

Title: Quantum Many-Body Scars and Space-Time Crystalline Order from Magnon Condensation

Speakers:

Series: Condensed Matter

Date: April 30, 2019 - 3:30 PM

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Abstract: We study the eigenstate properties of a nonintegrable spin chain that was recently realized experimentally in a Rydberg-atom quantum simulator. In the experiment, long-lived coherent many-body oscillations were observed only when the system was initialized in a particular product state. This pronounced coherence has been attributed to the presence of special "scarred" eigenstates with nearly equally-spaced energies and putative nonergodic properties despite their finite energy density. In this paper we uncover a surprising connection between these scarred eigenstates and low-lying quasiparticle excitations of the spin chain. In particular, we show that these eigenstates can be accurately captured by a set of variational states containing a macroscopic number of magnons with momentum $\hbar\epsilon$. This leads to an interpretation of the scarred eigenstates as finite-energy-density condensates of weakly interacting $\hbar\epsilon$ -magnons. One natural consequence of this interpretation is that the scarred eigenstates possess long-range order in both space and time, providing a rare example of the spontaneous breaking of continuous time-translation symmetry. We verify numerically the presence of this space-time crystalline order and explain how it is consistent with established no-go theorems precluding its existence in ground states and at thermal equilibrium.

Quantum many-body scars and space-time crystalline order from magnon condensation

Thomas Iadecola

Perimeter Institute
30 April 2019

TI, M. Schecter, and S. Xu, arXiv:1903.10517



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Quantum dynamics


Fundamental: Many-body physics beyond ground states

Many new experiments, important questions for quantum info:

When and how is quantum information lost?
How can it be retained?

Quantum many-body scars

A new regime of non-ergodic quantum dynamics?

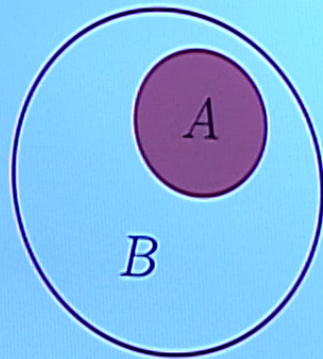
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Quantum ergodicity

Isolated system: "quantum quench"



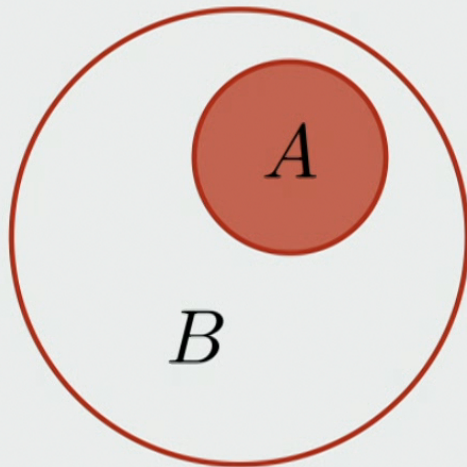
"Ergodic" dynamics: relaxes to a locally thermal state regardless of the initial condition



Quantum ergodicity

Key metric: entanglement entropy

$$S_A = -\text{tr} \rho_A \ln \rho_A$$



ETH prediction:

$$\rho_A \sim e^{-\beta H}$$

\Downarrow (if $\beta < \infty$)

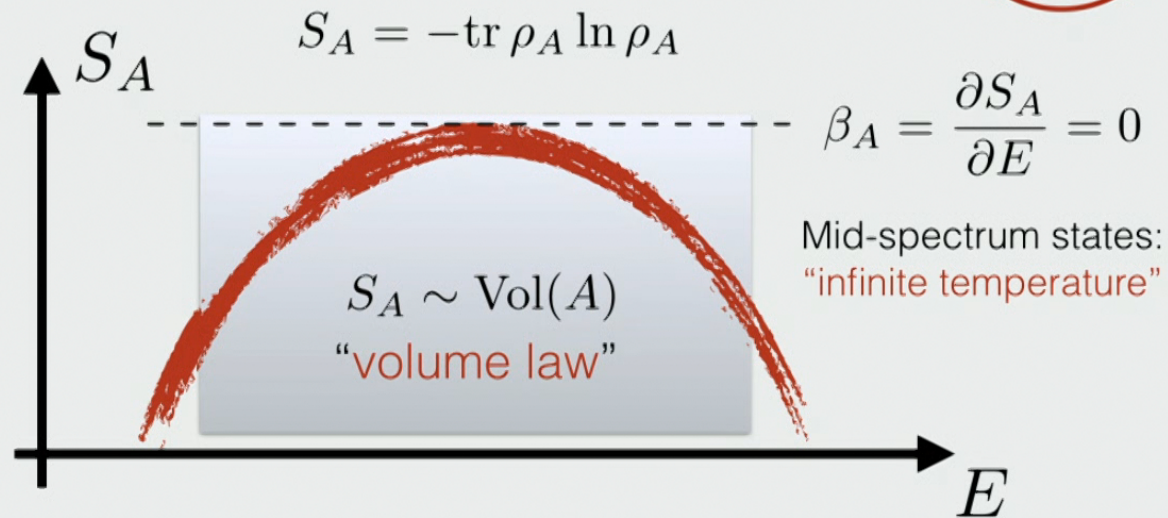
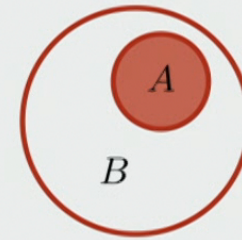
$$S_A \sim S_{\text{thermal}} \sim \text{Vol}(A)$$

(i.e., thermal entropy is extensive at finite temperature)

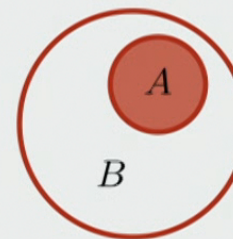


Quantum ergodicity

Key metric: entanglement entropy

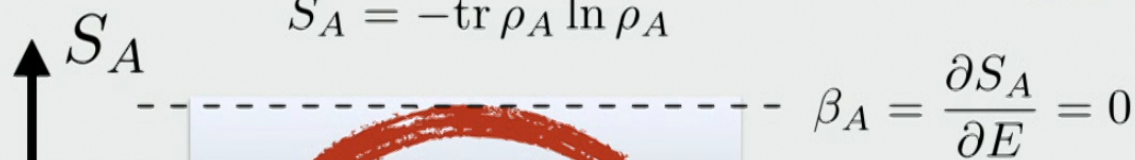


Quantum ergodicity



Key metric: entanglement entropy

$$S_A = -\text{tr} \rho_A \ln \rho_A$$



Rest of this talk: 1D systems

volume law



$$S_A \sim \text{Vol}(\partial A)$$

“area law”
($T \rightarrow 0^\pm$)

Ex: gapped ground states of 1D systems, $S_A \sim \text{const.}$

Hastings, JSTAT P08024 (2007)



ETH: *Every* eigenstate is “thermal,”
all finite-energy-density eigenstates
are “volume-law”

Appears to be quite common for
sufficiently “generic” many-body
Hamiltonians

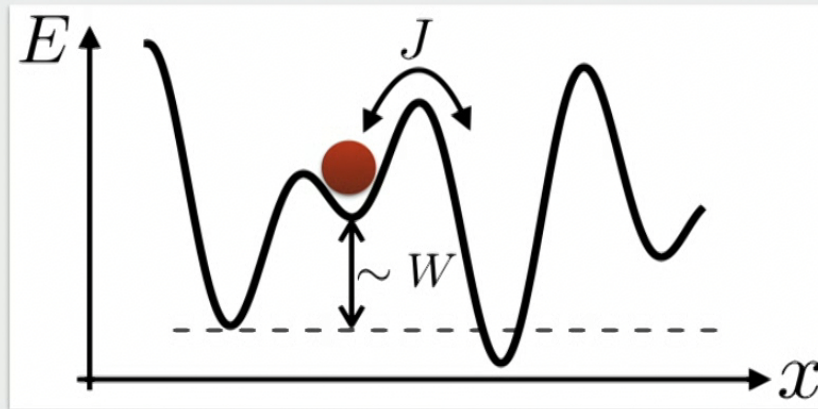
Are there exceptions to this “rule?”

