Title: Phase transition of fractional Chern insulators: QED3 and beyond

Speakers: Yin-Chen He

Collection: Quantum Matter: Emergence & Entanglement 3

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Abstract: Recent experiments in graphene heterostructures have observed Chern insulators - integer and fractional Quantum Hall states made possible by a periodic substrate potential. Here we study theoretically that the competition between different Chern insulators, which can be tuned by the amplitude of the periodic potential, leads to a new family of quantum critical points described by QED3-Chern-Simons theory. At these critical points, Nf flavors of Dirac fermions interact through an emergent U(1) gauge theory at Chern-Simons level K, and remarkably, the entire family (with any Nf or K) can be realized at special values of the external magnetic field. I will talk about the physical properties and microscopic realization of those critical points. We propose experiments on Chern insulators that could resolve open questions in the study of 2+1 dimensional conformal field theories and test recent duality inspired conjectures.

Phase transitions of fractional Chern Insulators: QED3 and beyond

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Quantum Matter: Emergence & Entanglement 3
Apr. 22-26, 2019

PERIMETER INSTITUTE

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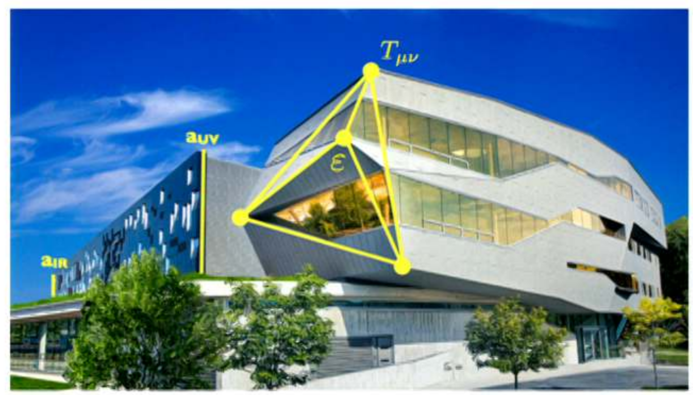


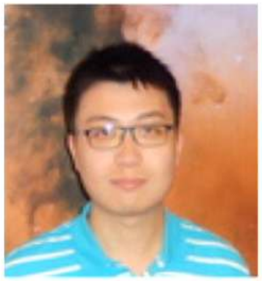
Collaborators

QED3

Lee, Wang, Zaletel, Vishwanath, YCH, PRX 8, 031015 (2018)

Jong Yeon Lee (Harvard)

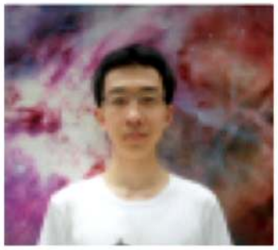
Chong Wang (Perimeter)



Ashvin Vishwanath (Harvard)

Mike Zaletel (Berkeley)

Beyond… in prepare



Ruochen Ma (Perimeter)

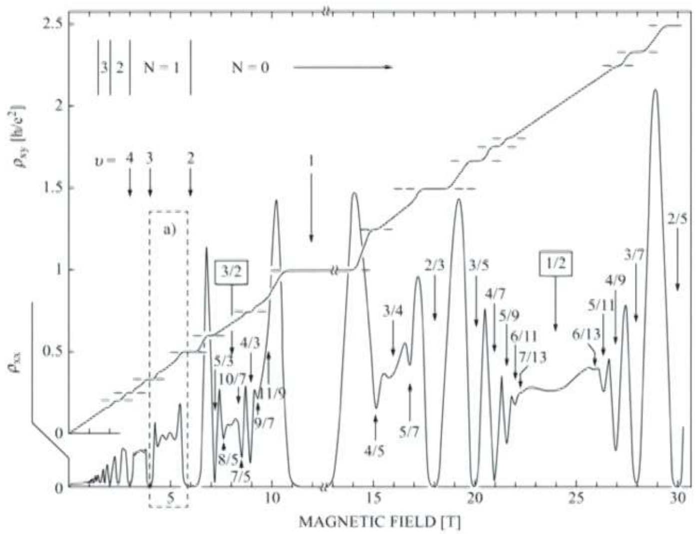
Plan

- Overview of motivation and results
- Abelian phase transitions: whole family of QED3
- Non-Abelian phase transitions: QCD3

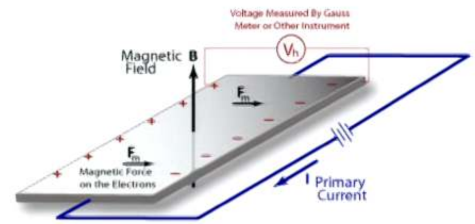
Exotic phases and phase transitions

Fractional quantum Hall effect

(topological/fractionalized phase)



