

Title: Finite Correlation Length Scaling in Lorentz-Invariant Gapless iPEPS Wave Functions

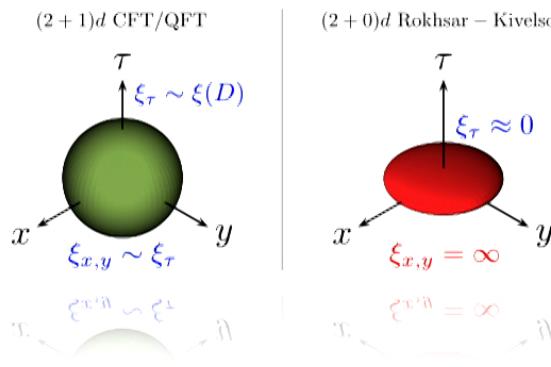
Speakers: Andreas Lauchli

Collection: Quantum Matter: Emergence & Entanglement 3

Date: April 25, 2019 - 2:00 PM

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Abstract: It is an open question how well tensor network states in the form of an infinite projected entangled-pair states (iPEPS) tensor network can approximate gapless quantum states of matter. In this talk we address this issue for two different physical scenarios: (i) a conformally invariant (2+1)d quantum critical point in the incarnation of the transverse-field Ising model on the square lattice and (ii) spontaneously broken continuous symmetries with gapless Goldstone modes exemplified by the S=1/2 antiferromagnetic Heisenberg and XY models on the square lattice. We find that the energetically best wave functions display finite correlation lengths and we introduce a powerful finite correlation length scaling framework for the analysis of such finite bond dimension (finite-D) iPEPS states. The framework is important (i) to understand the mild limitations of the finite-D iPEPS manifold in representing Lorentz-invariant, gapless many-body quantum states and (ii) to put forward a practical scheme in which the finite correlation length  $\hat{l}^{3/4}(D)$  combined with field theory inspired formulas can be used to extrapolate the data to infinite correlation length, i.e., to the thermodynamic limit. The finite correlation length scaling framework opens the way for further exploration of quantum matter with an (expected) Lorentz-invariant, massless low-energy description, with many applications ranging from condensed matter to high-energy physics.



## Quantum Criticality with iPEPS (&iMPS) Tensor Networks

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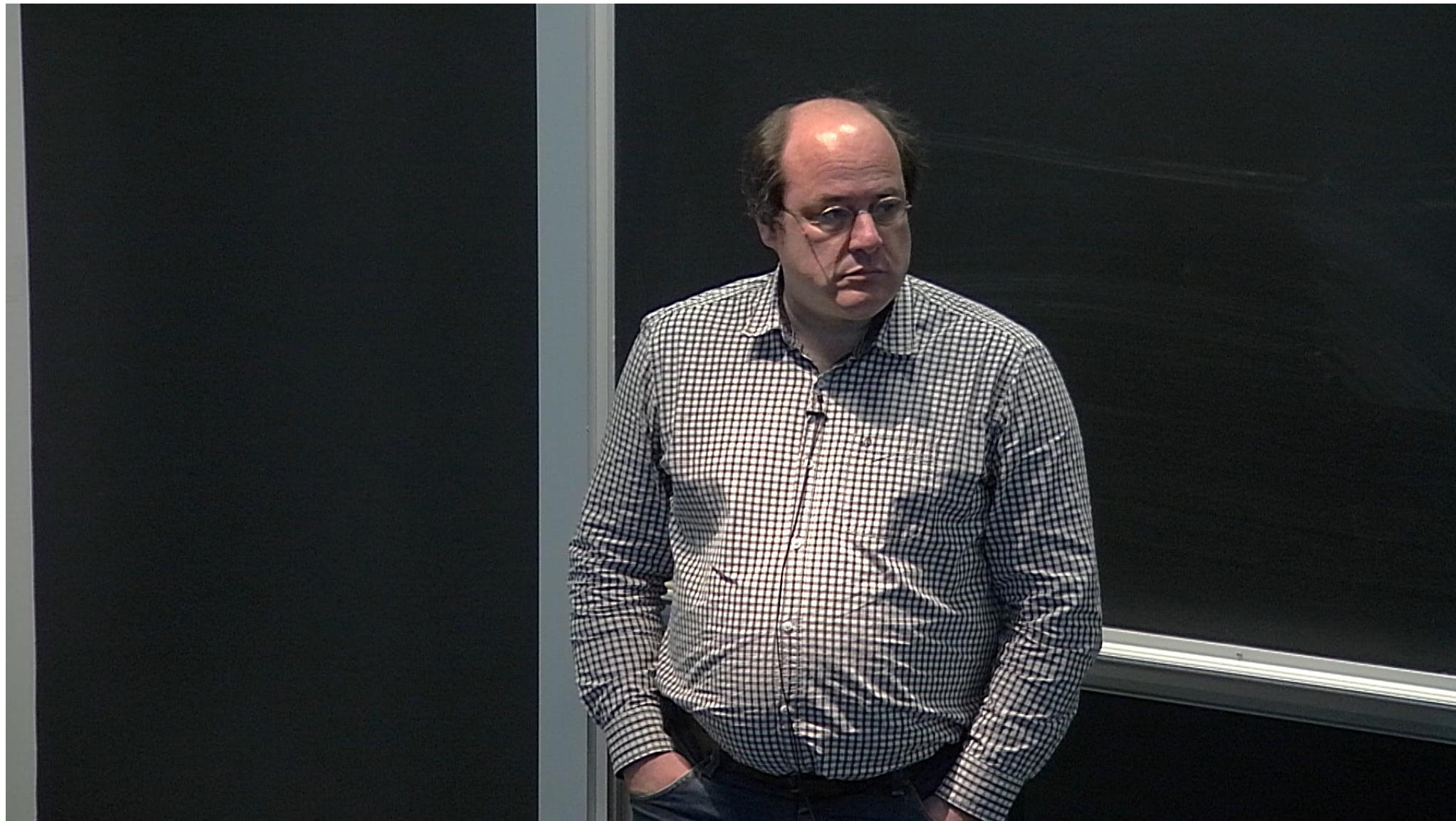
[andreas.laeuchli@uibk.ac.at](mailto:andreas.laeuchli@uibk.ac.at)  
<http://www.uibk.ac.at/th-physik/laeuchli-lab>