Yes! Many-body localization (**MBL**):
All states are area-law!



What is MBL?

Starting point: Anderson localization (single particle)



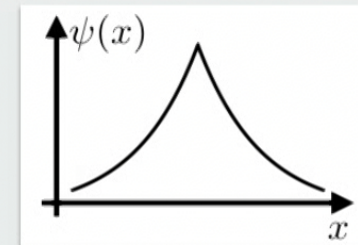
J : hopping

W : disorder strength



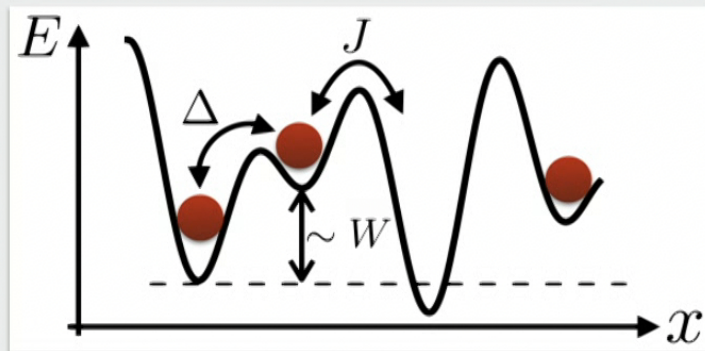
In 1D, for *any finite* W :
All eigenstates are **localized**

Anderson, Phys. Rev. **109**, 1492 (1958)



What is MBL?

Now add interaction Δ (for hardcore bosons):



Persists at finite Δ
when $\Delta, J \ll W$!

Basko, Aleiner, and Altshuler,
Ann. Phys. **321**, 1126 (2006);
Gornyi, Mirlin, and Polyakov,
PRL **95**, 206603 (2005);
Pal and Huse, PRB **82**, 174411 (2010)

Current perspective:
complete set of **emergent local conserved quantities**

$$[H, \tilde{n}_i] = 0, \quad [\tilde{n}_i, \tilde{n}_j] = 0$$



$$|E\rangle = |\{\tilde{n}_i\}_{i=1}^L\rangle$$

Serbyn, Papić, and Abanin, PRL **111**, 127201 (2013);
Huse, Nandkishore, and Oganesyan, PRB **90**, 174202 (2014);
Swingle, arXiv:1307.0507



Consequences: MBL phenomenology

$$[H, \tilde{n}_i] = 0, [\tilde{n}_i, \tilde{n}_j] = 0 \implies |E\rangle = |\{\tilde{n}_i\}_{i=1}^L\rangle$$

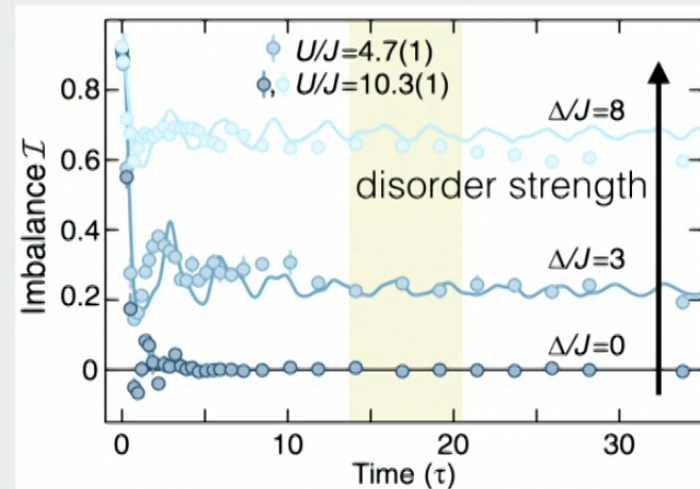
- All eigenstates have **area-law entanglement** $S_A \sim \text{const.}$

Bauer and Nayak, JSTAT P09005 (2013)

- Dynamics: **Retains memory** of initial state (nonergodicity)

EX: $|\Psi_0\rangle = |\bullet \circ \bullet \circ \dots\rangle$

(or any local density product state)



Bloch Group, Science **349**, 842 (2015)



Localization-protected “eigenstate order”

- All eigenstates have **area-law entanglement** $S_A \sim \text{const.}$
Bauer and Nayak, JSTAT P09005 (2013)

Enables stable infinite-temperature quantum phases!

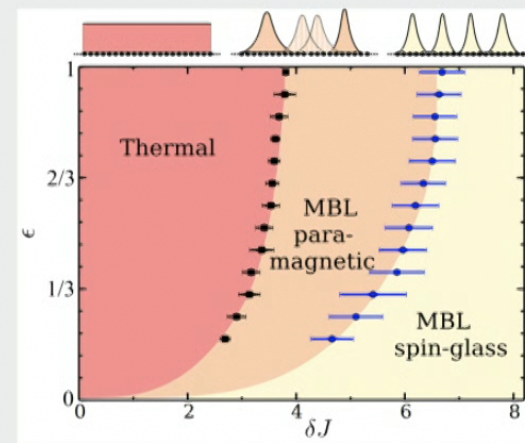
- Spontaneous symmetry breaking

Huse *et al.*, PRB **88**, 014206 (2013)
Kjäll *et al.*, PRL **113**, 107204 (2014)
Pekker *et al.*, PRX **4**, 011052 (2014)
Vasseur *et al.*, PRB **93**, 134207 (2016)

...

- Symmetry-protected topological (SPT) order

Chandran *et al.*, PRB **89**, 144201 (2014)
Slagle *et al.*, arXiv:1505.05147
Potter and Vishwanath, arXiv:1506.00592
...

