Title: PSI 2018/2019 - Explorations in Quantum Information - Lecture 13

Speakers: Eduardo Martin-Martinez

Collection: PSI 2018/2019 - Explorations in Quantum Information (Martin-Martinez)

Date: May 02, 2019 - 9:00 AM

URL: http://pirsa.org/19050003

Quantum Energy Teleportation

(Masahiro Hotta)

$$\begin{tikzcd} A \arrow[rr, dash] & & B \\ M(\alpha) \arrow[u] \arrow[r] & \alpha \arrow[r] & U_B(\alpha) \arrow[u] \end{tikzcd}$$

$$\Delta E_{U_\alpha} < 0$$

$$\hat{H}_A =$$

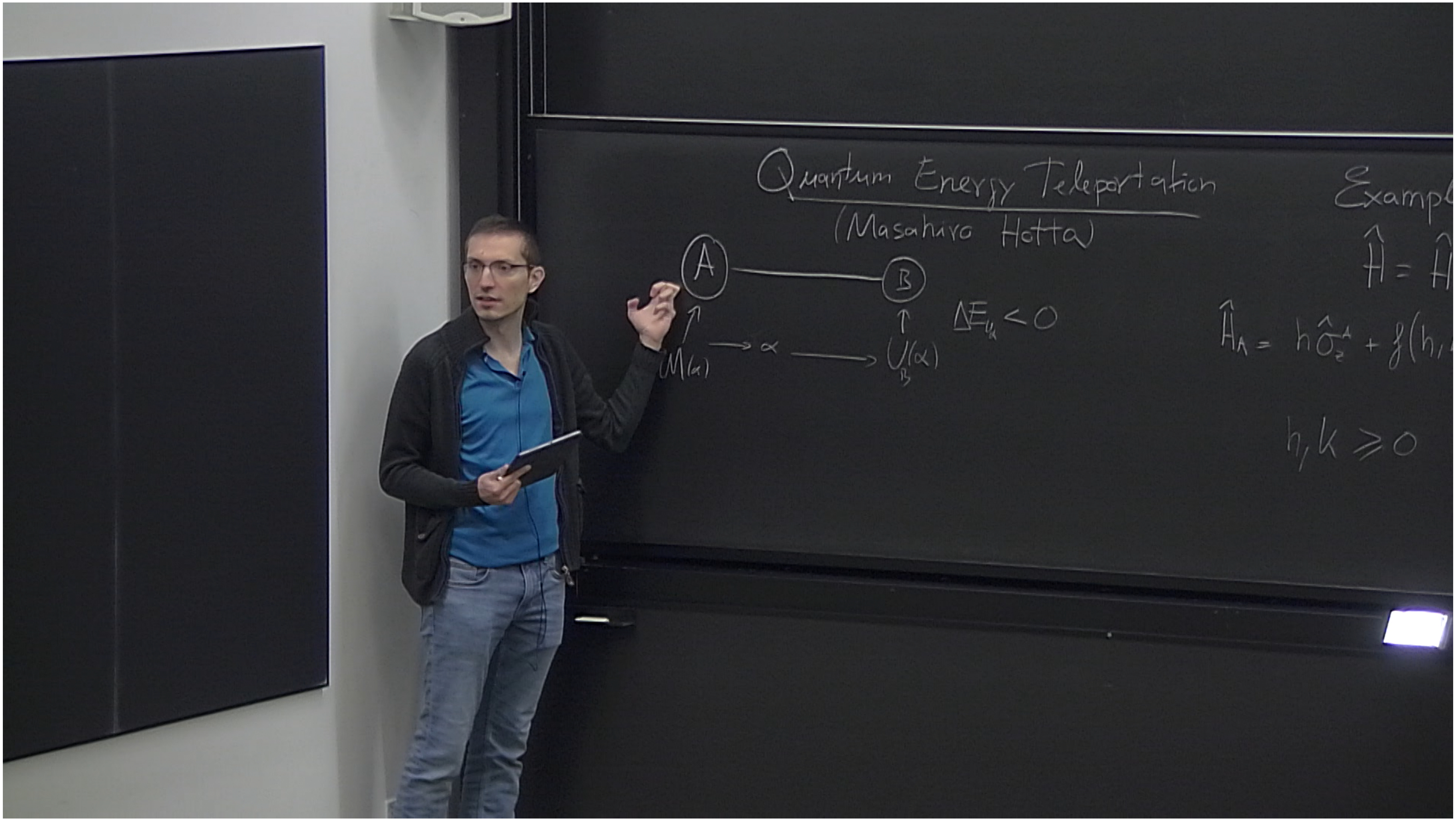
...tion

Example (Minimal QET)   2 two-level systems

$$\hat{H} = \hat{H}_A + \hat{H}_B + \hat{V}$$

$$\hat{H}_A = h\hat{\sigma}_z^A + f(h,k)\,\mathbb{1} \qquad \hat{H}_B = h\hat{\sigma}_z^B + f(h,k)\,\mathbb{1} \qquad \hat{V} = 2\left[k\hat{\sigma}_x^A\hat{\sigma}_x^B + \frac{k^2}{h^2} f(h,k)\,\mathbb{1}\right]$$

$$h,k \geqslant 0 \qquad f(h,k) = \frac{h^2}{\sqrt{h^2+k^2}}$$


Quantum Energy Teleportation
(Masahiro Hotta)
A
B
ΔE < 0
M(α) → α → U(α)
h, k ≥ 0

nal QFT)