Stormer, Tsui, Laughlin



Classical

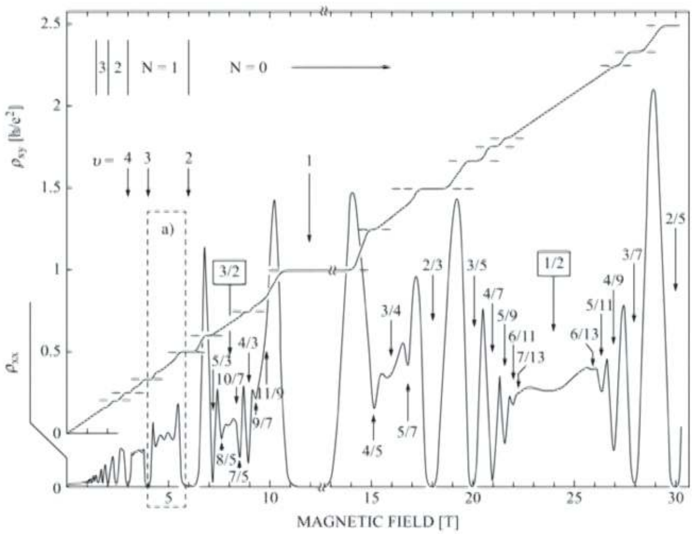
$$R_{xy} = 1/\sigma_{xy} \sim B$$

Quantum $\sigma_{xy} = \nu \frac{e^2}{h}$

Exotic phases and phase transitions

Fractional quantum Hall effect

(topological/fractionalized phase)



Stormer, Tsui, Laughlin

topological/fractionalized phase transition?

Eg. phase transition between different fractions?

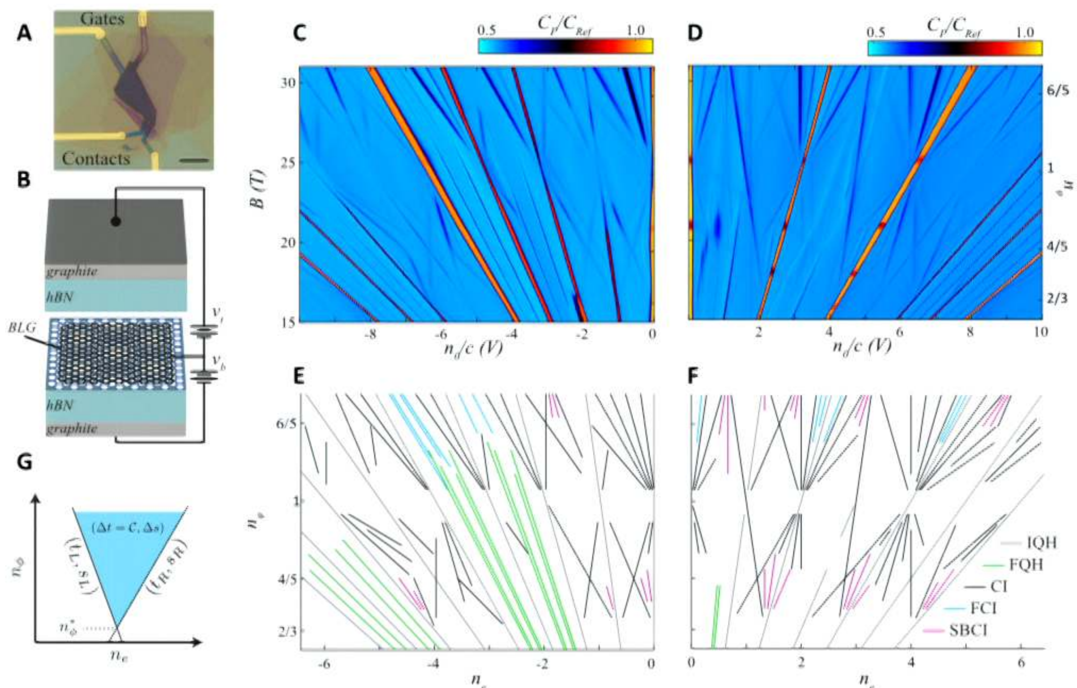
Kivelson, Zhang, Lee, …

finite disorder, super-universality…

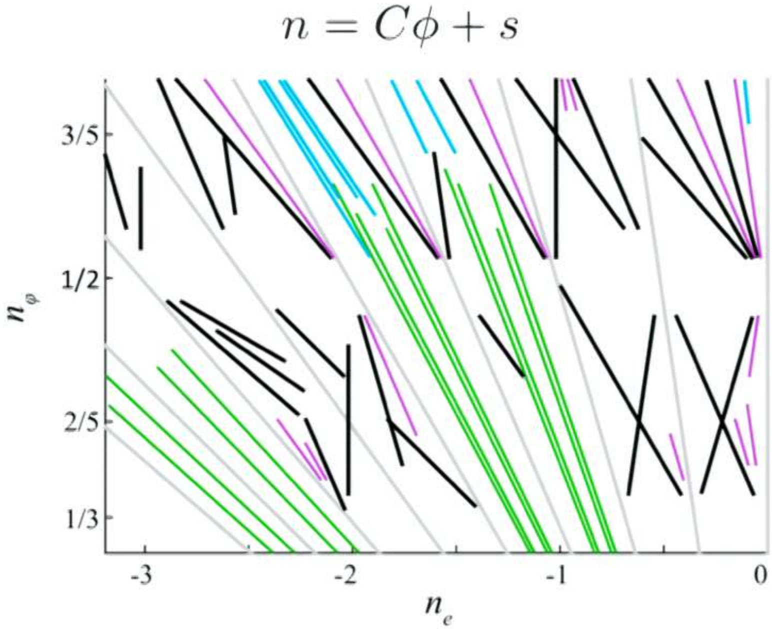
Fractional Chern insulator (FCI)