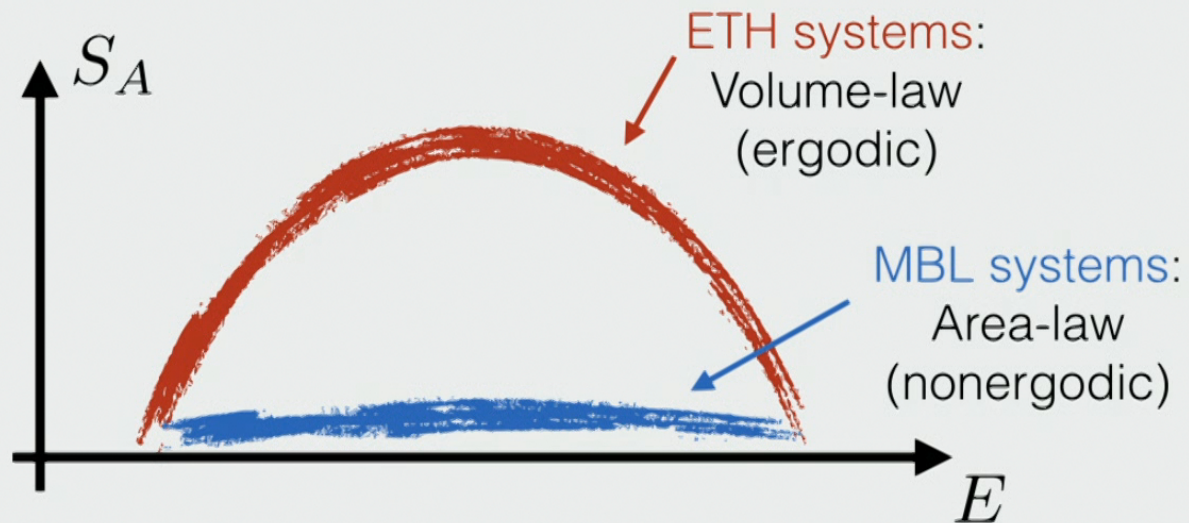


Kjäll *et al.*, PRL **113**, 107204 (2014)



A tale of two entanglements?

Generic highly excited many-body eigenstates:



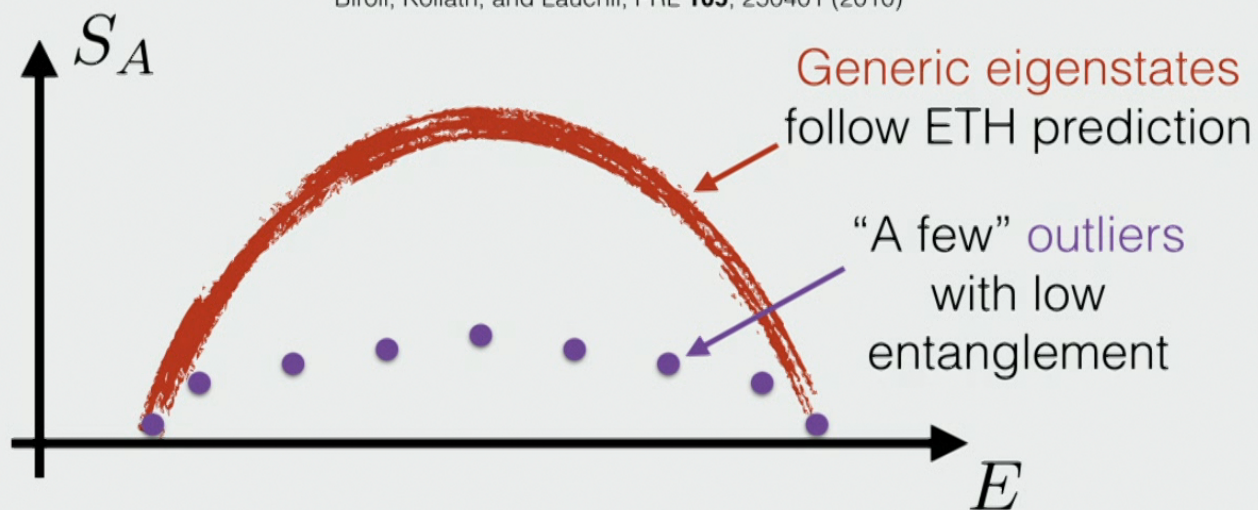
The only two possibilities?

Quantum many-body scars

“Weak ETH”: “Almost all” eigenstates are thermal

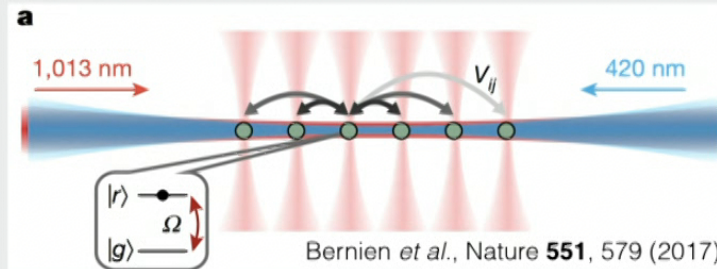
$$\lim_{L \rightarrow \infty} \left(\frac{1}{\dim \mathcal{H}} \times \mathcal{N}_{\text{non-thermal}} \right) = 0$$

Biroli, Kollath, and Läuchli, PRL **105**, 250401 (2010)



Experimental realization

Setup: ^{87}Rb atoms in an optical tweezer array



View as “spins”

$$|r\rangle \sim |\uparrow\rangle$$

$$|g\rangle \sim |\downarrow\rangle$$

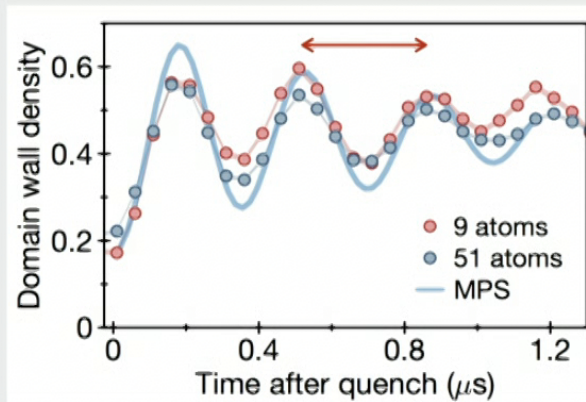
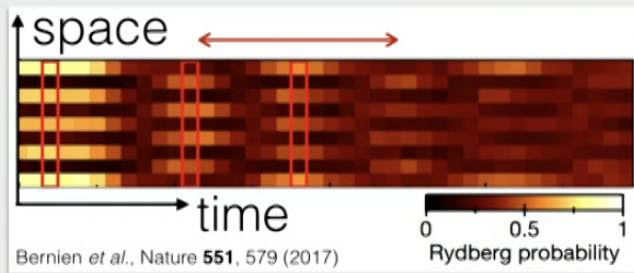
Drive with $\Omega \ll V_{i,i+1} \implies H = \frac{\Omega}{2} \sum_i P_{i-1} X_i P_{i+1}$

$$P_i = \frac{\mathbb{1} - Z_i}{2}$$



Constrained-paramagnet or “PXP” model

A dynamical PXPuzzle



Strong coherent revivals after quench from Néel state

$$|\mathbb{Z}_2\rangle = |\uparrow\downarrow\uparrow \dots\rangle$$

No revivals for generic initial product states

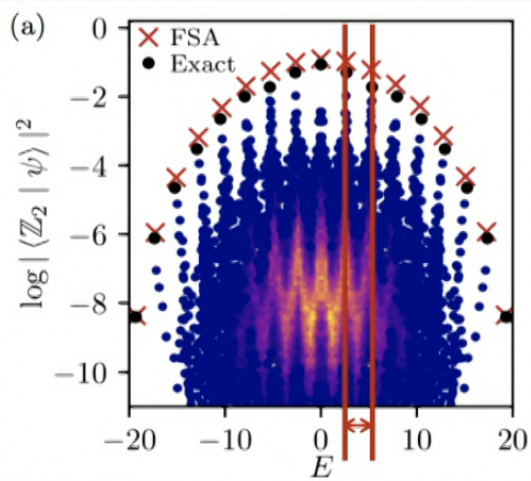
Highly unexpected! Model “should” satisfy ETH

Strong initial-state dependence:

~~ETH~~ ~~MBL~~

(still consistent with weak ETH)





Turner *et al.*, Nat. Phys. **14**, 745 (2018)

Turner *et al.*, Nat. Phys. **14**, 745 (2018) — Krylov subspace description
 Turner *et al.*, PRB **98**, 155134 (2018) — ETH violation

What are these states?
 Stability to perturbations?
 Mechanism for their existence?



