

Title: Emergent O(4) Symmetry and Signatures of Deconfined Quantum Critical Point in Shastry-Sutherland Lattice Material SrCu₂(BO₃)₂

Speakers:

Series: Condensed Matter

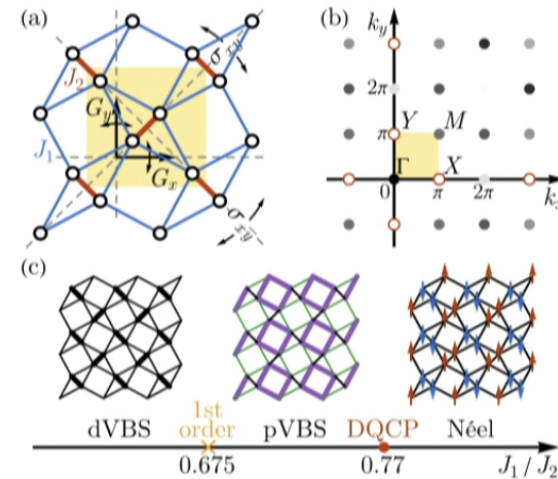
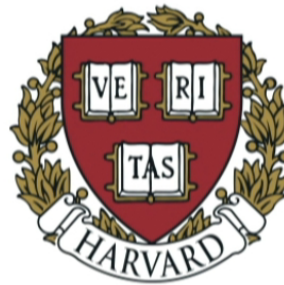
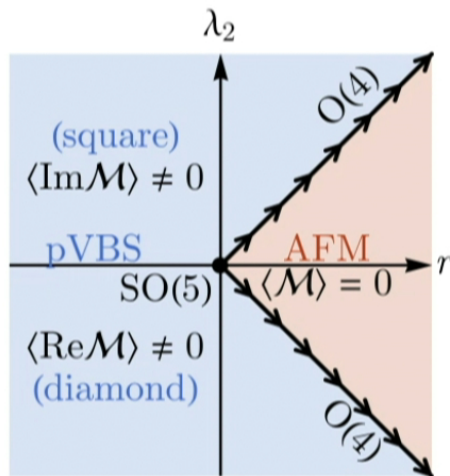
Date: April 16, 2019 - 3:30 PM

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Abstract: We study the possibility of a deconfined quantum phase transition in a realistic model of a two dimensional Shastry-Sutherland quantum magnet, using both numerical and field theoretic techniques. We argue that the quantum phase transition between a two fold degenerate plaquette valence bond solid (pVBS) order and N\`eel ordered phase may be described by a deconfined quantum critical point (DQCP) with emergent O(4) symmetry. Further, using the infinite density matrix renormalization group (iDMRG) numerical technique, we verify the emergence of an intermediate pVBS order, between the dimer and Neel ordered phases. By analyzing the correlation length spectrum, we provide evidence for deconfinement and emergent O(4) symmetry at the phases transition between the pVBS and N\`eel orders. Such a phase transition has been reported in the recent pressure tuned experiments in the Shastry-Sutherland lattice material SrCu₂(BO₃)₂. The non-symmorphic lattice structure of the Shastry-Sutherland compound leads to extinction points in the scattering, where we predict sharp signatures of a DQCP in both the phonon and magnon spectra associated with the spinon continua. Our results can guide future experimental studies of DQCP in quantum magnets.

Signatures of a Deconfined Phase Transition on the Shastry-Sutherland Lattice: Applications to Quantum Critical $\text{SrCu}_2(\text{BO}_3)_2$

Perimeter Institute
4.16.2019



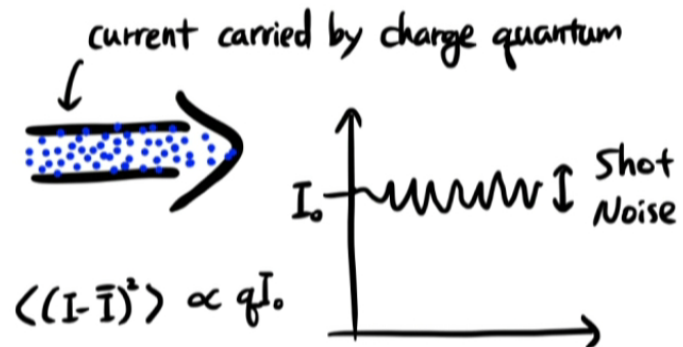
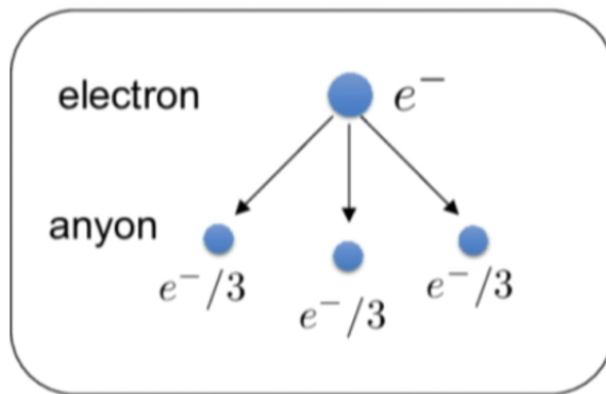
Jong Yeon Lee

With Yi-Zhuang You, Subir Sachdev, and Ashvin Vishwanath

Review: Deconfined Quantum Criticality

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What is deconfinement?



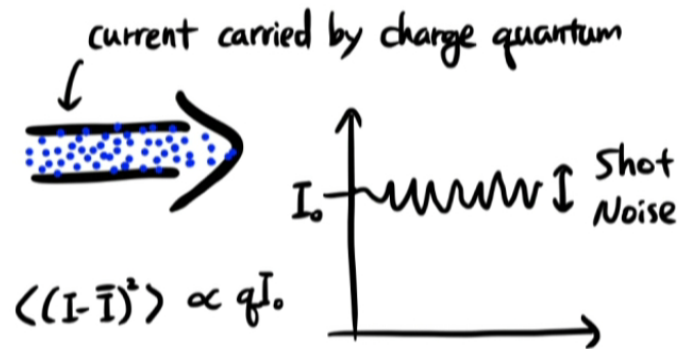
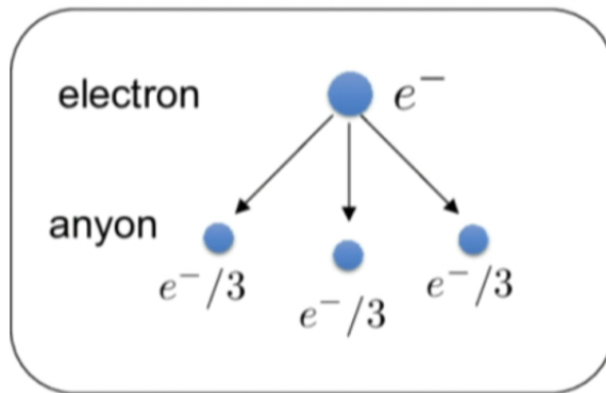
Fractional Quantum Hall at 1/3-filling

Fractionalized excitations are unbounded and move independently! — no longer confined

Figure from Xie Chen's Talk

Review: Deconfined Quantum Criticality

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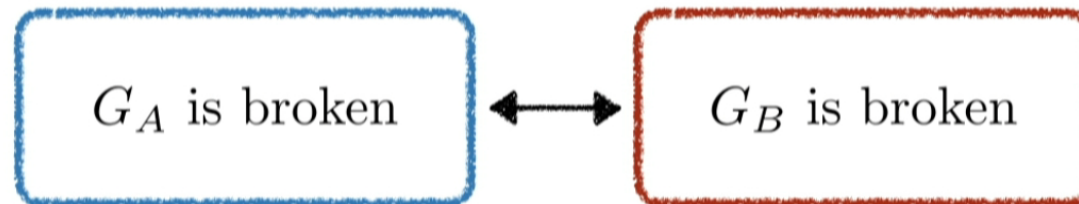
Landau-Ginzburg Theory

Two order parameters: Ψ and Φ

$$F \sim r_1 |\Psi_1|^2 + s_1 |\Psi_1|^4 + r_2 |\Phi_1|^2 + s_2 |\Phi_1|^4 + t |\Psi_1|^2 |\Phi_1|^2 + \dots$$

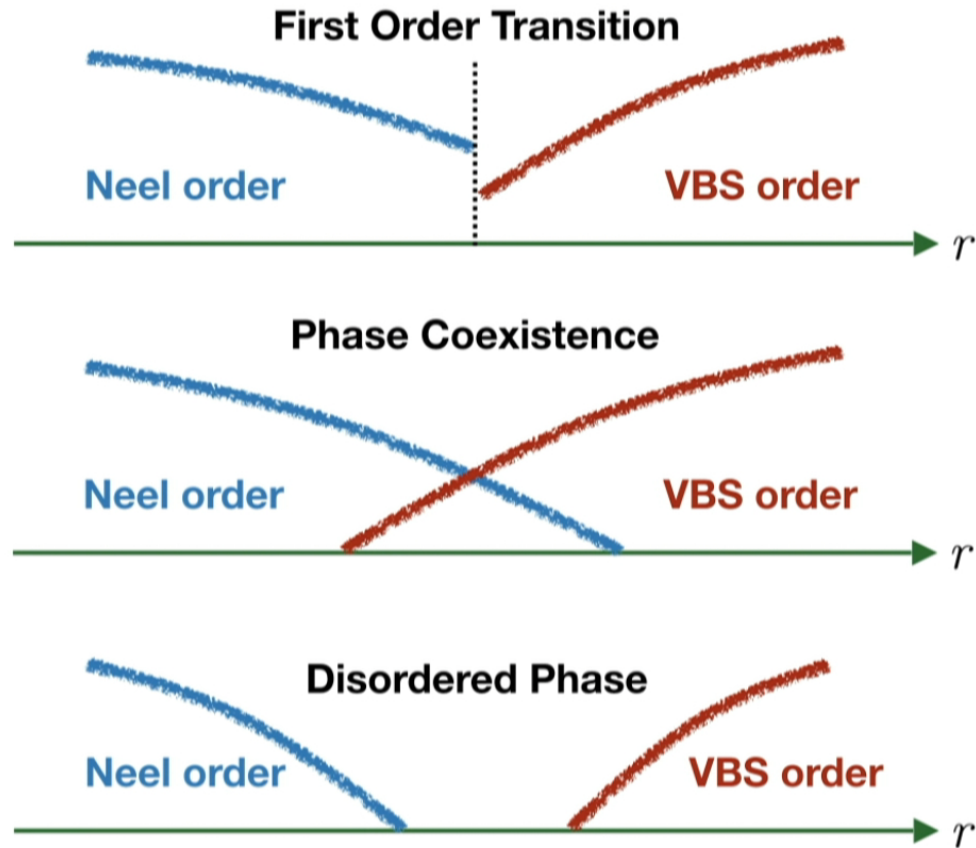
What is the most generic scenario for the phase transition between two different symmetry breaking phases?