Fractional quantum Hall state on the lattice

A. Young group, arXiv: 1706.06116, Science



Phase transition between FCI/FQH



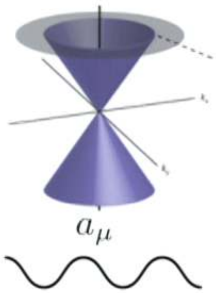
- Disorder is not necessary
- Lattice symmetry

New family of quantum critical points

$$\mathcal{L} = \sum_{I=1}^{N_f} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/ - m)\psi_I + \frac{K}{4\pi} ada + \cdots$$

Lee, Wang, Zaletel, Vishwanath, YCH, PRX 8, 031015 (2018)

QED3-Chern-Simons theory



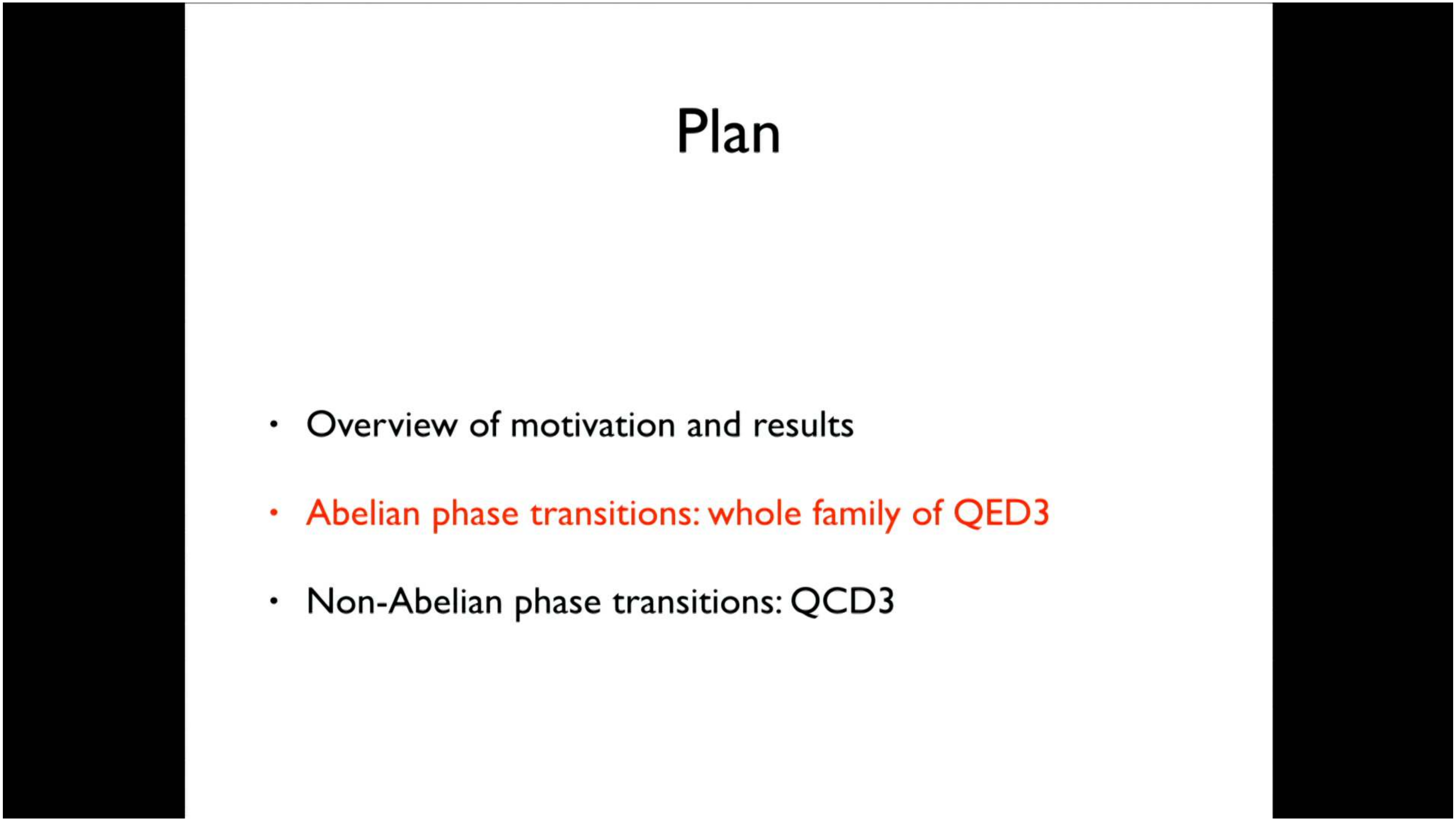
$$\mathcal{L} = \sum_{I=1}^{N_f} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/ - m)\psi_I + \frac{K}{4\pi} ada + \cdots$$

Simulate the entire family in condensed matter experiments!

