Title: Delocalized quantum clocks and relativistic time dilation

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Series: Quantum Foundations

Date: April 25, 2019 - 3:30 PM

URL: http://pirsa.org/19040127

Abstract: The theory of relativity associates a proper time with each moving object via its spacetime trajectory. In quantum theory on the other hand, such trajectories are forbidden. I will discuss an operation approach to exploring this conflict, considering the average time measured by a quantum clock in the weak-field, low-velocity limit. Considering the role of the clockâ€™s state of motion, one finds that all ``good'' quantum clocks experience the time dilation prescribed by general relativity for the most classical states of motion. For nonclassical states of motion, on the other hand, one finds that quantum interference effects give rise to a discrepancy between the proper time and the time measured by the clock. I will also describe how ignorance of the clock's state of motion leads to a larger uncertainty in the time as measured by the clock, a consequence of entanglement between the clock time and its center-of-mass degrees of freedom.

Delocalized quantum clocks and relativistic time dilation

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Perimeter Institute, April 2019

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General relativistic time dilation and increased uncertainty in generic quantum clocks

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(Dated: April 5, 2019)

The theory of relativity associates a proper time with each moving object via its world line. In quantum theory however, such well-defined trajectories are forbidden. After introducing a general characterisation of quantum clocks, we demonstrate that, in the weak-field, low-velocity limit, all "good" quantum clocks experience time dilation as dictated by general relativity when their state of motion is classical (i.e. Gaussian). For nonclassical states of motion, on the other hand, we find that quantum interference effects may give rise to a significant discrepancy between the proper time and the time measured by the clock. We also show how ignorance of the clock's state of motion leads to a larger uncertainty in the time measured by the clock — a consequence of entanglement between the clock time and its center-of-mass degrees of freedom. We demonstrate how this lost precision can be recovered by performing a measurement of the clock's state of motion alongside its time reading.

I. INTRODUCTION

One of the most important programs in theoretical physics is the pursuit of a successful theory unifying quantum mechanics and general relativity. Arguably, many of the difficulties arising in this pursuit stem from a lack of understanding of the nature of time [1–3], partic-

operational approach has revealed fundamental limitations to time-keeping in non-relativistic quantum systems [6, 9–11].

The present work concerns slowly-moving generic quantum clocks embedded in a weakly-curved spacetime. We are interested in the phenomenon of time dilation, and how quantizing the clock may result in predictions

arXiv: 1904.02178

Contents

- Operationalism
- Time in general relativity and quantum mechanics
- Setting a quantum clock into a curved background
- Time dilation in quantum clocks
- Relativistic effects on a clock's precision

Operationalism:

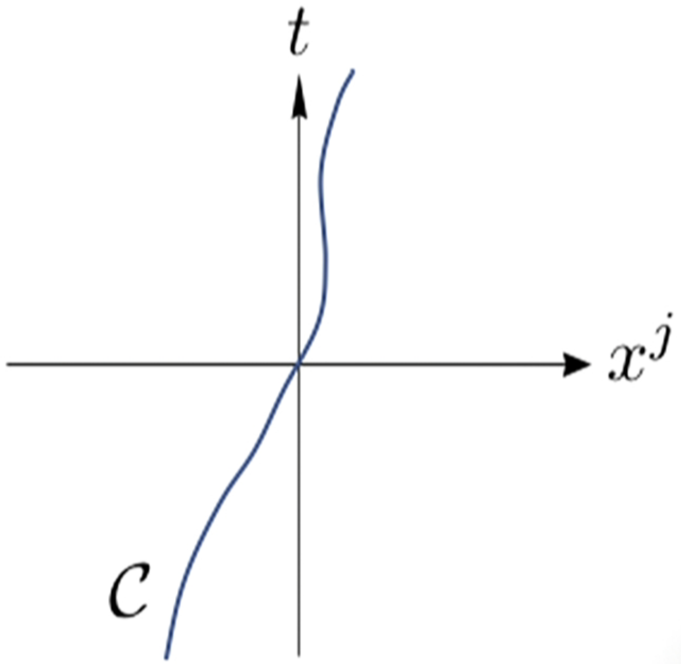
A concept is defined by the set of operations by which it is measured or determined

