Title: Shortcuts in Real and Imaginary Time

Speakers: Timothy Hsieh

Collection: Quantum Matter: Emergence & Entanglement 3

Date: April 25, 2019 - 11:30 AM

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Abstract: In the first half, I will demonstrate an efficient and general approach for realizing non-trivial quantum states, such as quantum critical and topologically ordered states, in quantum simulators. In the second half, I will present a related variational ansatz for many-body quantum systems that is remarkably efficient. In particular, representing the critical point of the one-dimensional transverse field Ising model only requires a number of variational parameters scaling logarithmically with system size. Though optimizing the ansatz generally requires Monte Carlo sampling, our ansatz potentially enables a partial mitigation of the sign problem at the expense of having to optimize a few parameters.

Shortcuts in Real and Imaginary Time

Tim Hsieh

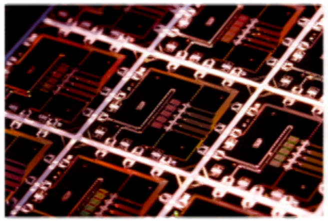
Perimeter Institute

Emergence and Entanglement III
April 25, 2019

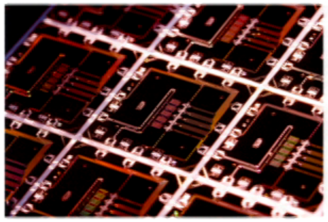
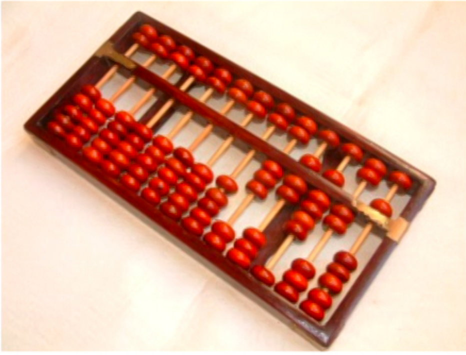
Accessing Quantum Many-Body States

Accessing Quantum Many-Body States

On Quantum Computers

Trapped Ions	Superconducting Circuits
	
e.g. Monroe group U. Maryland/IonQ	e.g. Martinis group UCSB/Google

Accessing Quantum Many-Body States

On Quantum Computers		On Classical Computers
Trapped Ions	Superconducting Circuits	
		
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Prepare nontrivial quantum states
using real time evolution

Accessing Quantum Many-Body States

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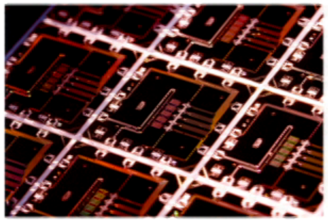
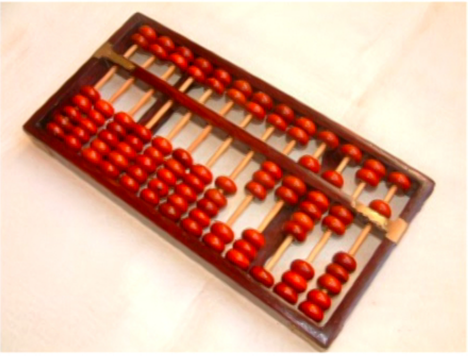




e.g. Monroe group
U. Maryland/IonQ

e.g. Martinis group
UCSB/Google

Prepare nontrivial quantum states using real time evolution → Efficient variational wavefunction using imaginary time evolution

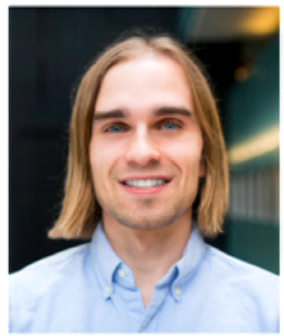
Collaborators

Real time

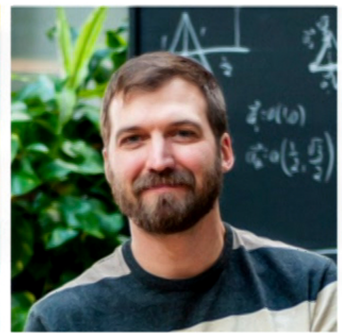
Wen Wei Ho
(Harvard)

SciPost Phys. 6, 029 (2019)

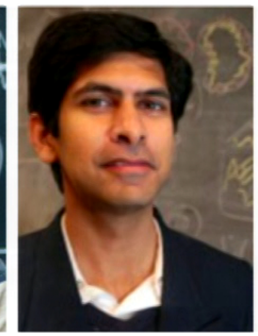
Imaginary time



Matt Beach
(UW/Perimeter)



Roger Melko
(UW/Perimeter)



Tarun Grover
(UCSD)

arXiv: 1904.00019 (2019)

Part I

General protocol for preparing nontrivial quantum states

Source: Pinterest

Motivation: QAOA

Quantum approximate optimization algorithm (QAOA)

Farhi, Goldstone, Gutmann (2014)

Motivation: QAOA

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Original goal: find good solution to a classical optimization problem

Motivation: QAOA

Quantum approximate optimization algorithm (QAOA)

Farhi, Goldstone, Gutmann (2014)

Original goal: find good solution to a classical optimization problem

Find bit string satisfying as many constraints on bits as possible
(find configuration minimizing energy)

QAOA: *Not* Adiabatic

Simple Hamiltonian

H_X

$|+\rangle$

Target Hamiltonian

H_t

$|\psi_t\rangle$

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Simple Hamiltonian

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H_t

$|\psi_t\rangle$

Quantum Approximate Optimization Algorithm:

Farhi, Goldstone, Gutmann (2014)
Wecker, Hastings, Troyer (2015)

$$|\psi\rangle = e^{-i\beta_p H_X} e^{-i\gamma_p H_t} \cdots e^{-i\beta_1 H_X} e^{-i\gamma_1 H_t} |+\rangle$$