$$G_{\text{tot}} = G_A \times G_B$$



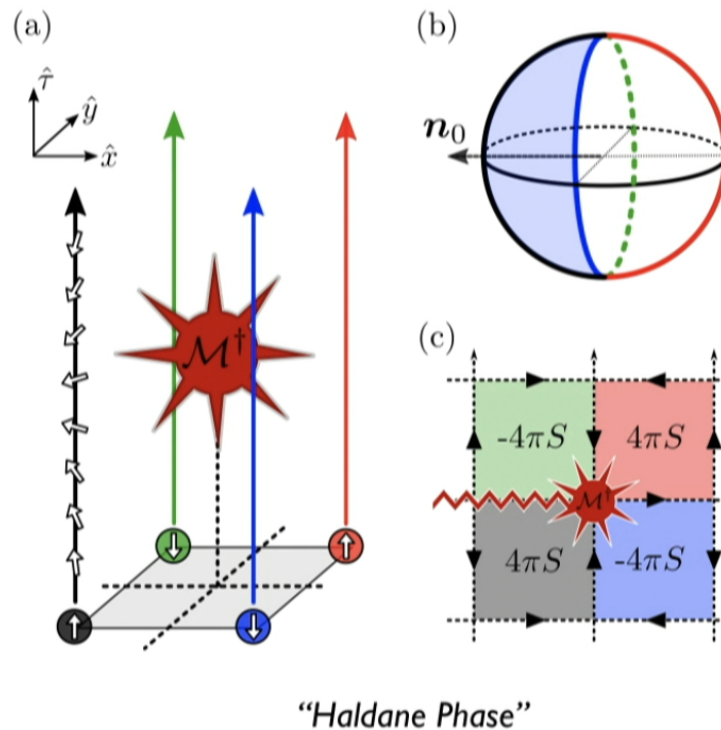
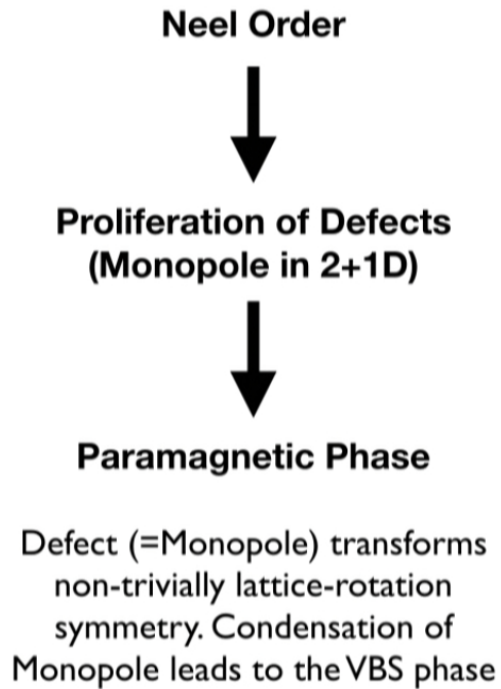
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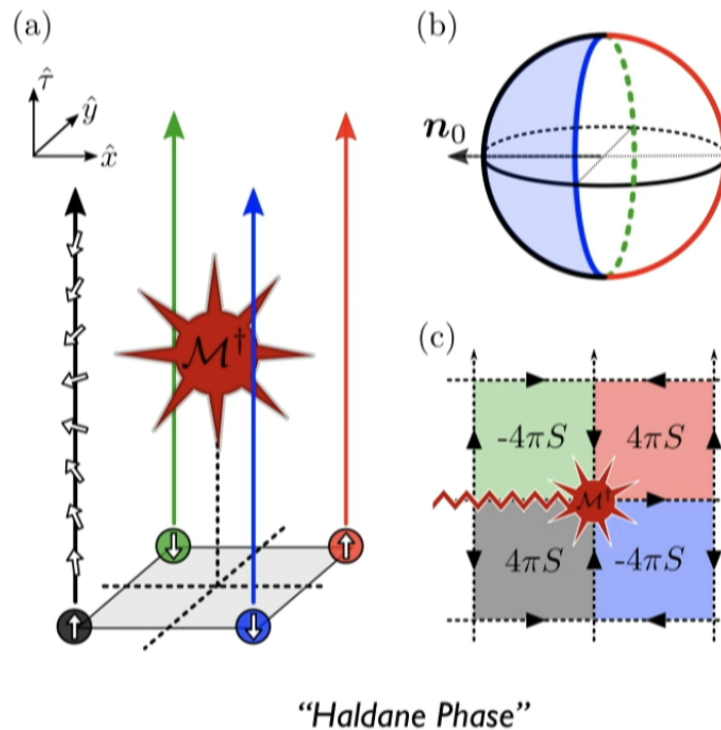
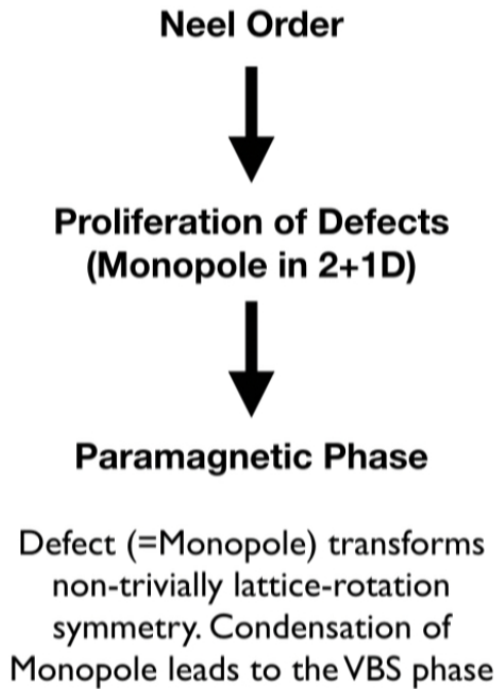
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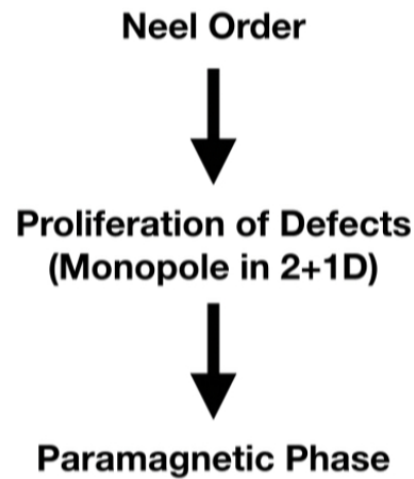
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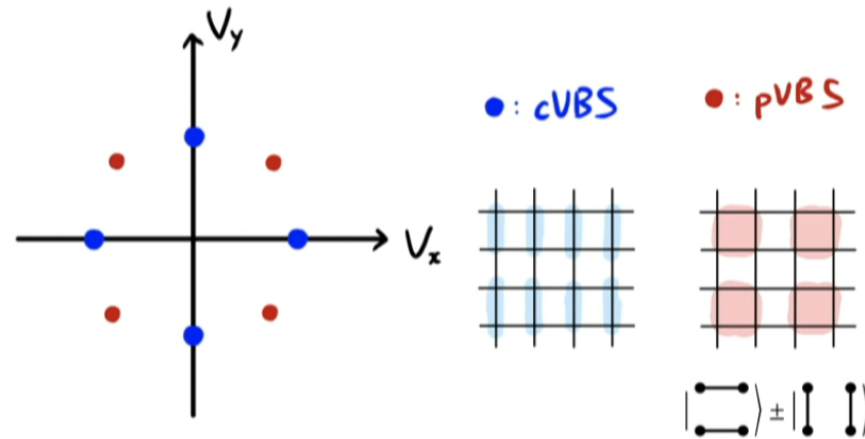
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Condensation of Monopole
leads to the VBS phase

$$\mathcal{M}^\dagger = \frac{1}{\sqrt{2}}((V_x + V_y) + i(V_x - V_y)),$$

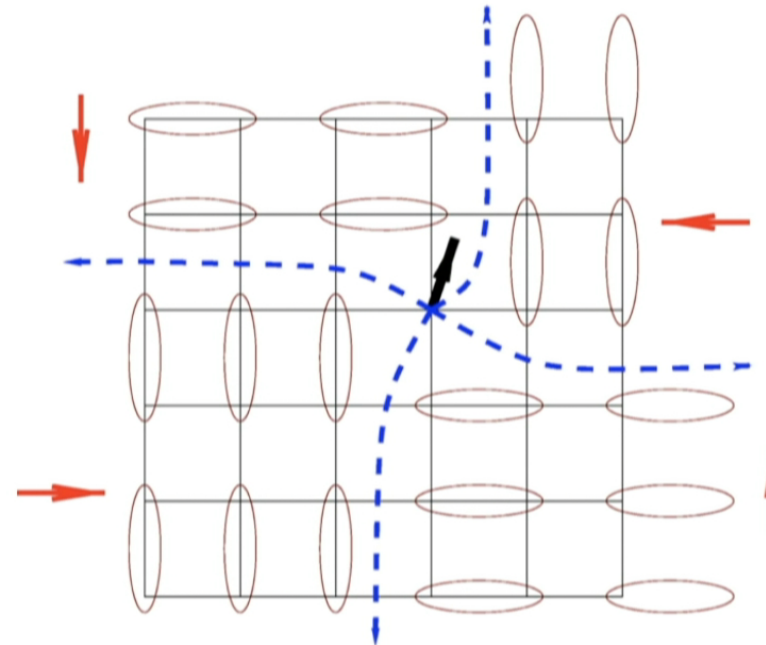
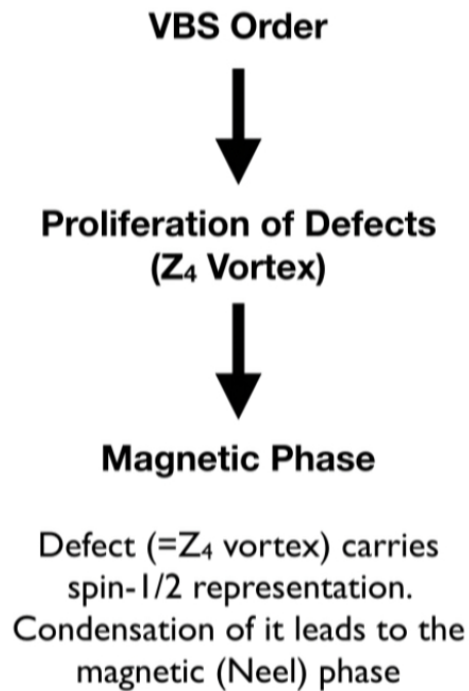


Which VBS phase?