Time is that which is measured by clocks

P. W. Bridgman, "The logic of modern physics" (1927)

Time in general relativity

$$\tau \propto \int_{\mathcal{C}} \sqrt{g_{\mu\nu}(x)\mathrm{d}x^{\mu}\mathrm{d}x^{\nu}}$$



Evolution parameter

$$i\hbar \frac{\partial}{\partial t} |\psi\rangle = \hat{H} |\psi\rangle$$

"Idealized" clock

If we wish to construct a time observable:

$$\langle \hat{T}_{\mathrm{c}} \rangle = t \qquad \Leftrightarrow \qquad \left[\hat{T}_{\mathrm{c}}, \hat{H}_{\mathrm{c}} \right] = i$$

Given a finite-energy system, how well can it function as a clock?

Generic quantum clock

$$\left\{\hat{T}_{\mathrm{c}}, \hat{H}_{\mathrm{c}}, \rho_{\mathrm{c}}(0)\right\}$$

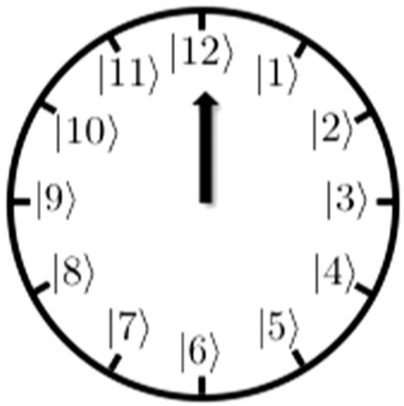
Clock "error operator"

$$-i\left[\hat{T}_{\mathrm{c}}, \hat{H}_{\mathrm{c}}\right]\rho_{\mathrm{c}}(t) = \rho_{\mathrm{c}}(t) + \hat{E}(t)$$

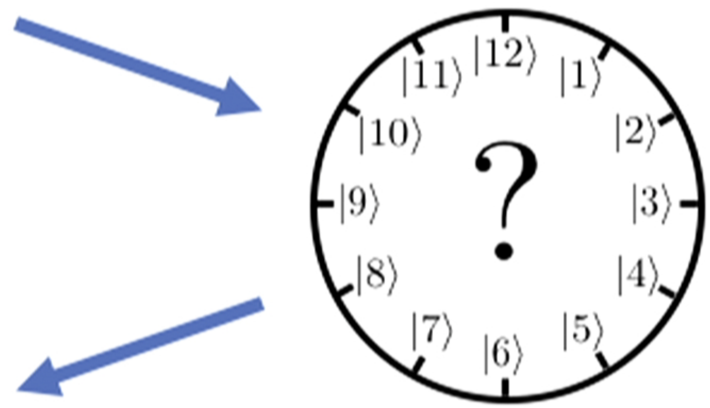
$$\langle\hat{T}_{\mathrm{c}}\rangle_{\mathrm{NR}}(t) = t + \int_0^t dt' \mathrm{tr}\left[\hat{E}(t')\right]$$

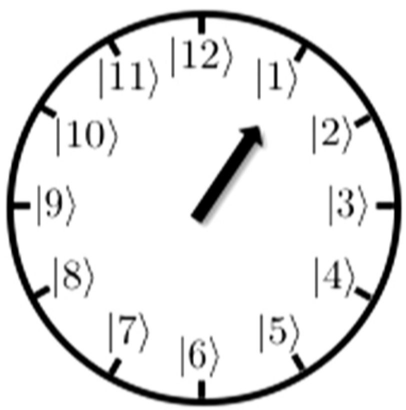
"Good" clock: $\mathrm{tr}\left[\hat{E}(t)\right] \approx 0$