Old problem with new excitements:

- A family of interacting 2+1D conformal field theories.
- Open problems after several decades efforts.
- Interesting duality properties.

  see review: Senthil, Son, Wang, Xu (2018)
- 2+1D version of 1+1D SU(N) WZW CFTs.


Plan
• Overview of motivation and results
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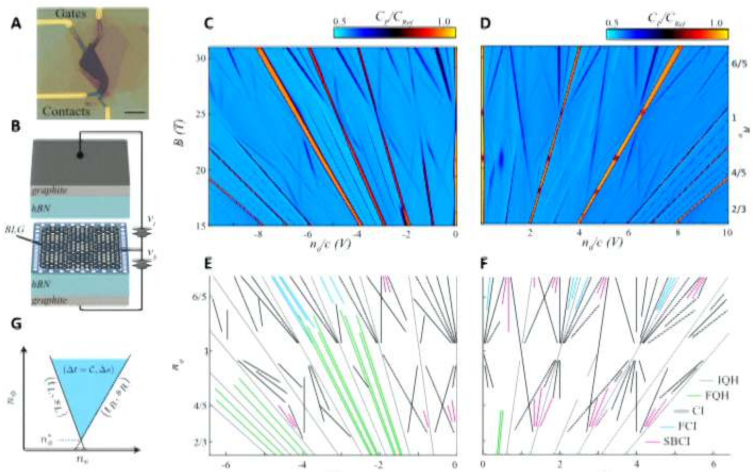
• Non-Abelian phase transitions: QCD3

Artificial superlattice on graphene

Potential strength is experimentally tunable

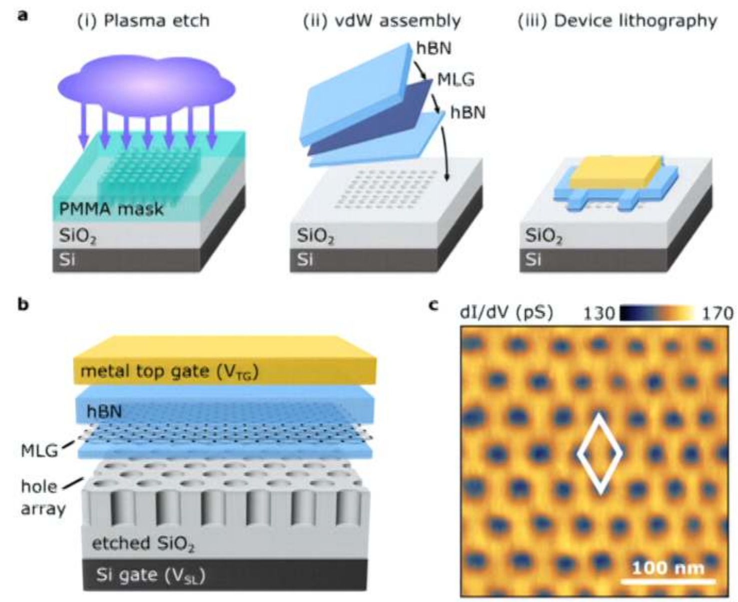
A. Young group, arXiv: 1706.06116

Moire potential



C. Dean group, arXiv:1710.01365

Electrostatic gate patterning



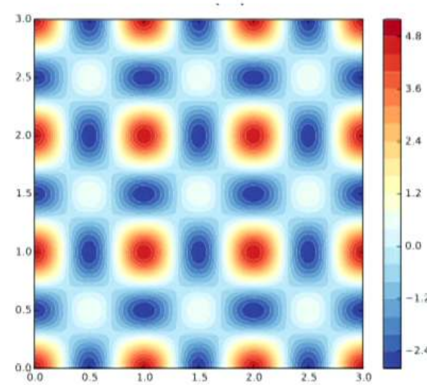
Microscopic model

Landau level with superlattice potential

$$\mu(\vec{r}) = U_0 \int d\vec{r} \sum_m (V_m e^{i\vec{r}\cdot\vec{G}_m} n_{\vec{r}} + h.c.),$$