QAOA: *Not* Adiabatic

Simple Hamiltonian

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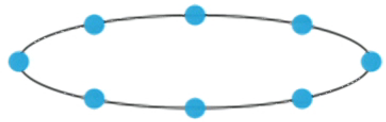
Choose evolution times to minimize energy $\langle\psi|H_t|\psi\rangle$

Transverse Field Ising Model



$$H_{\mathrm{TFIM}} = -\sum_{i=1}^{L} Z_i Z_{i+1} - g \sum_{i=1}^{L} X_i$$

$|\uparrow\uparrow\uparrow \ldots\rangle + |\downarrow\downarrow\downarrow \ldots\rangle$ $\quad$ $|+++\ldots\rangle$

Warmup: GHZ (Cat) State

Target: $|\psi_t\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\uparrow ...\rangle + |\downarrow\downarrow\downarrow ...\rangle)$

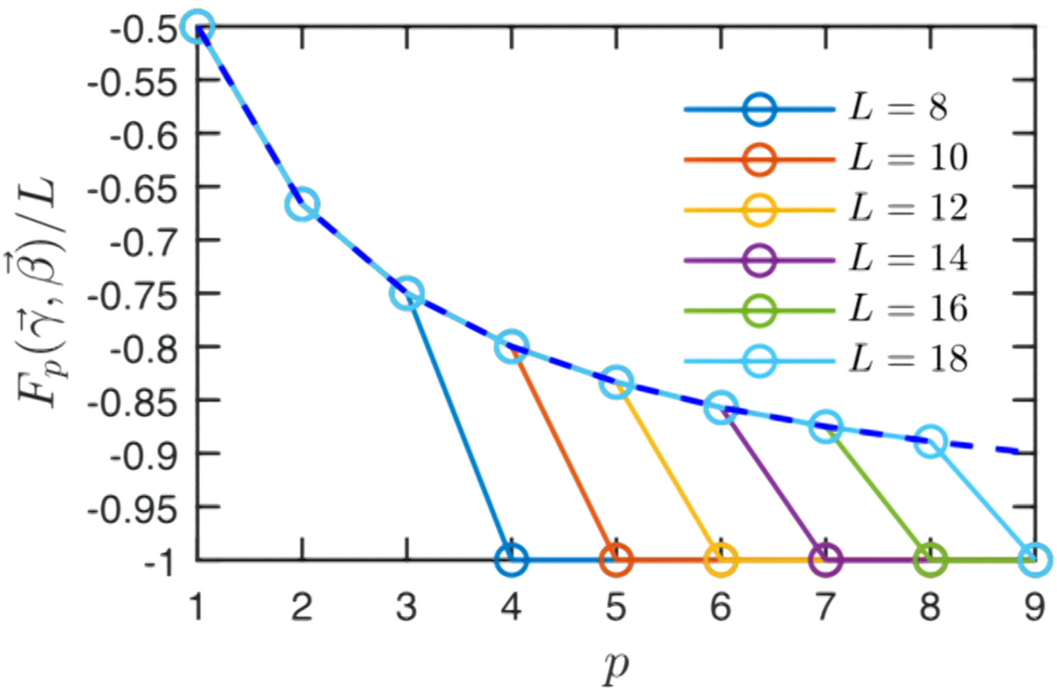
$$H_t = -\sum_{i=1}^{L} Z_i Z_{i+1}$$

$$H_X = -\sum_i X_i$$

$$H_I = -\sum_{i=1}^{L} Z_i Z_{i+1}$$

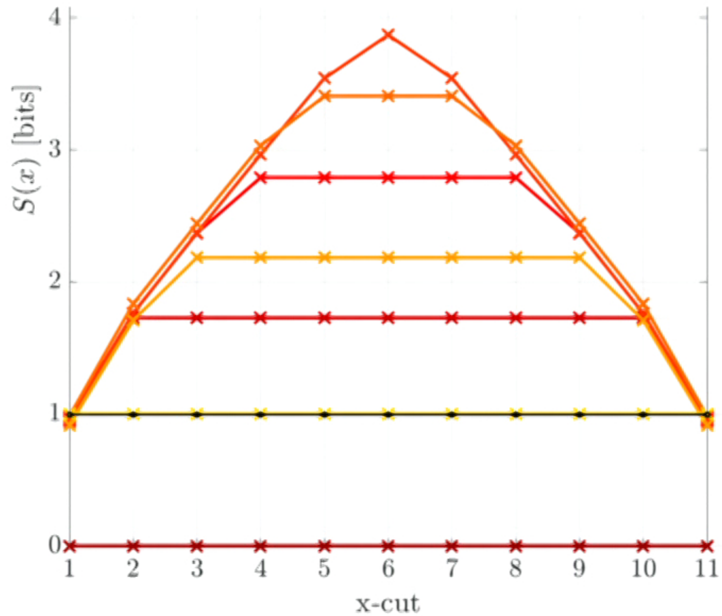
$$|\psi(\vec{\gamma}, \vec{\beta})\rangle_p = e^{-i\beta_p H_X} e^{-i\gamma_p H_I} \dots e^{-i\beta_1 H_X} e^{-i\gamma_1 H_I} |+\rangle$$