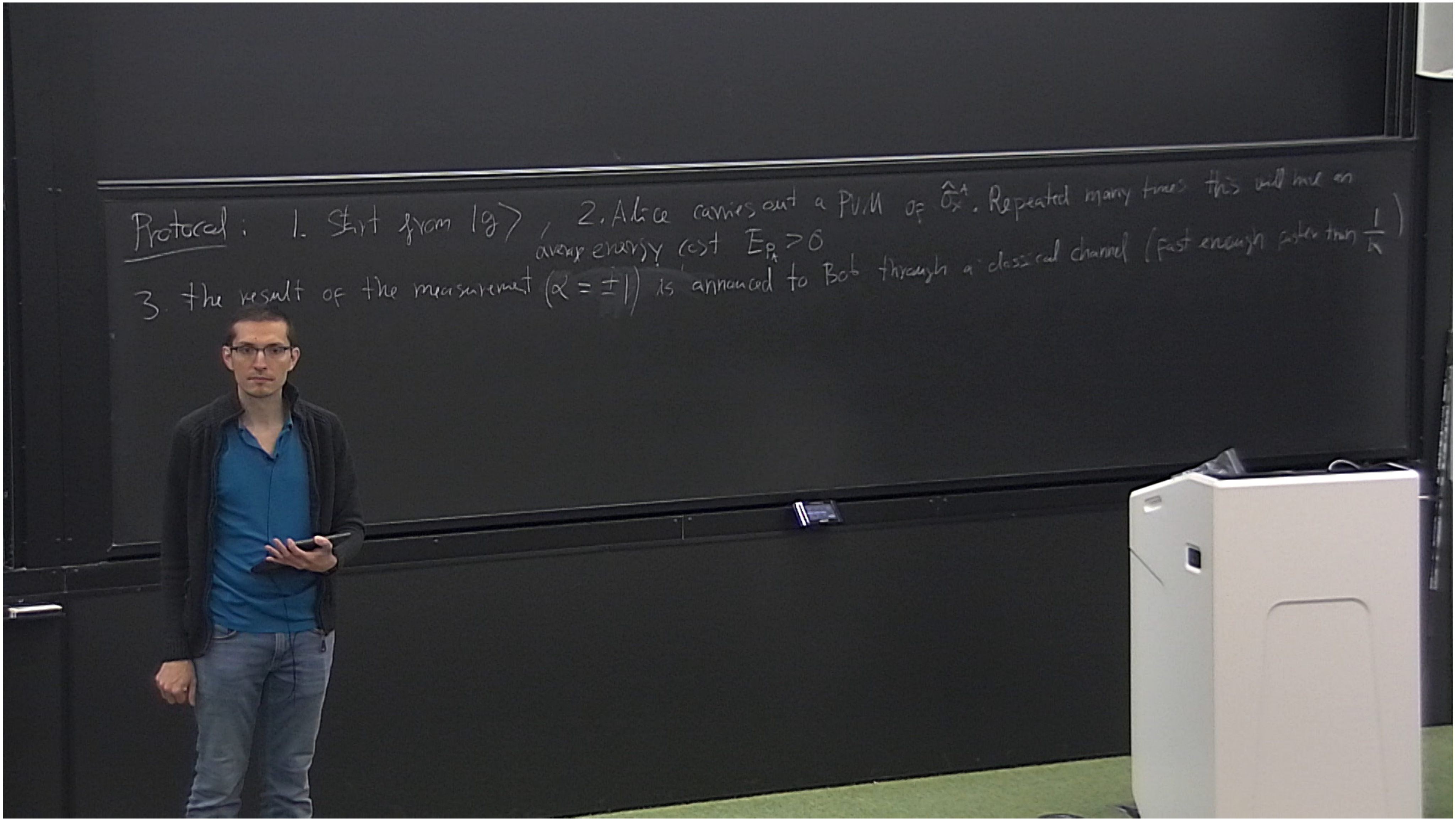
$\hat{V}$

$$\hat{H}_B = h\,\hat{\sigma}_z^B + f(h,k)\,\mathbb{1} \qquad \hat{V} = 2\left[k\,\hat{\sigma}_x^A\,\hat{\sigma}_x^B + \frac{k^2}{h^2} f(h,k)\,\mathbb{1}\right]$$

$$= \frac{h^2}{\sqrt{h^2+k^2}}$$

$$|g\rangle = \frac{1}{\sqrt{2}}\left(\sqrt{1-\frac{f(h,k)}{h}}\,|1\rangle_A|1\rangle_B - \sqrt{1+\frac{f(h,k)}{h}}\,|0\rangle_A|0\rangle_B\right)$$

$$\sigma_z^\nu|1\rangle_\nu = |1\rangle_\nu \qquad \sigma_z^\nu|0\rangle_\nu = -|0\rangle_\nu$$