Depending on the phase of monopole condensate

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“Levin-Senthil Picture”

Figure from Levin and Senthil, PHYSICAL REVIEW B **70**, 220403(R) (2004)

Review: Deconfined Quantum Criticality

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Non-Linear Sigma Model for Neel order

$$\mathcal{S} = \int d\tau d^2r \left[\left(\frac{\partial \mathbf{n}}{\partial \tau} \right)^2 + (\nabla \mathbf{n})^2 \right] + iS \sum_r (-1)^{r_x+r_y} \omega(\mathbf{n}(r))$$

Introduce $\mathbb{C}P^1$ theory with 'spinons'

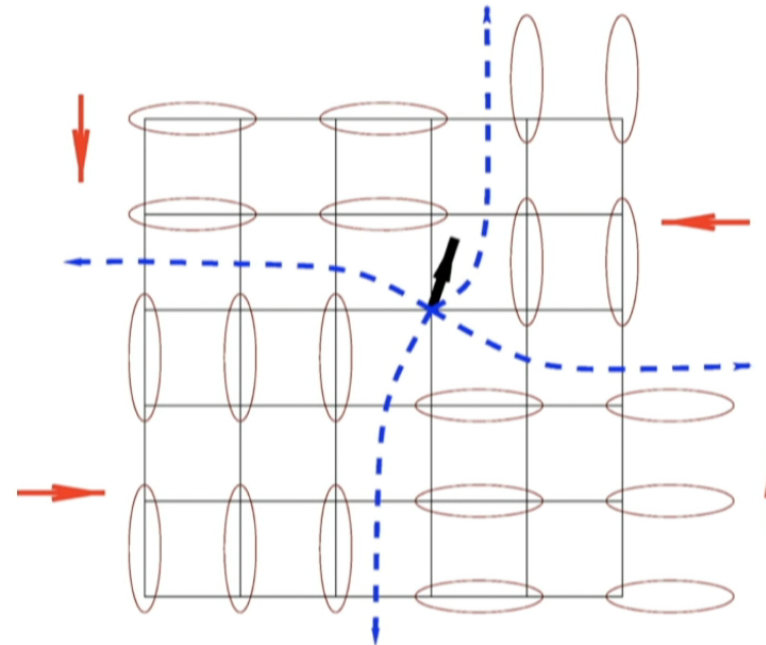
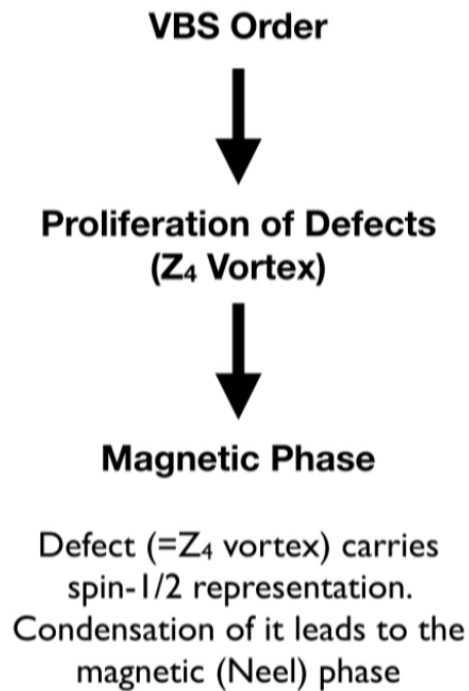
Neel order parameter is expressed in terms of fractional degrees of freedom

$$\mathbf{n} = z^\dagger \vec{\sigma} z, \quad \text{where } (z_1, z_2)^T$$

$$\mathcal{L}_{\mathbb{C}P^1} = \sum_i |(\partial - ia)z_i|^2 + \kappa(\nabla \times a)^2 + \mu \sum_i |z_i|^2 + \lambda \left(\sum_i |z_i|^2 \right)^2$$

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	$V_x + iV_y$	$n_x + in_y$	n_z
in \mathcal{L}	\mathcal{M}_a	$2z_1^* z_2$	$ z_1 ^2 - z_2 ^2$

Both Neel and VBS order parameters can be expressed in terms of spinons + gauge field

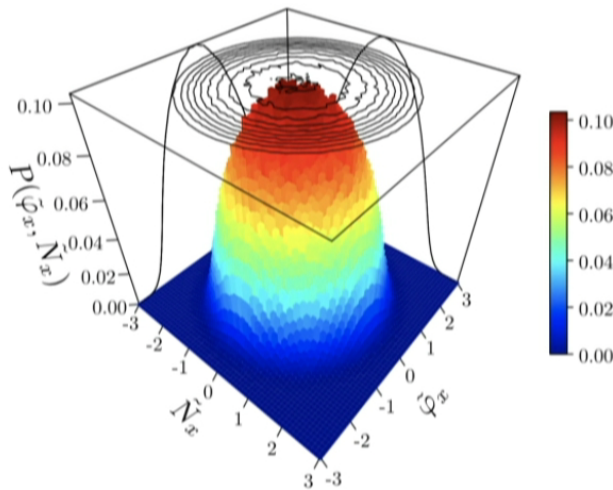
Review: Deconfined Quantum Criticality

Anders Sandvik, PRL **98**, 227202 (2007)
Nahum et al., PRL **115**, 267203 (2015)

Symmetry Enhancement: $SO(3) \times Z_4 \longrightarrow SO(3) \times U(1) \longrightarrow SO(5)$

Both VBS and Neel order parameter have similar
anomalous dimension at the critical point $\eta \sim 0.26$

$$\langle N_i(r) \cdot N_i(0) \rangle \sim \langle V_i(r) \cdot V_i(0) \rangle \sim \frac{1}{r^{1+\eta}}$$



Evidence of Symmetry Enhancement

- Both operators have the same scaling dimension
- Fluctuations of both order parameters are scaling identically with system size
- Duality conjectures and fermionic PSG ansatz implies $SO(5)$ emergent symmetry at the DQCP

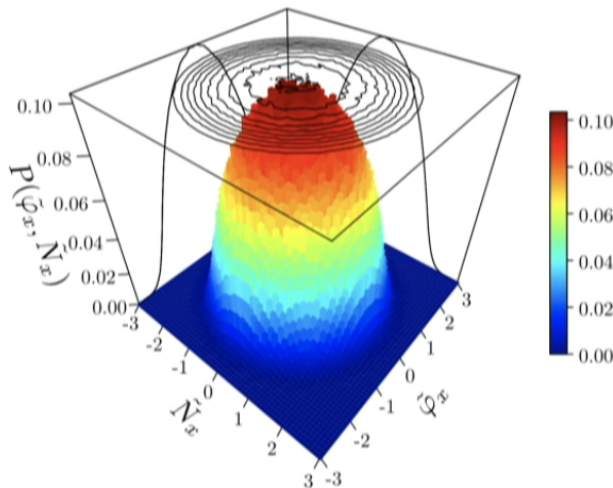
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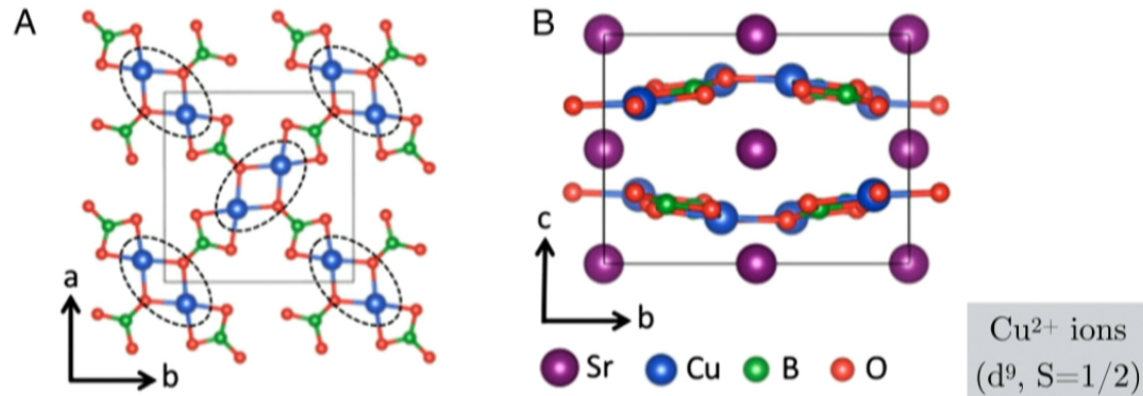
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$$\mathcal{L} \sim a + |\partial M|^2 + b|M|^2 + c|M|^4 +$$

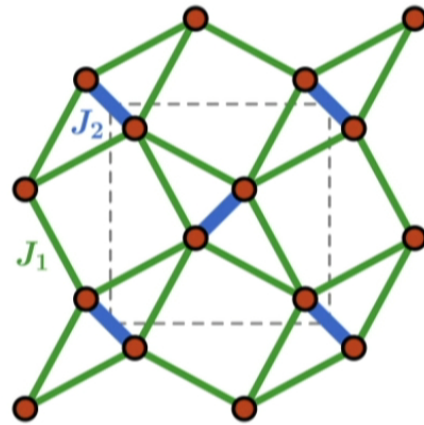
N	0	1	2	3
γ	1.1596 ± 0.0020	1.2396 ± 0.0013	1.3169 ± 0.0020	1.3895 ± 0.0050
ν	0.5882 ± 0.0011	0.6304 ± 0.0013	0.6703 ± 0.0015	0.7073 ± 0.0035
η	0.0284 ± 0.0025	0.0335 ± 0.0025	0.0354 ± 0.0025	0.0355 ± 0.0025
β	0.3024 ± 0.0008	0.3258 ± 0.0014	0.3470 ± 0.0016	0.3662 ± 0.0025
ω	0.812 ± 0.016	0.799 ± 0.011	0.789 ± 0.011	0.782 ± 0.0013