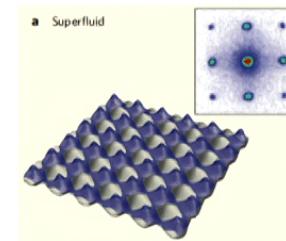
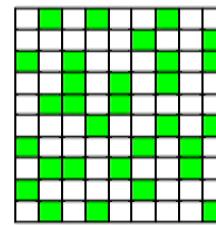
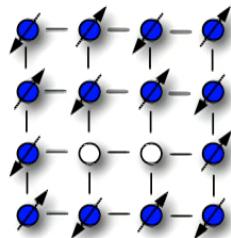
Support: **FWF**



PI Workshop “Quantum Matter: Emergence & Entanglement 3”, 2019/4/25, Waterloo



# Quantum Matter

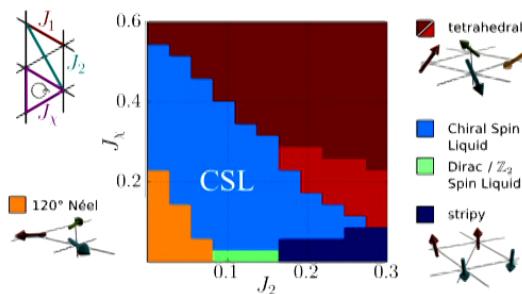


- We would like to understand phase diagrams of complex systems, but whose Hamiltonians are often reasonably well known.
- Quantum phase transitions occur. What is their universality class & field theoretical description ?
- New tools welcome to diagnose/characterize QFTs at phase transitions

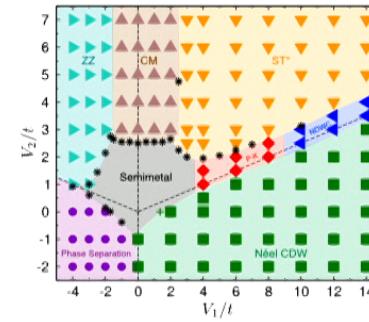


# Example of Microscopic Condensed Matter Models

- From microscopic models:



Phys. Rev. B **95**, 035141 (2017)

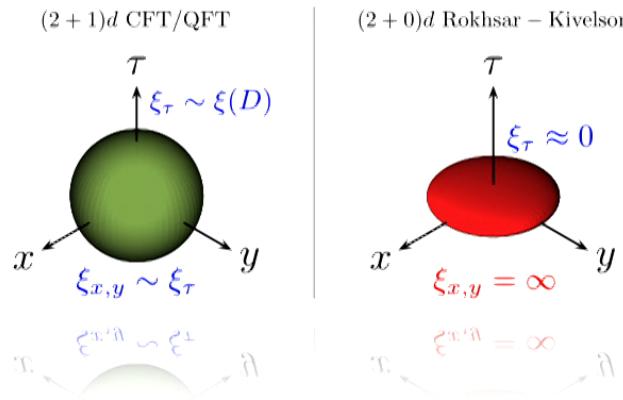


Phys. Rev. B **92**, 085146 (2015)

$$\mathcal{H} = J_1 \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \vec{S}_i \cdot \vec{S}_j + J_x \sum_{i,j,k \in \Delta} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$

$$\begin{aligned} \mathcal{H} = & -t \sum_{\langle ij \rangle} (c_i^\dagger c_j + h.c.) \\ & + V_1 \sum_{\langle ij \rangle} (n_i - 1/2)(n_j - 1/2) \\ & + V_2 \sum_{\langle\langle ij \rangle\rangle} (n_i - 1/2)(n_j - 1/2) \end{aligned}$$

- To quantum phase transitions: Wilson Fisher CFTs, QED<sub>3</sub>, Gross Neveu, ...



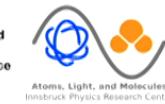
## Quantum Criticality of iPEPS Tensor Networks

M. Rader & A.M. Läuchli, Phys. Rev. X 8, 031030 (2018).

see also Ph. Corboz et al., Phys. Rev. X 8, 031031 (2018).

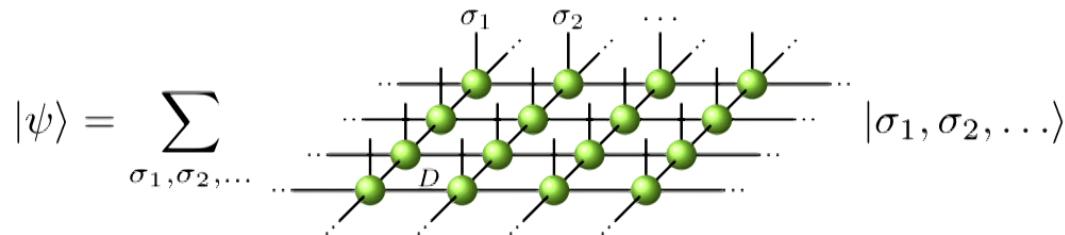


Michael Rader  
PhD Student



## new tool: iPEPS

- infinite (system size) projected entangled pair states (iPEPS) tensor network  
F. Verstraete & I. Cirac 2004, J. Jordan et al 2008