Protocol: 1. Start from $|g\rangle$, 2. Alice carries out a PVM of $\hat{\sigma}_x^A$. Repeated many times this will have an average energy cost $E_{P_A} > 0$
3. the result of the measurement $(\alpha = \pm 1)$ is announced to Bob through a classical channel (fast enough faster than $\frac{1}{h}$)

Example (Minimal QET) 2 two-level systems $E \langle \Delta U \rangle = tr(\rho_2 \hat{H}) - tr(\rho_1 \hat{H})$

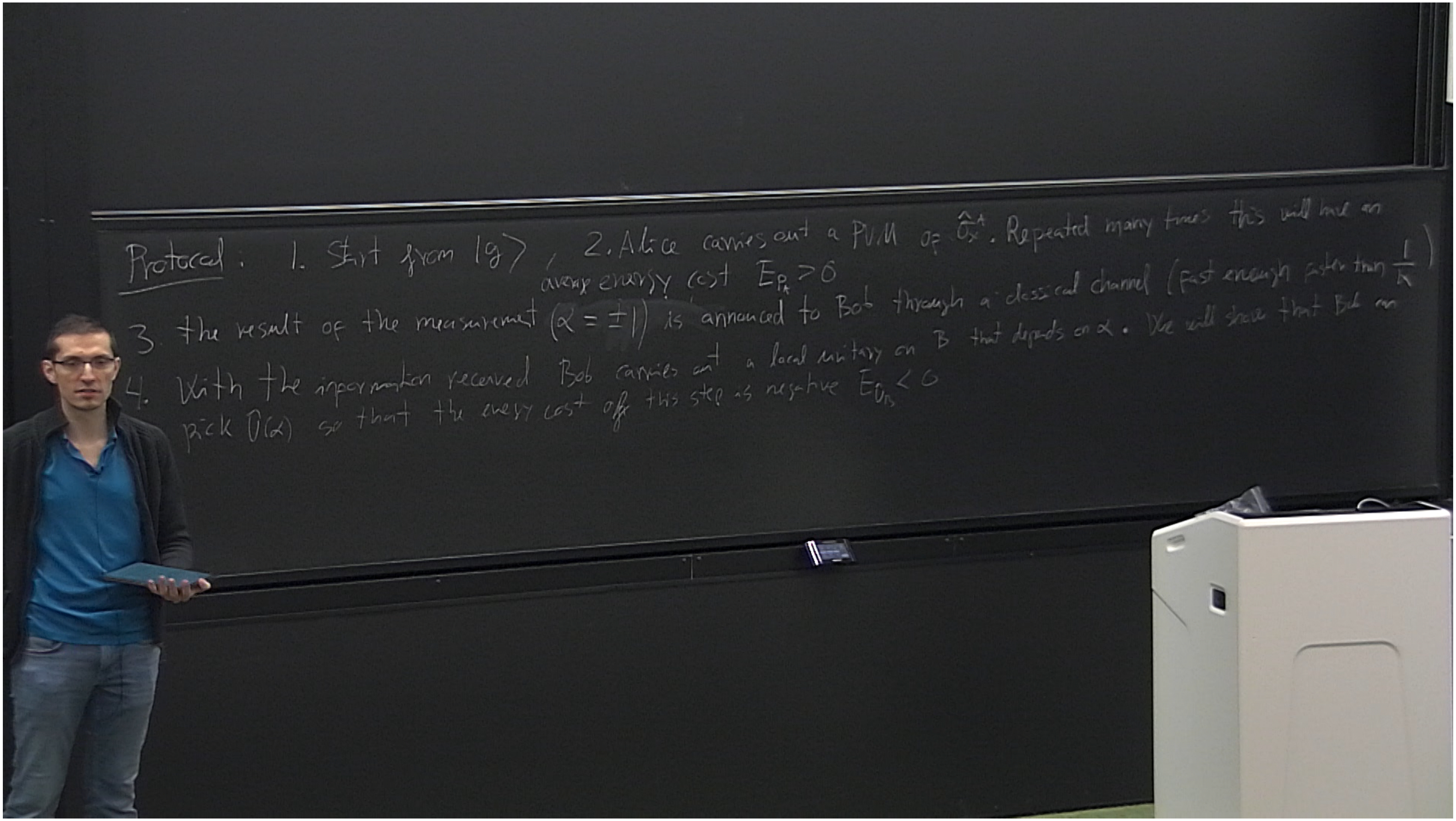
$$\hat{H} = \hat{H}_A + \hat{H}_B + \hat{V}$$