$$\vec{G}_1 = \frac{2\pi}{a}(1,0), \quad \vec{G}_2 = \frac{2\pi}{a}(0,1)$$

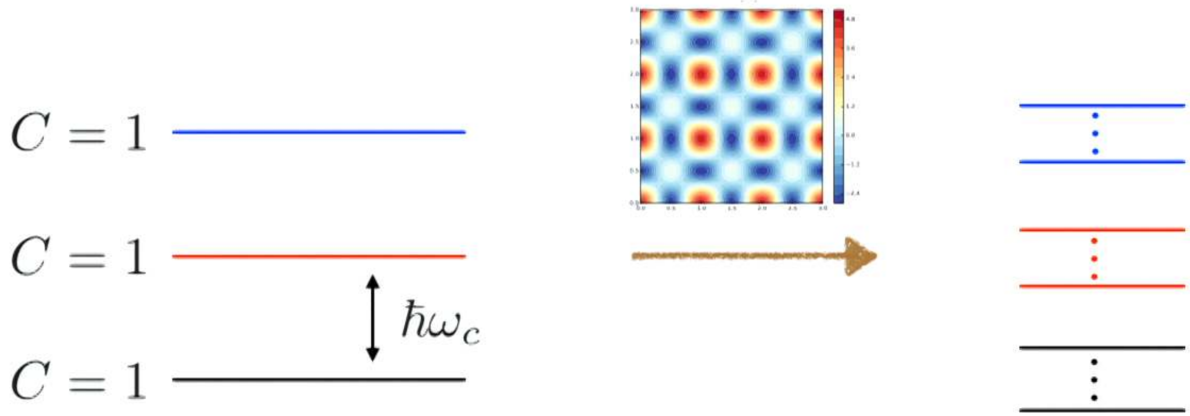
$$\vec{G}_3 = \frac{2\pi}{a}(1,1), \quad \vec{G}_2 = \frac{2\pi}{a}(1,-1), \cdots$$



Weak potential limit: potential strength smaller/comparable to cyclotron gap

Physical effects: Landau level gets broadened and splitted

Landau levels under lattice potential

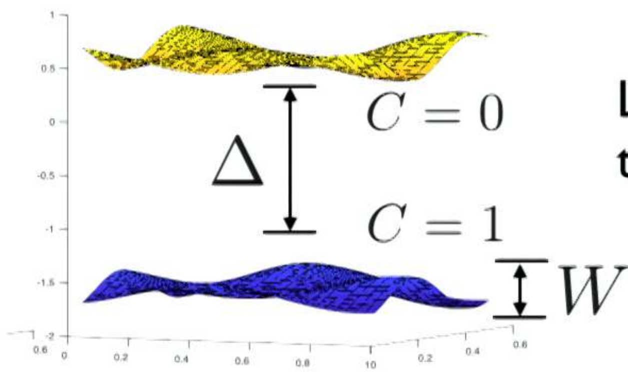


$\phi = 2\pi \frac{p}{q}$ Each Landau level splits into p sub-bands

Streda, MacDonald, Pfannkuche, Gerhardts, Usov,...

1/3 FQH to 2/3 FCI transition

Flux: $\phi = Ba^2 = 4\pi$



LLL splits to two sub-bands

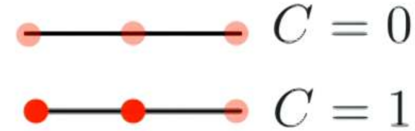
Energy scale

Cyclotron gap: ω_0
Coulomb energy: E_c
band width: W
band gap: Δ

$C = 1$

Flux: $\phi = Ba^2 = 4\pi$
Density: $n = 2/3$

$\Delta, W \propto U_0$



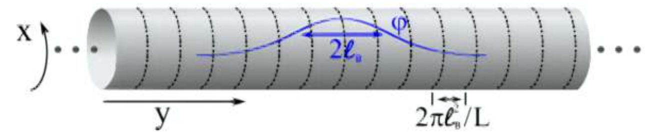
1/3 FQH state:
$\omega_0 \gg E_c \gg \Delta, W$

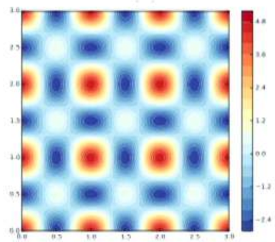
2/3 FCI state:
$\omega_0 \gg \Delta \gg E_c \gg W$