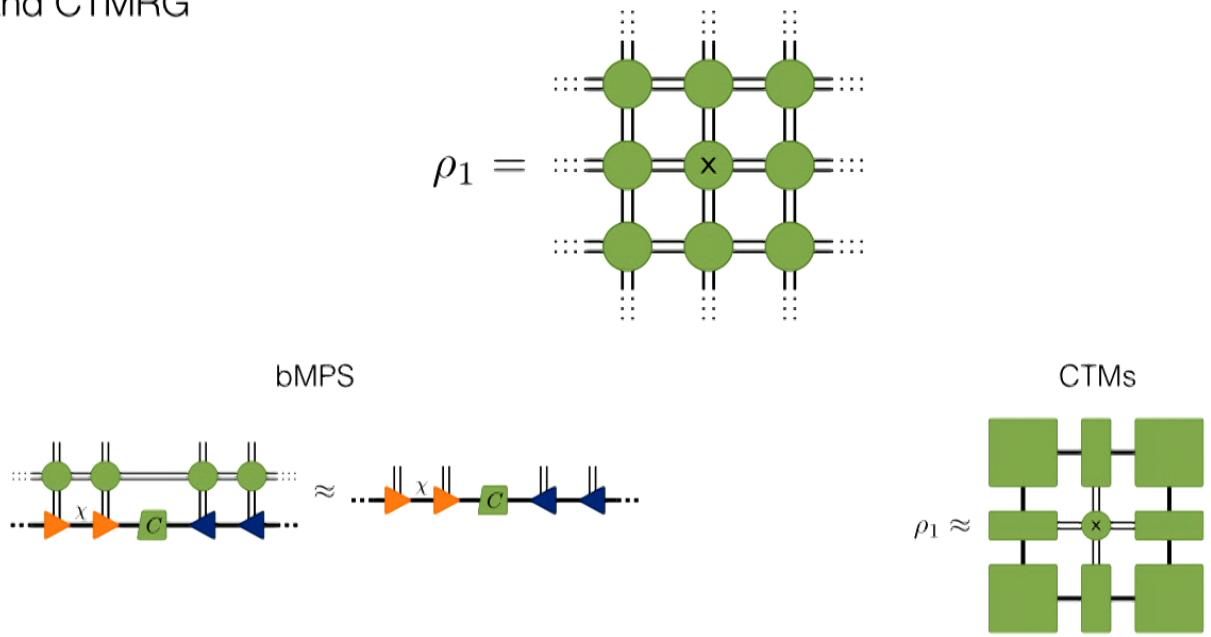
- Twofold new approach to quantum many body systems:
  - particular tensor network structure
  - directly in the thermodynamic limit, i.e. infinite system.  
**How does that work well for quantum criticality ?**  
(rather well understood for iMPS, c.f. Tagliacozzo et al. 2008, Pollmann et al. 2009)

## Brief iPEPS technicalities

- Optimization by new gradient descent methods

P. Corboz 2016, L. Vanderstraeten et al 2016

- Contractions using boundary MPS and CTMRG



## Brief iPEPS technicalities

- Correlation functions:

$$c(r) = \begin{array}{c} \text{Diagram showing a central green circle connected to two vertical bars, } u \text{ (blue) and } v \text{ (orange), which are further connected to horizontal lines. A bracket labeled } r-1 \text{ spans the distance between the } u \text{ bar and the } v \text{ bar. Blue and orange arrows point towards the central circle from the bars.} \end{array} = \mathbf{u}^T \mathbf{A}^{r-1} \mathbf{v}$$

$$\begin{array}{c} \text{Diagram showing a central green circle connected to two vertical bars, both labeled } j, \text{ which are further connected to horizontal lines. Blue and orange arrows point towards the central circle from the bars.} \end{array} = \lambda_j \quad j =$$

$$|\lambda_0| \geq |\lambda_1| \geq \dots$$

$$\xi = -\frac{1}{\log |\lambda_1|}$$

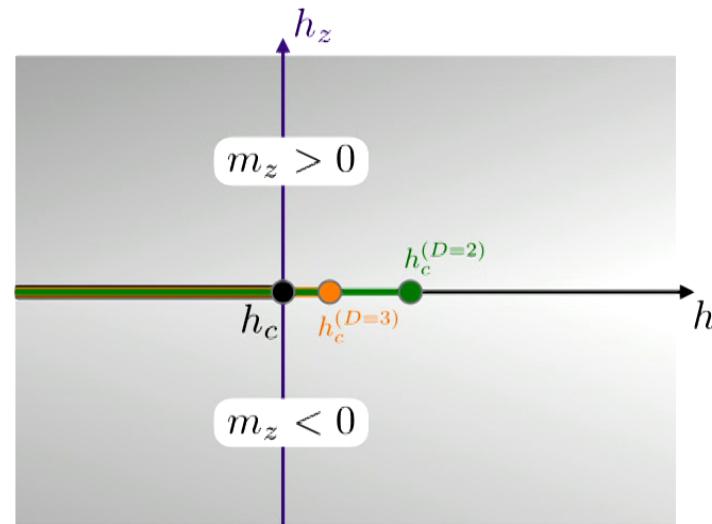
$$\frac{1}{\xi(\chi)} = \frac{1}{\xi(\infty)} + k \log \left| \frac{\lambda_1(\chi)}{\lambda_2(\chi)} \right|$$

