Title: Extracting conformal and superconformal data from critical quantum spin chains

Speakers: Ashley Milsted, Yijian Zou

Collection: Quantum Matter: Emergence & Entanglement 3

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Abstract: Key to characterizing universality in critical systems is the identification of the RG fixed point, which is very often a conformal field theory (CFT). We show how to use lattice operators that mimic the Virasoro generators of conformal symmetry to systematically extract, from a generic critical quantum spin chain, a complete set of the conformal data (central charge, scaling dimensions of primary fields, OPE coefficients) specifying a 2D CFT. We further show that, in the case of an extended superconformal symmetry, one can construct lattice operators that mimic the generators of the superconformal algebra, which allows us to identify superconformal primary states.

extracting **conformal** and superconformal data from critical quantum spin chains

Quantum Matter: Emergence & Entanglement 3

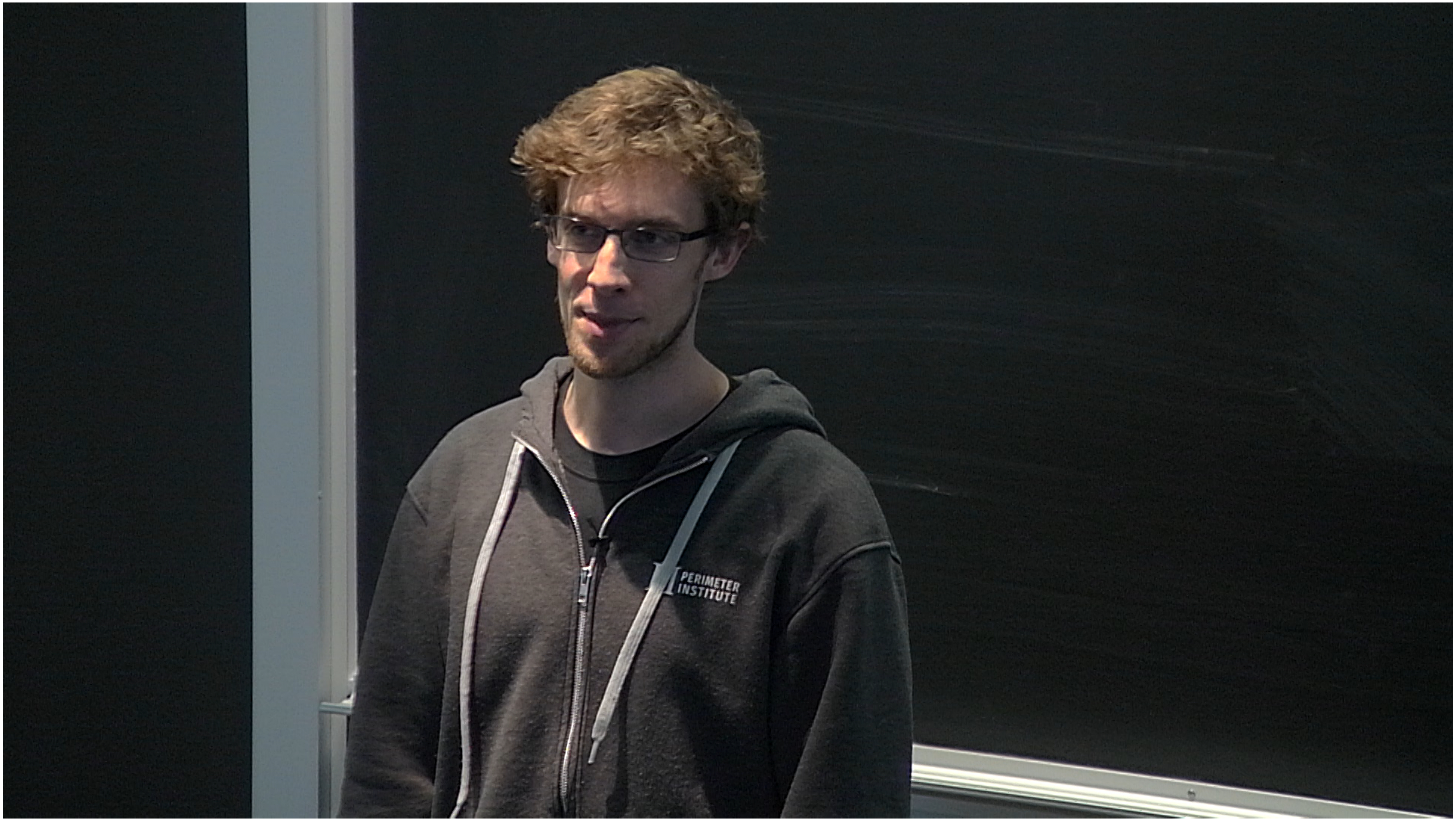
April 2019

Ashley Milsted Yijian Zou
Guifre Vidal

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

TENSOR NETWORKS INITIATIVE

Simons Collaboration on the Many Electron Problem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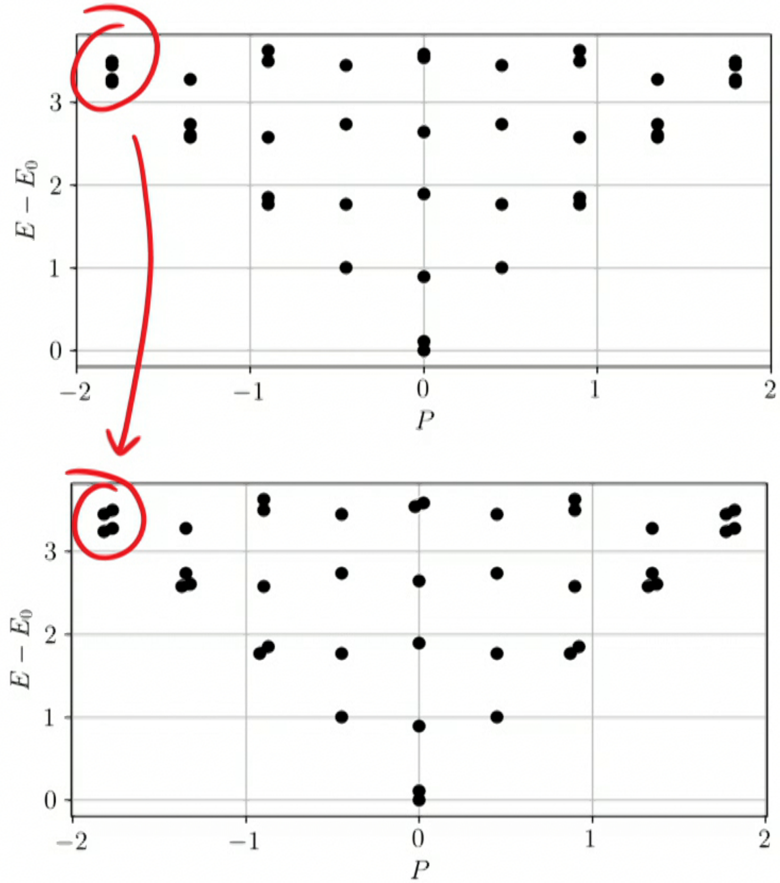
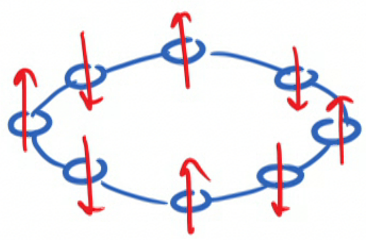


ABUSE OF X-AXES FOR THE NEXT 33 SLIDES

critical spin chain

$$H = \sum_{j=1}^{N} h_j$$

on the circle



Belavin, Polyakov, Zamolodchikov, Nucl. Phys. B **241**, 333 (1984)

critical spin chain

$$H = \sum_{j=1}^{N} h_j$$

on the circle

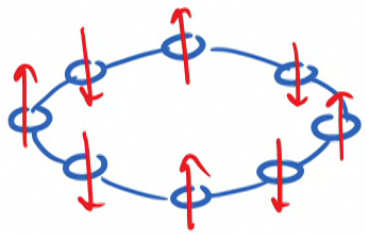
1+1D CFT

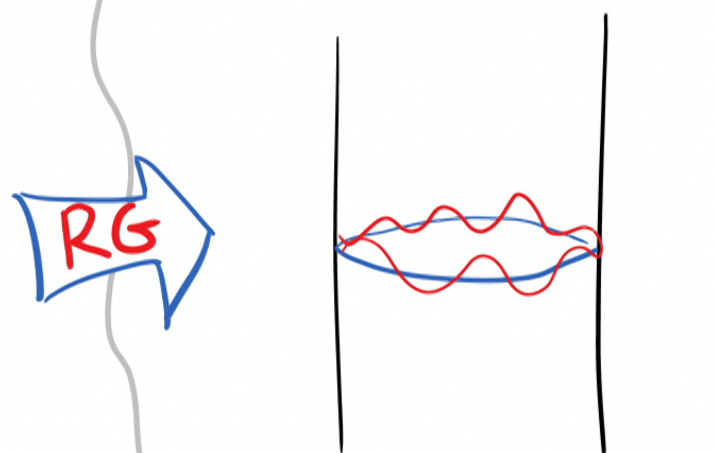
central charge: c

primary fields: Δ_ϕ, s_ϕ

OPE coefficients: $C_{\phi_2 \phi_3}^{\phi_1}$

(3-point correlators)