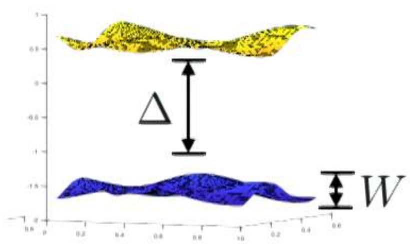
Numerical results: DMRG

DMRG: White

FQH DMRG:
Zaletel, Mong, Pollmann

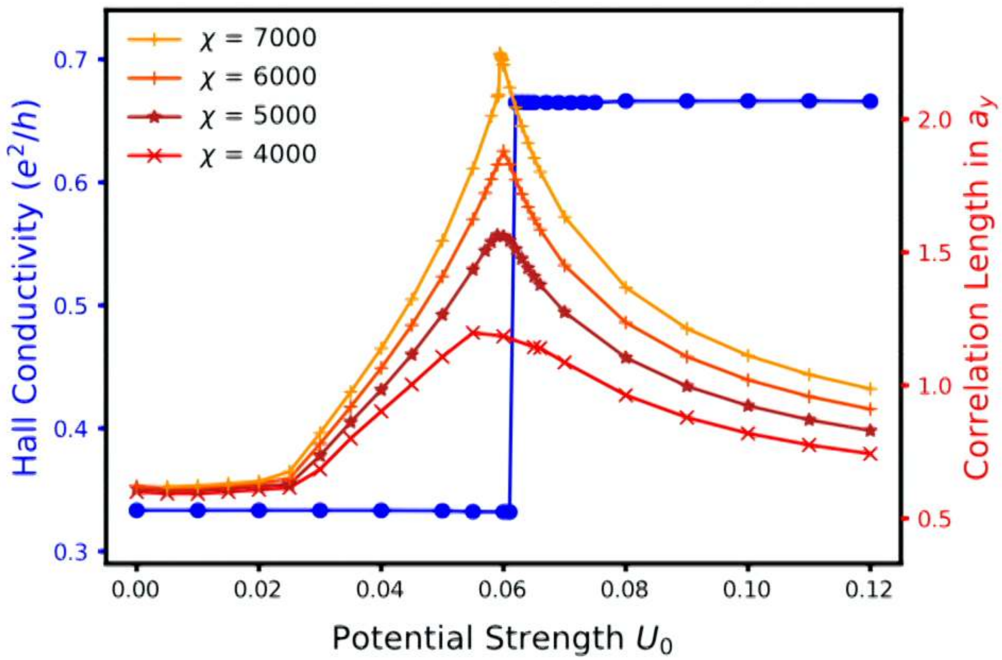






$E_c = 1$

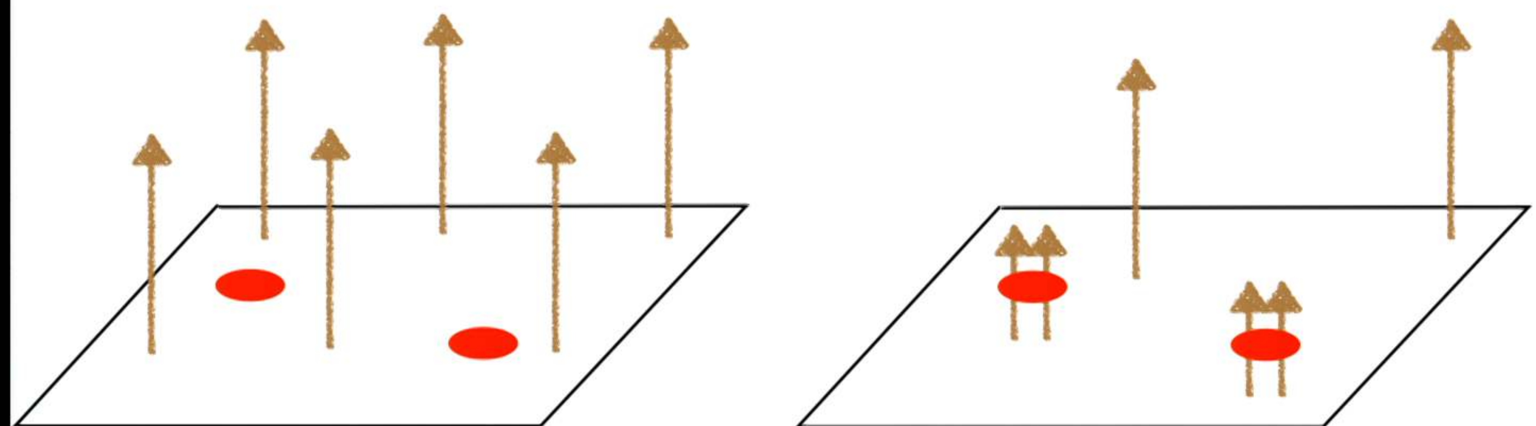
Phase transition from $\sigma_{xy} = 1/3$ to $\sigma_{xy} = 2/3$



Composite fermion for the FQH/FCI

Eg: 1/3 Fractional quantum Hall state

$n/B = 1/3$ $\quad$ $n/B_{\text{eff}} = 1$



Jain; Lopez, Fradkin; Halperin, Lee, Read;...

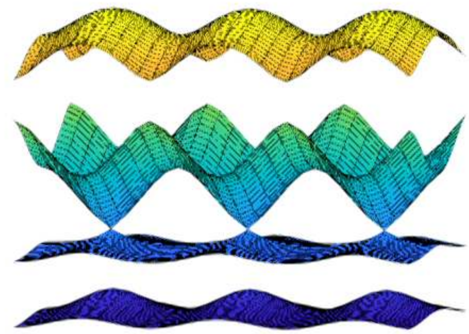
Composite fermion for the transition

Composite fermion
$\nu_{\mathrm{CF}} = C$ Integer QH

Original electron
$\sigma^{xy} = \dfrac{C}{2C+1}$ FQH/FCI

Electron:
$\sigma^{xy} = 1/3 \leftrightarrow \sigma^{xy} = 2/3$

Composite fermion:
$\nu_{\mathrm{CF}} = 1 \leftrightarrow \nu_{\mathrm{CF}} = -2$



Critical theory:

$$\mathcal{L} = \sum_{I=1}^{3} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/)\psi_I - \frac{1}{8\pi} a da + \frac{1}{8\pi}(a-A)d(a-A)$$

$$= \sum_{I=1}^{3} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/)\psi_I - \frac{1}{4\pi} A da + \frac{1}{8\pi} AdA.$$