E.g. Salecker-Wigner-Peres clock



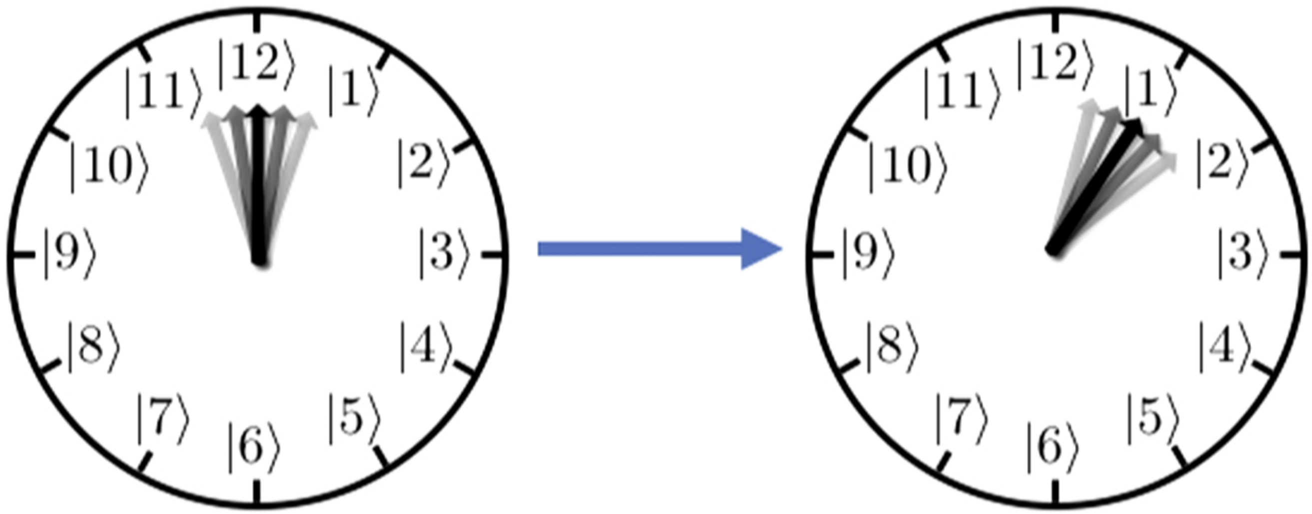
$$\hat{H}_{\mathrm{c}} = \sum_{j=0}^{d-1} j\omega \, |e_j\rangle\langle e_j|$$





$$|m\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} e^{-i2\pi jm/d} \, |e_j\rangle$$

E.g. "Quasi-ideal" clock



$$\mathrm{tr}\left[\hat{E}(t)\right] \sim \mathrm{e}^{-d}$$

M. P. Woods *et al*. "Autonomous quantum machines and finite-sized clocks." *Annales Henri Poincaré*. 20, 1 (2019)

Post-Newtonian expansion

$$g_{00} = -\left[1 + \frac{2\Phi(r)}{c^2} + \frac{2\Phi(r)^2}{c^4}\right] + \mathcal{O}\left(\left(\frac{\Phi(r)}{c^2}\right)^3\right)$$

$$g_{ij} = \delta_{ij}\left[1 - \frac{2\Phi(r)}{c^2}\right] + \mathcal{O}\left(\left(\frac{\Phi(r)}{c^2}\right)^3\right)$$

$$\Phi(r) \approx \Phi(r_0) + gx$$

Proper time of a moving clock

$$\tau = \left[1 - \frac{v_0^2}{2c^2} + \frac{g x_0}{c^2} + \frac{v_0 g t}{c^2} - \frac{1}{3}\left(\frac{gt}{c}\right)^2\right] t$$