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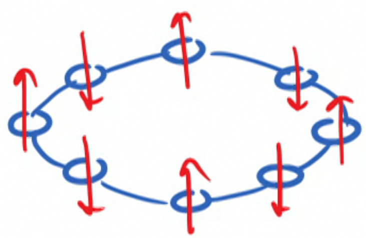
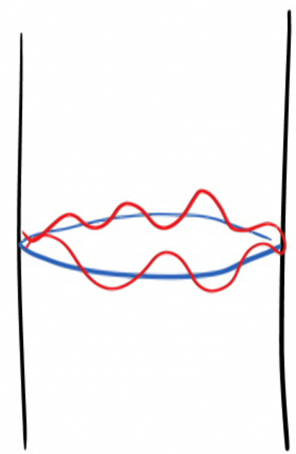




Belavin, Polyakov, Zamolodchikov, Nucl. Phys. B **241**, 333 (1984)

outline of part 1

critical quantum spin chain Hamiltonian

⇓

low-energy eigenstates + lattice Virasoro generators

⇓

complete conformal data

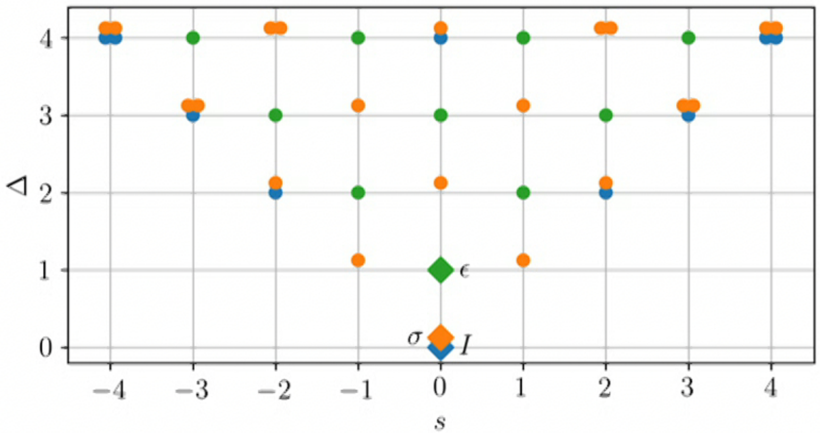
conformal data: 1+1D Ising CFT

central charge: $c = \frac{1}{2}$

conformal data: 1+1D Ising CFT

central charge: $c = \frac{1}{2}$

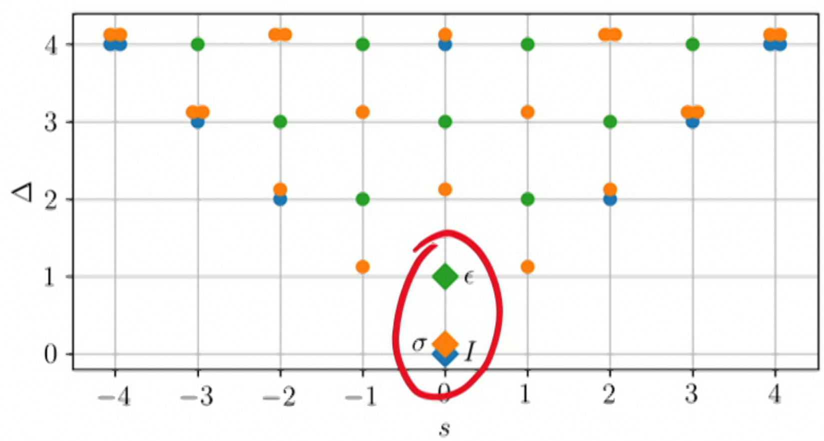
scaling operator dimensions



conformal data: 1+1D Ising CFT

central charge: $c = \frac{1}{2}$

scaling operator dimensions



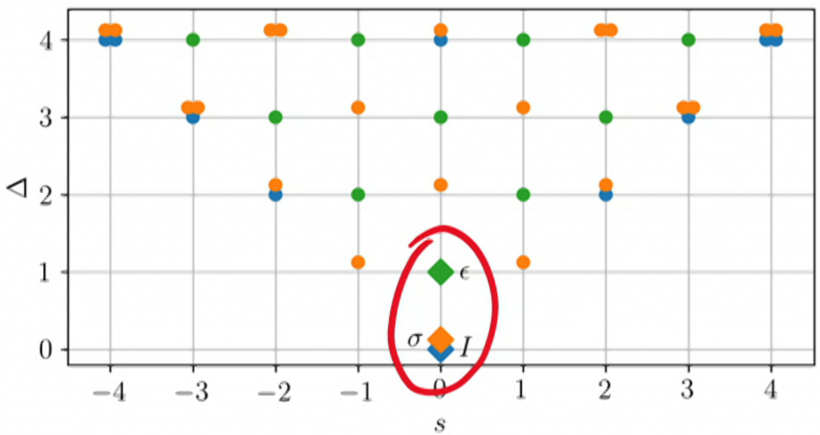
primary fields

ϕ	Δ	s
I	0	0
σ	$1/8$	0
ϵ	1	0

conformal data: 1+1D Ising CFT

central charge: $c = \frac{1}{2}$

scaling operator dimensions



primary fields

ϕ	Δ	s
I	0	0
σ	$1/8$	0
ϵ	1	0

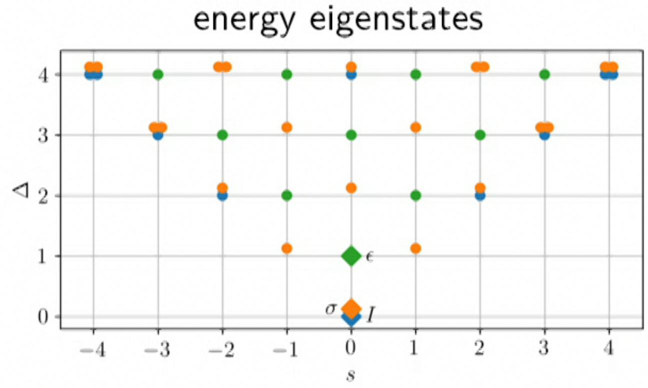
nonzero OPE coefficients: $C^{\epsilon}_{\sigma\sigma} = \frac{1}{2}$

(not involving I)

lattice low-energy spectrum (Ising model)

critical spin chain

1+1D CFT



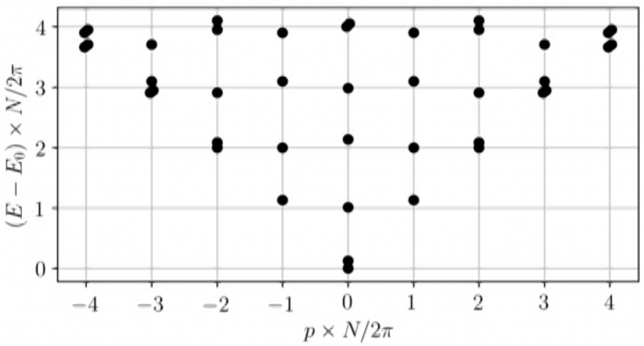
$$E_\alpha = \frac{2\pi}{L}(\Delta_\alpha - \frac{c}{12})$$

$$P_\alpha = \frac{2\pi}{L} s_\alpha$$

lattice low-energy spectrum (Ising model)

critical spin chain

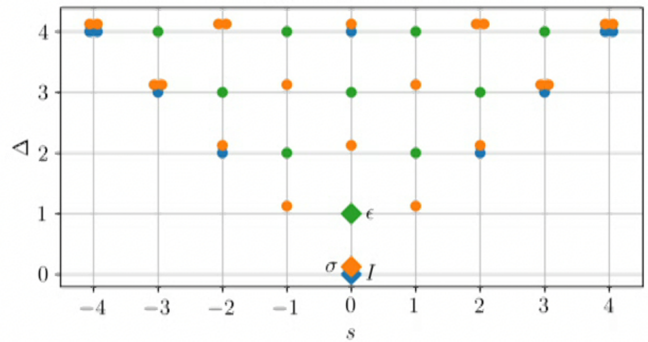
energy eigenstates



$\approx$

1+1D CFT

energy eigenstates



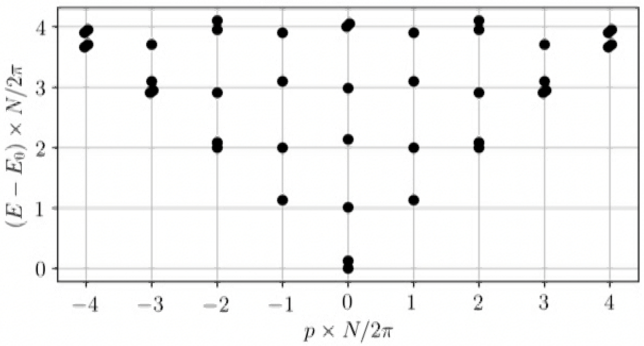
$$E_\alpha = \frac{2\pi}{L}\left(\Delta_\alpha - \frac{c}{12}\right)$$

$$P_\alpha = \frac{2\pi}{L} s_\alpha$$

lattice low-energy spectrum (Ising model)

critical spin chain

energy eigenstates



$$E_\alpha = A + \frac{B}{N}(\Delta_\alpha - \frac{c}{12}) + \mathcal{O}(N^{-x})$$