Nf=3 QED3

Critical theory

$$\mathcal{L} = \sum_{I=1}^{3} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/)\psi_I - \frac{1}{4\pi} Ada + \frac{1}{8\pi} AdA$$

Tuning parameter: $m \sum_{I=1}^{3} \bar{\psi}_I \psi_I \Longrightarrow \mathrm{sgn}(m) \frac{3}{8\pi} ada$

1. m>0, $\mathcal{L}_{\mathrm{res}} = \frac{1}{3}\frac{1}{4\pi} AdA$

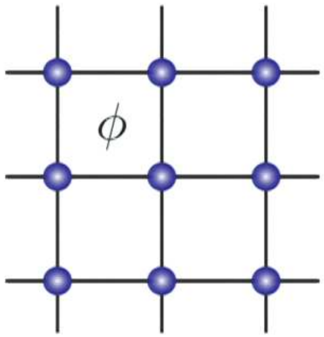
2. m<0, $\mathcal{L}_{\mathrm{res}} = \frac{2}{3}\frac{1}{4\pi} AdA$

We need symmetry to forbid other relevant term.

eg. Barkeshli, McGreevy 2014

Magnetic translation symmetry

$$T_x T_y = e^{i\phi} T_y T_x$$



$$\mathcal{L} = \sum_{I=1}^{3} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/)\psi_I - \frac{1}{4\pi} Ada + \frac{1}{8\pi} AdA$$

Composite fermions see a flux $2\pi/3$

$$T_x : \psi_I \to e^{i2\pi I/3}\psi_I \qquad T_y : \psi_I \to \psi_{I+1}$$

Only symmetry allowed mass term: $\sum_{I=1}^{3} \bar{\psi}_I \psi_I$

Fundamental operators

$$\mathcal{L} = \sum_{I=1}^{3} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/)\psi_I - \frac{1}{4\pi} Ada + \frac{1}{8\pi} AdA$$

Field theory

Fermion bilinears

SU(3) adjoint

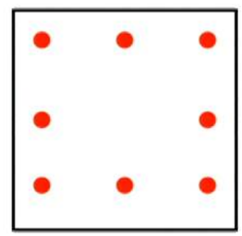
$\bar{\psi}_I M_{IJ} \psi_J$

Monopole

spin-1/2, SU(3) adjoint

spin-3/2, SU(3) singlet

Experiment



Charge-density-wave

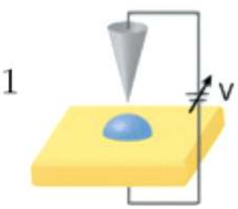
$\vec{k}_\alpha = (\frac{2l_x\pi}{3}, \frac{2l_y\pi}{3})$

$(l_x, l_y) \neq (0,0)$

Local electron

STM

$dI/dV \propto V^{2\Delta-1}$



The entire family of transition

Between two composite fermion states:  bound k flux quanta

$$\sigma^{xy} = \frac{C_1}{kC_1 + 1} \quad \longleftrightarrow \quad \sigma^{xy} = \frac{C_2}{kC_2 + 1}$$

Bosonic system has odd k, fermionic system has even k

$$\mathcal{L} = \sum_{I=1}^{|C_2 - C_1|} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/ - m)\psi_I + \frac{C_2 + C_1}{8\pi} ada + \frac{1}{4k\pi}(a - A)d(a - A).$$

Protected by (magnetic) translation symmetry

Even flavor QED3 theory

Between bosonic particle-hole partner:

$$\sigma^{xy} = \frac{N-1}{N} \quad \longleftrightarrow \quad \sigma^{xy} = \frac{N+1}{N}$$

Critical theory:

$$\mathcal{L} = \sum_{I=1}^{2N} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/)\psi_I - \frac{1}{2\pi} A da + \frac{1}{4\pi} A dA$$

i) 0-2 transition: Nf=2, SPT transition Grover, Vishwanath; Lu, Lee
Self-dual, possible emergent O(4) symmetry
Senthil, Fisher; Xu, You; Tong, Karch; Wang, Nahum, Metlitski, Xu, Senthil

ii) 1/2-3/2 transition: Nf=4

iii) …

Odd flavor QED3 theory

Between fermionic particle-hole partner

$$\sigma^{xy} = \frac{N}{2N+1} \longleftrightarrow \sigma^{xy} = \frac{N+1}{2N+1}$$

Critical theory:

$$\mathcal{L} = \sum_{I=1}^{2N+1} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/)\psi_I - \frac{1}{4\pi} Ada + \frac{1}{8\pi} AdA$$

i) 0-1 transition: Nf=1, vortex dual of free Dirac

Son; Wang & Senthil; Metlitski & Vishwanath…

ii) 1/3-2/3 transition: Nf=3

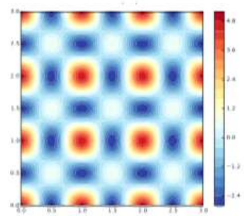
iii) ….