Clock rest frame "Laboratory" frame

Clock energy

$$p^\mu = (E/c, \vec{p})$$

(norm is a scalar) $\downarrow$

$$E_{\text{lab}} = \sqrt{-g_{00}\,(E_{\text{rest}}^2 + p_j p^j c^2)}$$

Assume the clock has some internal structure. Then by mass-energy equivalence:

$$E_{\text{rest}} = mc^2 + E_{\text{c}}$$

Quantization

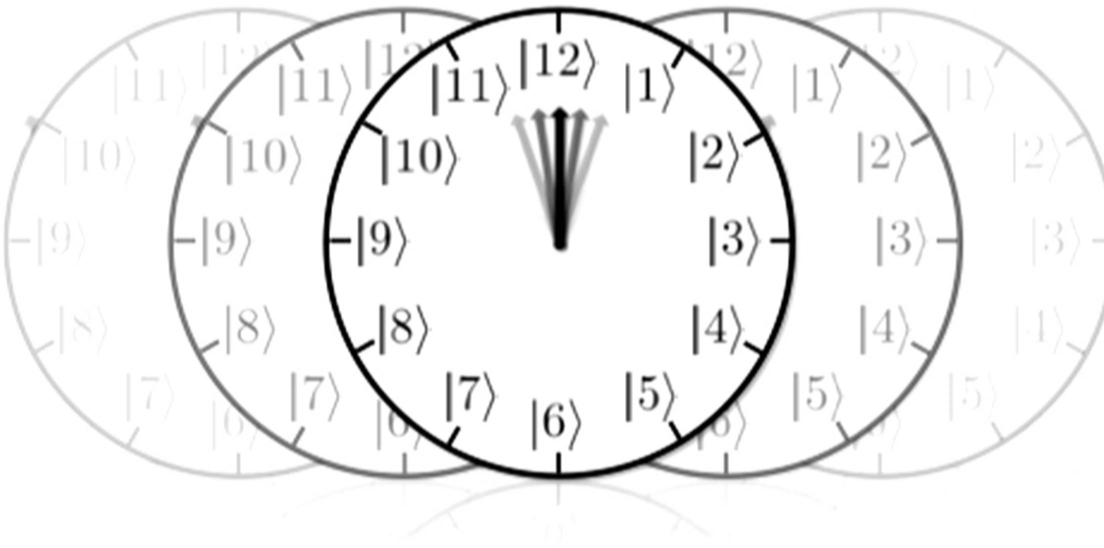
$$E_{\mathrm{lab}} = \sqrt{-g_{00}\left(E_{\mathrm{rest}}^2 + p_j p^j c^2\right)}$$

Assumptions:

- Low velocity
- Weak gravity
- Internal energy << rest-mass energy

M. Zych, PhD Thesis (Springer 2017)

After quantization



$$\mathcal{H}_\mathrm{k} \otimes \mathcal{H}_\mathrm{c}$$

After quantization

Total Hamiltonian:

$$\hat{H} = \hat{H}_{\mathrm{c}} + \hat{H}_{\mathrm{k}} + \hat{H}_{\mathrm{ck}}$$

"Kinematic" part:

$$\hat{H}_{\mathrm{k}} = mc^2 + mg\hat{x} + \frac{\hat{p}^2}{2m} - \frac{\hat{p}^4}{8m^3c^2}$$

Interaction term:

$$\hat{H}_{\mathrm{ck}} = \hat{H}_{\mathrm{c}} \otimes \left(-\frac{\hat{p}^2}{2m^2c^2} + \frac{g\hat{x}}{c^2} \right)$$

Neglected terms

$$\mathcal{O}(1/c^4) := \quad (\hat{H}_{\rm c}/mc^2)^2 \qquad (\hat{p}/mc)^4 \qquad (g\hat{x})^2/c^4 \qquad g\hat{x}\hat{p}^2/m^2c^4$$

Time dilation in a quantum clock

Average clock time:

$$\langle \hat{T}_\mathrm{c} \rangle(t) = \langle \hat{T}_\mathrm{c} \rangle_\mathrm{NR}(t) + \underbrace{t R(t) \left\{ 1 + \mathrm{tr}\left[\hat{E}(t) \right] \right\}}_{\text{Time dilation}}$$

$$R(t) := \mathrm{tr}\left[\left(-\frac{\hat{p}^2}{2m^2c^2} + \frac{g\hat{x}}{c^2} + \frac{\hat{p}gt}{mc^2} - \frac{g^2t^2}{3c^2} \right) \rho_\mathrm{k}(0) \right]$$