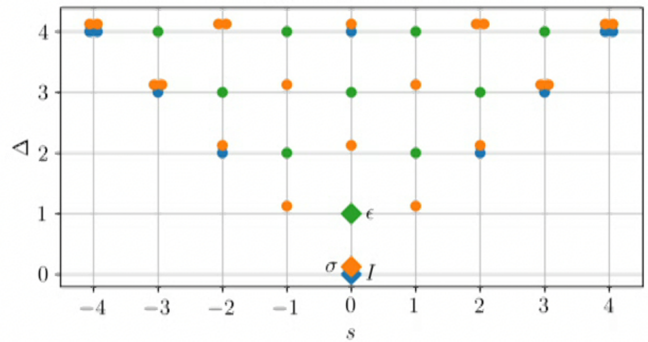
$$P_\alpha = \frac{2\pi}{N} s_\alpha$$

Cardy, Blöte, Nightingale, Affleck (1986)

$\approx$

1+1D CFT

energy eigenstates



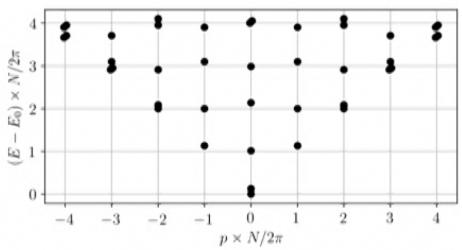
$$E_\alpha = \frac{2\pi}{L}(\Delta_\alpha - \frac{c}{12})$$

$$P_\alpha = \frac{2\pi}{L} s_\alpha$$

extracting conformal data

critical spin chain

low-energy spectrum



$$E_\alpha = A + \frac{B}{N}(\Delta_\alpha - \frac{c}{12}) + \mathcal{O}(N^{-x})$$

$$P_\alpha = \frac{2\pi}{N} s_\alpha$$

1+1D CFT

central charge: c

some Δ_ϕ, s_ϕ

primary fields: Δ_ϕ, s_ϕ

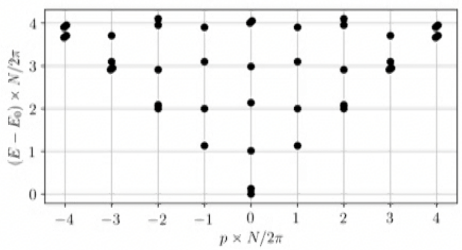
OPE coefficients: $C^{\phi_1}_{\phi_2\phi_3}$

(3-point correlators)

extracting conformal data

critical spin chain

low-energy spectrum



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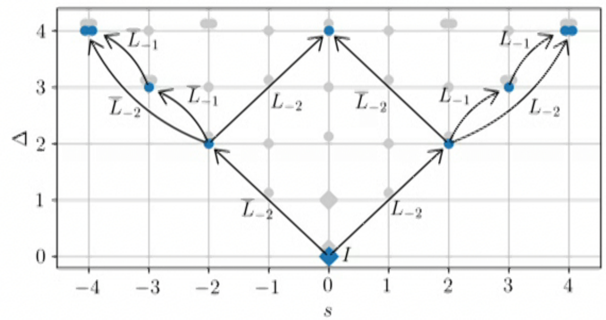
(3-point correlators)

identifying primary states on the lattice

critical spin chain

1+1D CFT



Virasoro generators
(ladder operators)
distinguish primary states

identifying primary states on the lattice

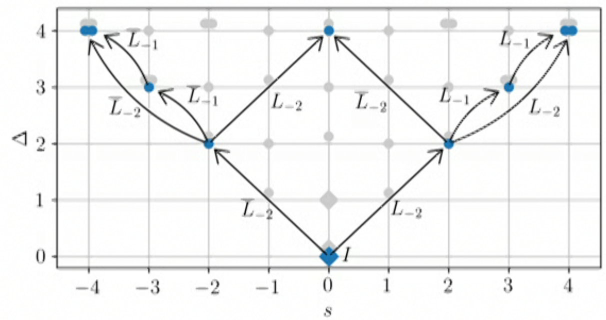
critical spin chain

$$H_n \propto \sum_{j=1}^{N} e^{-inj\frac{2\pi}{N}} h_j$$

"lattice Virasoro generators"
distinguish
"lattice primary states"

Koo & Saleur (1994)
AM & G. Vidal, PRB 96, 245105 (2017)

1+1D CFT



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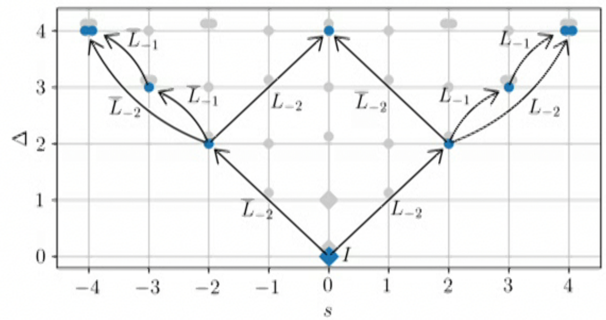
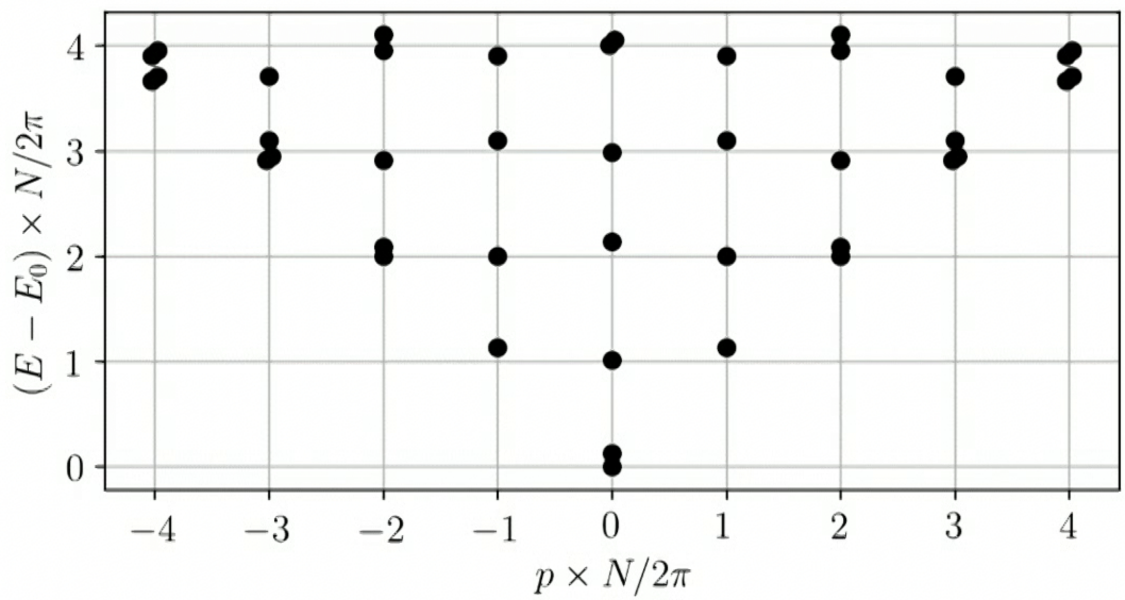


Virasoro generators
(ladder operators)
distinguish primary states

$$H_n^{CFT} \equiv L_n^{CFT} + \overline{L}_{-n}^{CFT}$$

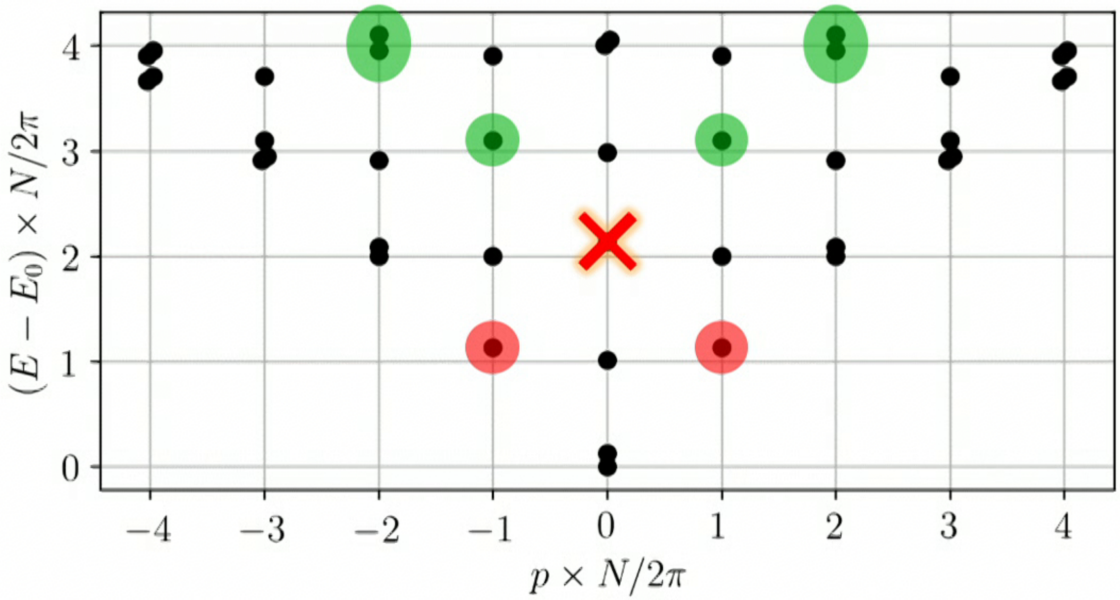
identifying primary states on the lattice