Shastry-Sutherland material: $\text{SrCu}_2(\text{BO}_3)_2$



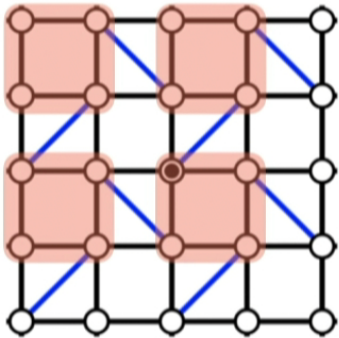
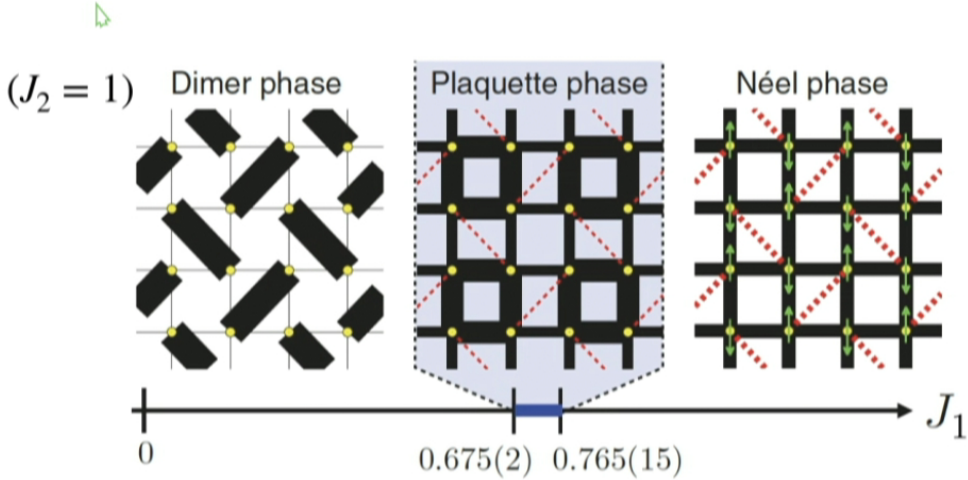
Kageyama, H. et al. Exact dimer ground state and quantized magnetization plateaus in the two-dimensional spin system $\text{SrCu}_2(\text{BO}_3)_2$. *Phys. Rev. Lett.* 82, 3168–3171 (1999).

Miyahara, S. & Ueda, K. Theory of the orthogonal dimer Heisenberg spin model for $\text{SrCu}_2(\text{BO}_3)_2$. *J. Phys. Condens. Matter* 15, R327 (2003).



$$H = J_1 \sum_{ij \in \text{n.n.}} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{ij \in \text{dimer}} \mathbf{S}_i \cdot \mathbf{S}_j$$

2D Phase Diagram (IPEPS / Perturbation Theory)



Z_2 glide-reflection
 symmetry broken
 ground states

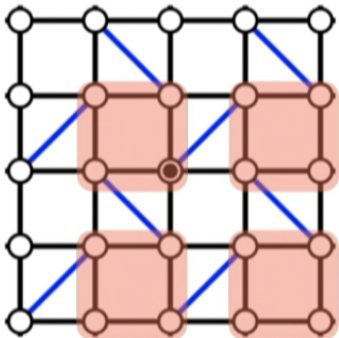
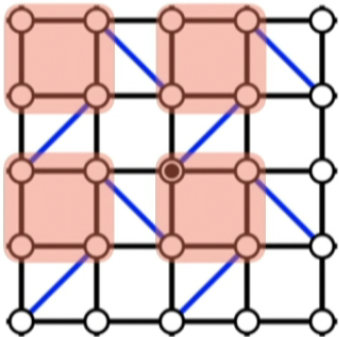
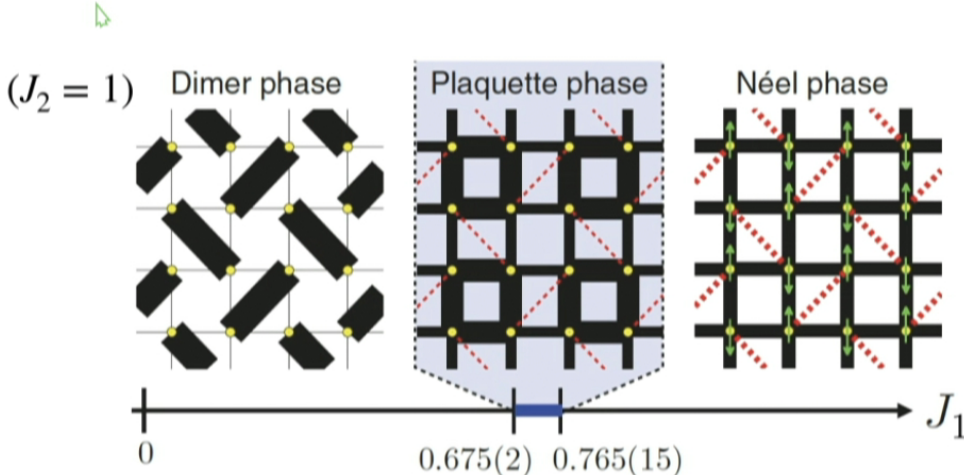


Figure from Corboz and Mila, PHYSICAL REVIEW B **87**, 115144 (2013)

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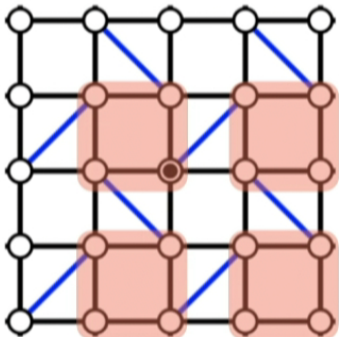
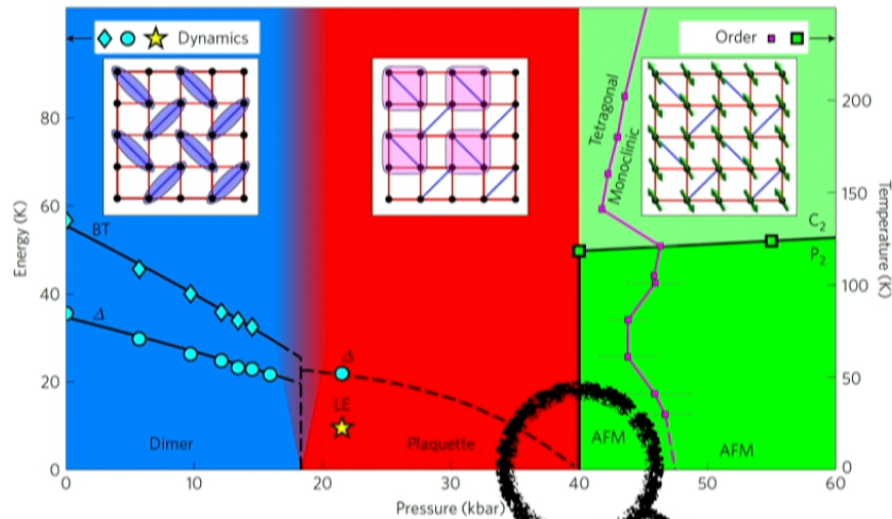


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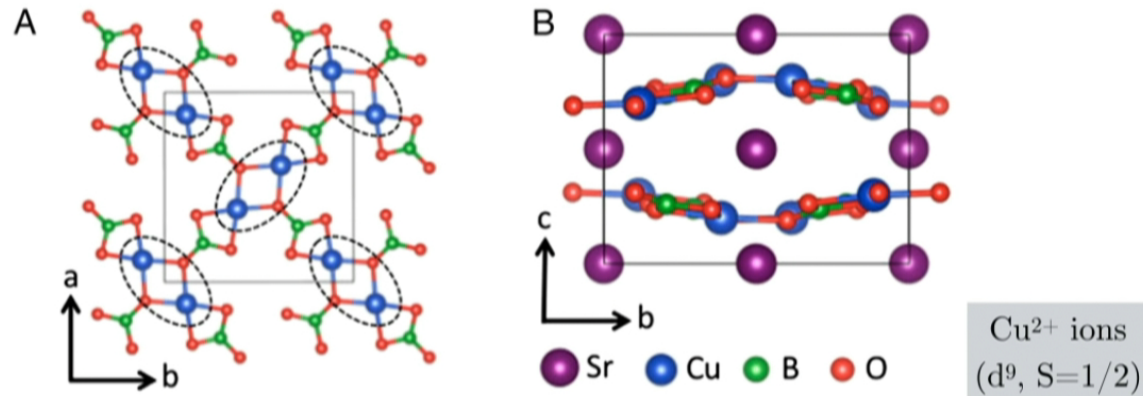
Recent Experiment with External Pressure



Candidate of Deconfined
Quantum Critical Point!

Zayed et al. (2017), DOI: 10.1038/NPHYS4190

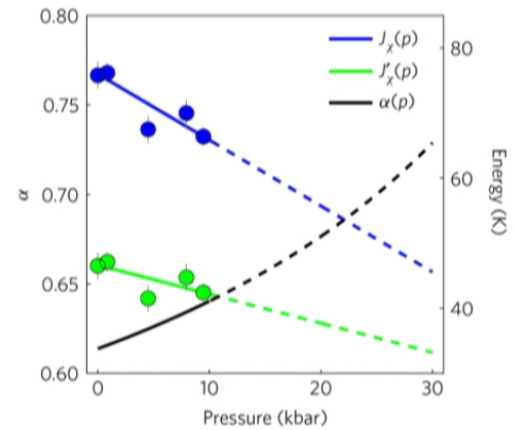
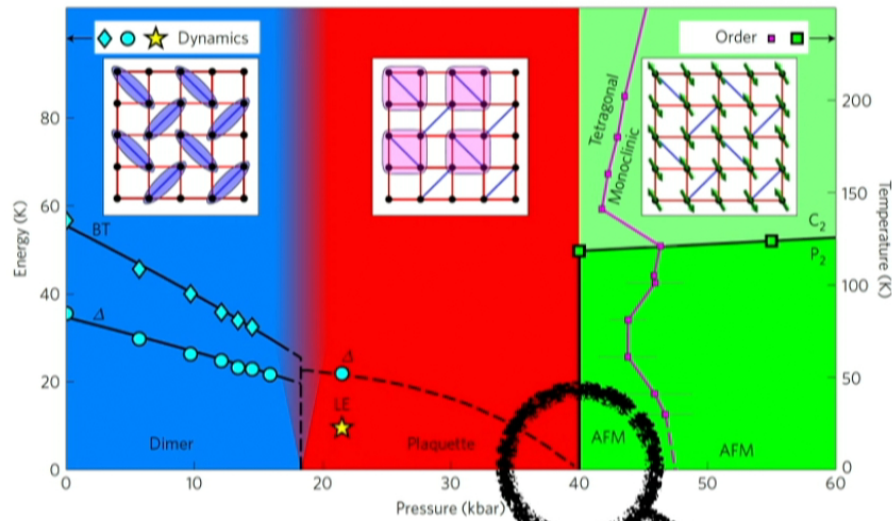
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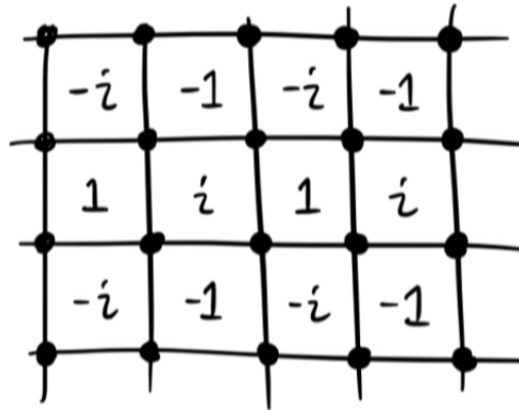
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Symmetry Analysis — Monopole