M. Rams et al, PRX 2018

## iPEPS for (2+1)d Ising criticality

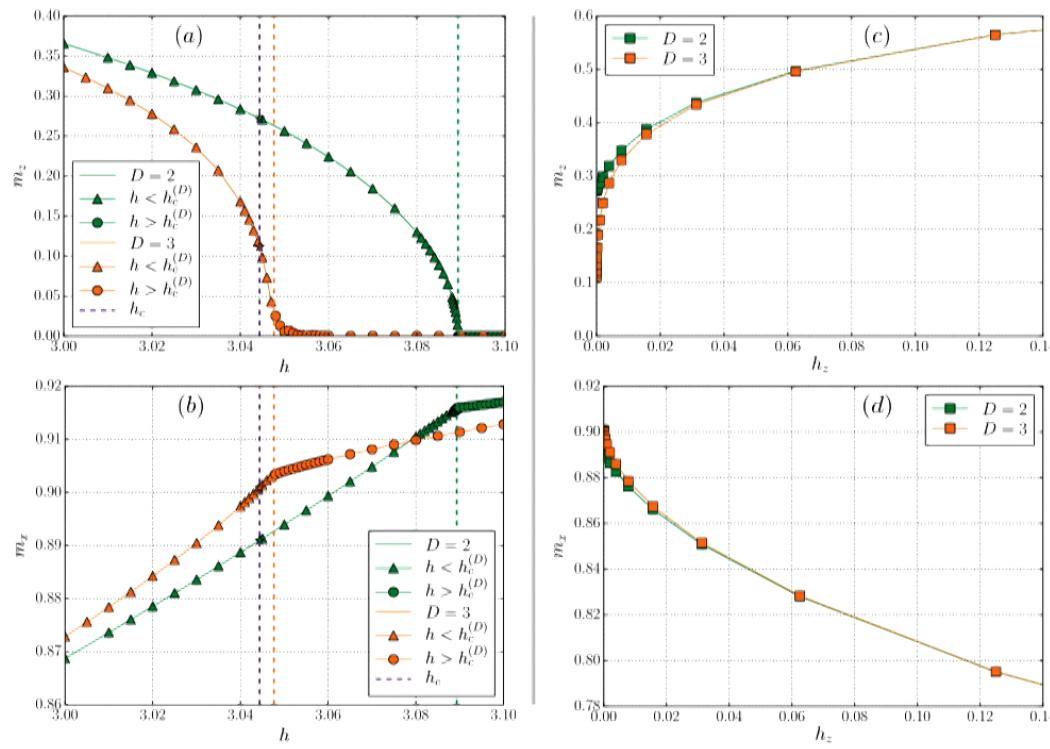
- Transverse field Ising model (Hamiltonian formulation)

$$H_{\text{TFI}} = -J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x - h_z \sum_i \sigma_i^z$$

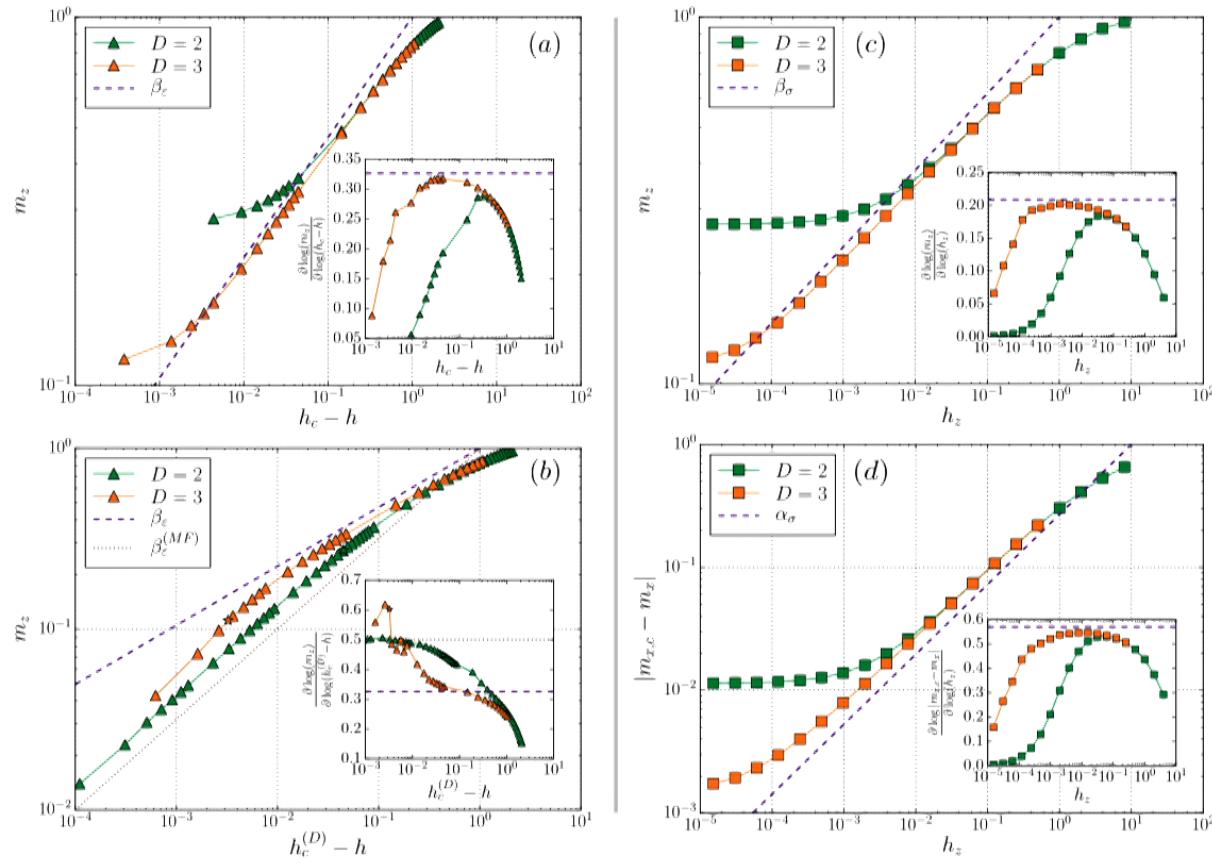


# Variationally optimised iPEPS tensors

- Ising criticality perturbed in two relevant directions ( $h-h_c / h_z$ ):