Choi *et al.*, arXiv:1812.05561 — emergent SU(2) symmetry?

Khemani, Laumann, and Chandran, arXiv:1807.02108 — proximity to integrability?

Lin and Motrunich, arXiv:1810.00888 — exact MPS in middle of spectrum

Surace *et al.*, arXiv:1902.09551 — Lattice gauge theory

Ho, Choi, Pichler, and Lukin, PRL **122**, 040603 (2019) — Semiclassics

Bull, Martin, and Papić, arXiv:1903.10491 — role of constraints

Generalizations

Stability



Aside on stability: A useful deformation of PXP

$$H' = H + \delta H_R$$

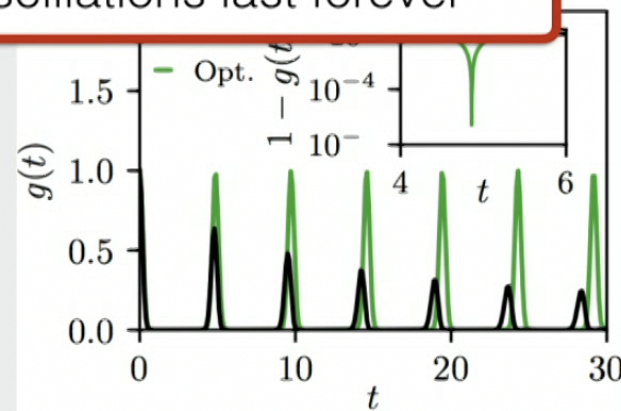
$$\delta H_R = - \sum_i \sum_{d=2}^R h_d P_{i-1} X_i P_{i+1} (Z_{i-d} + Z_{i+d})$$

Suggests proximity to some “perfect point” where revivals become exact, oscillations last forever

$$h_d = h_0 \left(\varphi^{d-1} - \varphi^{-(d-1)} \right)$$

$$\varphi = \frac{1 + \sqrt{5}}{2}$$

$$R = 10$$



Choi *et al.*, arXiv:1812.05561

Khemani, Laumann, and Chandran, arXiv:1807.02108 — proximity to integrability?

Choi *et al.*, arXiv:1812.05561 — emergent SU(2) symmetry?



Do we know of any other non-integrable
Hamiltonians with ETH-violating
eigenstates?

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Exact non-thermal states in the AKLT chain

$$|S_{2N}\rangle \propto \left(\sum_i (-1)^i (S_i^+)^2 \right)^N |G\rangle$$

When N/L is finite:
 Finite energy density!
 Entanglement $\sim \ln L!$

Exact energies:

$$H |S_{2N}\rangle = 2N |S_{2N}\rangle$$

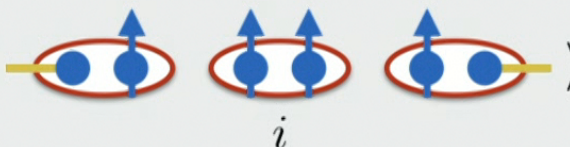
Moudgalya *et al.*, PRB **98**, 235155 (2018)

Entanglement entropy:

$$S_A \sim \ln N$$

Moudgalya *et al.*, PRB **98**, 235156 (2018)

What are these states?

$$|S_2\rangle = \sum_i (-1)^i | \text{---} \text{---} \text{---} \rangle$$


Spin-2
 magnons w/
 $k = \pi$

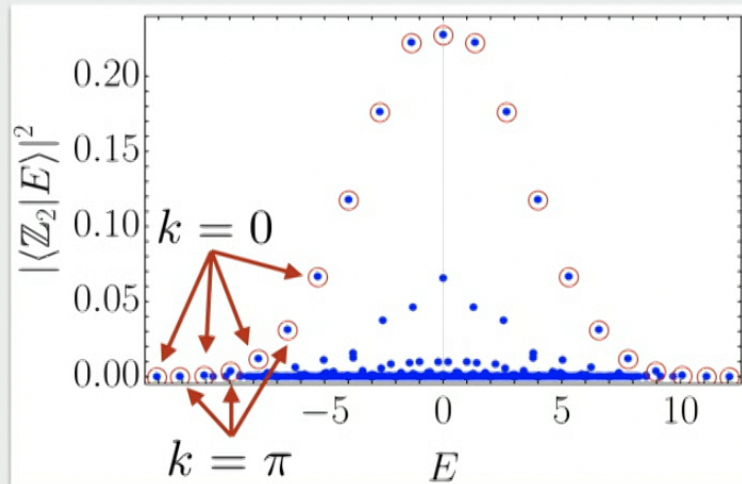
N magnons: scattering interferes
 destructively when all magnons
 have momentum π



“ π magic”



Could something like this be going on in the PXP model?



Correct momentum structure

Numerically obtained entanglement $\sim \ln(L)$



Can we describe the scar states in terms of (nearly-)free magnons with momentum π ?

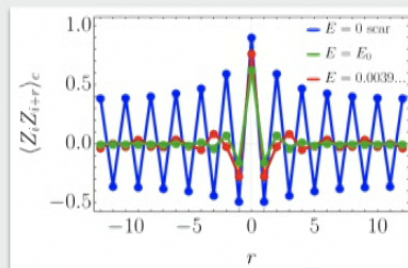
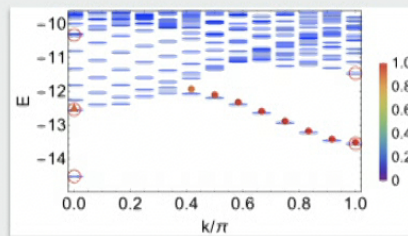


Can we describe the scar states
in terms of (nearly-)free magnons
with momentum π ?

Yes.

This work:

TI, M. Schechter, and S. Xu, arXiv:1903.10517



- 1) Develop a magnon description of the PXP scar states
- 2) Explore consequences: long-range order in **both space and time**
("eigenstate order" in highly excited states, w/o MBL!)