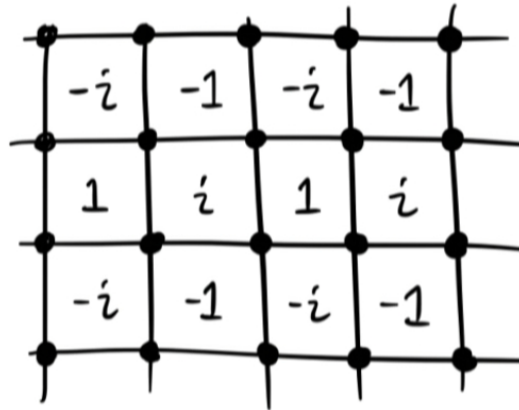


**Monopole Berry phase
= Lattice Effect**

Not considered in the CP^1 theory! —
Its effect on the continuum field theory
can be studied by symmetry analysis

Symmetries	Transformations
T_x	$\mathcal{M}^\dagger \mapsto -i\mathcal{M} \quad \mathbf{n} \mapsto -\mathbf{n}$
T_y	$\mathcal{M}^\dagger \mapsto i\mathcal{M} \quad \mathbf{n} \mapsto -\mathbf{n}$
$R_{\pi/4}$	$\mathcal{M}^\dagger \mapsto i\mathcal{M}^\dagger \quad \mathbf{n} \mapsto \mathbf{n}$
σ_x	$\mathcal{M}^\dagger \mapsto i\mathcal{M} \quad \mathbf{n} \mapsto \mathbf{n}$
σ_y	$\mathcal{M}^\dagger \mapsto -i\mathcal{M} \quad \mathbf{n} \mapsto \mathbf{n}$
\mathcal{T}	$\mathcal{M}^\dagger \mapsto \mathcal{M} \quad \mathbf{n} \mapsto -\mathbf{n}$

Symmetry Analysis — Monopole

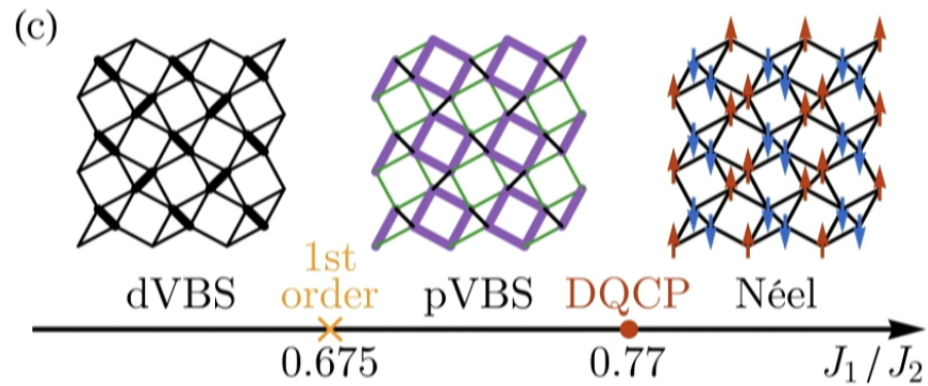


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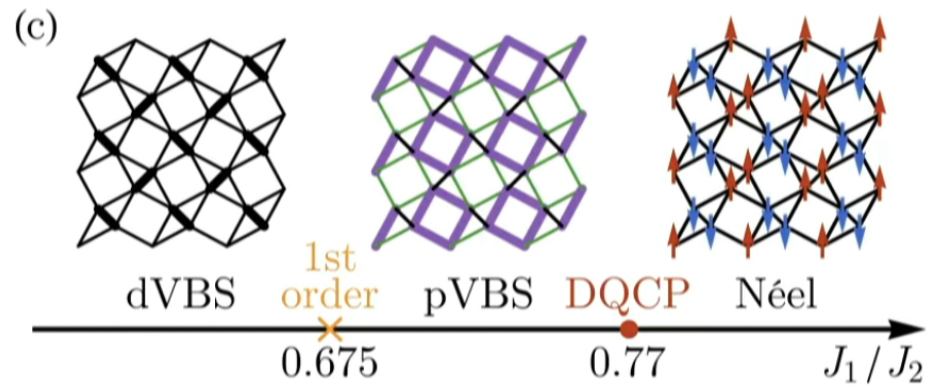
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Symmetry Analysis — Monopole



$$\mathcal{L}_M \sim \mathcal{M}^\dagger + \mathcal{M} \quad \text{or} \quad i(\mathcal{M}^\dagger - \mathcal{M})$$

Symmetry Analysis — Monopole

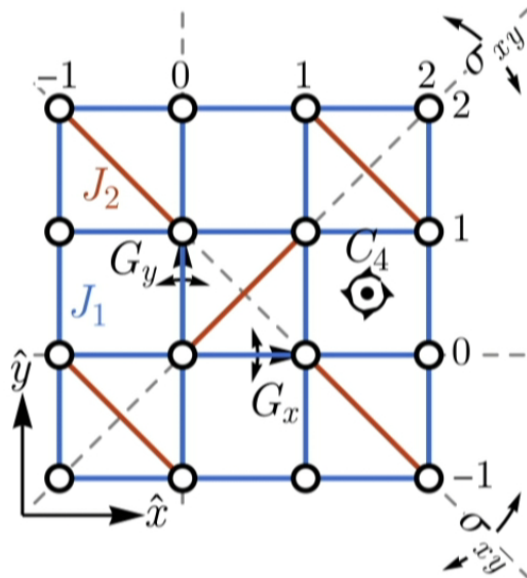


~~$$\mathcal{L}_M \sim \mathcal{M}^\dagger \partial_t \mathcal{M} + i(\mathcal{M}^\dagger - \mathcal{M})$$~~

Symmetry Analysis — Monopole



Strategy: starting from Neel order in the square lattice (p4m), one can deduce how monopoles transform under the lattice symmetries of Shastry-Sutherland lattice (p4g)



G_{p4g}	G_{p4m}	Action
T_x	T_x^2	$\mathcal{M}^\dagger \mapsto \mathcal{M}^\dagger \quad \mathbf{n} \mapsto \mathbf{n}$
T_y	T_y^2	$\mathcal{M}^\dagger \mapsto \mathcal{M}^\dagger \quad \mathbf{n} \mapsto \mathbf{n}$
σ_{xy}	$R_{\pi/2}\sigma_x$	$\mathcal{M}^\dagger \mapsto \mathcal{M} \quad \mathbf{n} \mapsto \mathbf{n}$
$\sigma_{x\bar{y}}$	$T_x T_y R_{\pi/2}\sigma_y$	$\mathcal{M}^\dagger \mapsto \mathcal{M} \quad \mathbf{n} \mapsto \mathbf{n}$
g_x	$T_x\sigma_x$	$\mathcal{M}^\dagger \mapsto -\mathcal{M}^\dagger \quad \mathbf{n} \mapsto -\mathbf{n}$
g_y	$T_y\sigma_y$	$\mathcal{M}^\dagger \mapsto -\mathcal{M}^\dagger \quad \mathbf{n} \mapsto -\mathbf{n}$
$R_{\pi/4}$	$R_{\pi/4}^{\text{plaq}}$	$\mathcal{M}^\dagger \mapsto -\mathcal{M} \quad \mathbf{n} \mapsto -\mathbf{n}$
\mathcal{T}	\mathcal{T}	$\mathcal{M}^\dagger \mapsto \mathcal{M} \quad \mathbf{n} \mapsto -\mathbf{n}$

Symmetry Analysis — Continuum Action

Square Lattice

$$\mathcal{L}_{\text{CP}^1} = \sum_i |(\partial - ia)z_i|^2 + \dots + \mathcal{L}_{\mathcal{M}} = r\mathcal{M}^\dagger\mathcal{M} + \lambda_4 \text{Re } \mathcal{M}^4$$

IRRELEVANT at DQCP

All other lower-order monopole terms are suppressed by C_4 rotation symmetry

Shastry-Sutherland Lattice

$$\mathcal{L}_{\text{CP}^1} = \sum_i |(\partial - ia)z_i|^2 + \dots + \mathcal{L}_{\mathcal{M}} = r\mathcal{M}^\dagger\mathcal{M} + \lambda_2 \text{Re } \mathcal{M}^2$$

RELEVANT at DQCP

All other monopole terms are suppressed by time-reversal-glide-reflection symmetry

Symmetry Analysis — Continuum Action

Square Lattice

$$\mathcal{L}_{\text{CP}^1} = \sum_i |(\partial - ia)z_i|^2 + \dots + \mathcal{L}_{\mathcal{M}} = r\mathcal{M}^\dagger\mathcal{M} + \lambda_4 \text{Re } \mathcal{M}^4$$

IRRELEVANT at DQCP

All other lower-order monopole terms are suppressed by C_4 rotation symmetry

Shastry-Sutherland Lattice

$$\mathcal{L}_{\text{CP}^1} = \sum_i |(\partial - ia)z_i|^2 + \dots + \mathcal{L}_{\mathcal{M}} = r\mathcal{M}^\dagger\mathcal{M} + \lambda_2 \text{Re } \mathcal{M}^2$$

RELEVANT at DQCP

All other monopole terms are suppressed by time-reversal-glide-reflection symmetry

Symmetry Analysis — Continuum Action

Relevant Perturbation! Would it lead to a first order transition?