Preparation of GHZ State



Trajectory in Hilbert Space

L = 12, optimal GHZ preparation sequence

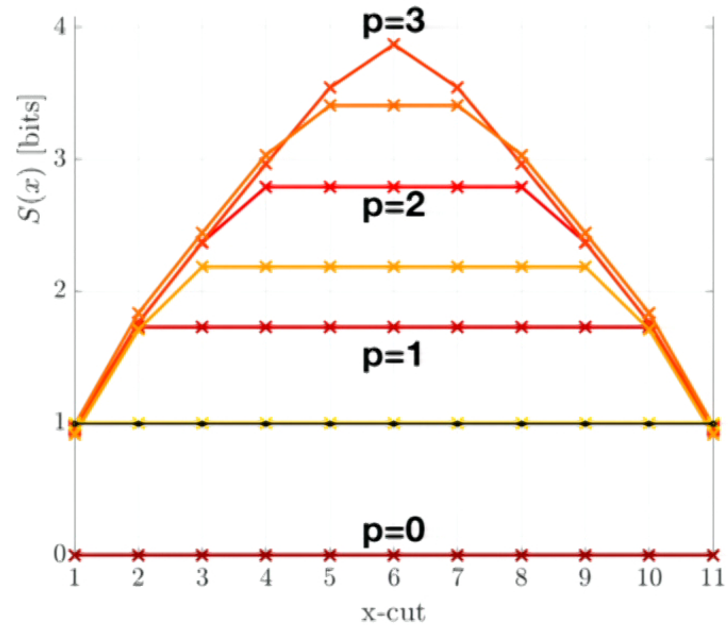


(thanks to C. Jonay)

Entanglement growth during evolution of state

Trajectory in Hilbert Space

L = 12, optimal GHZ preparation sequence



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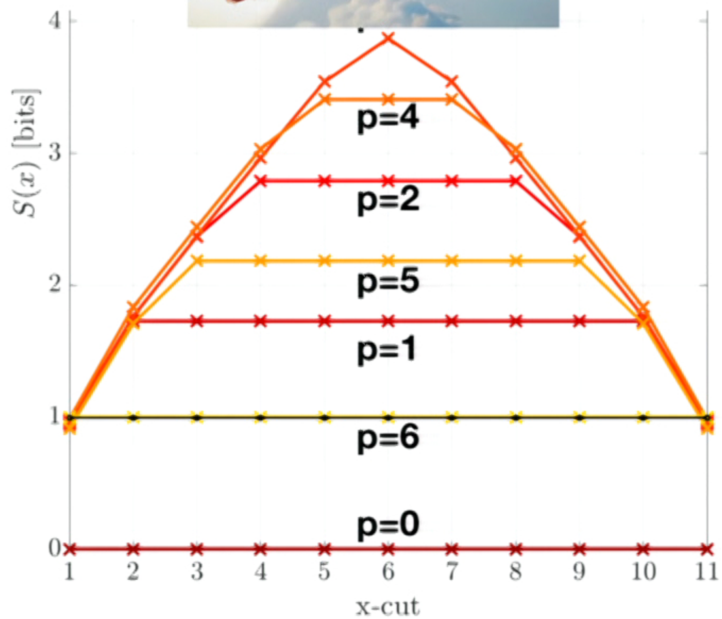

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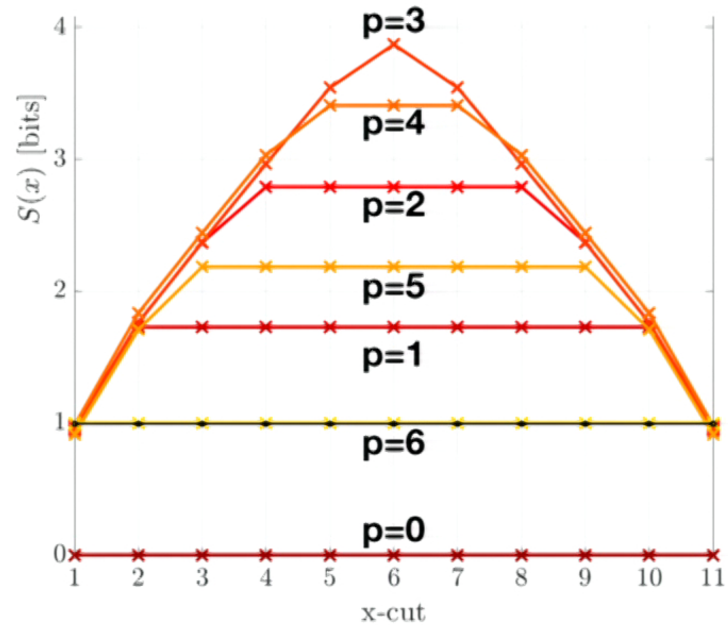

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Entanglement growth during evolution of state

Quantum Critical State

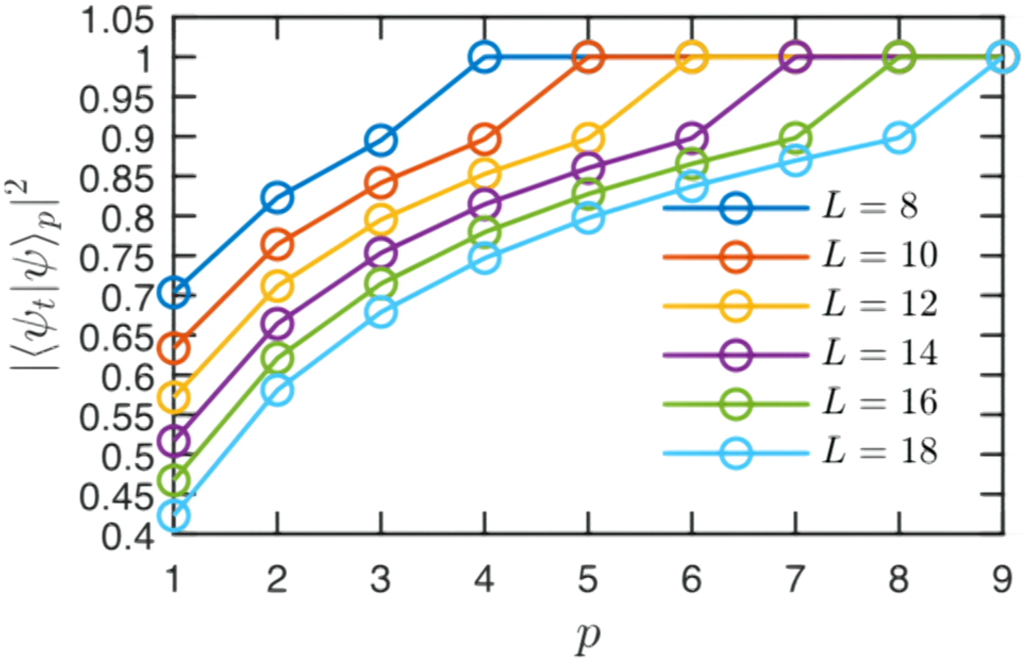
Target: ground state of

$$H_t = -\sum_{i=1}^{L} Z_i Z_{i+1} - \sum_{i=1}^{L} X_i$$

$$H_X = -\sum_i X_i$$

$$H_I = -\sum_{i=1}^{L} Z_i Z_{i+1}$$

Quantum Critical State Preparation



Toric Code

$$H_t = -\sum_{i=1}^{L}\sum_{j=1}^{L}\sigma^x_{i,j+1}\sigma^y_{i+1,j+1}\sigma^x_{i+1,j}\sigma^y_{i,j}$$