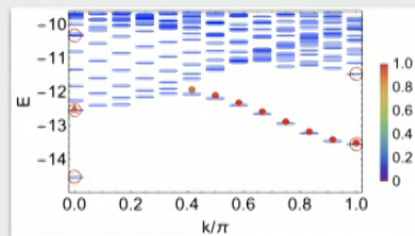
Single-mode approximation for magnons

Build low-energy excitations (magnons) above GS using

$$Z_k = \sum_{r=1}^L e^{-ikr} Z_r$$

See also Bijl (1940),
Feynman (1954),
and Girvin-McDonald-
Platzman (1986)





1) Develop a magnon description of the PXP scar states

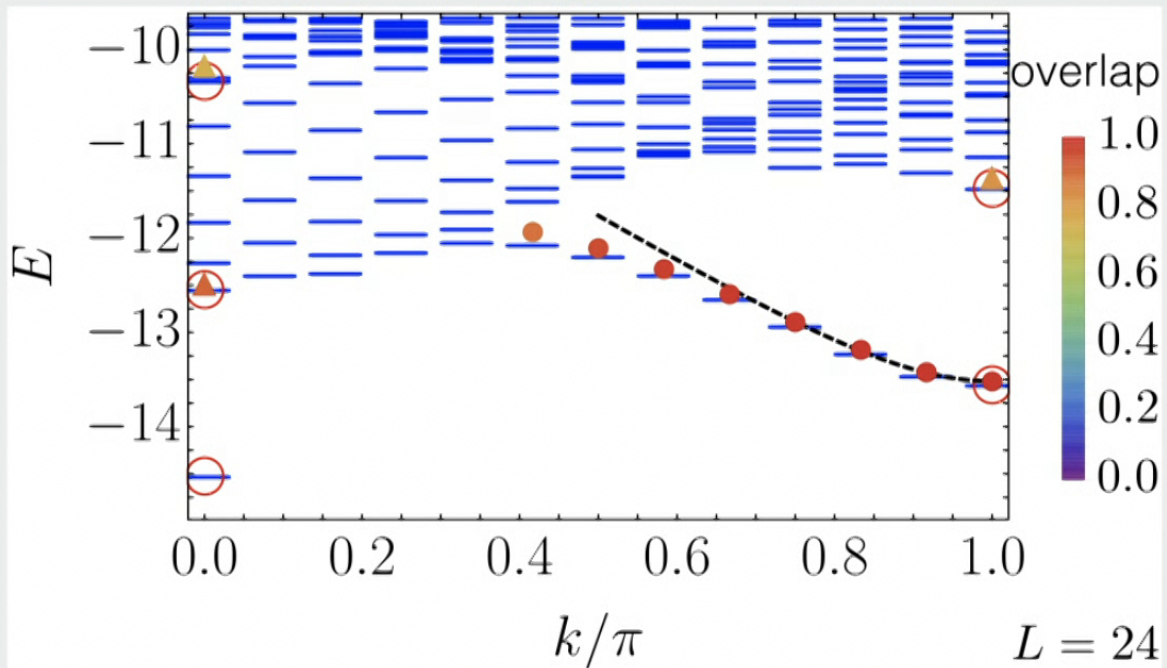


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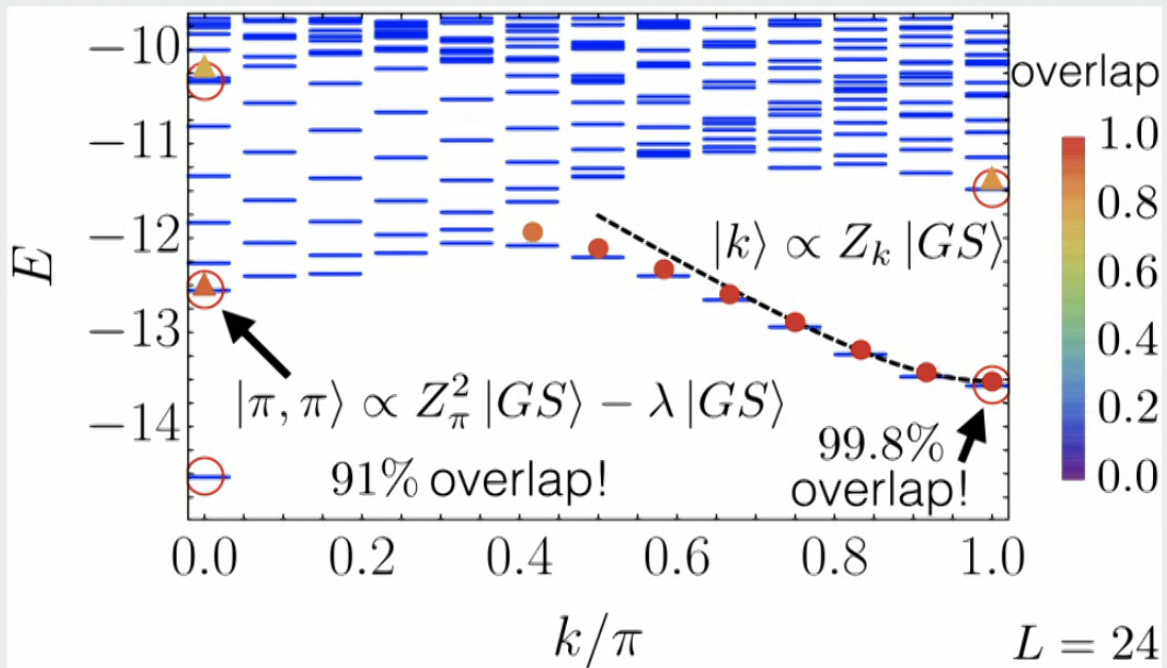


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Feynman (1954),
and Girvin-McDonald-
Platzman (1986)



Next step: Find π -magnon creation operator

Numerical search for an operator

$$S_{\pi}^{\pm}(\alpha) = \frac{Z_{\pi} \mp i\alpha Y_{\pi}}{2} \quad \alpha \in \mathbb{R} \quad (\text{form constrained by symmetries})$$

$$Y_{\pi} = \sum_j (-1)^j P_{j-1} Y_j P_{j+1}$$

Simultaneously optimize

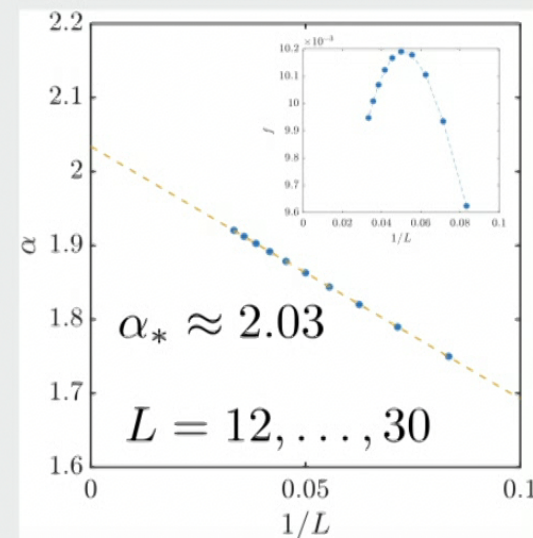
$$S_{\pi}^{+}(\alpha_{*}) |GS\rangle \approx |\text{first excited state}\rangle$$

and

$$[S_{\pi}^{+}(\alpha_{*})]^{\dagger} |GS\rangle \equiv S_{\pi}^{-}(\alpha_{*}) |GS\rangle \approx 0$$

From now on, use

$$S_{\pi}^{\pm} = \frac{Z_{\pi} \mp i2Y_{\pi}}{2}$$



Magnon description of scar states?