$$\langle \phi_{\rm out} | H_1 + H_{-1} + H_2 + H_{-2} | \phi_{\rm in} \rangle$$



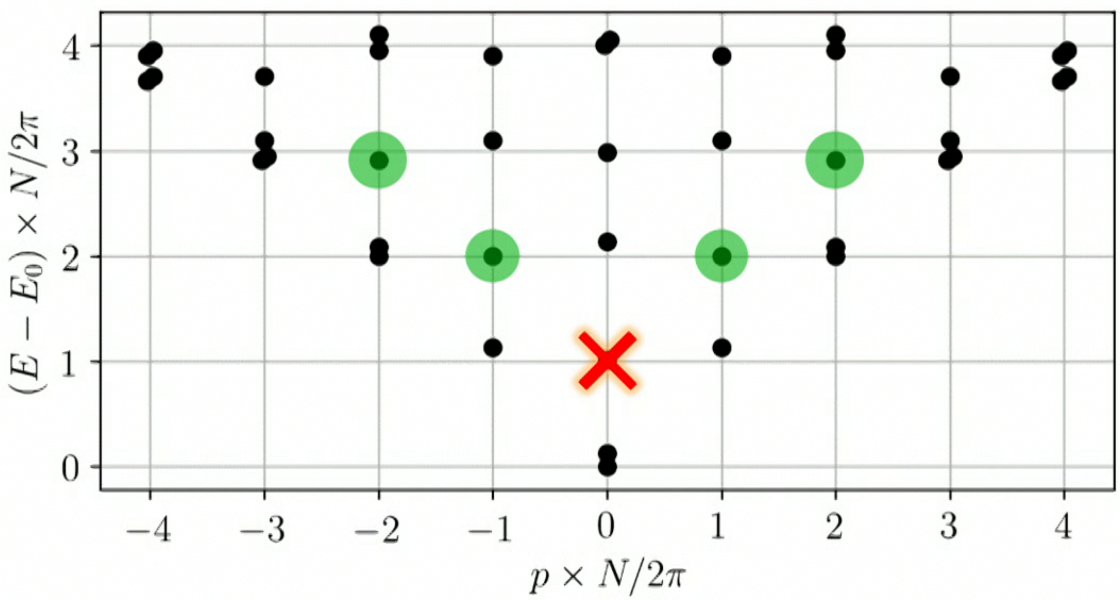
identifying primary states on the lattice

$$\langle \phi_{\rm out} | H_1 + H_{-1} + H_2 + H_{-2} | \phi_{\rm in} \rangle$$



identifying primary states on the lattice

$$\langle \phi_{\rm out} | H_1 + H_{-1} + H_2 + H_{-2} | \phi_{\rm in} \rangle$$

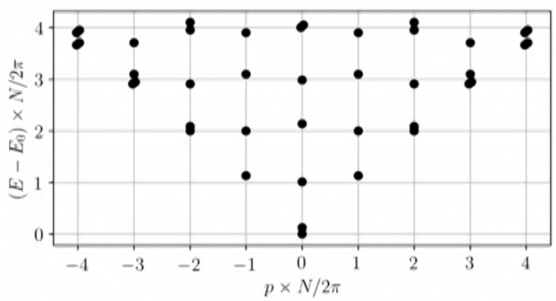


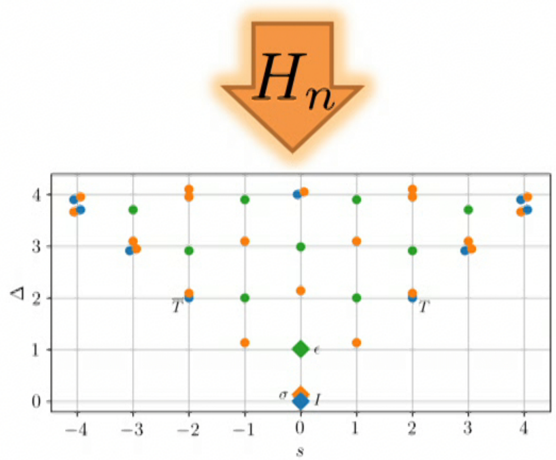
a primary!

extracting conformal data

critical spin chain

low-energy spectrum





1+1D CFT

central charge: c

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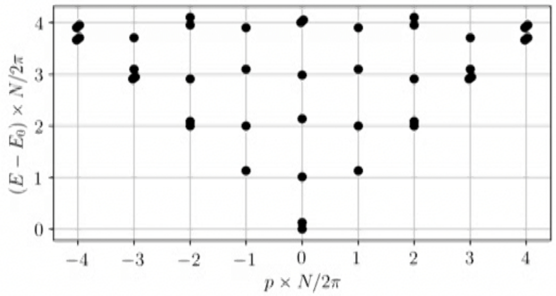
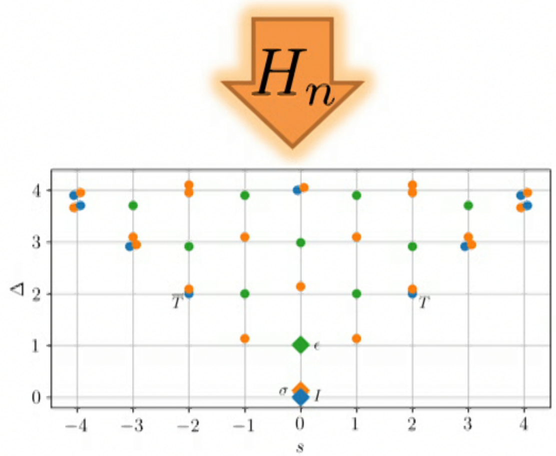
(3-point correlators)

extracting conformal data

critical spin chain

low-energy spectrum





1+1D CFT

central charge: c

primary fields: Δ_ϕ, s_ϕ

OPE coefficients: $C^{\phi_1}_{\phi_2\phi_3}$

(3-point correlators)

OPE coefficients via lattice primary operators

critical spin chain

$$C^{\phi_1}_{\phi_2\phi_3} \approx \langle \phi_1 | \phi_2 | \phi_3 \rangle$$

1+1D CFT

$$C^{\phi_1}_{\phi_2\phi_3} = \langle \phi_1^{CFT} | \phi_2^{CFT} | \phi_3^{CFT} \rangle$$

YZ, AM, G. Vidal, arXiv:1901.06439 (2019)

OPE coefficients via lattice primary operators

critical spin chain

$$C^{\phi_1}_{\phi_2\phi_3} \approx \langle\phi_1|\phi_2|\phi_3\rangle$$

=?

1+1D CFT

$$C^{\phi_1}_{\phi_2\phi_3} = \langle\phi_1^{CFT}|\phi_2^{CFT}|\phi_3^{CFT}\rangle$$

$$\phi^{CFT}(x)$$

primary field operators

YZ, AM, G. Vidal, arXiv:1901.06439 (2019)

OPE coefficients via lattice primary operators

critical spin chain

$$C^{\phi_1}_{\phi_2\phi_3} \approx \langle\phi_1|\phi_2|\phi_3\rangle$$

=?

$$\phi_j$$

find **lattice** primary field operators **variationally**

$$\langle\phi^{(n,m)}|\ \phi\ |I\rangle$$

$\approx$

1+1D CFT

$$C^{\phi_1}_{\phi_2\phi_3} = \langle\phi_1^{CFT}|\phi_2^{CFT}|\phi_3^{CFT}\rangle$$

$$\phi^{CFT}(x)$$

primary field operators

$$\langle\phi^{(n,m)\,CFT}|\ \phi^{CFT}\ |I^{CFT}\rangle$$

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OPE coefficients via lattice primary operators

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≈

1+1D CFT

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$$\phi^{CFT}(x)$$

primary field operators

$$\langle\phi^{(n,m)\,CFT}|\ \phi^{CFT}\ |I^{CFT}\rangle$$

accuracy ~ 10^{-7}

(for Ising model)

YZ, AM, G. Vidal, arXiv:1901.06439 (2019)

extracting conformal data

critical spin chain

1+1D CFT

low-energy spectrum

lattice Virasoro generators

AM & G. Vidal (2017)

central charge: c

primary fields: Δ_ϕ, s_ϕ

OPE coefficients: $C^{\phi_1}_{\phi_2\phi_3}$

(3-point correlators)

extracting conformal data

critical spin chain | **1+1D CFT**

low-energy spectrum

lattice Virasoro generators

AM & G. Vidal (2017)

lattice primary operators

YZ, **AM,** G. Vidal, arXiv:1901.06439 (2019)

central charge: c

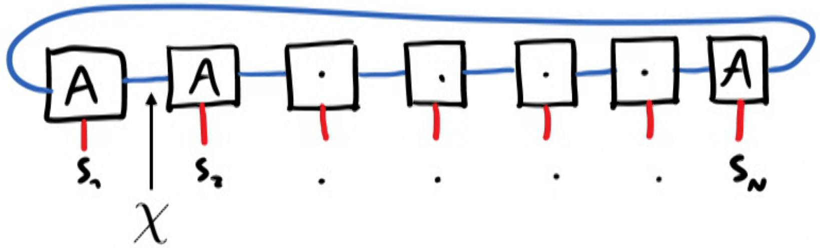
primary fields: Δ_ϕ, s_ϕ

OPE coefficients: $C^{\phi_1}_{\phi_2\phi_3}$

(3-point correlators)