$$H_X = -\sum_i X_i$$

$$H_I = -\sum_{i=1}^{L}\sum_{j=1}^{L}\sigma^x_{i,j+1}\sigma^y_{i+1,j+1}\sigma^x_{i+1,j}\sigma^y_{i,j}$$

An operator duality maps each diagonal into a TFIM chain

Diagonals can be prepared in parallel

Use optimal angles from GHZ prep

Perfect fidelity at depth p = L / 2

Concrete Protocol

$$|\psi(\vec{\gamma}, \vec{\beta})\rangle_p = e^{-i\beta_p H_X} e^{-i\gamma_p H_I} \cdots e^{-i\beta_1 H_X} e^{-i\gamma_1 H_I} |+\rangle$$

$L = 10, T = 5.250$:

$$(0.2473, 0.6977, 0.4888, 0.6783, 0.5559, 0.6567, 0.5558, 0.6029, 0.4598, 0.3068)$$

$L = 12, T = 6.7651$:

$$(0.2809, 0.6131, 0.6633, 0.4537, 0.8653, 0.4663, 0.6970, 0.6829, 0.4569, 0.7990, 0.3565, 0.4304)$$

$L = 14, T = 8.1604$:

$$(0.3090, 0.5710, 0.6923, 0.5648, 0.5391, 0.9684, 0.3979, 0.6852, 0.8235, 0.4474, 0.6930, 0.6465, 0.4120, 0.4104)$$

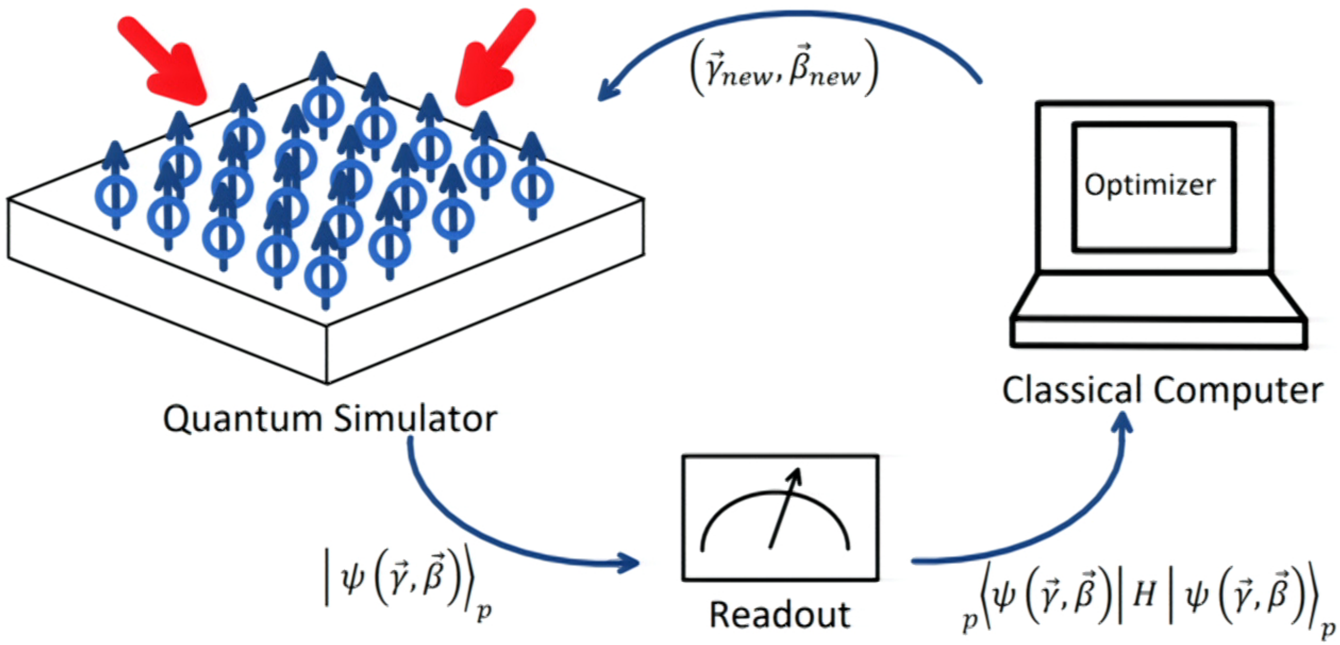
$L = 16, T = 9.8198$:

$$(0.3790, 0.5622, 0.5638, 0.7101, 0.9046, 0.3210, 0.6738, 0.8377, 0.8616, 0.4004, 0.5624, 0.9450, 0.5224, 0.6466, 0.4119, 0.5172)$$

$L = 18, T = 11.1485$:

$$(0.3830, 0.4931, 0.7099, 0.7010, 0.5330, 0.6523, 0.6887, 1.0405, 0.3083, 0.6215, 0.9607, 0.5977, 0.6209, 0.5597, 0.7850, 0.5851, 0.4132, 0.4948)$$

Hybrid Quantum-Classical Simulation



Farhi, Goldstone, Gutmann (2014)
Wecker, Hastings, Troyer (2015)