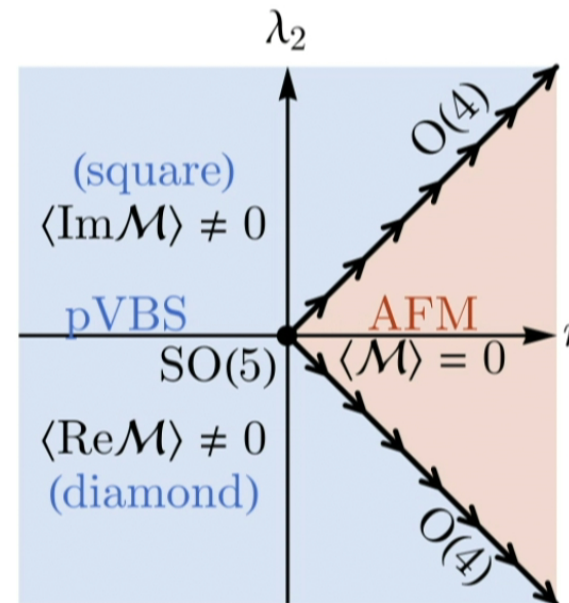
$$\mathcal{L}_{\mathcal{M}} = r\mathcal{M}^\dagger\mathcal{M} + \lambda_2 \text{Re } \mathcal{M}^2 + \dots$$



$$\mathcal{L}_{\mathcal{M}} = \tilde{r}(\text{Im } \mathcal{M})^2 + \tilde{\lambda}_2(\text{Re } \mathcal{M})^2$$

$$\tilde{r} = r - \lambda_2 \text{ and } \tilde{\lambda}_2 = r + \lambda_2$$

Perturbation gaps out imaginary or real part of monopole fluctuations



Symmetry Analysis — Continuum Action

$$\mathcal{L}_{\mathcal{M}} = \tilde{r}(\text{Im } \mathcal{M})^2 + \tilde{\lambda}_2(\text{Re } \mathcal{M})^2$$

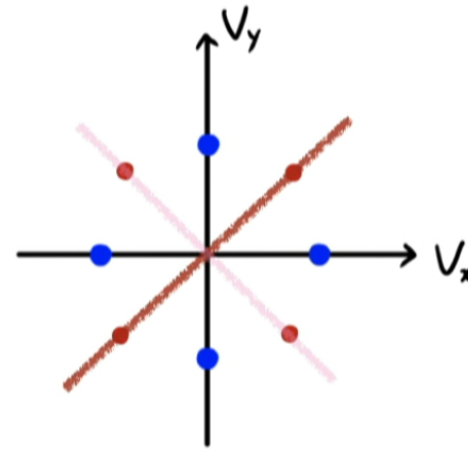
Perturbation gaps out imaginary or real part of monopole fluctuations

$$n = (N_x, N_y, N_z, V_x, V_y)$$

$$\downarrow \lambda_2 > 0$$

$$\tilde{n} = (N_x, N_y, N_z, \text{Im } \mathcal{M})$$

$$\left(\text{Im } \mathcal{M} \sim V_x - V_y \right)$$



Symmetry Analysis — Continuum Action

Relevant Perturbation! Would it lead to a first order transition?

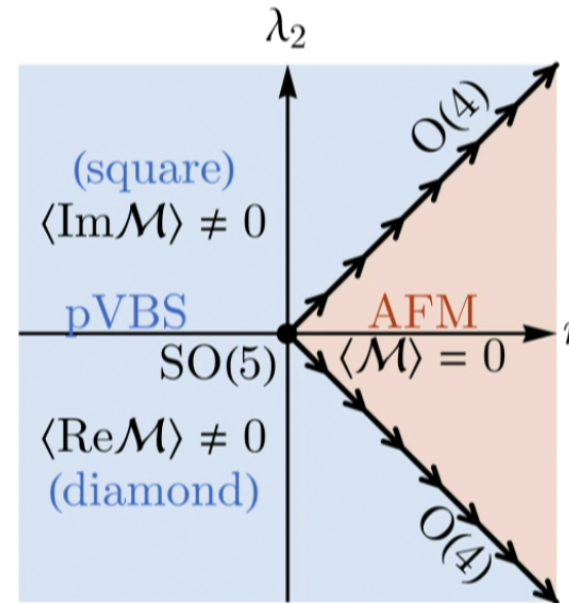
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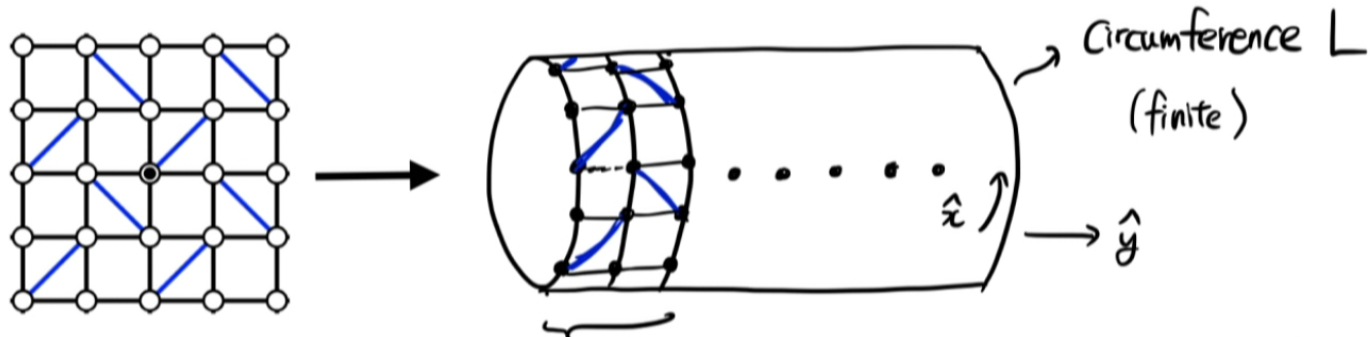
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Perturbation gaps out imaginary or real part of monopole fluctuations

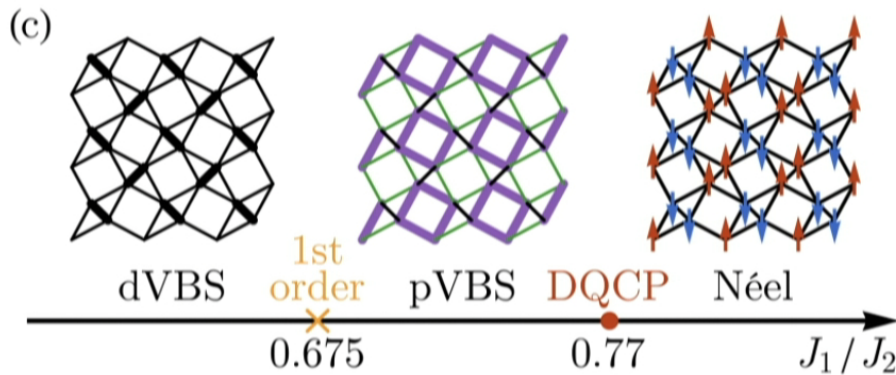


IDMRG study of phase transition



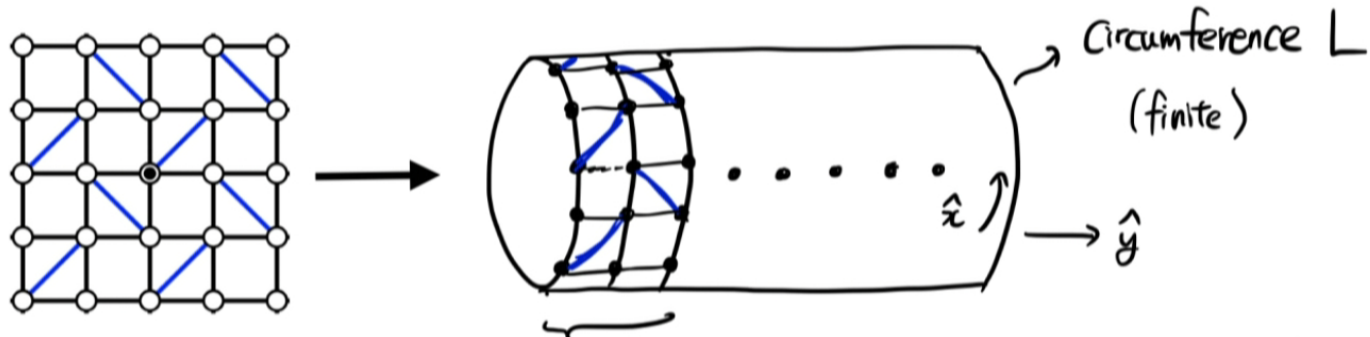
$$H = J_1 \sum_{ij \in \text{n.n.}} S_i \cdot S_j + J_2 \sum_{ij \in \text{dimer}} S_i \cdot S_j$$

DMRG unit cell \rightarrow repeated in infinite direction



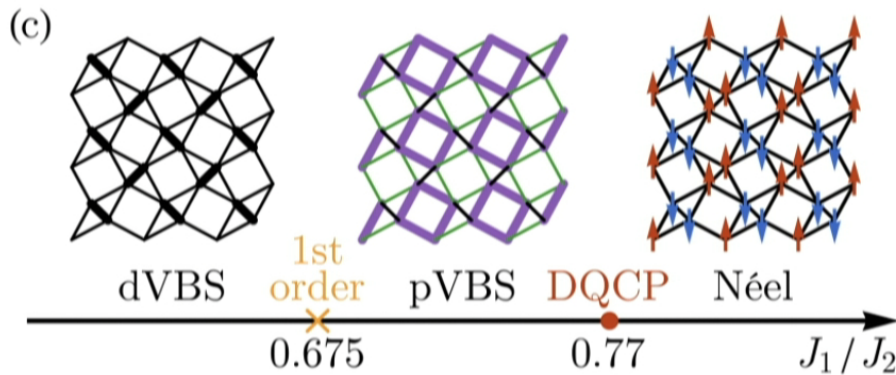
- $L = 6, 8, 10$ (double columns)
- Bond dimension up to 5000

IDMRG study of phase transition



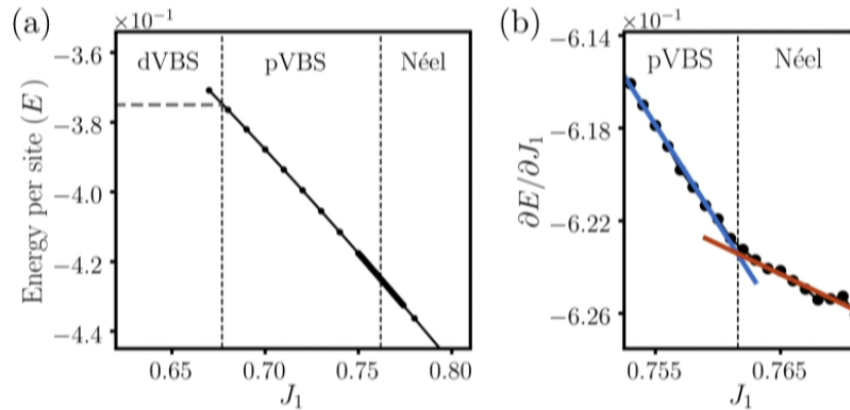
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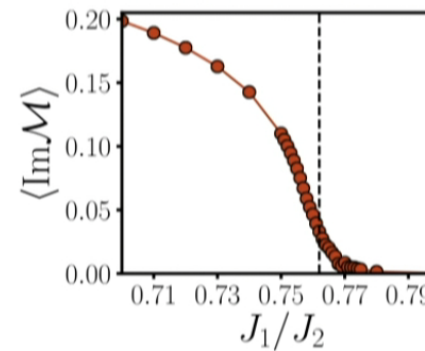
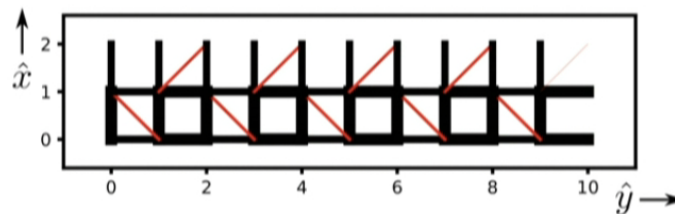
IDMRG study of phase transition