extracting *accurate, precise* conformal data

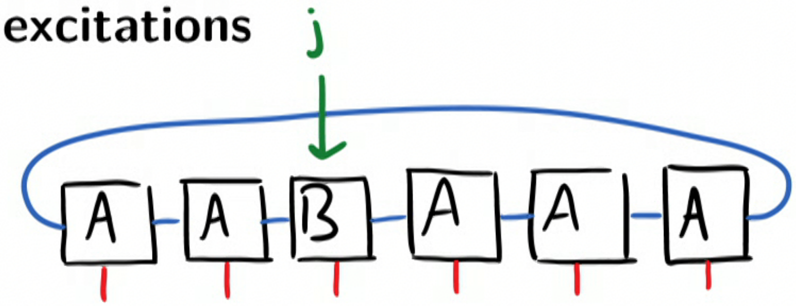
puMPS for the ground state



Rommer & Östlund (1997)
Verstraete, Porras, Cirac (2004)
Pippan, White, Evertz (2010)
Pirvu, Verstraete, Vidal (2011)

puMPS Bloch states for excitations

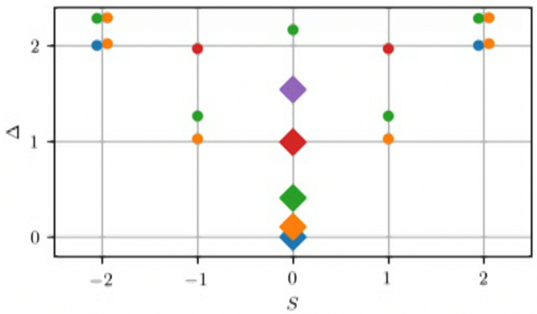
$$\sum_{j=1}^{N} e^{ipj\frac{2\pi}{N}} T^{j}$$



Rommer & Östlund (1997)
Pirvu, Haegeman, Verstraete (2012)

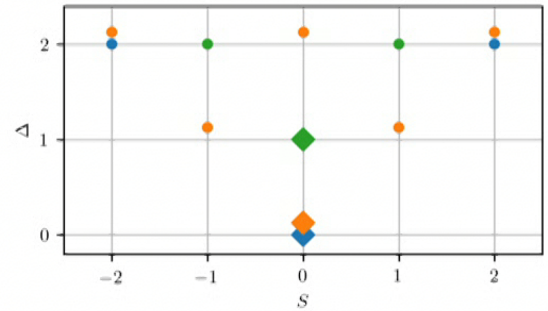
extracting *accurate* conformal data

critical spin chain

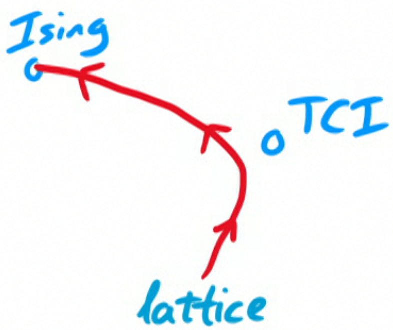


$N = 32$

1+1D CFT



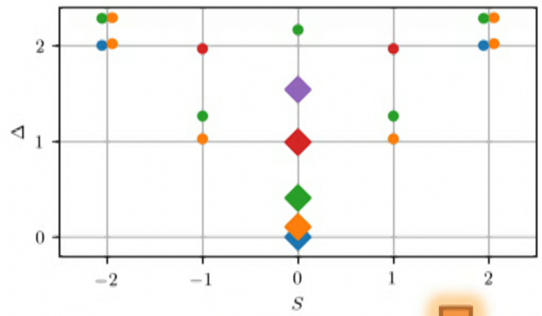
Ising CFT



YZ, **AM,** G. Vidal, PRL 121, 230402 (2018)
[model: O'Brien & Fendley, PRL 120, 206403 (2018)]

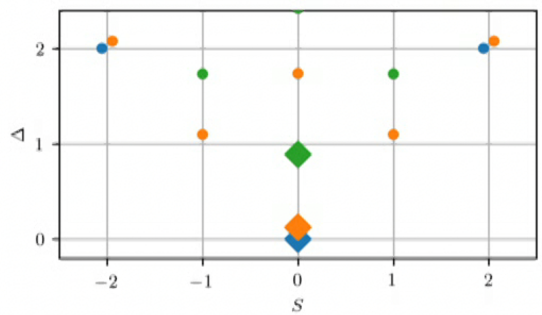
extracting *accurate* conformal data

critical spin chain



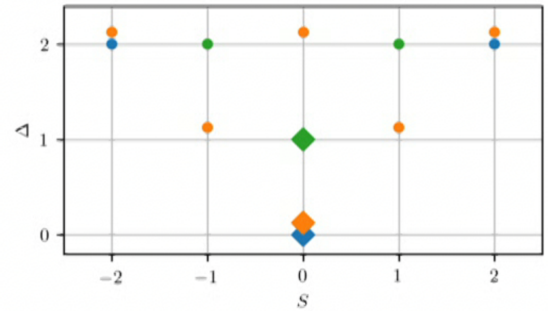
$N = 32$

puMPS

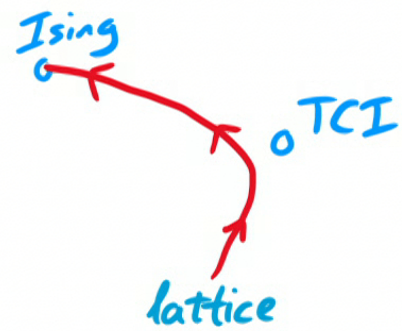


$N = 256$

1+1D CFT

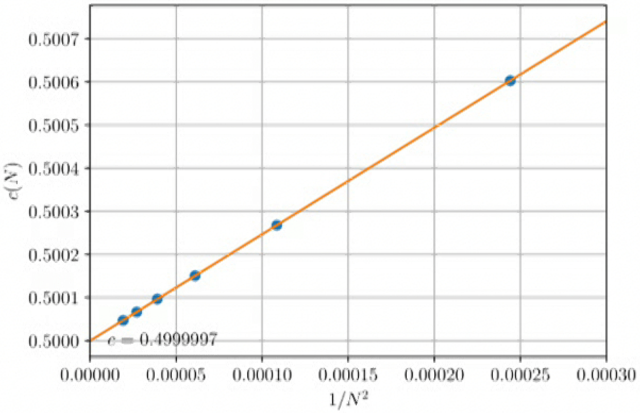


Ising CFT



YZ, **AM**, G. Vidal, PRL 121, 230402 (2018)
[model: O'Brien & Fendley, PRL 120, 206403 (2018)]

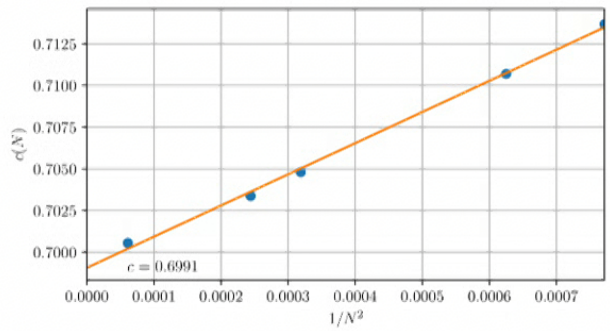
extracting *precise* conformal data



Critical Ising model

	exact	puMPS	error
c	0.5	0.4999997	10^{-7}
Δ_σ	0.125	0.1249995	10^{-7}
Δ_ε	1	0.9999994	10^{-7}
$\Delta_{\partial\bar{\partial}\sigma}$	2.125	2.12501	10^{-5}
$\Delta_{\partial\bar{\partial}\varepsilon}$	3	3.00002	10^{-5}
$\Delta_{T\bar{T}}$	4	4.007	10^{-3}

$N \leq 228$



OF model, TCI point

	exact	puMPS	error
c	0.7	0.6991	10^{-4}
Δ_σ	0.075	0.07492	10^{-5}
Δ_ε	0.2	0.2001	10^{-4}
$\Delta_{\sigma'}$	0.875	0.8747	10^{-4}
$\Delta_{\varepsilon'}$	1.2	1.203	10^{-3}
$\Delta_{\varepsilon''}$	3.0	3.002	10^{-3}

$N \leq 128$

YZ, **AM**, G. Vidal, PRL 121, 230402 (2018)

the story so far

critical quantum spin chain Hamiltonian

⇩

lattice Virasoro generators + low-energy eigenstates

⇩

complete conformal data

extension to richer symmetry

we assumed only conformal symmetry:

$$[L_n^{CFT}, L_m^{CFT}] = (n-m)L_{n+m}^{CFT} + \frac{c}{12}n(n^2-1)\delta_{n+m,0}$$

extracting conformal and **superconformal data** from critical quantum spin chains

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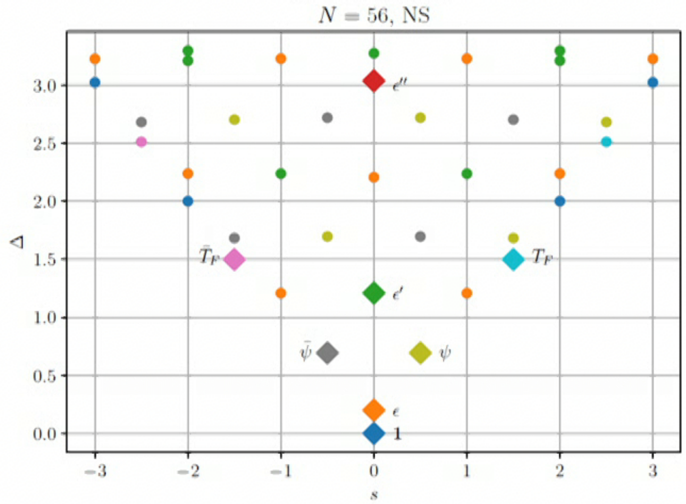
TENSOR NETWORKS INITIATIVE