E.g. Gaussian ("classical") motion

$$|\psi\rangle_{\mathrm{k}} = \int dp\, \psi(p)\, |p\rangle_{\mathrm{k}}$$

$$\psi(p) := \frac{1}{(2\pi\sigma_p^2)^{1/4}} e^{-\left(\frac{p-\bar{p}_0}{2\sigma_p}\right)^2} e^{-i\bar{x}_0(p-\bar{p}_0)}$$

E.g. Gaussian ("classical") motion

$$\langle \hat{T}_{\mathrm{c}} \rangle(t) = \langle \hat{T}_{\mathrm{c}} \rangle_{\mathrm{NR}}(t) + tR(t) \left\{ 1 + \mathrm{tr}\left[\hat{E}(t) \right] \right\}$$

$$R(t) = -\frac{\bar{p}_0^2 + \sigma_p^2}{2m^2c^2} + \frac{g\bar{x}_0}{c^2} + \frac{\bar{p}_0 g t}{mc^2} - \frac{1}{3}\left(\frac{gt}{c}\right)^2$$

Consider a classical clock whose position and momentum follow Gaussian distributions:

$$\tau = \left[1 - \frac{v_0^2}{2c^2} + \frac{gx_0}{c^2} + \frac{v_0 gt}{c^2} - \frac{1}{3}\left(\frac{gt}{c}\right)^2\right] t$$

$$\Downarrow$$

$$\langle\tau\rangle = \left[1 - \frac{\bar{p}_0^2 + \sigma_p^2}{2m^2c^2} + \frac{g\bar{x}_0}{c^2} + \frac{\bar{p}_0 gt}{mc^2} - \frac{1}{3}\left(\frac{gt}{c}\right)^2\right] t$$

E.g. Gaussian ("classical") motion, idealized clock

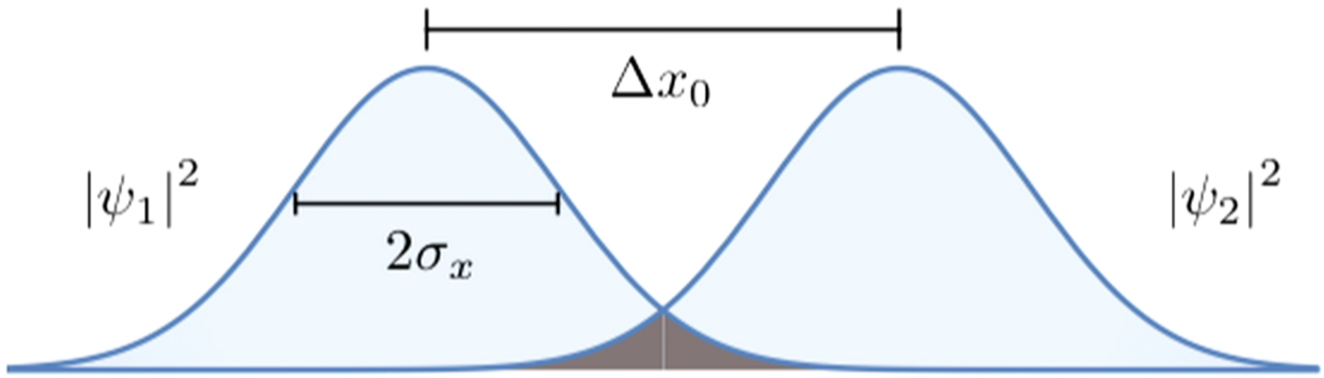
Quantum clock:

$$\langle \hat{T}_{\rm c} \rangle (t) = \left[1 - \frac{\bar{p}_0^2 + \sigma_p^2}{2m^2c^2} + \frac{g\bar{x}_0}{c^2} + \frac{\bar{p}_0 g t}{mc^2} - \frac{1}{3}\left(\frac{gt}{c}\right)^2 \right] t$$