First order derivative of the energy seems continuous
— weakly first order or second order transition

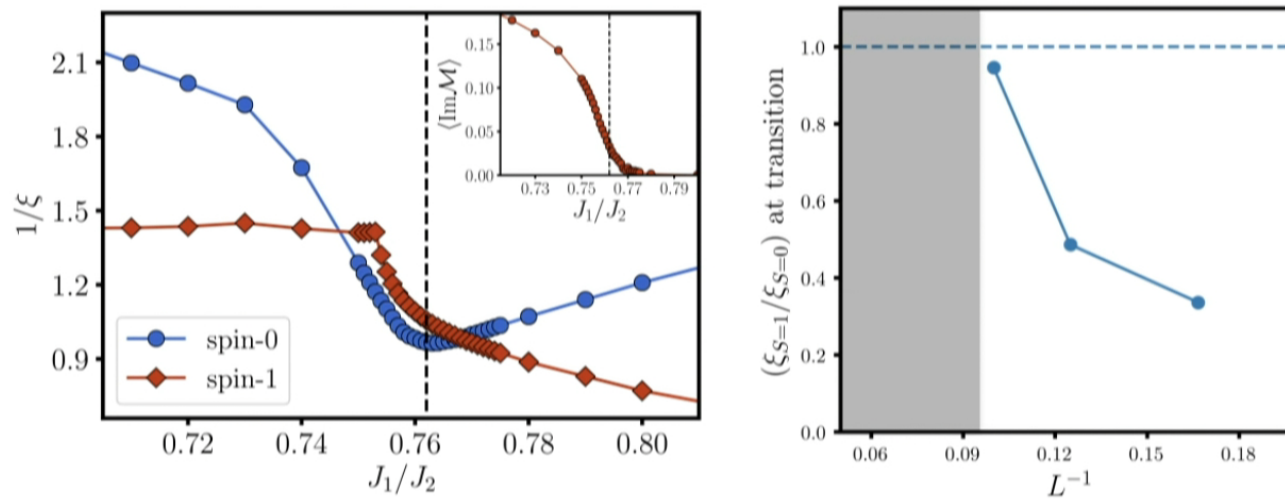
$$\mathcal{M}^\dagger = \frac{1}{\sqrt{2}} ((V_x + V_y) + i(V_x - V_y)),$$

VBS has a singlet plaquette at the empty square



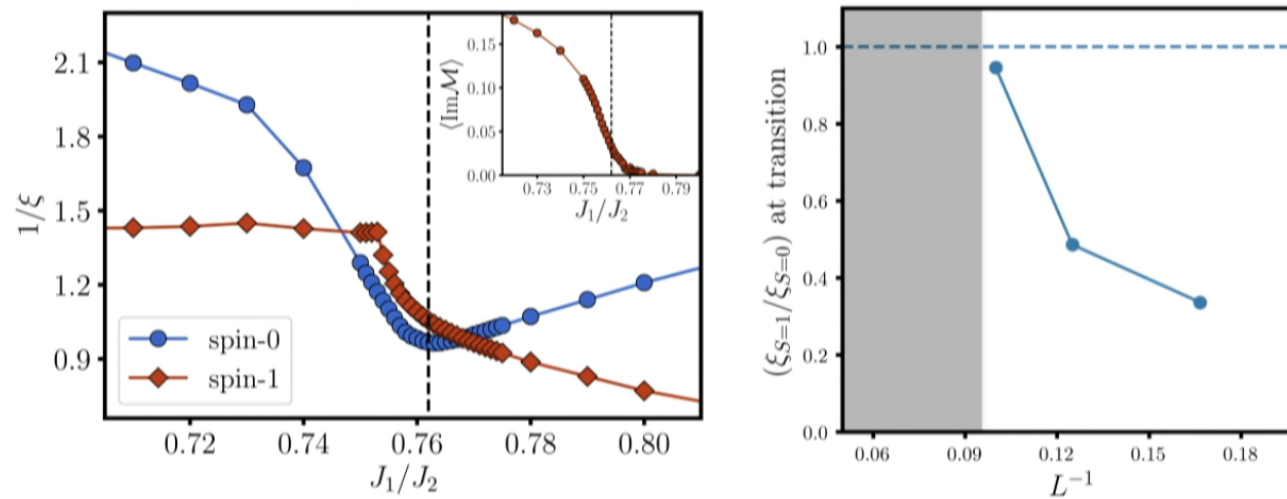
IDMRG study of phase transition

Correlation Length Spectra



IDMRG study of phase transition

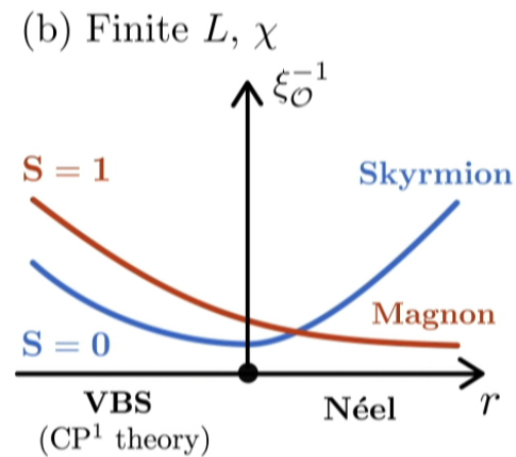
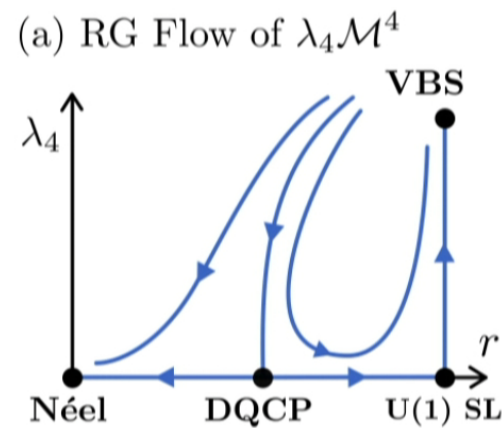
Correlation Length Spectra



Absence of Dangerously Irrelevant Operator

Conventional SO(5) or easy-plane O(4) DQCP

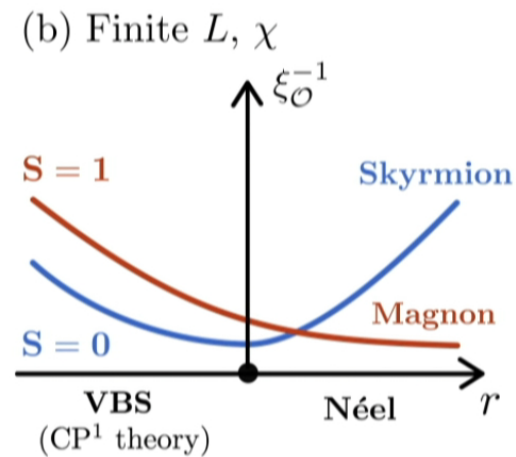
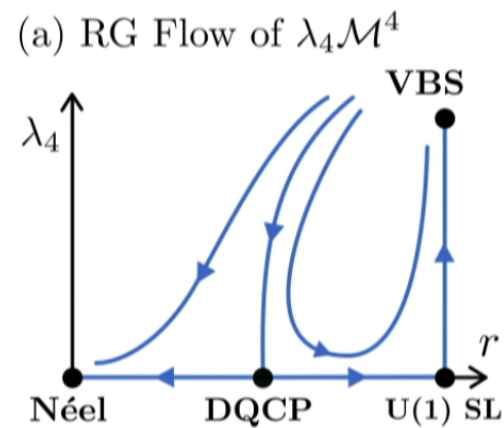
There is a separation of length scales between correlation length for spin and VBS fluctuations due to the emergent U(1) symmetry



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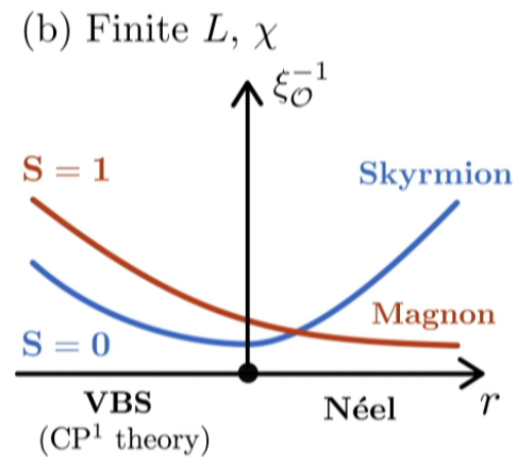
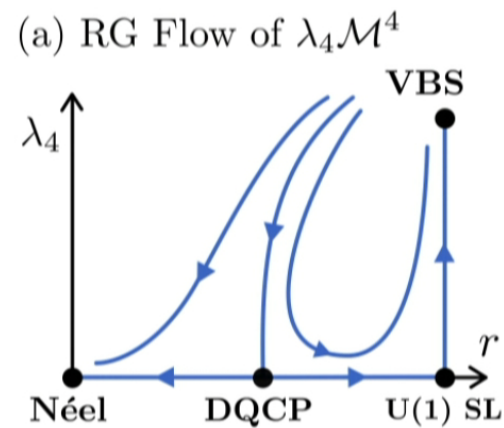
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Absence of Dangerously Irrelevant Operator

Conventional SO(5) or easy-plane O(4) DQCP

There is a separation of length scales between correlation length for spin and VBS fluctuations due to the emergent U(1) symmetry



Identify Noether Currents

- Emergent **O(4) symmetry** rotates the O(4) vector

$$\mathbf{n} = (n_1, n_2, n_3, n_4)$$

$$\mathcal{L}[\mathbf{n}] = \frac{1}{g} (\partial_\mu \mathbf{n})^2 + \frac{i\Theta}{2\pi^2} \epsilon^{abcd} n_a \partial_\tau n_b \partial_x n_c \partial_y n_d$$

- Noether theorem
 - Each **generator** T_{ab} : $\mathbf{n} \rightarrow e^{i\theta T_{ab}} \mathbf{n}$ ($a, b = 1, 2, 3, 4$)
 - Associate with a **conserved current** J_{ab}^μ with $\partial_\mu J_{ab}^\mu = 0$

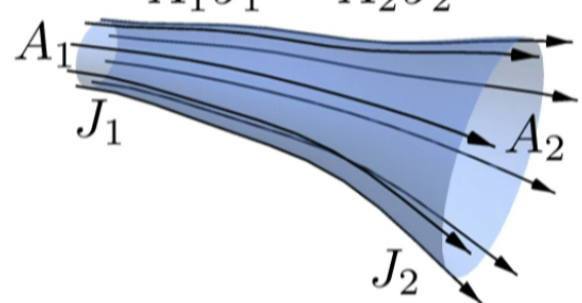
$$J_{ab}^\mu = \frac{\delta \mathcal{L}}{\delta (\partial_\mu \mathbf{n})} \cdot (iT_{ab} \mathbf{n}) = n_a \partial_\mu n_b - n_b \partial_\mu n_a$$

- SO(4) has 6 generators, each current has (2+1) space-time components \rightarrow altogether **18 components**
- Not all of them appear in the **spin** excitation spectrum ...