Natural basis of magnon states: $|n\rangle = \mathcal{N}_n (S_\pi^+)^n |\text{GS}\rangle$

Hamiltonian has energy-reflection symmetry:

$$C H C = -H, C^2 = \mathbb{1}$$

So we can also use the reflected states:

$$\begin{aligned} |\tilde{n}\rangle &= \mathcal{N}_n (S_\pi^-)^n |\text{CS}\rangle = C |n\rangle \\ |\text{CS}\rangle &= C |\text{GS}\rangle \end{aligned}$$

Problem! $\{|n\rangle, |\tilde{n}\rangle\}$ is not an orthogonal set

Use these states to make a “caricature” of each scar state



Source: KidZone Party Rentals



Magnon description of scar states?

Define the vector space

$$\mathcal{V}_\pi = \text{span} \{ |n\rangle, |\tilde{n}\rangle \mid n = 0, \dots, L/2 \} \quad \dim \mathcal{V}_\pi = L + 2$$

w/ orthonormal basis $\{ |e_i\rangle \}_{i=1}^{L+2}$

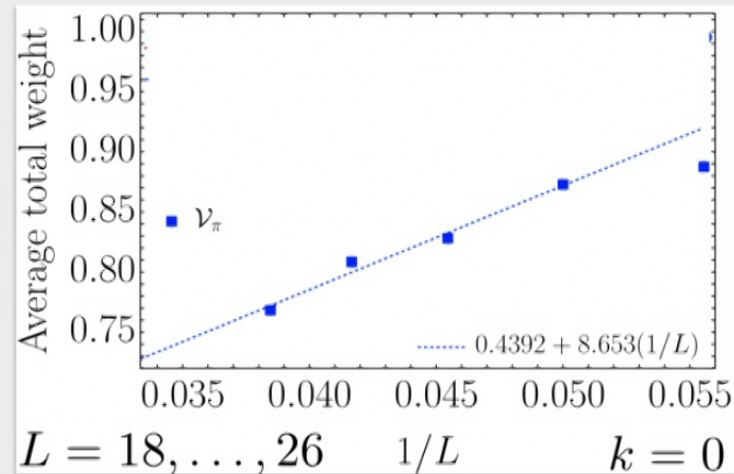
Compute the weight

$$W_{\mathcal{V}_\pi}(m) = \sum_{i=1}^{\dim \mathcal{V}_\pi} |\langle e_i | m \rangle|^2 \leq 1$$

for each scar state

$|m\rangle$, $m = 0$ (GS), \dots , $L + 1$ (CS)

and average over m



Compare to a random state: $\dim \mathcal{V}_\pi / \mathcal{D} = 0.002676 \dots$ at $L=26$!



Magnon description of scar states?

Systematic improvements: use π -magnon scattering states

$$|n, \delta k\rangle = \mathcal{N}_{n, \delta k} (S_{\pi}^+)^{n-2} (S_{\pi+\delta k}^+ S_{\pi-\delta k}^+ + S_{\pi-\delta k}^+ S_{\pi+\delta k}^+) |\text{GS}\rangle$$

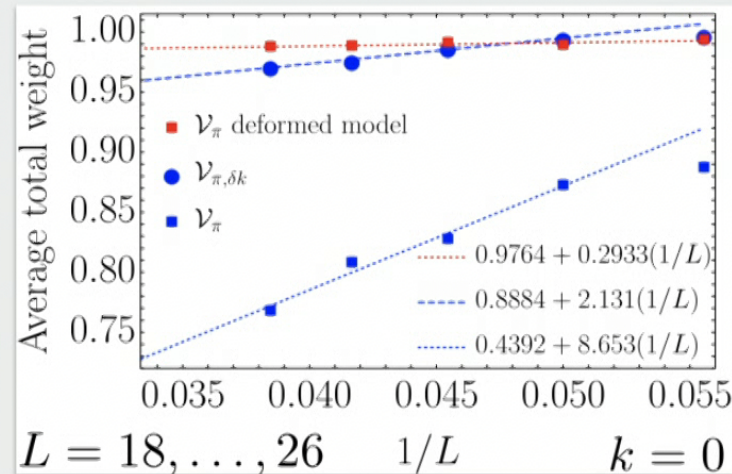
$$n \geq 2, \delta k = 0, \frac{2\pi}{L}, \dots, \pi$$

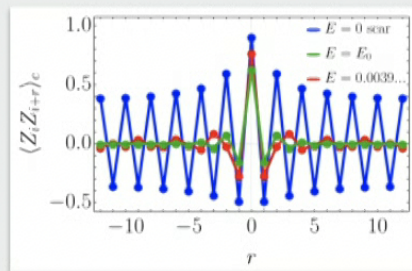
to build vector space $\mathcal{V}_{\pi, \delta k}$ w/ dimension $\sim L^2$

Accounts for unknown magnon interactions

Note: For deformed model, \mathcal{V}_{π} is enough!

Suggests magnons interact more weakly as “perfect point” is approached





2) Explore consequences: long-range order in **both space and time**

(“eigenstate order” in highly excited states, w/o MBL!)

Magnon description of scar states?

Define the vector space

$$\mathcal{V}_\pi = \text{span} \{ |n\rangle, |\tilde{n}\rangle \mid n = 0, \dots, L/2 \} \quad \dim \mathcal{V}_\pi = L + 2$$

w/ orthonormal basis $\{ |e_i\rangle \}_{i=1}^{L+2}$

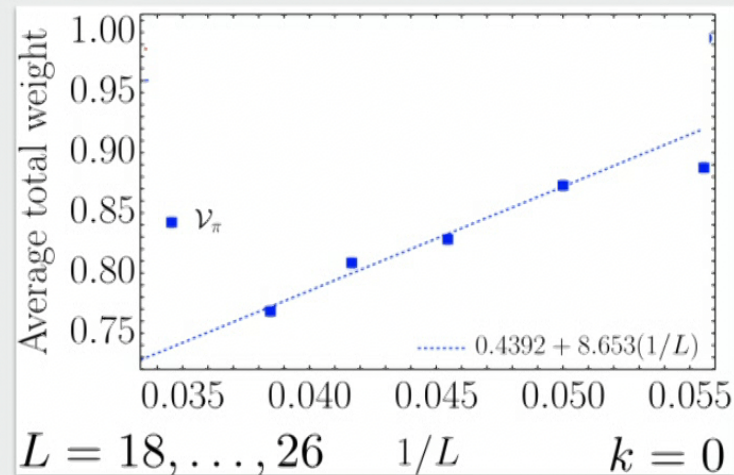
Compute the weight

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for each scar state

$|m\rangle$, $m = 0$ (GS), \dots , $L + 1$ (CS)