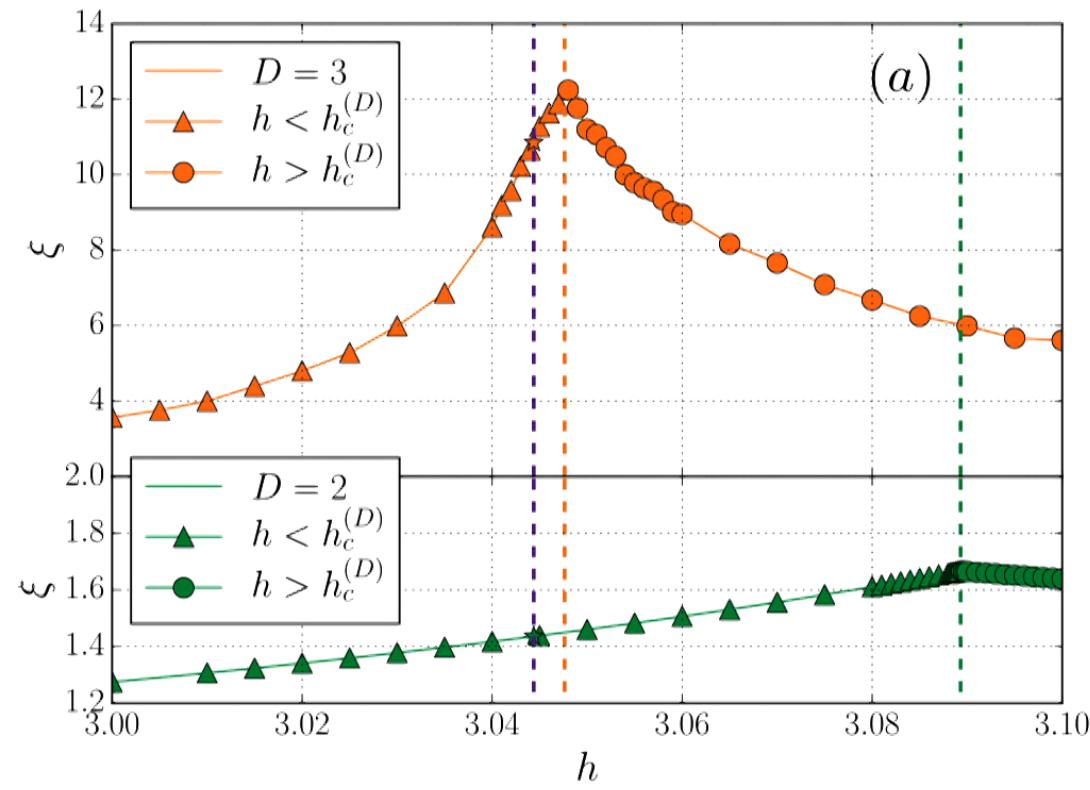


# Approach to criticality: local observables as a function of perturbing coupling



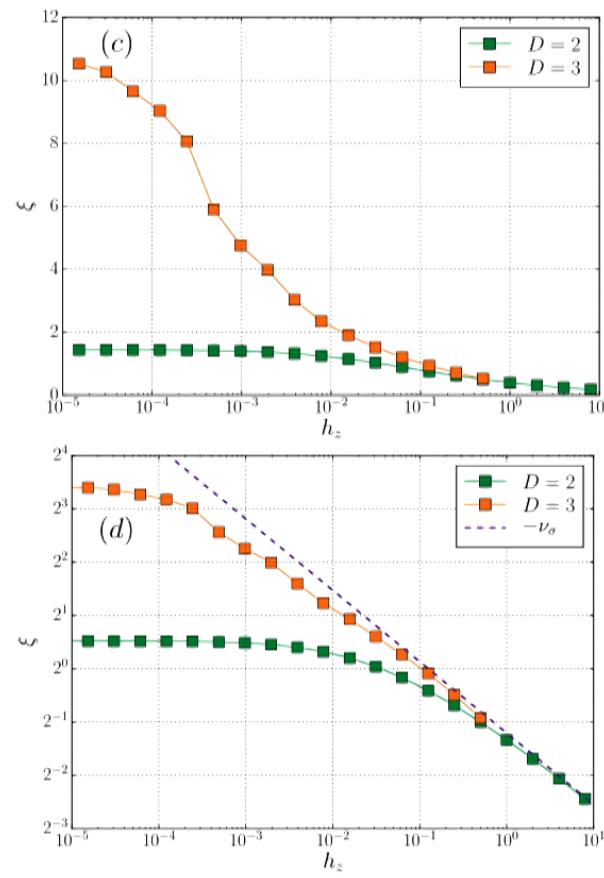
## Now: correlation lengths

- tuning  $h-h_c$  : correlation lengths grow, but do not diverge !