Simons Collaboration on the Many Electron Problem

Primary fields may be related

$$h_j = -(X_j X_{j+1} + Z_j) + \lambda^*(X_j X_{j+1} Z_{j+2} + Z_j X_{j+1} X_{j+2})$$

$$\lambda^* \approx 0.428$$



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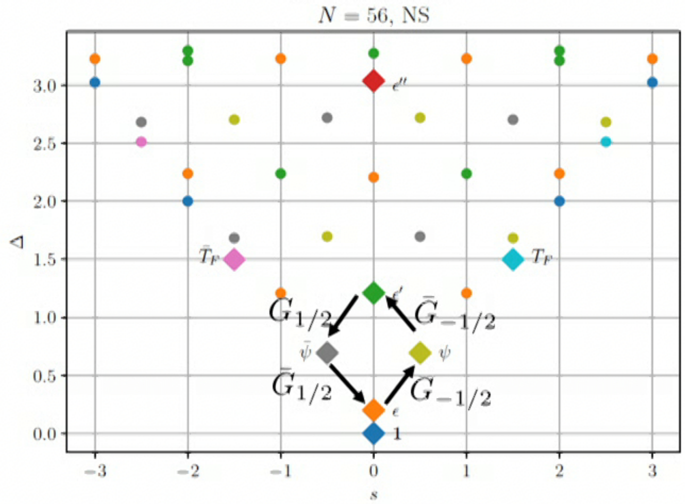
$$\lambda^* \approx 0.428$$



Emergent extended symmetry

- Emergent extended symmetry provides extra structure that relates different primary states
- Example: conformal symmetry + supersymmetry = superconformal
  - Other examples include Kac-Moody algebra and W algebra

- *Our goal:*
- *1. find lattice operators that correspond to generators of the superconformal algebra*
- *2. Identify superconformal primary states*

1. Superconformal symmetry in the tricritical Ising (TCI) CFT

Supervirasoro algebra

Virasoro algebra

Conserved currents $T^{CFT}\,(\bar{T}^{CFT})$

$$\Delta = \pm s = 2$$

Generators $L_n^{CFT} \propto \int_0^L dx\, T^{CFT}(x) e^{-i2\pi n x/L}$

$$[L_n^{CFT}, L_m^{CFT}] = (n-m)L_{n+m}^{CFT} + \frac{c^{CFT}}{12} n(n^2-1)\delta_{n+m,0}$$

Supervirasoro algebra

Virasoro algebra

Conserved currents $T^{\text{CFT}}\,(\bar{T}^{\text{CFT}})$ ⟷ *superpartner* ⟷ $T_F^{\text{CFT}}\,(\bar{T}_F^{\text{CFT}})$

$$\Delta = \pm s = 2 \qquad\qquad \Delta = \pm s = 3/2$$

Generators $L_n^{\text{CFT}} \propto \int_0^L dx\, T^{\text{CFT}}(x) e^{-i2\pi n x/L}$ $\qquad$ $G_m^{\text{CFT}} \propto \int_0^L dx\, T_F^{\text{CFT}}(x) e^{-i2\pi m x/L}$

$$[L_n^{\text{CFT}}, L_m^{\text{CFT}}] = (n-m)L_{n+m}^{\text{CFT}} + \frac{c^{\text{CFT}}}{12} n(n^2-1)\delta_{n+m,0} \qquad [L_n^{\text{CFT}}, G_m^{\text{CFT}}] = \left(\frac{1}{2}n - m\right) G_{n+m}^{\text{CFT}}$$

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$$[L_n^{\text{CFT}}, G_m^{\text{CFT}}] = \left(\frac{1}{2}n - m\right) G_{n+m}^{\text{CFT}}$$

$$\{G_n^{\text{CFT}}, G_m^{\text{CFT}}\} = 2L_{n+m}^{\text{CFT}} + \frac{c^{\text{CFT}}}{3}\left(n^2 - \frac{1}{4}\right)\delta_{n+m,0}$$

Supervirasoro primary states

- Neveu-Schwarz (NS) sector: $m \in \mathbf{Z} + 1/2$
- Lowering operators $m > 0$ Raising operators $m < 0$
- Supervirasoro primary states are a subset of Virasoro primary states

$$L_n^{CFT}|\Phi_\alpha^{CFT}\rangle = 0,\ \bar{L}_n^{CFT}|\Phi_\alpha^{CFT}\rangle = 0\ (\forall n > 0)$$

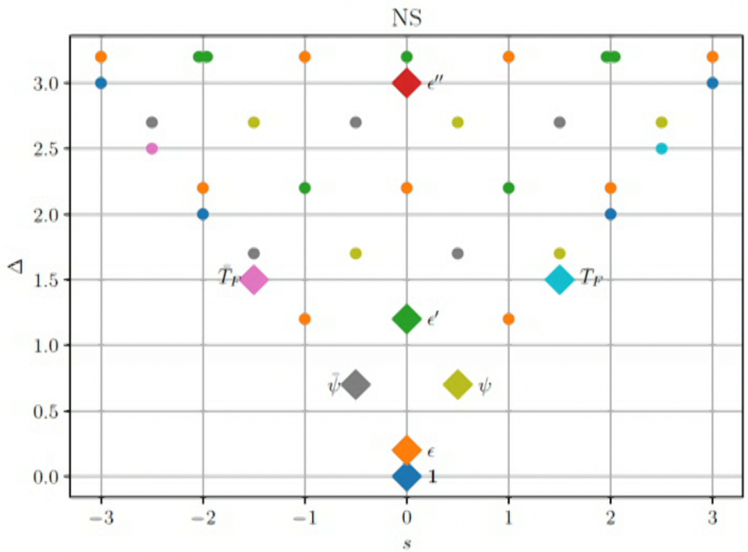
$$G_m^{CFT}|\Phi_\alpha^{CFT}\rangle = 0,\ \bar{G}_m^{CFT}|\Phi_\alpha^{CFT}\rangle = 0\ (\forall m > 0)$$

TCI CFT (NS sector)

- supersymmetric CFT

[Friedan,Qiu,Shenker 1984]

$$c^{CFT} = 7/10$$



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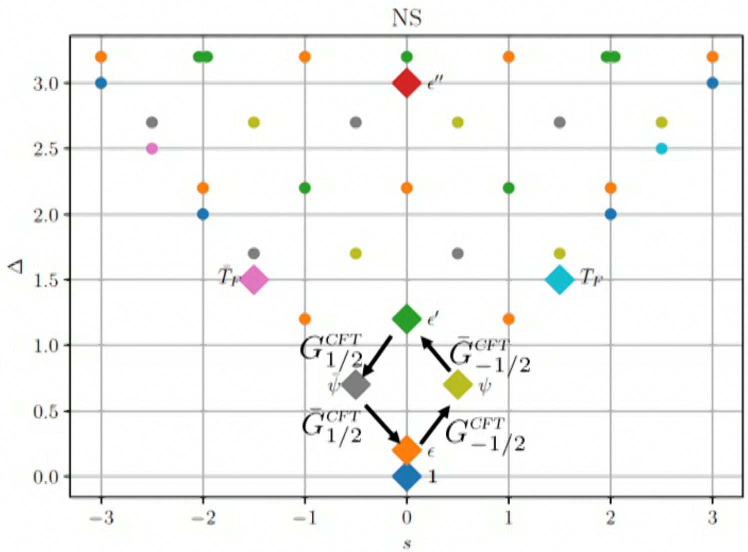

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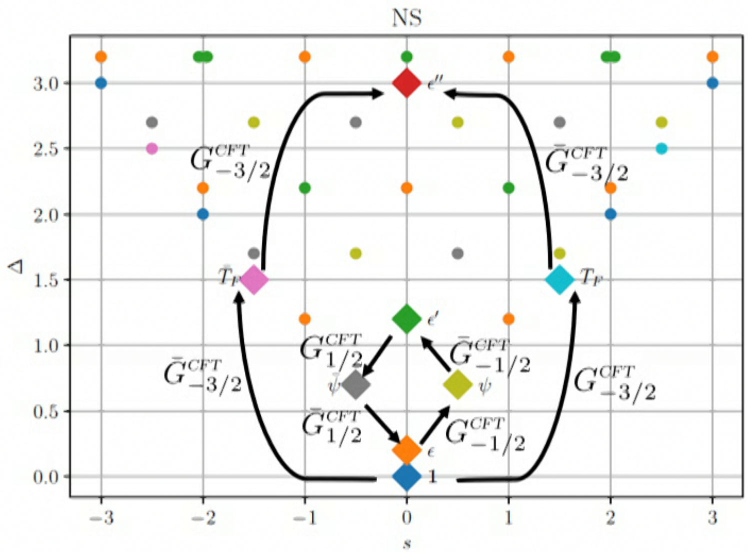