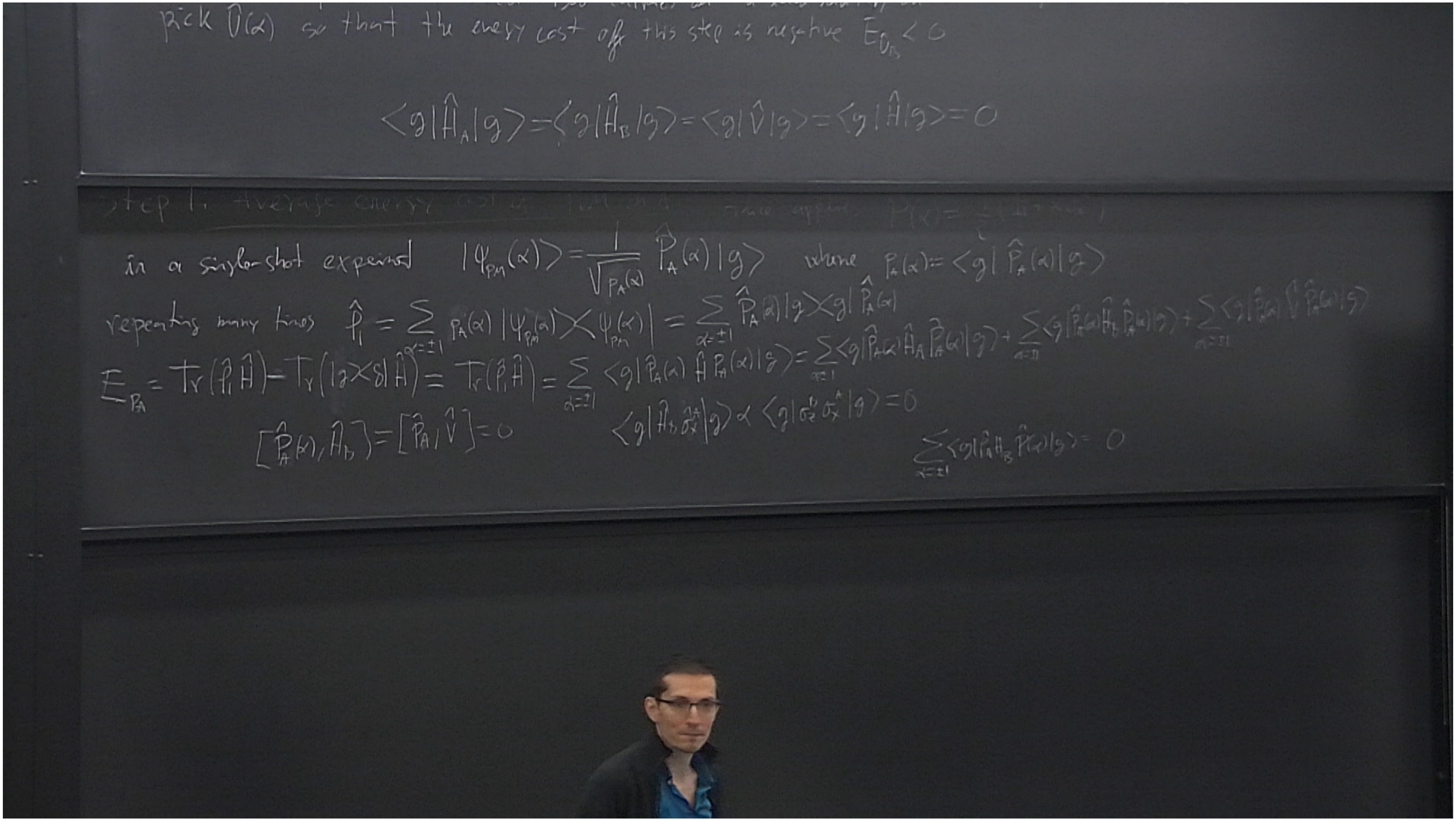
$$\hat{\sigma}_z^A + f(h,k)\mathbb{1} \qquad \hat{H}_B = h\hat{\sigma}_z^B + f(h,k)\mathbb{1} \qquad \hat{V} = 2\left[k\hat{\sigma}_x^A\hat{\sigma}_x^B + \frac{k^2}{h^2} f(h,k)\mathbb{1}\right]$$

$$k \geq 0 \qquad f(h,k) = \frac{h^2}{\sqrt{h^2+k^2}}$$

$$|g\rangle = \frac{1}{\sqrt{2}}\left(\sqrt{1 - \frac{f(h,k)}{h}}\,|1\rangle_A|1\rangle_B - \sqrt{1 + \frac{f(h,k)}{h}}\,|0\rangle_A|0\rangle_B\right)$$

$$\sigma_z^\nu|1\rangle_\nu = |1\rangle_\nu \qquad \sigma_z^\nu|0\rangle_\nu = -|0\rangle_\nu$$