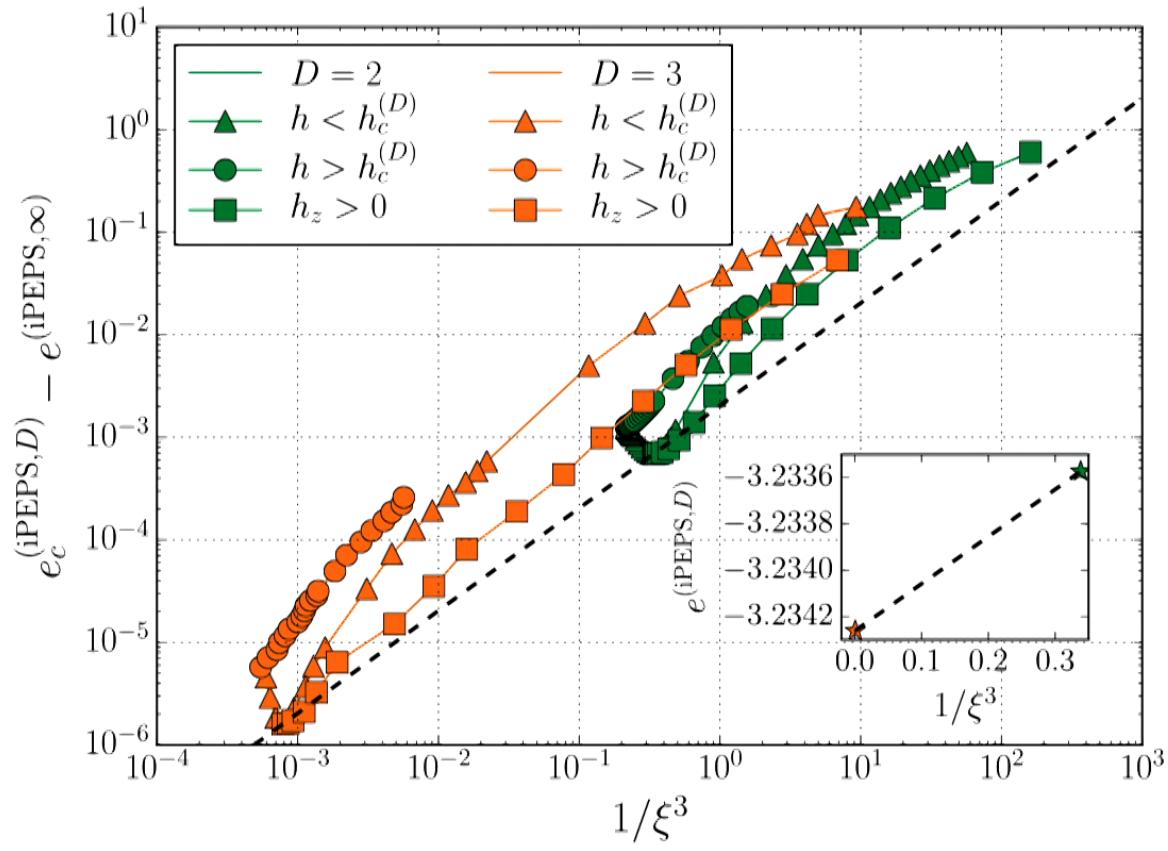


## Now: correlation lengths

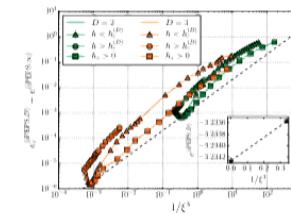
● tuning  $h_z$  at  $h_c$ :



## Correlation lengths: “Casimir effect”



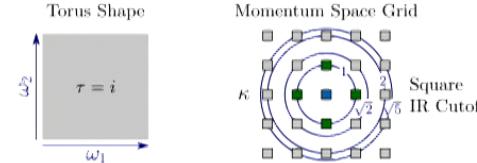
## Correlation lengths: “Casimir effect”



- Finite volume ( $L$ ):

$$e(L) = e(\infty) - \frac{\alpha_{\tau}^{\text{QCP}} \times v}{L^3}$$

$$\alpha_{\tau=i}^{(3d \text{ Ising CFT})} = +0.35(2)$$



- Our conjecture: Finite correlation length scaling of variational energy:

$$e(\xi) = e(\infty) - \frac{\alpha_{\text{iPEPS}}^{(3d \text{ Ising CFT})} \times v}{\xi^3}$$

$$\alpha_{\text{iPEPS}}^{(3d \text{ Ising CFT})} \approx -0.00061, \quad e(\infty) \approx -3.2342623.$$

And now for something completely different:  
Continuous symmetry breaking

## Different scenario: Continuous symmetry breaking (i.e. magnetic order, superfluids, ...)

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- hydrodynamic description based on O(N) non-linear sigma model  
(collinear order): parameters: velocity, spin stiffness, ordered moment
- finite volume effects:

$$e(L) = e(\infty) - \left[ \alpha_{\text{shape/bc}}^{\text{NLSM}} \left( \frac{N-1}{2} \right) v \right] \frac{1}{L^3} + \frac{(N-1)(N-2)}{8} \frac{v^2}{\rho_s L^4} + \mathcal{O}\left(\frac{1}{L^5}\right)$$

$$\frac{m^2(L)}{m^2(\infty)} = 1 + \left[ \mu_{\text{shape/bc}}^{\text{NLSM}} \left( \frac{N-1}{2} \right) \frac{v}{\rho_s} \right] \frac{1}{L} + \mathcal{O}\left(\frac{1}{L^2}\right)$$

## Continuous symmetry breaking (i.e. magnetic order, superfluids, ...)

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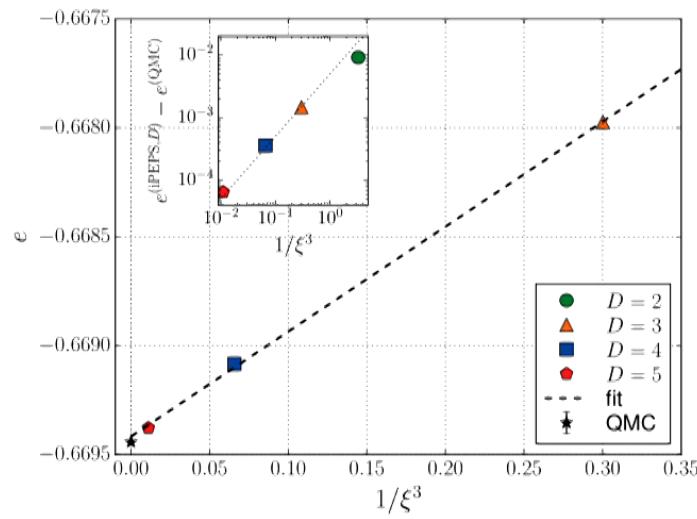
- hydrodynamic description based on O(N) non-linear sigma model (collinear order): parameters: velocity, spin stiffness, ordered moment
- Conjectured finite correlation length effects:

$$e(\xi) = e(\infty) - \left[ \alpha_{\text{iPEPS}}^{\text{NLSM}} \left( \frac{N-1}{2} \right) v \right] \boxed{\frac{1}{\xi^3}} + \mathcal{O} \left( \frac{1}{\xi^4} \right)$$