and average over m



Long-range correlations in scar states

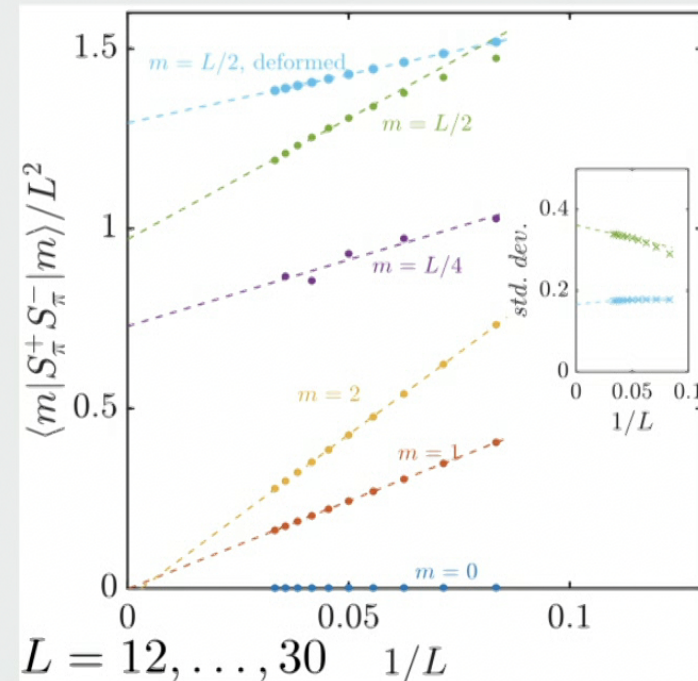
Scar states appear well-described by states of many magnons. Does this mean they are “magnon condensates”?

Specifically, look for “off-diagonal long-range order” (ODLRO), i.e.

$$\langle S_{\pi}^{+} S_{\pi}^{-} \rangle / L^2 \rightarrow \text{const.} \\ \text{as } L \rightarrow \infty$$

(Note: ETH predicts zero!)

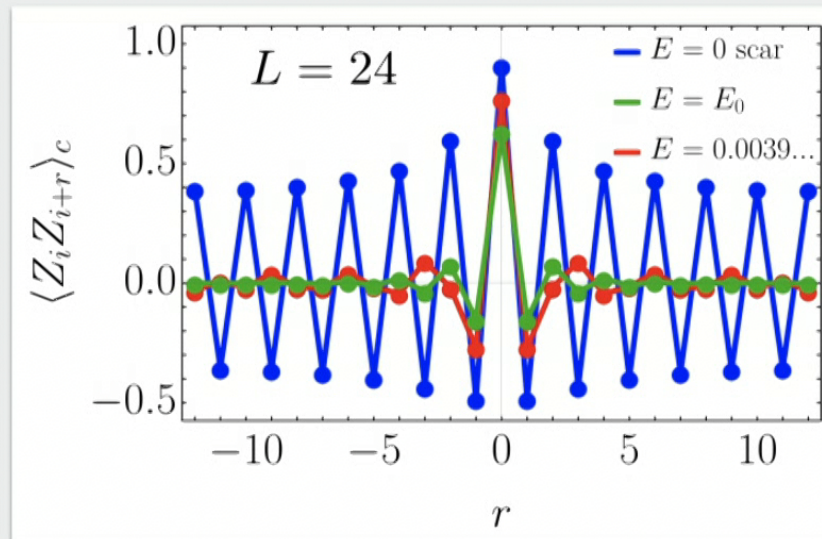
Finite-energy-density scar states have finite π -magnon density for both pure and deformed PXP models



Long-range correlations in scar states

This translates to translation-symmetry breaking in real space (a “magnon density wave” w/ wavenumber π)

Note:
similar for
 $\langle Y_i Y_{i+r} \rangle_c$



Compare ETH prediction

$$\langle Z_i Z_{i+r} \rangle_{c, \text{ETH}} \sim (-1)^r \varphi^{-2r}, \quad r \rightarrow \infty$$

Can be measured
experimentally! Test w/
quench dynamics



Space-time crystalline order in scar states

What are time crystals?

Wilczek (2012): Spontaneous breaking of time-translation symmetry in **ground states**, analogous to spatial crystal formation

Watanabe and Oshikawa (2015):

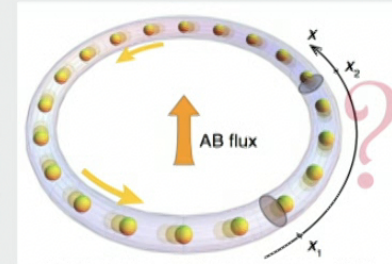
PRL **114**, 251603 (2015)

$$\langle \Phi_{\mathbf{G}}(t) \Phi_{-\mathbf{G}}(0) \rangle / V^2 \rightarrow f(t) \equiv \sum_{\nu \in \mathbb{Z}} e^{i\nu\Omega t} f_{\nu}$$

as $V \rightarrow \infty$

Proven to be impossible in **ground states** or at **thermal equilibrium**

But possible in highly excited states!



Watanabe and Oshikawa, PRL **114**, 251603 (2015)

Space-time crystalline order in scar states

Time crystals so far:

Floquet time crystals—break *discrete* time-translation symmetry

Theory: Sacha, PRA **91**, 033617 (2015)
Khemani *et al.*, PRL **116**, 250401 (2016)
Else, Bauer, and Nayak, PRL **117**, 090402 (2016)
von Keyserlingk, Khemani, and Sondhi, PRB **94**, 085112 (2016)
Else, Bauer, and Nayak, PRX **7**, 011026 (2017)
and many more...

Expt.: Choi *et al.*, Nature **543**, 221 (2017)
Zhang *et al.*, Nature **543**, 217 (2017)
Pal, Nishad, Mahesh, and Sreejith, PRL **120**, 180602 (2018)
Rovny, Blum, and Barrett, PRL **120**, 180603 (2018)
Smits *et al.*, PRL **121**, 185301 (2018)
Rovny, Blum, and Barrett, PRB **97**, 184301 (2018)



Continuous time crystals? Still controversial

Systems with ODLRO

Wilczek, PRL **111**, 250402 (2013)
Volovik, JETP Lett. **98**, 491 (2013)
Watanabe and Oshikawa, PRL **114**, 251603 (2015)

Prethermalization

Else, Bauer, and Nayak,
PRX **7**, 011026 (2017)

Argued to be impossible in non-Floquet MBL systems

Khemani, von Keyserlingk, and Sondhi,
PRB **96**, 115127 (2017)