Protocol: 1. Start from $|g\rangle$, 2. Alice carries out a PVM of $\hat{\sigma}_x^A$. Repeated many times this will have an average energy cost $E_{P_A} > 0$
3. the result of the measurement $(\alpha = \pm 1)$ is announced to Bob through a classical channel (fast enough faster than $\frac{1}{h}$)
4. With the information received Bob carries out a local unitary on B that depends on α. We will show that Bob can pick $\hat{U}(\alpha)$ so that the energy cost of this step is negative $E_{U_B} < 0$


pick $\hat{U}(\alpha)$ so that the energy cost of this step is negative $E_{U_B} < 0$
$\langle g|\hat{H}_A|g\rangle = \langle g|\hat{H}_B|g\rangle = \langle g|\hat{V}|g\rangle = \langle g|\hat{H}|g\rangle = 0$
in a single-shot experiment $|\psi_{PM}(\alpha)\rangle = \frac{1}{\sqrt{p_A(\alpha)}}\hat{P}_A(\alpha)|g\rangle$ where $p_A(\alpha) := \langle g|\hat{P}_A(\alpha)|g\rangle$
repeating many times $\hat{\rho}_1 = \sum_{\alpha=\pm1} p_A(\alpha)|\psi_{PM}(\alpha)\rangle\langle\psi_{PM}(\alpha)| = \sum_{\alpha=\pm1}\hat{P}_A(\alpha)|g\rangle\langle g|\hat{P}_A(\alpha)$
$E_{P_A} = \mathrm{Tr}(\hat{\rho}_1\hat{H}) - \mathrm{Tr}(|g\rangle\langle g|\hat{H}) = \mathrm{Tr}(\hat{\rho}_1\hat{H}) = \sum_{\alpha=\pm1}\langle g|\hat{P}_A(\alpha)\hat{H}\hat{P}_A(\alpha)|g\rangle = \sum_{\alpha=\pm1}\langle g|\hat{P}_A(\alpha)\hat{H}_A\hat{P}_A(\alpha)|g\rangle + \sum_{\alpha=\pm1}\langle g|\hat{P}_A(\alpha)\hat{H}_B\hat{P}_A(\alpha)|g\rangle + \sum_{\alpha=\pm1}\langle g|\hat{P}_A(\alpha)\hat{V}\hat{P}_A(\alpha)|g\rangle$
$[\hat{P}_A(\alpha), \hat{H}_B] = [\hat{P}_A, \hat{V}] = 0$
$\langle g|\hat{H}_B\sigma_x^A|g\rangle \propto \langle g|\sigma_z^B\sigma_x^A|g\rangle = 0$
$\sum_{\alpha=\pm1}\langle g|\hat{P}_A\hat{H}_B\hat{P}_A(\alpha)|g\rangle = 0$

Step 1. Average energy cost of

in a single-shot experiment $|\psi_{PM}(\alpha)\rangle = \frac{1}{\sqrt{p_A(\alpha)}} \hat{P}_A(\alpha)|g\rangle$ where $p_A(\alpha) := \langle g|\hat{P}_A(\alpha)|g\rangle$

repeating many times $\hat{\rho}_1 = \sum_{\alpha=\pm1} p_A(\alpha)|\psi_{PM}(\alpha)\rangle\langle\psi_{PM}(\alpha)| = \sum_{\alpha=\pm1} \hat{P}_A(\alpha)|g\rangle\langle g|\hat{P}_A(\alpha)$

$E_{P_A} = \mathrm{Tr}(\hat{\rho}_1\hat{H}) - \mathrm{Tr}(|g\rangle\langle g|\hat{H}) = \mathrm{Tr}(\hat{\rho}_1\hat{H}) = \sum_{\alpha=\pm1}\langle g|\hat{P}_A(\alpha)\hat{H}\hat{P}_A(\alpha)|g\rangle = \sum_{\alpha=\pm1}\langle g|\hat{P}_A(\alpha)\hat{H}_A\hat{P}_A(\alpha)|g\rangle + \sum_{\alpha=\pm1}\langle g|\hat{P}_A(\alpha)\hat{H}_B\hat{P}_A(\alpha)|g\rangle + \sum_{\alpha=\pm1}\langle g|\hat{P}_A(\alpha)\hat{V}\hat{P}_A(\alpha)|g\rangle$

$[\hat{P}_A(\alpha), \hat{H}_B] = [\hat{P}_A, \hat{V}] = 0$ $\qquad \langle g|\hat{H}_B\hat{\sigma}_x^A|g\rangle \propto \langle g|\sigma_z^B\sigma_x^A|g\rangle = 0$

$\sum_{\alpha=\pm1}\langle g|\hat{P}_A\hat{H}_B\hat{P}(\alpha)|g\rangle = 0$

$\boxed{E_{P_A} = \mathrm{Tr}(\hat{\rho}_1\hat{H}) = f(h,k) > 0}$

Steps 2, 3 LOCC Bob performs a α-dependent unitary $\hat{U}_B(\alpha) = \cos\theta\,\mathbb{1} - i\alpha\sin\theta\,\sigma_y^B$