From Yi-Zhuang You's Talk

Conservation Law from Scaling Dimension

- In (2+1)D space-time, conserved current must scale as

$$A_1 J_1 = A_2 J_2 \quad \Rightarrow \quad J \sim 1/A \sim 1/r^2$$


Current-current correlation must decay with exact power

$$\langle J_{ab}^\mu(r) J_{ab}^\mu(0) \rangle \sim \frac{1}{r^4}$$

- Non-conserved current will not follow the precise scaling

$$\langle J_{ab}^\mu(r) J_{ab}^\mu(0) \rangle \sim \frac{1}{r^{4+\eta}} \leftarrow \text{“anomalous” dimension}$$

- A **vanishing** η indicates the current **conservation**
- This exponent can be measured from **spin-spin correlation**, given their correspondence to the O(4) current

From Yi-Zhuang You's Talk

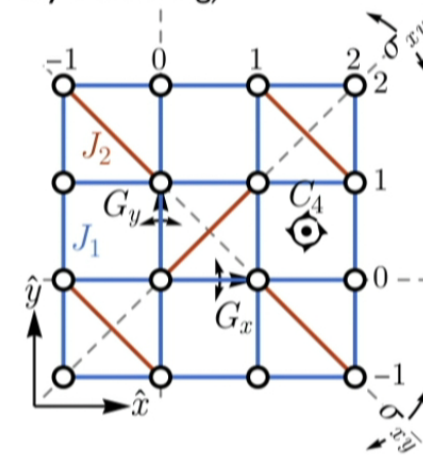
Spectral Signature of DQCP

(Inelastic Neutron Scattering or Resonant Inelastic X-ray Scattering)

Glide-reflection symmetry — Extinction of some Bragg peaks

$$S(Q_x, Q_y) = e^{iQ_x/2} S(Q_x, -Q_y) = e^{iQ_y/2} S(-Q_x, Q_y)$$

$$S = 0 \quad \text{if} \quad Q \in \pi(2\mathbb{Z} + 1, 0) \text{ or } \pi(0, 2\mathbb{Z} + 1)$$



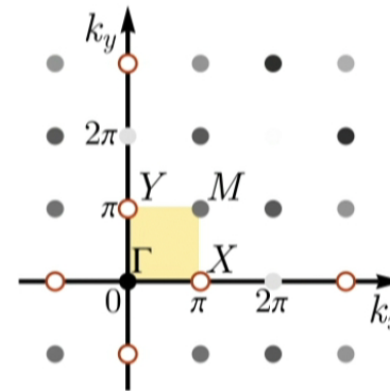
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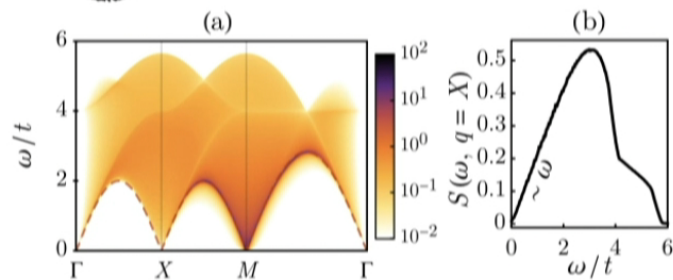
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1

Magnon Spectrum ($S=1$)



conserved current of $O(4)$ symmetry

$$\mathbf{J}_y = \mathbf{n} \partial_y \text{Im } \mathcal{M} - \text{Im } \mathcal{M} \partial_y \mathbf{n}$$

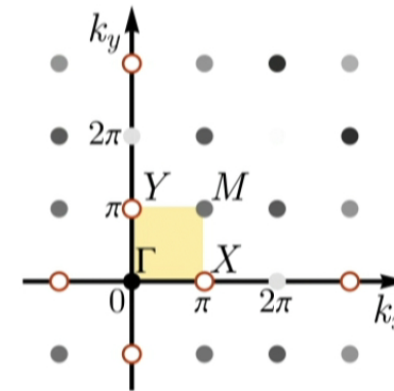
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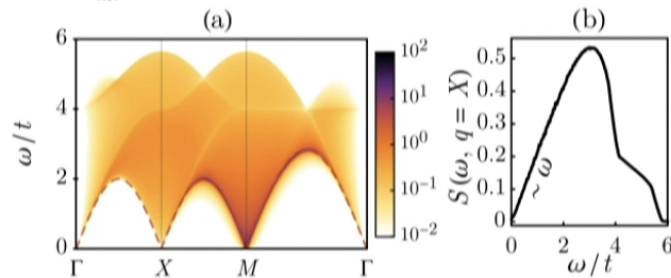
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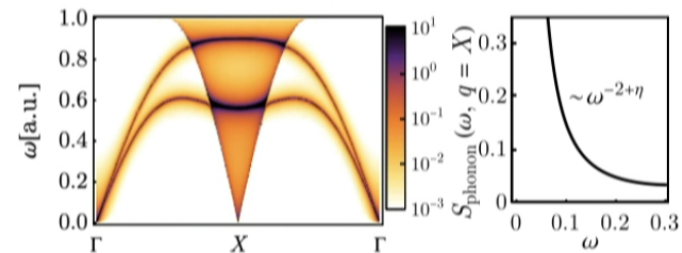


conserved current of $O(4)$ symmetry

$$\mathbf{J}_y = \mathbf{n} \partial_y \text{Im } \mathcal{M} - \text{Im } \mathcal{M} \partial_y \mathbf{n}$$

2

Phonon Spectrum ($S=0$)



$$\mathcal{L}_{\text{VBS-phonon}} \sim u_x V_x + u_y V_y$$

$$S_{\text{phonon}}(\omega, \mathbf{q} = X) \propto \omega^{-2+\eta}$$

Interlayer Coupling

$$\mathcal{S}_{\text{DQCP}}^{(l)} = \int d^3x \frac{1}{2g} (\partial_\mu \Phi^{(l)})^2 + 2\pi i \Gamma_{\text{WZW}}[\Phi^{(l)}]$$

$$\mathcal{S} = \sum_l \mathcal{S}_{\text{DQCP}}^{(l)} - \sum_l \int d^3x g^{ab} \Phi_a^{(l)} \Phi_b^{(l+1)}$$

Depending on the locking parameterized by g^{ab} , WZW-terms in neighboring neighbors would get canceled or added up

Canceled — Becomes Theta-term!

Added — System behaves as a classical system!

Interlayer Coupling

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Interlayer Coupling

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3+1D NLSM with O(5) vector and theta-term



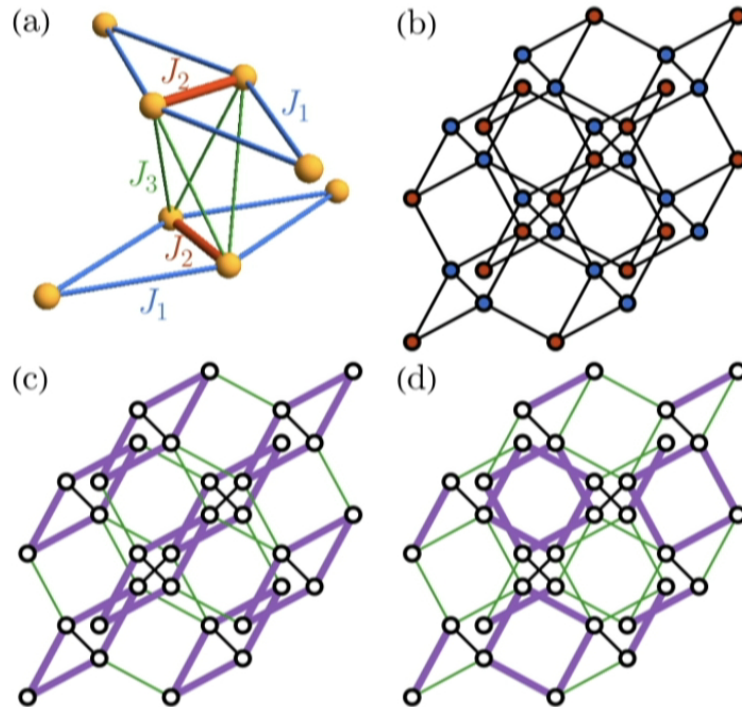
$$\mathcal{L} = \bar{\psi}(\gamma^\mu D_\mu + m + i\gamma^5 \Phi_a \Gamma^a)\psi.$$

SU(2) QCD with $m=0$

The number of Dirac fermion flavors ($N=2$) is not enough to avoid a chiral symmetry breaking in 3+1D.

Flow to first-order transition

Interlayer Coupling



$$\mathbf{n}^{(l)} = \mathbf{n}^{(l+1)}$$

$$\text{Im } \mathcal{M}^{(l)} = -\text{Im } \mathcal{M}^{(l+1)}$$

$$\text{Re } \mathcal{M}^{(l)} = \text{Re } \mathcal{M}^{(l+1)}$$

$$g^{aa} = (+, +, +, +, -)$$

Flow to first-order transition

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Summary

- Monopole symmetry analysis — $O(4)$ DQCP is possible
- Infinite DMRG study
 - Phase diagram confirmed
 - Absence of dangerously irrelevant scaling
 - Emergent $O(4)$ symmetry
- Predicted spectral features of the emergent $O(4)$ symmetry, which can be probed by INS, RIXS or NMR
- Coupling 2+1D DQCP into 3+1D — leads to the first order

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(Harvard)