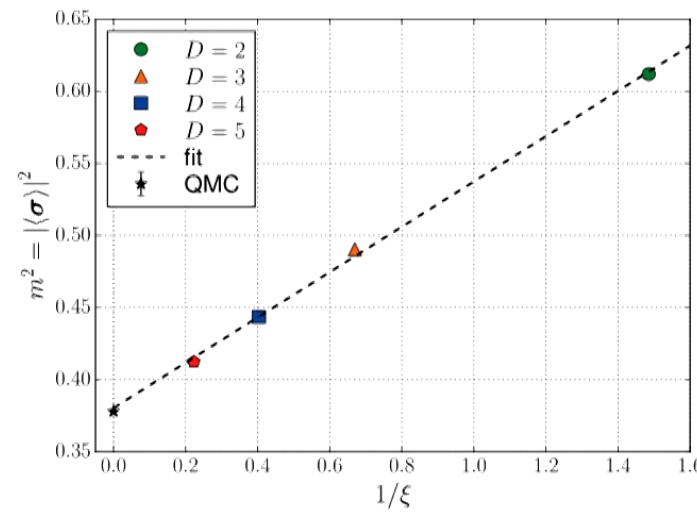
$$\frac{m^2(\xi)}{m^2(\infty)} = 1 + \left[ \mu_{\text{iPEPS}}^{\text{NLSM}} \left( \frac{N-1}{2} \right) \frac{v}{\rho_s} \right] \boxed{\frac{1}{\xi}} + \mathcal{O} \left( \frac{1}{\xi^2} \right)$$

## S=1/2 HB magnet / O(3)

$$H_{\text{HB}} = J \sum_{\langle i,j \rangle} (S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z)$$



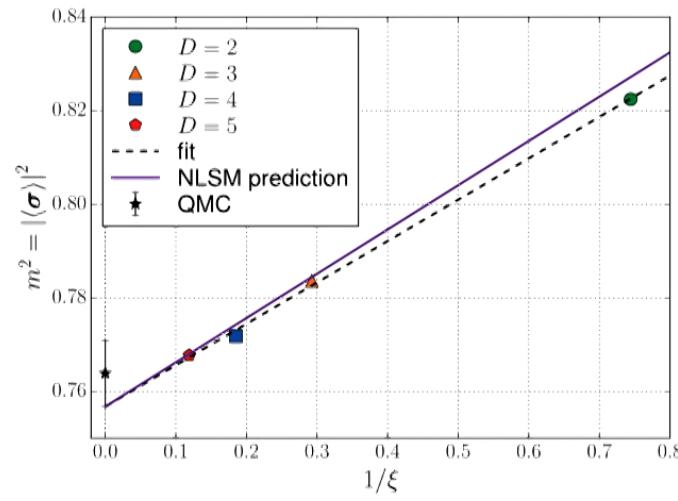
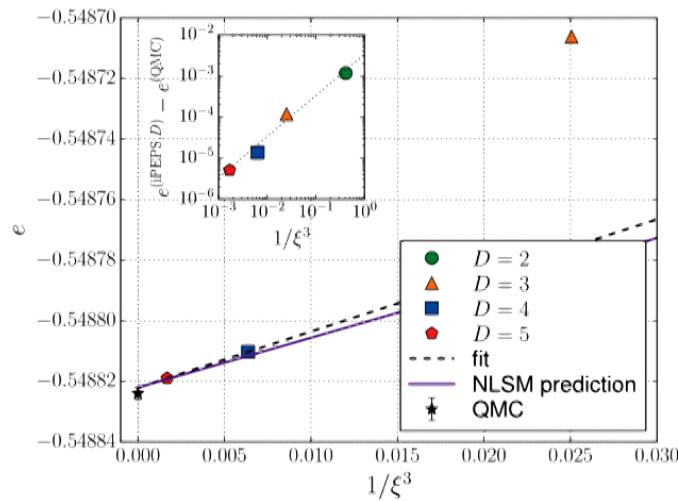
$$\alpha_{\text{iPEPS}}^{\text{NLSM}} \approx -0.0029$$



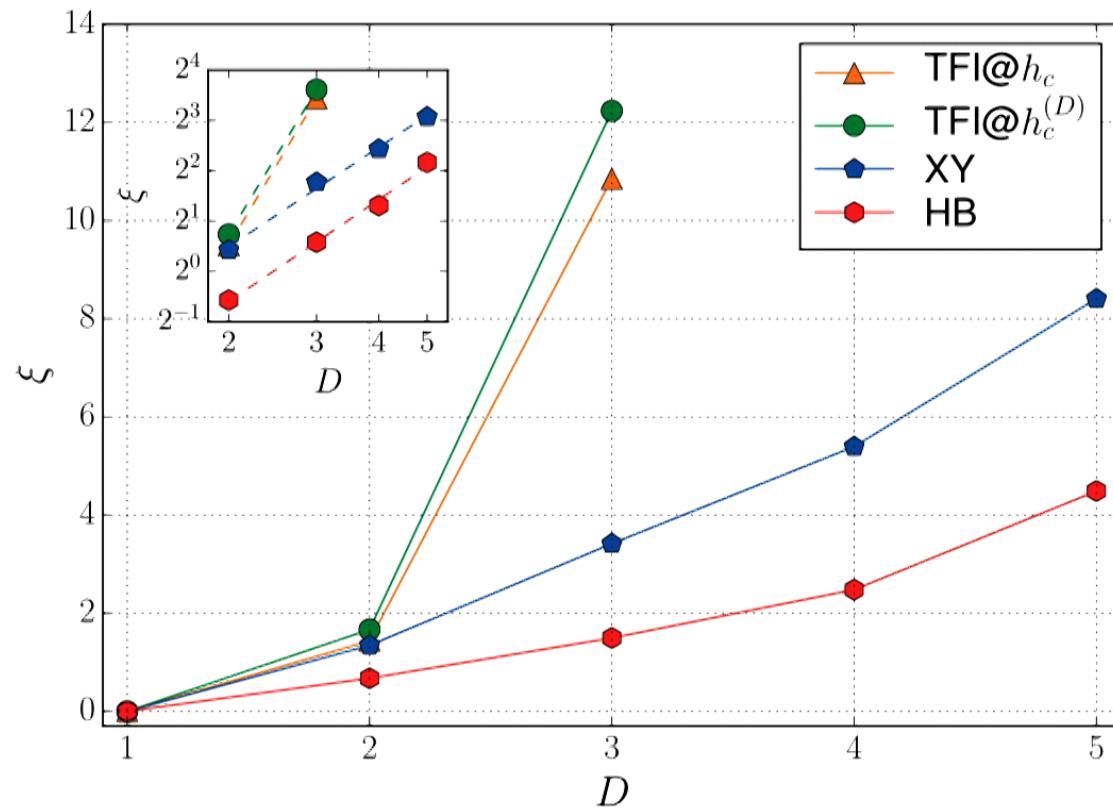
$$\mu_{\text{iPEPS}}^{\text{NLSM}} \approx +0.045$$