More about microscopic realization

$C = 1$

Partially filled Landau level

$\nu = \dfrac{p}{2p+1}$ FQH



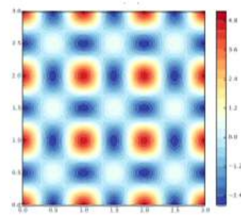
Potential strength

(Partially) filled sub-bands

Chern insulator

Transition to	N_f	K	ϕ	n
$\sigma_{xy} = \frac{p+1}{2p+1}$ FCI	$2p+1$	0	$2p$	$\frac{2p^2}{2p+1}$
$\sigma_{xy} = 1$ ICI	$p+1$	$p/2$	$\frac{2p+1}{p+1}$	$\frac{p}{p+1}$
$\sigma_{xy} = 0$ ICI	p	$(p+1)/2$	$\frac{2p+1}{p}$	1

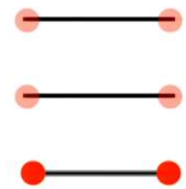
Fractional QH to Chern Insulator transition



$C = 1$

Flux: $\phi = Ba^2 = 3\pi$

Density: $n = 1/2$



$C = -1$

$C = 1$

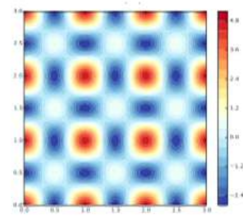
$C = 1$

$\Delta, W \propto U_0$

1/3 FQH state → $\sigma^{xy} = 1$ CI state

$$\mathcal{L} = \sum_{I=1}^{2} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/)\psi_I + \frac{1}{8\pi} ada - \frac{1}{4\pi} adA + \frac{1}{8\pi} AdA$$

Fractional QH to Chern Insulator transition



$C = 1$

Flux: $\phi = Ba^2 = 3\pi$

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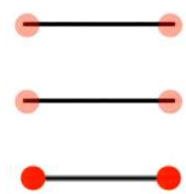

$C = 1$

$C = 1$

$C = -1$

1/3 FQH state $\xrightarrow{\Delta, W \propto U_0}$ $\sigma^{xy} = -1$ CI state

Composite fermion approach doesn't work.

$k = 1, p = 1$ $\qquad$ $\sigma^{xy} = \dfrac{p}{2kp + 1}$ $\qquad$ $k = -1, p = 1$

Plan

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- Non-Abelian phase transitions: QCD3

Non-Abelian transition between Abelian state

Fractionalize electron operator into three fermionic partons:

$$c = f_1 f_2 f_3$$

$\sigma^{xy} = \frac{1}{3}$ FQH: $C_1 = C_2 = C_3 = 1$

$\sigma^{xy} = -1$ CI: $C_1 = 1, C_2 = C_3 = -1$

Non-Abelian transition between Abelian state

Fractionalize electron operator into three fermionic partons:

$$c = \underbrace{f_1 \overbrace{f_2}^{SU(2)}}_{U(1)} f_3$$

$\sigma^{xy} = \frac{1}{3}$ FQH: $C_1 = C_2 = C_3 = 1$

$\sigma^{xy} = -1$ CI: $C_1 = 1, C_2 = C_3 = -1$

Transition: $N_f = 2$ ψ coupled to $U(2)_{0,-2}$ Chern-Simons

$$\mathcal{L}_{CS} = \frac{1}{4\pi} \mathrm{Tr}(a)\, d\, \mathrm{Tr}(a)$$

Parton of non-Abelian state

see also Wen; Barkeshli, et. al.

$c = f f_1 f_2$ Gauge symmetry: $U(2) = \frac{SU(2) \times U(1)}{Z_2}$

$\kappa_{xy} = -1/2$, Anti-Pfaffian

$f: \ 1/3$ FQH

$(f_1, f_2): \ C = -2$

$\kappa_{xy} = 5/2$, (122) state

$f: \ C = 1$

$(f_1, f_2): \ C = 2$

$c = f(f_1 f_4 - f_3 f_2)/\sqrt{2}$ Gauge symmetry: $\frac{USp(4) \times U(1)}{U(2) \times U(1)}$

$\kappa_{xy} = 1/2$, PH-Pfaffian

f: $1/3$ FQH

f_i: $C = -1$

$\kappa_{xy} = 3/2$, Pfaffian

f: $C = 1$

f_i: $C = 1$

A fun example

$\kappa_{xy} = 3/2$, Pfaffian

f: $C = 1$

f_i: $C = 1$

Transition $\longleftrightarrow$

$\kappa_{xy} = -1/2$, $p - ip$ SC

f: $C = 1$

f_i: $C = -1$

$c = f(f_1 f_4 - f_3 f_2)/\sqrt{2}$ Gauge symmetry: $USp(4) \times U(1)$

Critical theory:

$N_f = 2$ bi-fundamental of $USp(4) \times U(1)$ with $USp(4)_0 \times U(1)_{-1}$

Summary

- Fractional Chern insulator transitions can be directly realized in experiments by tuning the potential strength.
- The transition is described by the QED3-Chern-Simons theory, and remarkably the entire family can be realized.
- We discuss the non-Abelian transitions of Abelian and non-Abelian states.

Lee, Wang, Zaletel, Vishwanath, YCH, PRX 8, 031015 (2018)
Ma, YCH, in prepare.

Thank you and see you in future (2021?)!