Variational Imaginary Time Ansatz (VITA)

Given Hamiltonian $H = H_A + gH_B$

Approximate ground state by

$$|\psi_P(\boldsymbol{\alpha}, \boldsymbol{\beta})\rangle = \mathcal{N} \prod_{p=1}^{P} e^{-\beta_p H_B} e^{-\alpha_p H_A} |\psi_0\rangle$$

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Trotter limit of $e^{-\tau H}|\psi_0\rangle$ achieved by taking P to infinity

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Small P, finite and variable time steps

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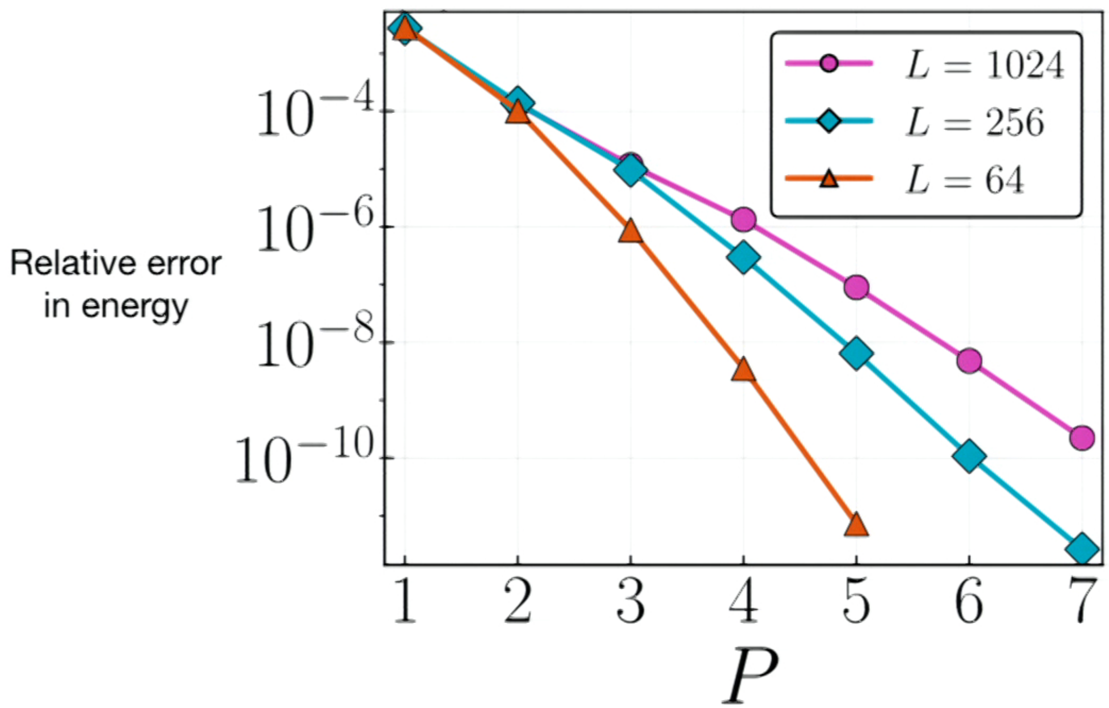
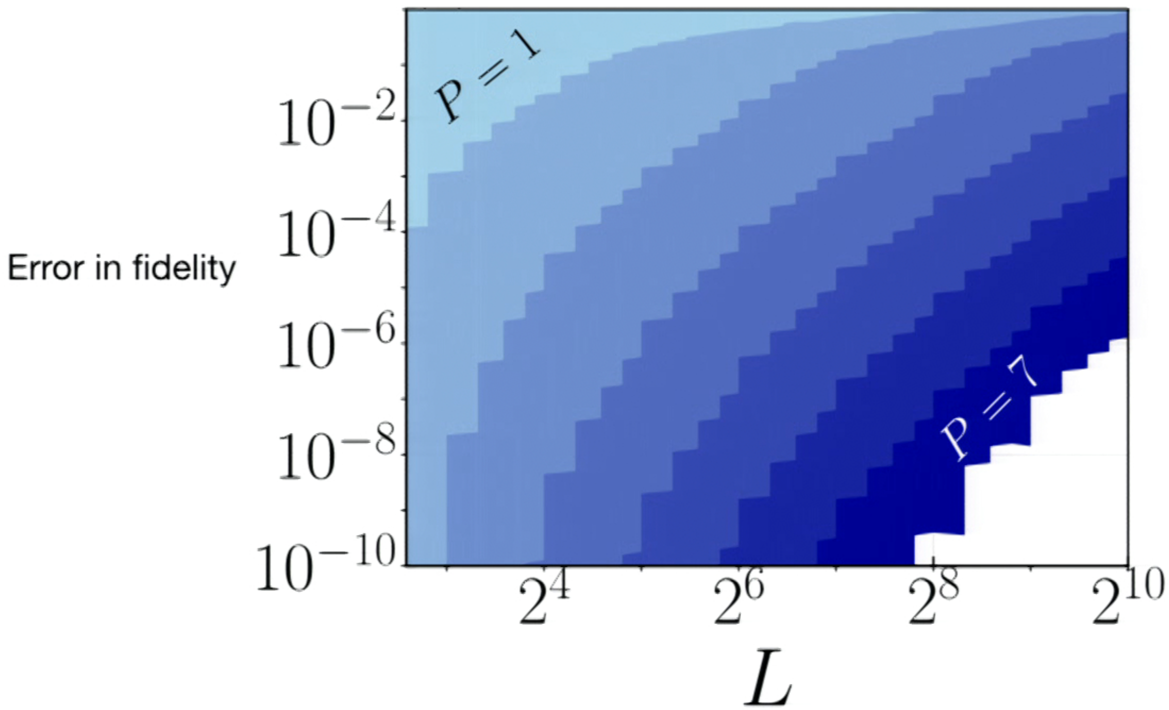
Small P, finite and variable time steps

for Hubbard model: P = 1 ansatz (Gutzwiller 1963, Baeriswyl 1987, Otsuka 1992)
Related ansatz (Vaezi and Vaezi 2018)