## S=1/2 XY model / O(2)

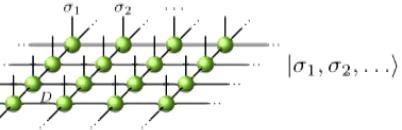
$$H_{\text{XY}} = -J \sum_{\langle i,j \rangle} (S_i^x S_j^x + S_i^y S_j^y)$$



## Overview: D-dependence of correlation lengths

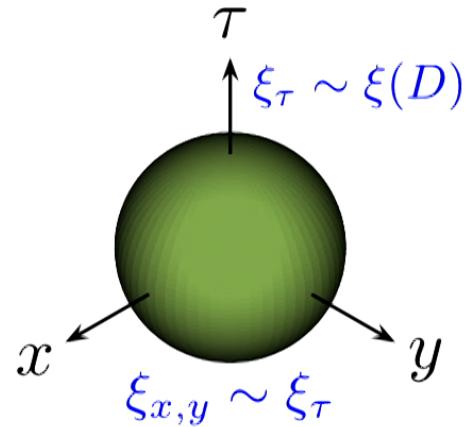


## Handwaving understanding:

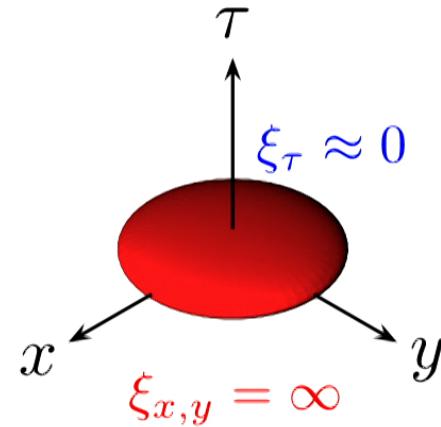
$$|\psi\rangle = \sum_{\sigma_1, \sigma_2, \dots} |\sigma_1, \sigma_2, \dots\rangle$$


- Space-time volume required to correctly describe correlation functions becomes D-limited in Lorentz invariant (2+1)d massless behaviour.

(2 + 1)d CFT/QFT



(2 + 0)d Rokhsar – Kivelson



- Entropic Area-law is not sufficient to decide representability by iPEPS.  
M. Rader & A.M. Läuchli, Phys. Rev. X 8, 031030 (2018).

## Conclusions

- rather accurate energies and critical exponents for  $d=3$  Ising already with bond dimension  $D=3$  !

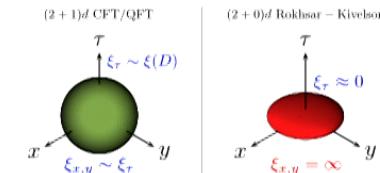
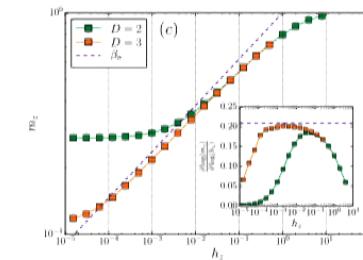
- correlation lengths stay **finite** for all couplings.

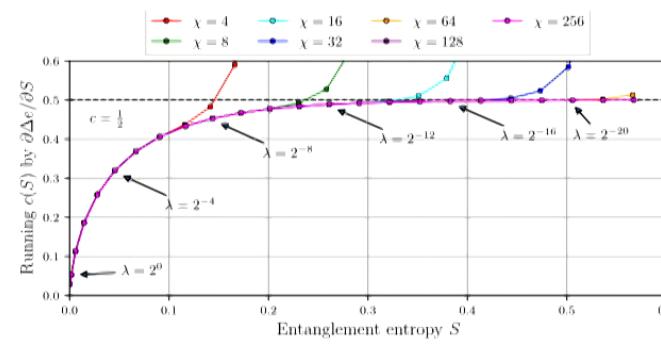
- Finite correlation length is a “blessing”, we can use it to extrapolate energies and order parameters to the “ $D \rightarrow \infty$ ” limit.

- Broader qualitative picture: it seems as though the variational iPEPS state is “self-detuned” by a relevant perturbation of the  $(2+1)d$  gapless QFT fixed point, rendering the state gapped.

- Space-time volume matters !

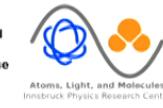
- Applications: Superfluids, Superconductors, Gapless Quantum Spin Liquids,...





## Finite Entanglement Scaling in iMPS revisited

A. Eberharter, M. Rader and AML, in preparation





Thank you for your attention !