Classical clock:

$$\langle \tau \rangle = \left[1 - \frac{\bar{p}_0^2 + \sigma_p^2}{2m^2c^2} + \frac{g\bar{x}_0}{c^2} + \frac{\bar{p}_0 g t}{mc^2} - \frac{1}{3}\left(\frac{gt}{c}\right)^2 \right] t$$

E.g. Non-classical motion, idealized clock



$$|\psi(0)\rangle_{\mathrm{k}} = \sqrt{\alpha}\,|\psi_1\rangle_{\mathrm{k}} + \sqrt{1-\alpha}\,|\psi_2\rangle_{\mathrm{k}}$$

E.g. Non-classical motion, idealized clock

Compare with mixed initial state:

$$\rho_{\mathrm{mix}} := \alpha\rho_{\psi_1} + (1-\alpha)\rho_{\psi_2}$$

which has mean clock time:

$$\langle \hat{T}_{\mathrm{c}} \rangle_{\mathrm{mix}}(t) = \alpha \langle \hat{T}_{\mathrm{c}} \rangle_{\psi_1}(t) + (1-\alpha)\langle \hat{T}_{\mathrm{c}} \rangle_{\psi_2}(t)$$

E.g. Non-classical motion, idealized clock

$$\langle \hat{T}_{\mathrm{c}} \rangle_{\mathrm{sup}}(t) = \langle \hat{T}_{\mathrm{c}} \rangle_{\mathrm{mix}}(t) + T_{\mathrm{coh}}(t)$$

$$T_{\mathrm{coh}}(t) := t \, \frac{\left(\frac{\Delta x_0}{2\sigma_x} \right)^2 \frac{\sigma_v^2}{c^2} - \frac{g \Delta x_0}{c^2}(1 - 2\alpha)}{\frac{\exp\left[\frac{1}{2} \left(\frac{\Delta x_0}{2\sigma_x} \right)^2 \right]}{\sqrt{(1-\alpha)\alpha}} + 2}$$

Relativistic effects on clock precision

An uncorrelated initial state of a general bipartite system will only typically remain in the set of separable states if:

- the eigenstates of the total Hamiltonian are separable, or
- the initial state is particularly mixed

K. Zyczkowski *et al*. "Volume of the set of separable states". *Phys. Rev. A,* 58, 883 (1998)

Standard deviation in clock time

For simplicity, we ignore gravity.

$$\sigma_T(t) = \sigma_{T,\mathrm{NR}}(t) + \sigma_{T,\mathrm{I}}(t) + \sigma_{T,\mathrm{NI}}(t)$$

$$\sigma_{T,\mathrm{I}}(t) := \frac{t^2}{8\sigma_{T,\mathrm{NR}}(t)} \frac{\langle \hat{p}^4 \rangle + \sigma_{p^2}^2}{m^4 c^4}$$

Note: we needed to work to one order higher in precision.