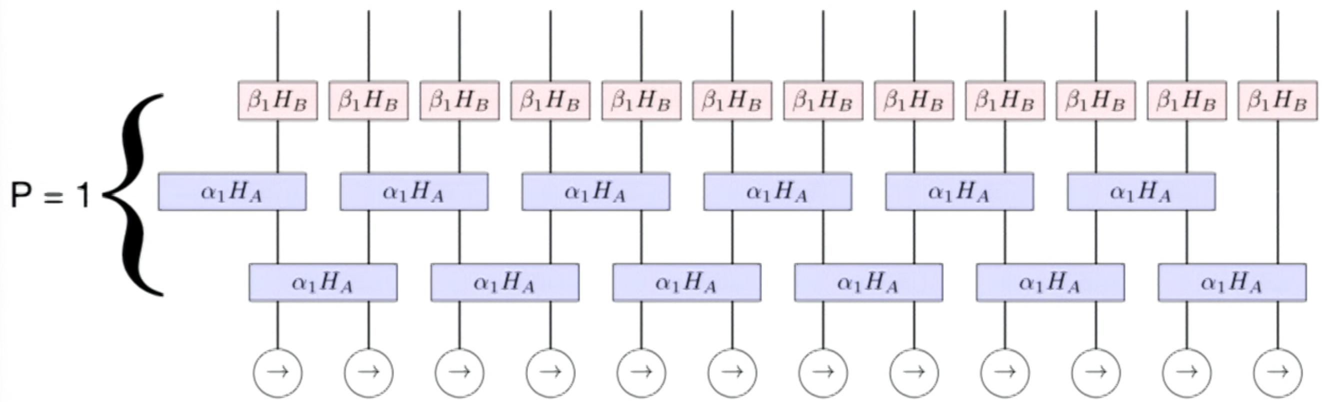
Efficient Representation of Critical TFIM



Efficient Representation of Critical TFIM



Lower Bound on Depth



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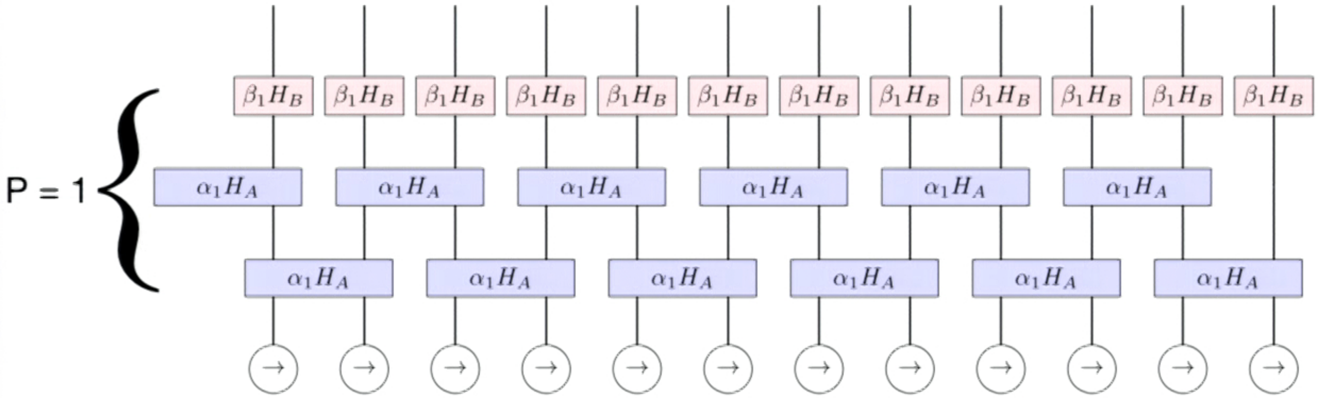

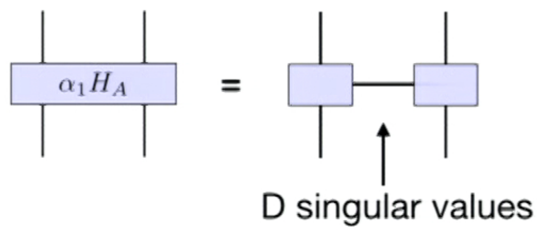

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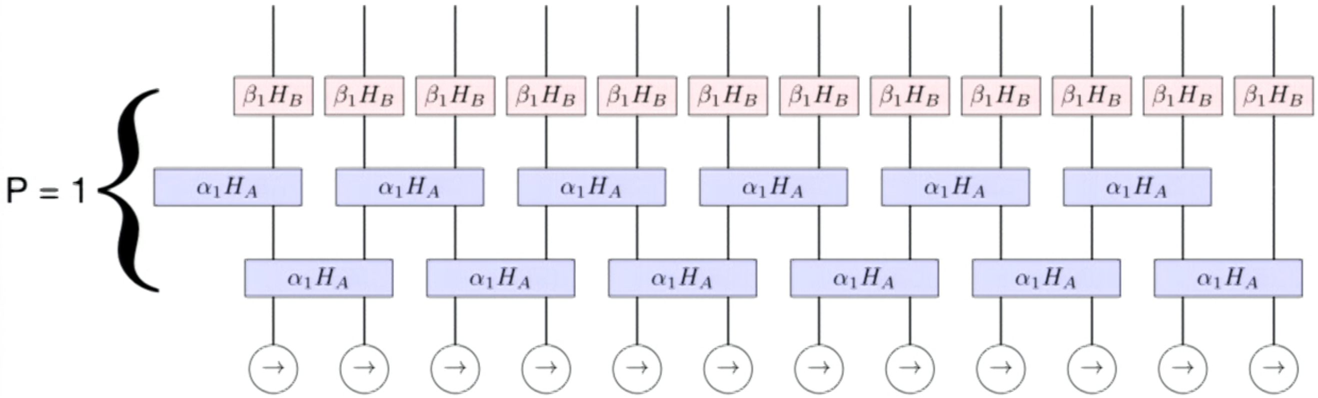
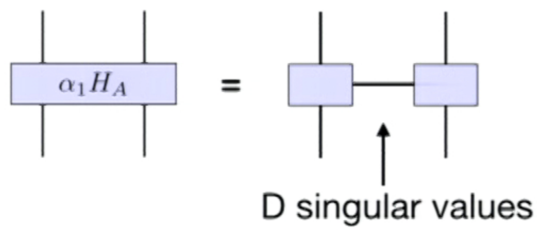