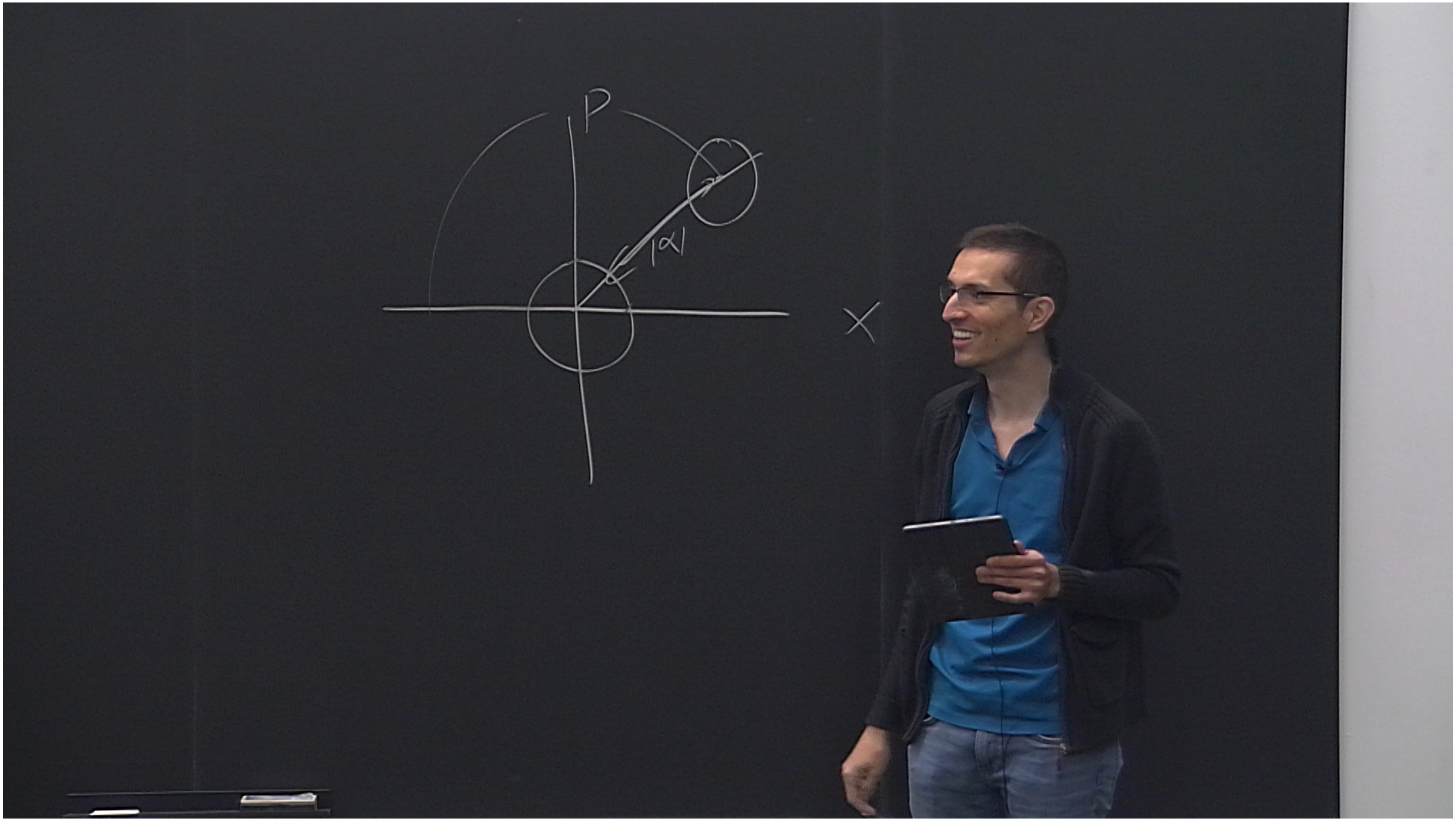
θ is such that $\cos(2\theta) = \frac{h^2+2k^2}{\sqrt{(h^2+2k^2)^2+h^2k^2}}$ $\qquad \sin(2\theta) = \frac{hk}{\sqrt{(\ \ )^2+h^2k^2}}$

in a single-shot experiment $\hat{U}_B(\alpha)|\psi_{PM}(\alpha)\rangle = \frac{1}{\sqrt{p_A(\alpha)}}\hat{U}_B(\alpha)\hat{P}_A(\alpha)|g\rangle \xRightarrow{\text{repeated many times}} \hat{\rho}_2 = \sum_{\alpha=\pm1} U_B(\alpha)\hat{P}_A(\alpha)|g\rangle\langle g|\hat{P}_A(\alpha)\hat{U}_B^\dagger(\alpha)$

$E_{U_B} = \mathrm{Tr}(\hat{\rho}_2\hat{H}) - \mathrm{Tr}(\hat{\rho}_1\hat{H}) = \mathrm{Tr}(\rho_2 H) - E_{P_A}$, using $[\hat{U}_B(\alpha), \hat{H}_A] = 0$ $\qquad [\hat{P}_A(\alpha), \hat{H}_B] = [P_A(\alpha), V] = 0$