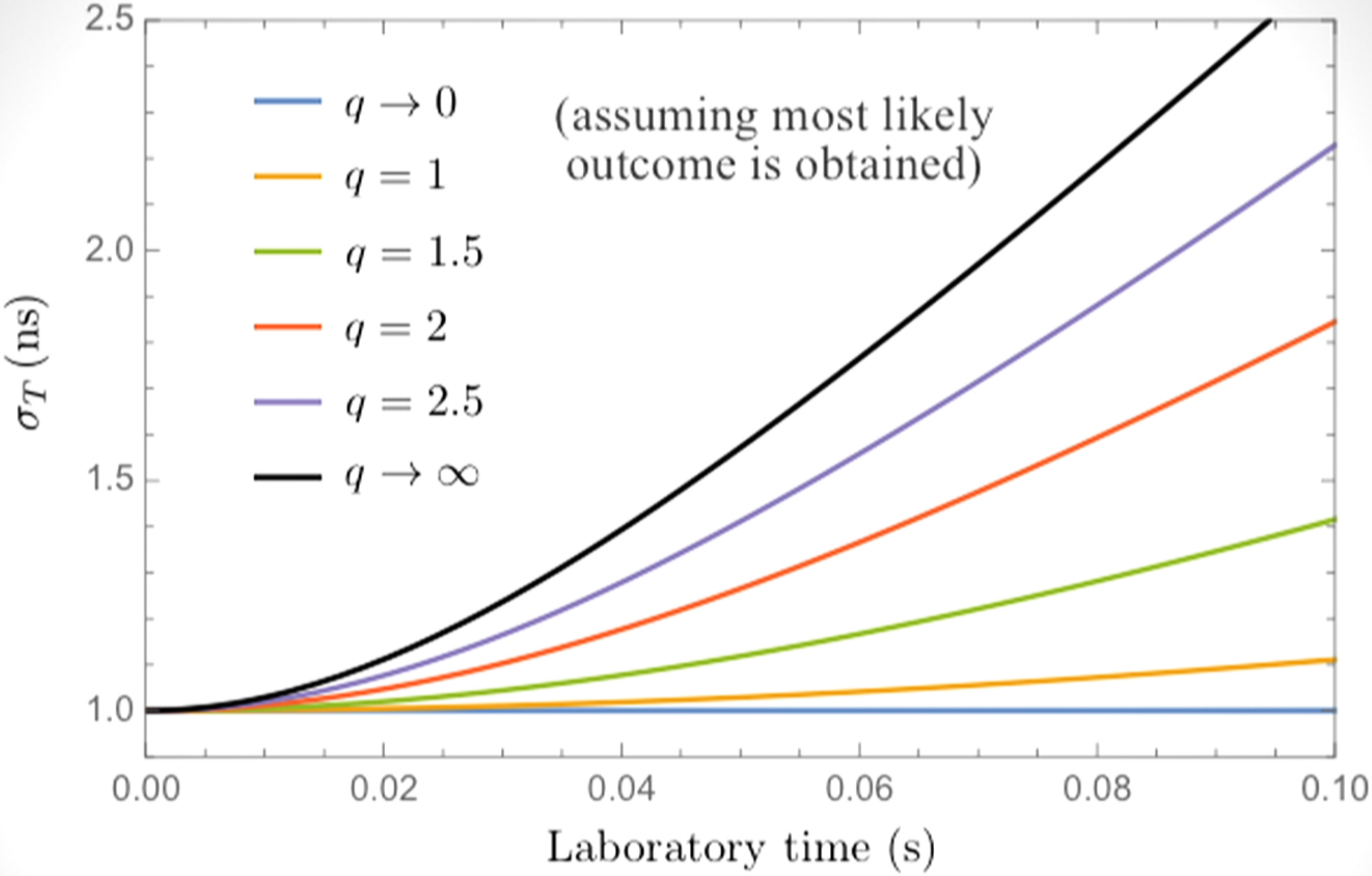
Recovering the lost precision by measuring the kinematic space

Consider a coarse-grained momentum measurement:

$$\{\hat{\Pi}_{n,\delta p}\}_n$$

$$\hat{\Pi}_{n,\delta p} := \int_{(n-1/2)\delta p}^{(n+1/2)\delta p} dp\, |p\rangle \langle p|_{\mathrm{k}}$$

What is the uncertainty in the clock time after performing the measurement of the kinematic space?



$q := \delta p/\sigma_p$ $\bar{p}_0 = 0$ $\sigma_{T,\mathrm{NR}}(0) = 1$ ns $\sigma_x = 1$ nm

Recovering the lost precision by measuring the kinematic space

Conclusion: temporal information bleeds into the kinematic degrees of freedom, and can be recovered by measuring them

Summary

- "Good" clocks experience classical time dilation (on average) for classical states of motion
- Non-classical states may result in a quantum contribution to time dilation, a consequence of interference
- The coupling that causes time dilation also entangles the clock time with its motion, encoding temporal information in the kinematic degrees of freedom

To do

- Translate into experiments
- Investigate recovery of clock precision in the case of non-zero gravitational field. Can we still recover all the lost precision?
- Go beyond conditioning kinematic measurements on the most likely outcome

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Thank you