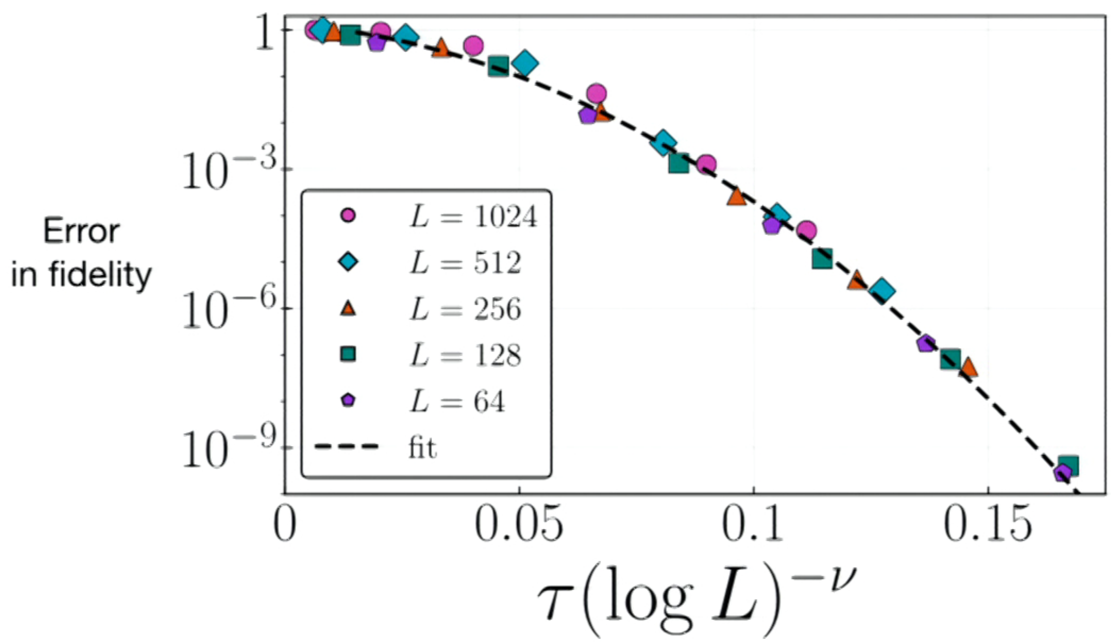


Maximum entanglement after P iterations is P log D

Scaling Collapse

For projector method: to get target fidelity, need total imaginary time $\tau \propto L$

VITA: $\tau \equiv \frac{1}{2}\sum_{p=1}^{P}(\alpha_p+\beta_p)$



Imaginary Time 101(i)

No Lieb-Robinson bound

$|GHZ\rangle \approx e^{-\tau H_{ZZ}}|+\rangle$ for τ constant, independent of L

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Local operator can generate entanglement far away

$$\eta_+|11\rangle + \eta_-|00\rangle$$

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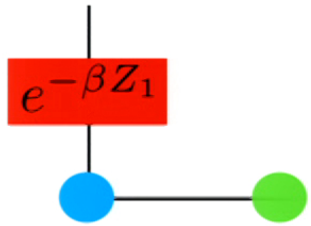

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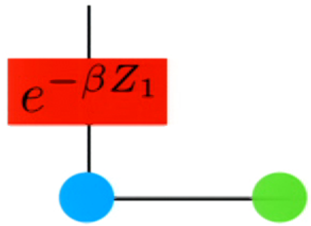
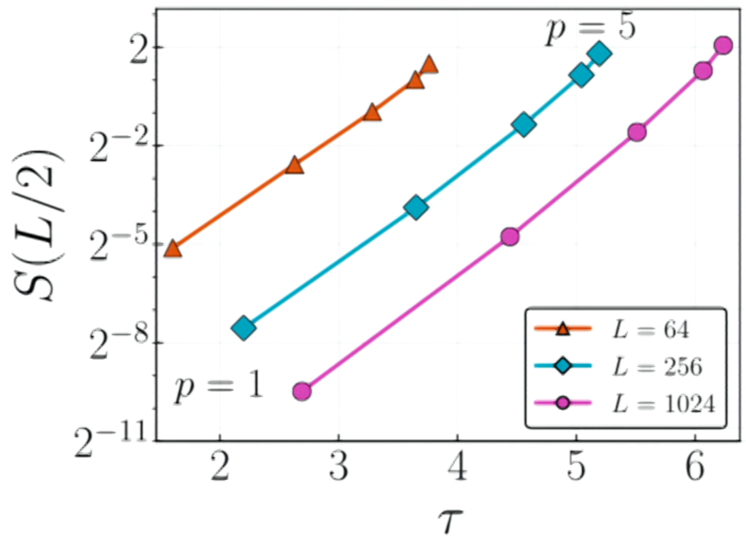