$E_{U_B} = \sum_{\alpha=\pm1}\langle g|P_A(\alpha)\hat{U}_B^\dagger(\alpha)[\hat{H}_B + \hat{V}]U(\alpha)|g\rangle$

$\Rightarrow E_{U_B} = \frac{-1}{h^2+k^2}\left[hk\sin(2\theta) - (h^2+2k^2)(1-\cos(2\theta))\right]$ if we pick $0 < \theta \ll 1$ $\qquad E_{U_B} \approx \frac{-2hk\theta}{h^2+k^2} < 0$


P
|α|
X

Step 1: Average energy cost of [illegible]

in a single-shot experiment $|\psi_{PM}(\alpha)\rangle = \frac{1}{\sqrt{p_A(\alpha)}} \hat{P}_A(\alpha)|g\rangle$ where $p_A(\alpha) := \langle g|\hat{P}_A(\alpha)|g\rangle$

repeating many times $\hat{\rho}_1 = \sum_{\alpha=\pm1} p_A(\alpha)\,|\psi_{PM}(\alpha)\rangle\langle\psi_{PM}(\alpha)| = \sum_{\alpha=\pm1} \hat{P}_A(\alpha)|g\rangle\langle g|\hat{P}_A(\alpha)$

$$E_{P_A} = \mathrm{Tr}(\hat{\rho}_1\hat{H}) - \mathrm{Tr}(|g\rangle\langle g|\hat{H}) = \mathrm{Tr}(\hat{\rho}_1\hat{H}) = \sum_{\alpha=\pm1}\langle g|\hat{P}_A(\alpha)\,\hat{H}\,\hat{P}_A(\alpha)|g\rangle = \sum_{\alpha=\pm1}\langle g|\hat{P}_A(\alpha)\hat{H}_A\hat{P}_A(\alpha)|g\rangle + \sum_{\alpha=\pm1}\langle g|\hat{P}_A(\alpha)\hat{H}_B\hat{P}_A(\alpha)|g\rangle + \sum_{\alpha=\pm1}\langle g|\hat{P}_A(\alpha)\hat{V}\hat{P}_A(\alpha)|g\rangle$$

$$[\hat{P}_A(\alpha), \hat{H}_B] = [\hat{P}_A, \hat{V}] = 0$$

$$\langle g|\hat{H}_B\hat{\sigma}_x^A|g\rangle \propto \langle g|\sigma_z^B\sigma_x^A|g\rangle = 0$$

$$\sum_{\alpha=\pm1}\langle g|\hat{P}_A\hat{H}_B\hat{P}(\alpha)|g\rangle = 0$$

$$\boxed{E_{P_A} = \mathrm{Tr}(\rho_1\hat{A}) = g(h,k) > 0}$$

Steps 2,3 LOCC Bob performs a α-dependent unitary $\hat{U}_B(\alpha) = \cos\theta\,\mathbb{1} - i\alpha\sin\theta\,\sigma_y^B$

θ is such that $\cos(2\theta) = \frac{h^2+2k^2}{\sqrt{(h^2+2k^2)^2 + h^2k^2}}$ $\sin(2\theta) = \frac{hk}{\sqrt{(\;)^2 + h^2k^2}}$

