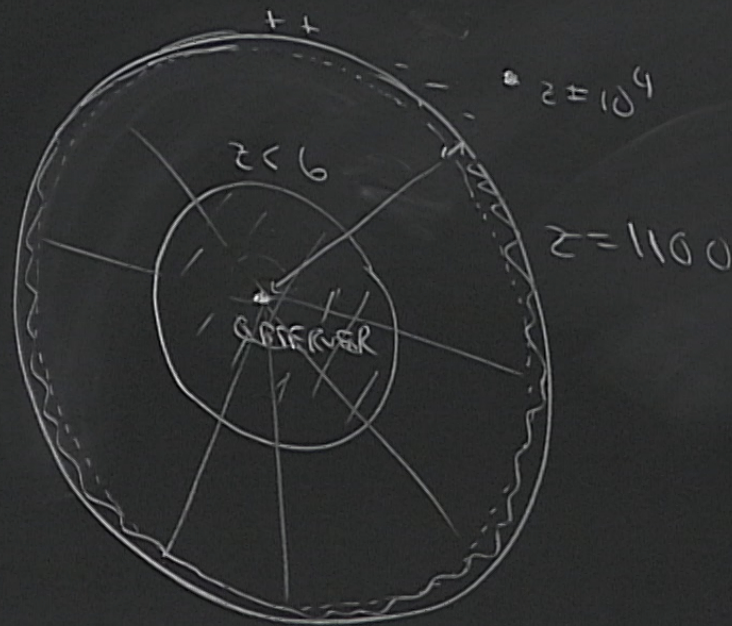
$$V_i(\vec{x}) = \partial_i \underbrace{g(x)}_{\text{SCALAR}} + \underbrace{\epsilon_{ij}}_{\text{PSEUDO SCALAR}} \partial_j \underbrace{c(x)}_{\text{PSEUDO SCALAR}}$$

CMB IS LINEARLY POLARIZED

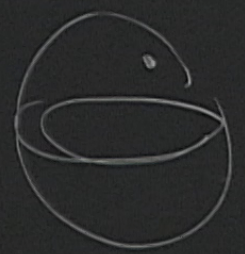
+  + "THOMSON" SCATTERING



+  + "THOMSON" SCATTERING



LINEAR POLARIZATION IS DESCRIBED
 COVARIANTLY AS A TRACELESS SYMMETRIC
 TENSOR $P_{ij}(\vec{k})$

$$I_{ij} = \begin{pmatrix} T+Q & U+iV \\ U-iV & T-Q \end{pmatrix} = \langle E_i(\omega)^* E_j(\omega) \rangle_\omega$$

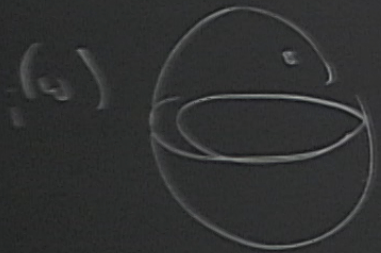
$E_i(\omega)$

THE POLARIZATION IS DESCRIBED

QUANTITATIVELY AS A TRACELESS SYMMETRIC

tensor $P_{ij}(\vec{k})$

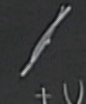
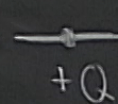
$$I_{ij} = \begin{pmatrix} T+Q & U+iV \\ U-iV & T-Q \end{pmatrix} = \langle E_i(\omega)^* E_j(\omega) \rangle_\omega$$



$P_{ij}(\hat{n})$ SYMMETRIC & TRACELESS ($g^{ij}P_{ij}=0$)

$$P_{ij}(x) = Q(x) \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} + U(x) \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$$

"Q + U ARE 'STOKES PARAMETERS'"



$$P_j(\vec{l}) = E(\vec{l}) \left[\hat{l}_i \hat{l}_j - \frac{1}{2} \delta_{ij} \right] + B(\vec{l}) \left[\frac{1}{2} \epsilon_{ik} \hat{l}_j \hat{l}_k + \frac{1}{2} \epsilon_{jk} \hat{l}_i \hat{l}_k \right]$$

$$\begin{pmatrix} Q(\vec{l}) \\ U(\vec{l}) \end{pmatrix} = \begin{pmatrix} \cos(2\varphi_e) & -\sin(2\varphi_e) \\ \sin(2\varphi_e) & \cos(2\varphi_e) \end{pmatrix} \begin{pmatrix} E(\vec{l}) \\ B(\vec{l}) \end{pmatrix}$$

POWER SPECTRA

$$\begin{pmatrix} C_e^{EE} & C_e^{EB} \\ C_e^{EB} & C_e^{EE} \end{pmatrix} \quad \text{ONLY DEPEND ON } l = |\vec{l}|$$

PARITY INVARIANCE $\Rightarrow C_l^{EB} = 0$

CMB POWER SPECTRA: T, E, B

$$\begin{pmatrix} C_l^{TT} & C_l^{TE} & 0 \\ C_l^{TE} & C_l^{EE} & 0 \\ 0 & 0 & C_l^{BB} \end{pmatrix}$$

IN REAL SPACE, E/B DECOMP IS

$$P_{ij}(x) = \left(\partial_i \partial_j - \frac{1}{2} \delta_{ij} \right) e(x) + \left(\frac{1}{2} \varepsilon_{ik} \partial_i \partial_k + \frac{1}{2} \varepsilon_{jk} \partial_j \partial_k \right) b(x)$$

↓
SCALAR

↘ PSEUDOSCALAR

CURVED SKY:

$$P_{ij}(\hat{n}) = \sum_{lm} a_{lm}^E \left(Y_{ij}^{E(lm)}(\hat{n}) \right) + \sum_{lm} a_{lm}^B \left(Y_{ij}^{B(lm)}(\hat{n}) \right)$$

$$Y_{ij}^{E(lm)}(\hat{n}) = \frac{1}{\sqrt{(l+2)(l+1)l(l-1)}} \left(\nabla_i \nabla_j - \frac{1}{2} g_{ij} \right) Y_{lm}(\hat{n})$$

$$Y_{ij}^{B(lm)}(\hat{n}) = \frac{1}{\sqrt{(l+2)(l+1)l(l-1)}} \left(\frac{1}{2} \varepsilon_{ik} \nabla_j \nabla^k + \frac{1}{2} \varepsilon_{jk} \nabla_i \nabla^k \right) Y_{lm}(\hat{n})$$

B_{ij}

$$\left[\begin{array}{c} \text{SCALAR INITIAL} \\ \text{CONDITIONS} \end{array} \right] + \left[\begin{array}{c} \text{LINEAR PERTURBATION} \\ \text{THEORY} \end{array} \right] \Rightarrow \left[\begin{array}{c} \text{NO B-MODES} \\ \text{IN CMB} \end{array} \right]$$

LINEAR THEORY $\Rightarrow$ CONSIDER $\vec{k} = k \hat{z}$

LINEAR THEORY $\Rightarrow$ CONSIDER $\vec{k} = k \hat{z}$

CMB POLARIZATION INVARIANT UNDER ROTATIONS AROUND $\hat{z}$
 + REFLECTION $x \rightarrow (-x)$

$$\Rightarrow P_{ij}(\theta) = E_{ij} e(\theta) + B_{ij} b(\theta)$$

$\begin{matrix} \text{DENSITY} & \text{SCALAR} & \text{PSEUDOSCALAR} \end{matrix}$

$$\Rightarrow \boxed{b=0}$$

$$\Rightarrow P_{ij}(\theta) = E_{ij} e(\theta) - B_{ij} b(\theta)$$