And no experiments so far



Space-time crystalline order in scar states

To have a spacetime crystal, we need

$$\langle Z_\pi(t) Z_\pi(0) \rangle_{\text{scar}} / L^2 \rightarrow f(t)$$

as $L \rightarrow \infty$

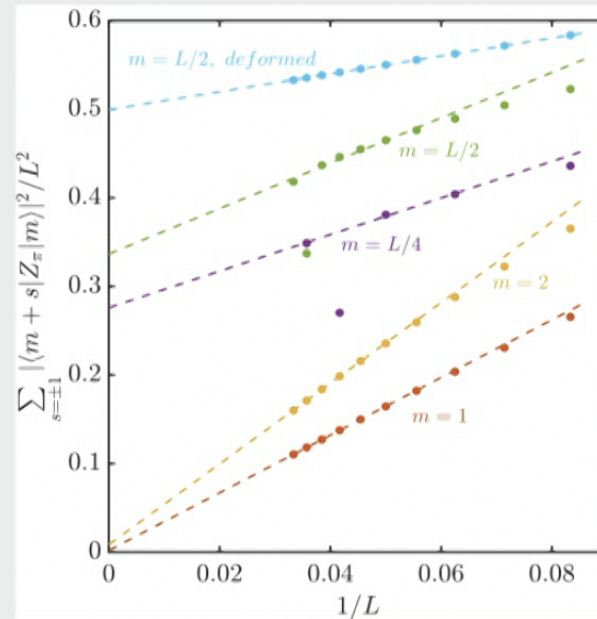
$$\begin{aligned} \langle m | Z_\pi(t) Z_\pi(0) | m \rangle = & \\ & e^{-i\Omega t} |\langle m+1 | Z_\pi | m \rangle|^2 \\ & + e^{+i\Omega t} |\langle m-1 | Z_\pi | m \rangle|^2 + \dots \end{aligned}$$

($\Omega \equiv E_{m+1} - E_m \approx \text{const.}$)



Need

$$\begin{aligned} \langle m \pm 1 | Z_\pi | m \rangle / L &\rightarrow \text{const.} \\ (\text{other matrix elements}) / L &\rightarrow 0 \end{aligned}$$



Plausible! Because

$$Z_\pi = S_\pi^+ + S_\pi^-$$



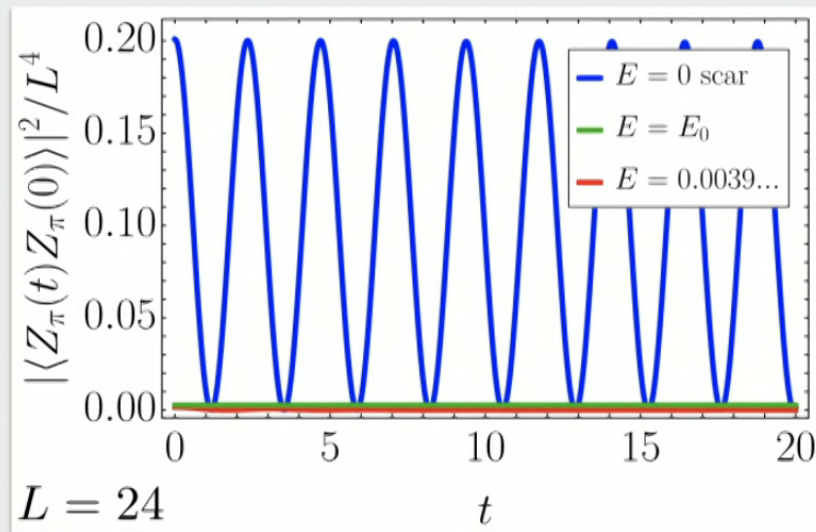
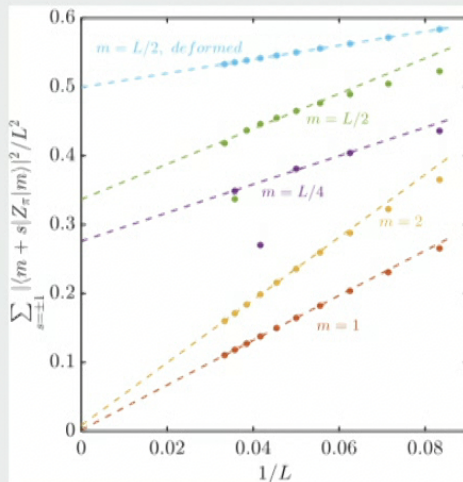
Space-time crystalline order in scar states

Example of a finite-size calculation in undeformed PXP:

Clear signature!

Oscillations do not decay as

$$t \rightarrow \infty$$

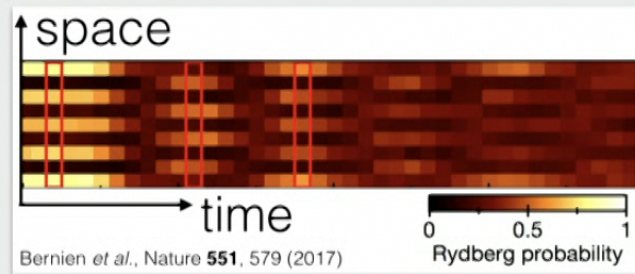


Matrix element scaling even improves substantially in the deformed model!



Space-time crystalline order in scar states

Suggests an intriguing reinterpretation of the Harvard experiment:



$$\sim \langle Z_i(t) \rangle \sim \langle Z_i(t) Z_j(0) \rangle$$

b/c initial state is a Z eigenstate

Initial state projects dynamics onto scar states

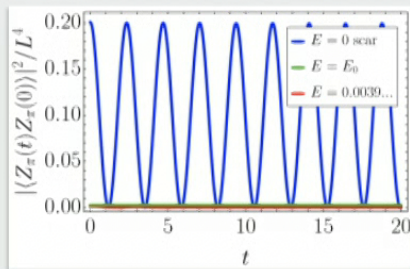
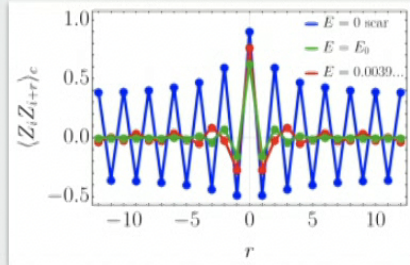
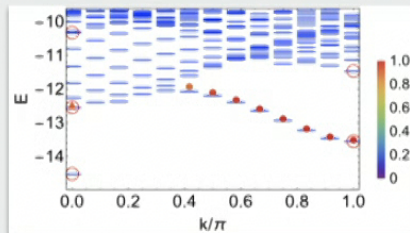
Spatiotemporal coherence is an **interaction effect**

Decoherence due to level anharmonicity of scar states, coupling to the environment

Test hypothesis in more detail by measuring, e.g., $\langle Y_i(t) Y_j(0) \rangle$

Summary

TI, M. Schechter, and S. Xu, arXiv:1903.10517



1) Develop a **magnon description** of the PXP scar states
Makes contact with recent AKLT results

2) Explore consequences: long-range order in **both space and time**
Systems with “scarred” eigenstates: new platform for eigenstate order

All results for PXP model are enhanced when model is deformed towards “perfect point”



Outlook

Much more work ahead to understand these states in
PXP model

(what about, e.g., magnon interactions?)
(are constraints really necessary?)

Is there really a “perfect point?” How to find it?

Relationship to other known mechanisms for
strong-ETH violation?

AKLT tower of states

Moudgalya, Rachel, Bernevig, and Regnault,
PRB **98**, 235155 (2018)

Moudgalya, Regnault, and Bernevig, PRB
98, 235156 (2018)

“Embedded Hamiltonians”

Shiraishi and Mori, PRL **119**, 030601 (2017)

Invariant subspaces in Hilbert space

TI and M. Žnidarič, arXiv:1811.07903

Sala, Rakovszky, Verresen, Knap,
and Pollmann, arXiv:1904.04266

Khemani and Nandkishore, arXiv:1904.04815



Thank you!

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