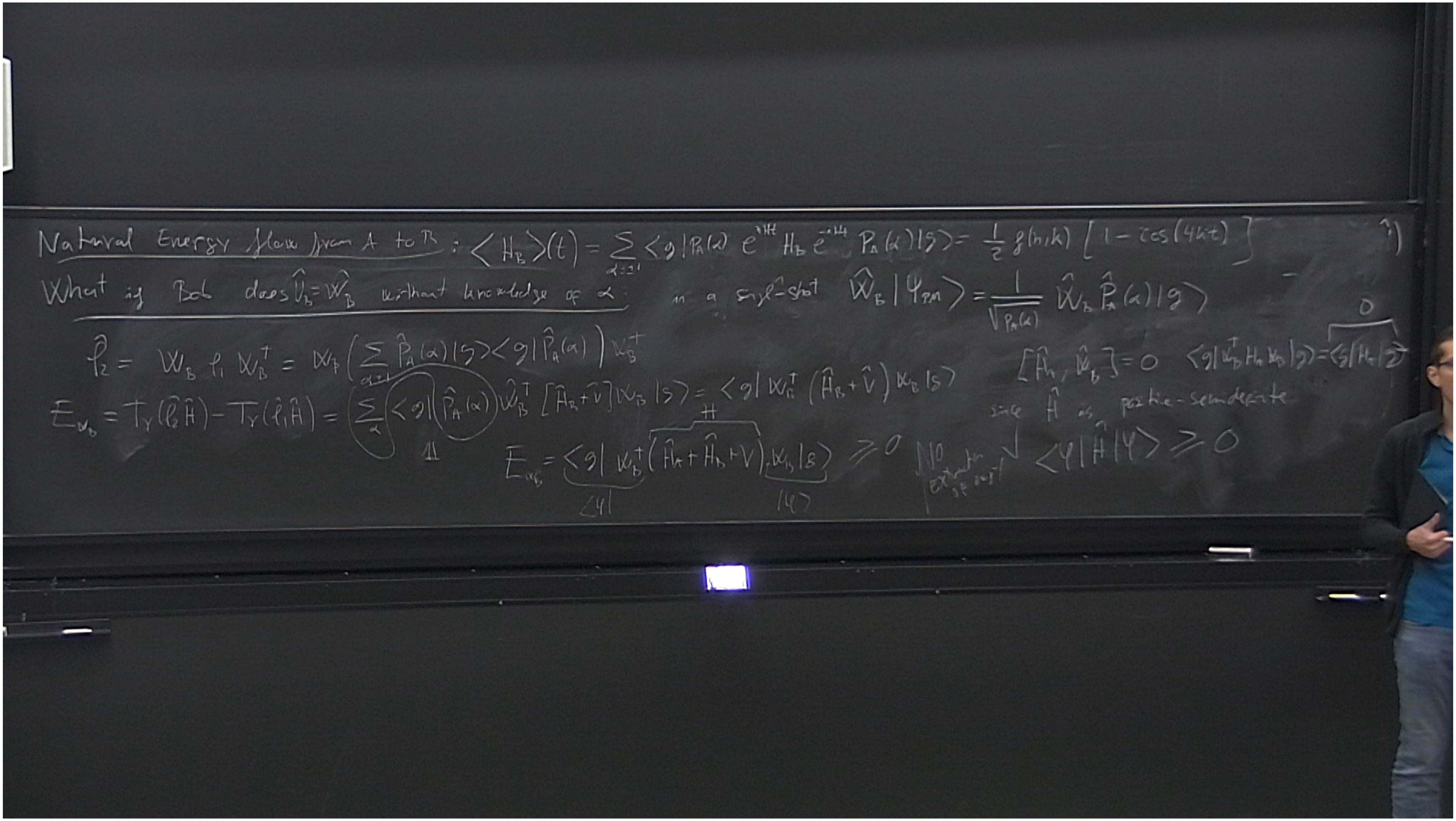
in a single-shot experiment $\hat{U}_B(\alpha)|\psi_{PM}(\alpha)\rangle = \frac{1}{\sqrt{p_A(\alpha)}}\hat{U}_B(\alpha)\hat{P}_A(\alpha)|g\rangle \overset{\text{repeated many times}}{\Longrightarrow} \hat{\rho}_2 = \sum_{\alpha=\pm1} U_B(\alpha)\hat{P}_A(\alpha)|g\rangle\langle g|\hat{P}_A(\alpha)\hat{U}_B^\dagger(\alpha)$

$$E_{U_B} = \mathrm{Tr}(\hat{\rho}_2\hat{H}) - \mathrm{Tr}(\hat{\rho}_1\hat{H}) = \mathrm{Tr}(\rho_2 H) - E_{P_A}\,,\quad \text{using } [\hat{U}_B(\alpha), \hat{H}_A] = 0 \quad [\hat{P}_A(\alpha), \hat{H}_B] = [P_A(\alpha), V] = 0$$

$$E_{U_B} = \sum_{\alpha=\pm1}\langle g|P_A(\alpha)\hat{U}_B^\dagger(\alpha)[\hat{H}_B + \hat{V}]U(\alpha)|g\rangle$$

$$\Rightarrow E_{U_B} = \frac{-1}{h^2+k^2}\left[hk\sin(2\theta) - (h^2+2k^2)(1-\cos(2\theta))\right]$$

if we pick $0 < \theta \ll 1$ $\quad E_{U_B} \approx \frac{-2hk\theta}{h^2+k^2} < 0$



Natural Energy flow from A to B : ⟨H_B⟩(t) = Σ_{α=±1} ⟨g|P_A(α) e^{iHt} H_B e^{-iHt} P_A(α)|g⟩ = 1/2 g(h,k) [1 − cos(4kt)]
What if Bob does Û_B = Ŵ_B without knowledge of α
in a single-shot Ŵ_B |Ψ_PM⟩ = 1/√(p_A(α)) Ŵ_B P̂_A(α)|g⟩
ρ̂_2 = W_B ρ_1 W_B^† = W_B (Σ_{α=±1} P̂_A(α)|g⟩⟨g|P̂_A(α)) W_B^†
[Ĥ_A, Ŵ_B] = 0
⟨g|W_B^† H_A W_B|g⟩ = ⟨g|H_A|g⟩
0
E_{W_B} = Tr(ρ̂_2 Ĥ) − Tr(ρ̂_1 Ĥ) = Σ_α ⟨g|P̂_A(α) Ŵ_B^† [Ĥ_B + V̂] W_B |g⟩ = ⟨g| W_B^† (Ĥ_B + V̂) W_B |g⟩
1
since Ĥ is positive-semidefinite
H
E_{W_B} = ⟨g| W_B^† (Ĥ_A + Ĥ_B + V) W_B |g⟩ ≥ 0
⟨ψ|
|ψ⟩
No extraction of energy!
⟨ψ|Ĥ|ψ⟩ ≥ 0