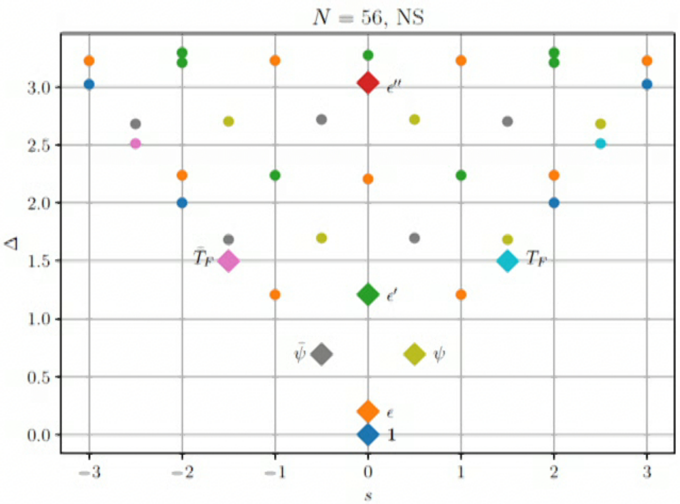
Lattice TCI model

$$h_j = -(X_j X_{j+1} + Z_j) + \lambda^*(X_j X_{j+1} Z_{j+2} + Z_j X_{j+1} X_{j+2})$$

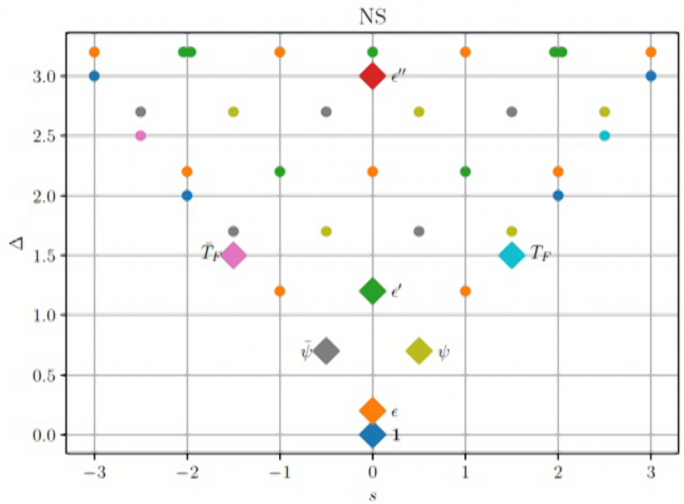
$\lambda^* \approx 0.428$ [O'Brien and Fendley 2018]

- Need low-energy states in both periodic boundary conditions (bosons) and anti-periodic boundary conditions (fermions) to see supersymmetry

Lattice spectrum



CFT spectrum



Lattice supervirasoro generators

- String operators $\mathcal{S}_{\mathcal{O},j} = \left(\prod_{i=1}^{j-1} Z_i\right) \mathcal{O}_j$

- Ansatz $T_{F,j} = \mathcal{S}_{X,j} + a_2 \mathcal{S}_{Y,j} + a_3 \mathcal{S}_{IX,j} + a_4 \mathcal{S}_{YZ,j}$

  Making use features of the lattice Hamiltonian, it was proposed that $a_2 = -1, a_3 = a_4 = -0.856$

- Lattice superconformal generators: $G_n = \eta \left(\frac{2\pi}{N}\right)^{-1/2} \sum_{j=1}^{N} T_{F,j} e^{-i2\pi nj/N}$

$$\langle \psi | G_{-1/2} | \mathbf{1} \rangle = 0$$
$$\langle \bar{\psi} | G_{1/2} | \mathbf{1} \rangle = 0$$
$$\langle \bar{T}_F | G_{3/2} | \mathbf{1} \rangle = 0$$
$$\langle T_F | G_{1/2} | T \rangle = \sqrt{3}$$

$\Rightarrow$

$$a_2 \approx -1.000$$
$$a_3 \approx a_4 \approx -0.861$$
$$\eta \approx 0.108$$

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$\Rightarrow$

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$$\eta \approx 0.108$$

Without additional assumptions of the model!

Central charge

$$c = 2|\langle T|L_{-2}|0\rangle|^2$$
$$c' = \frac{3}{2}|\langle T_F|G_{-3/2}|0\rangle|^2$$

$N \to \infty$ ⇒

$$c^{CFT} = 0.7$$

Central charge

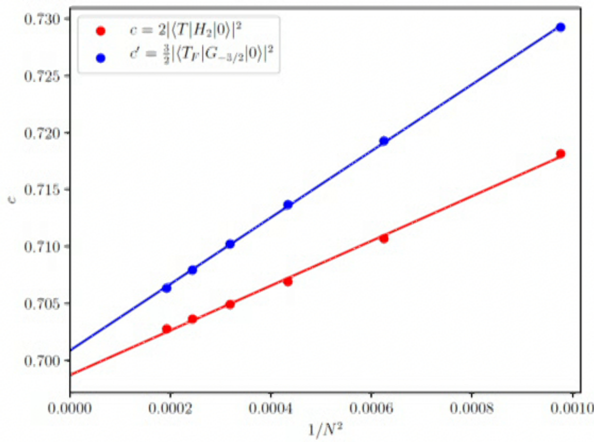
$$c = 2|\langle T|L_{-2}|0\rangle|^2$$
$$c' = \frac{3}{2}|\langle T_F|G_{-3/2}|0\rangle|^2$$

$N \to \infty$

$$c^{CFT} = 0.7$$

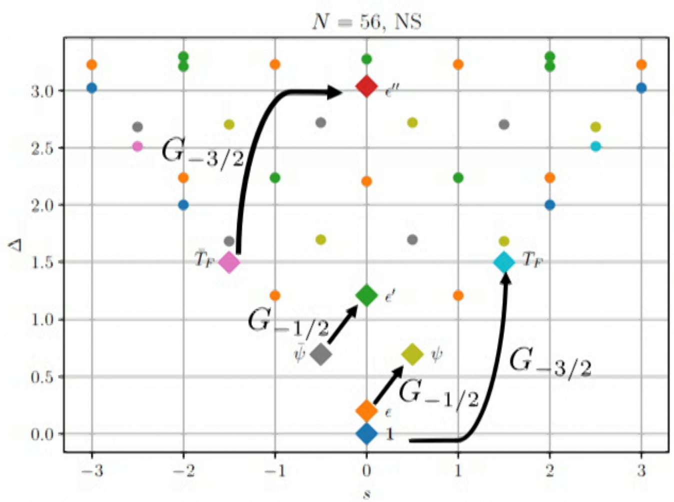
$c \approx 0.699$

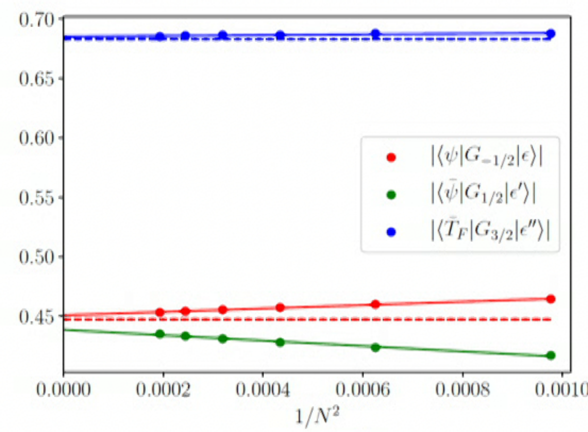
$c' \approx 0.701$



$32 \leq N \leq 72$

Superconformal primary states





$\psi, \epsilon', \epsilon''$ are not superconformal primaries!