$\eta_+|11\rangle + \eta_-|00\rangle$

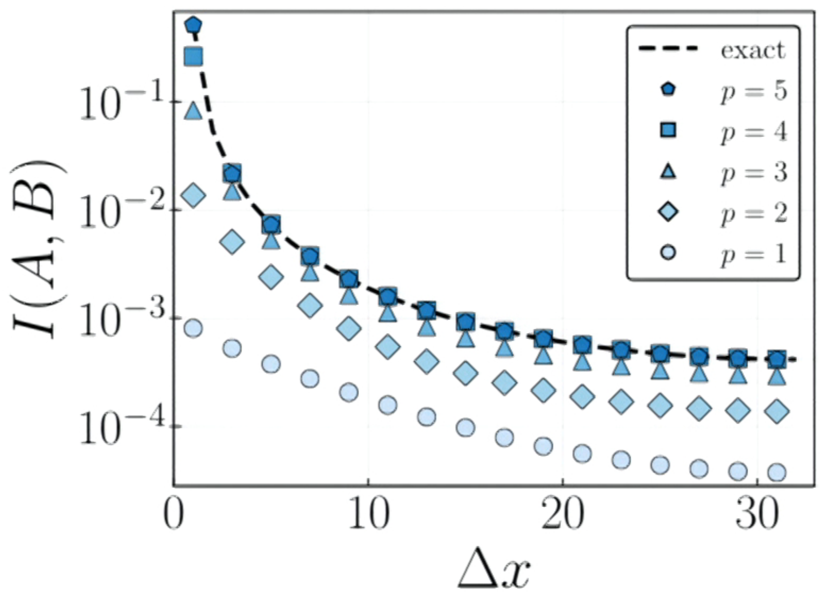
The more initial entanglement, the more imaginary time evolution can change the entanglement

Exponential Growth of Entanglement



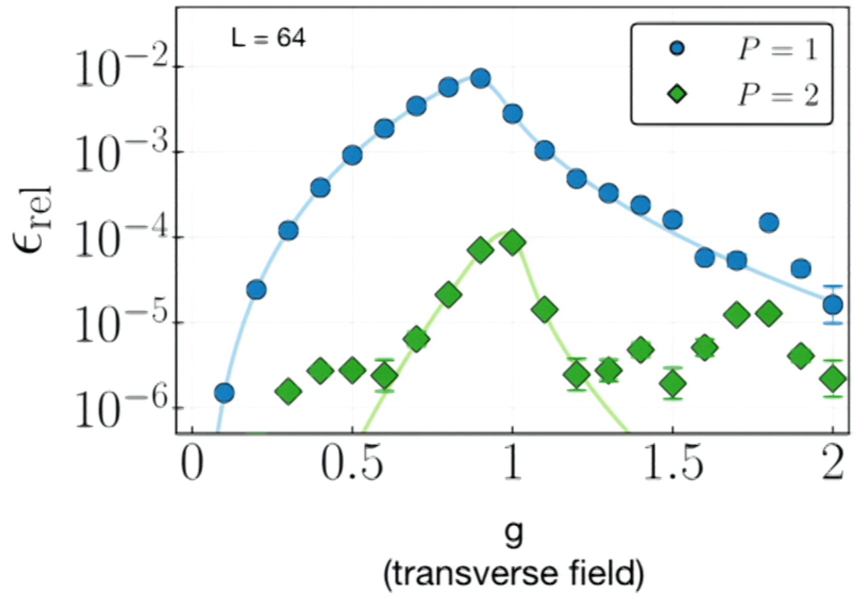
Long-Range Correlations in Intermediate States

Mutual Information
$(S_A + S_B - S_{AB})$



VITA via Monte Carlo

Quantum-Classical mapping: $\langle \psi_P(\boldsymbol{\alpha}, \boldsymbol{\beta}) | \mathcal{O} | \psi_P(\boldsymbol{\alpha}, \boldsymbol{\beta}) \rangle = \sum_{\{s\}} \tilde{\mathcal{O}}(s)\, p_{\boldsymbol{\alpha},\boldsymbol{\beta}}(s)$



Non-Integrable Model

$$H = -\sum_i^L Z_i Z_{i+1} - gX_i - hZ_i$$

Modify ansatz accordingly: $\alpha_p H_{ZZ} \rightarrow \alpha_p H_{ZZ} + \gamma_p H_Z$

Non-Integrable Model

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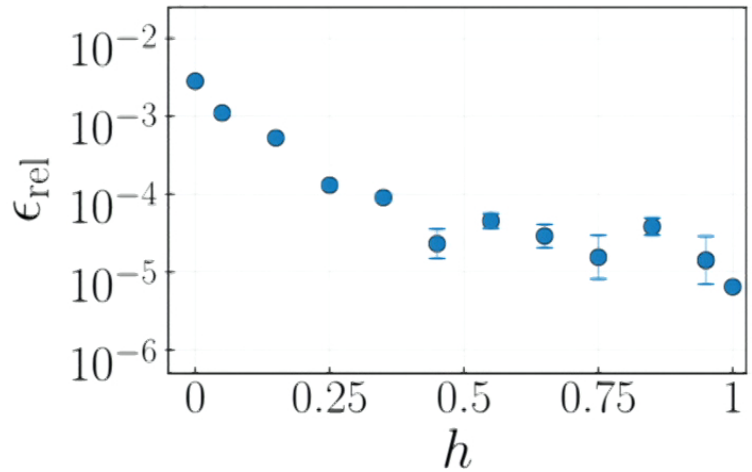

P = 1

Sign Problem?

Sign Problem?

Compressed imaginary time direction —> sampling with sign problem tractable?

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Protocols for preparing nontrivial quantum states	
Exactly prepare TFIM quantum critical state with L iterations	
In progress: experiment (Monroe group)	
W.W. Ho and TH SciPost Phys. 6, 029 (2019)	

Accessing Quantum Many-Body States

On Quantum Computers	On Classical Computers
Protocols for preparing nontrivial quantum states	Highly efficient variational imaginary time ansatz (VITA)
Exactly prepare TFIM quantum critical state with L iterations	Requires ~log L parameters to represent TFIM critical state
In progress: experiment (Monroe group)	In progress: application to harder models
W.W. Ho and TH SciPost Phys. 6, 029 (2019)	M. Beach, R. Melko, T. Grover, and TH arXiv: 1904.00019 (2019)