Summary of the two parts of this talk

Input

$$H = \sum_j h_j$$

Output

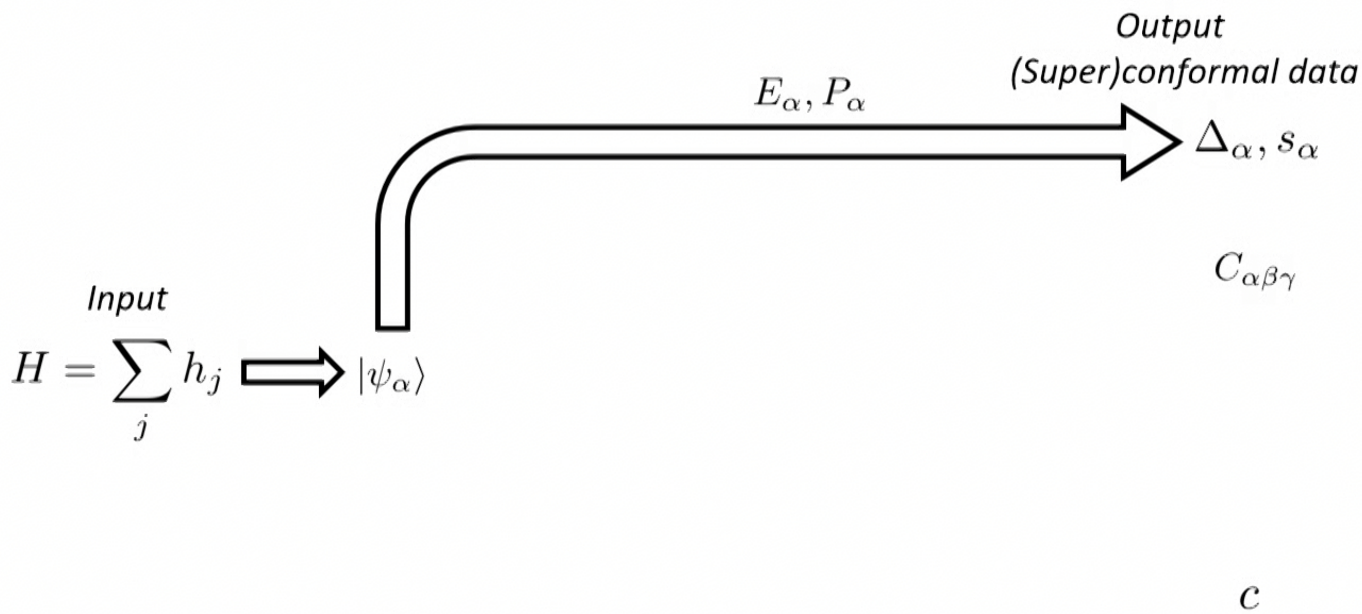
(Super)conformal data

$$\Delta_\alpha, s_\alpha$$

$$C_{\alpha\beta\gamma}$$

$$c$$

Summary of the two parts of this talk



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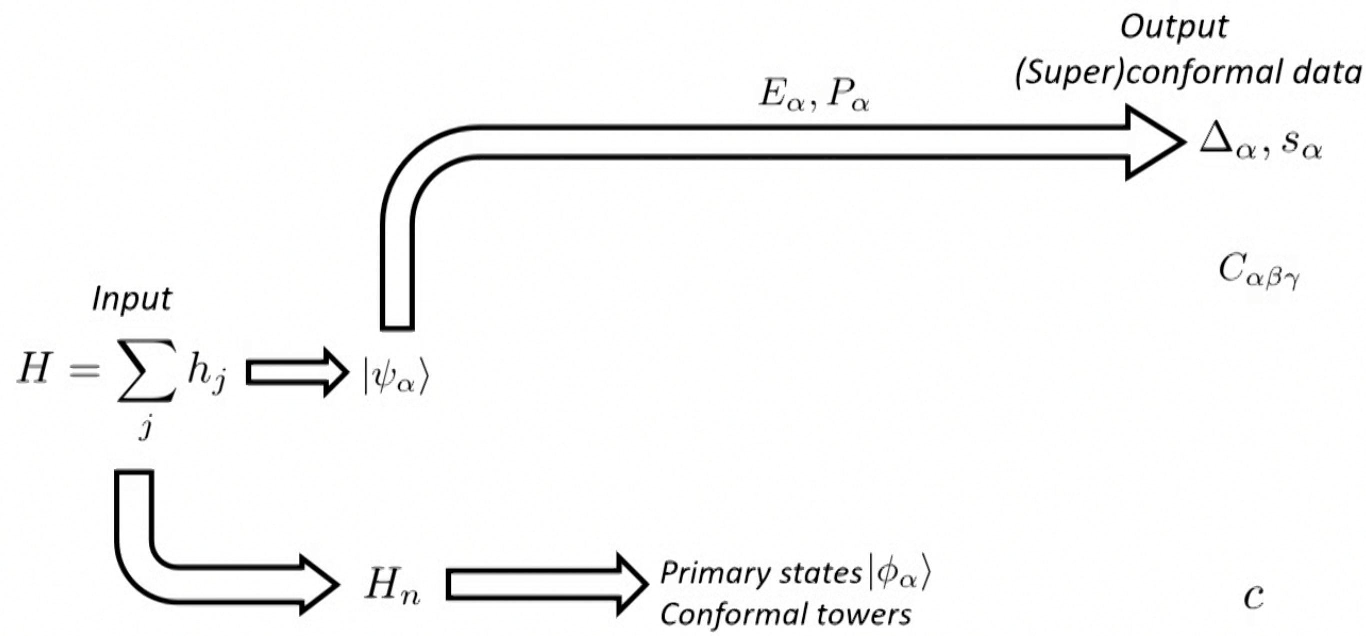

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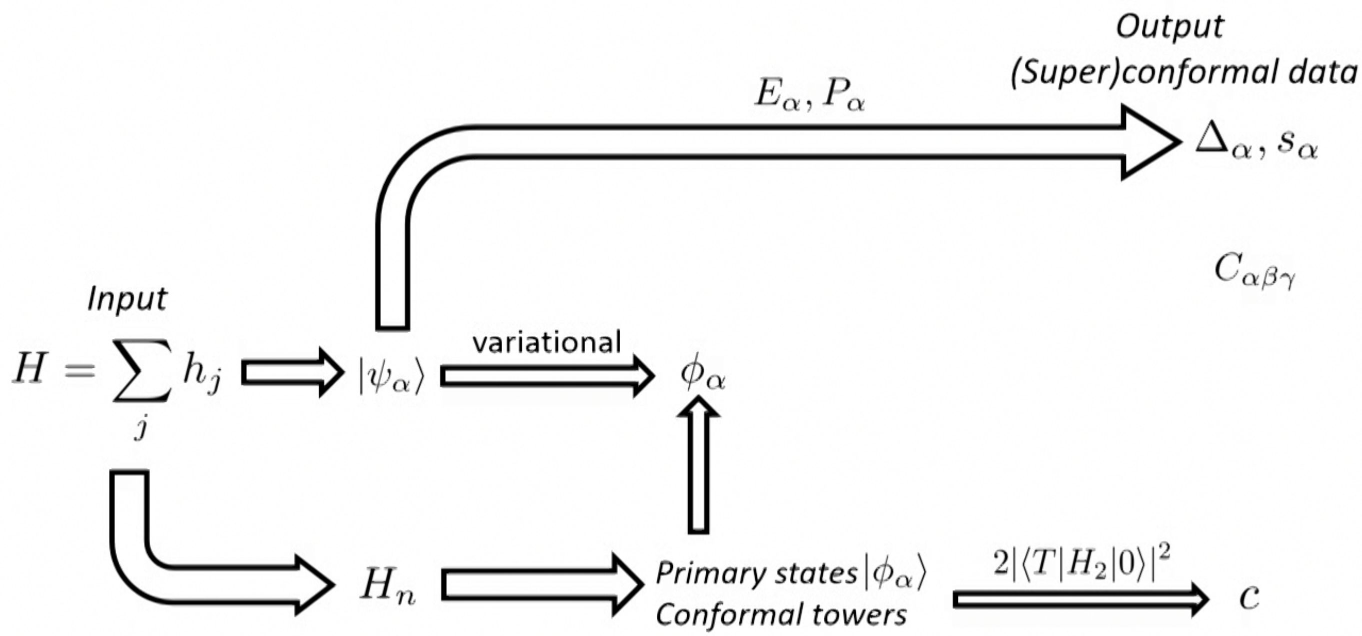

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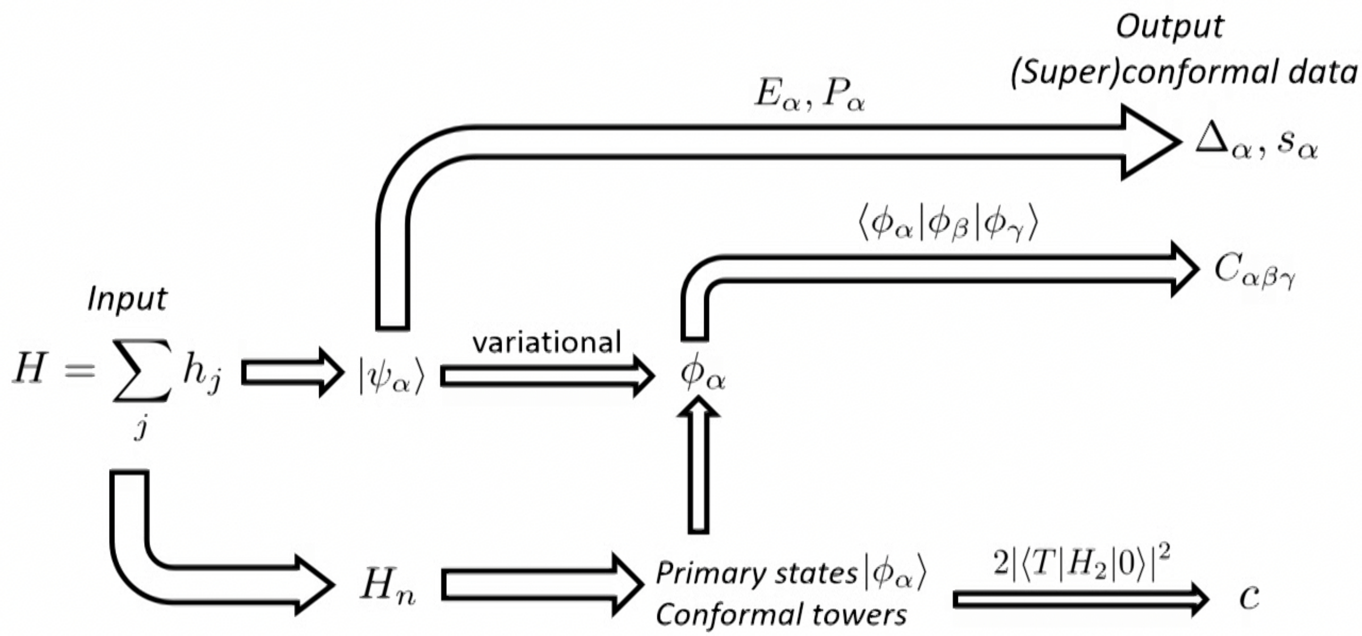

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