Title: Application of Tensor Network States to Lattice Field Theories

Speakers: Stefan Kuhn

Collection: Quantum Matter: Emergence & Entanglement 3

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Abstract: The conventional Euclidean time Monte Carlo approach to Lattice Field Theories faces a major obstacle in the sign problem in certain parameter regimes, such as the presence of a nonzero chemical potential or a topological theta-term. Tensor Network States, a family of ansatzes for the efficient description of quantum many-body states, offer a promising alternative for addressing Lattice Field Theories in the Hamiltonian formulation. In particular, numerical methods based on Tensor Network states do not suffer from the sign problem which makes it possible to study scenarios which are not accessible with standard Monte Carlo methods. In this talk I will present some recent work demonstrating this capability using two (1+1)-dimensional models as a test bed. Studying the O(3) nonlinear sigma model at nonzero chemical potential and the Schwinger model with topological theta-term, I will show how Tensor Networks States accurately describe the low-energy spectrum and that numerical errors can be controlled well enough to make contact with continuum predictions.

Application of Tensor Network States to Lattice Field Theories

Stefan Kühn

PERIMETER INSTITUTE

in collaboration with

Falk Bruckmann — Universität Regensburg
Lena Funcke — Perimeter Institute
Karl Jansen — NIC, DESY Zeuthen

Quantum Matter: Emergence & Entanglement 3

Motivation

Theory

- (Gauge) field theories
- Non-perturbative regime: Analytical access hard

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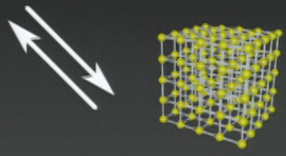

Classical simulation

- Monte Carlo methods in euclidean space-time

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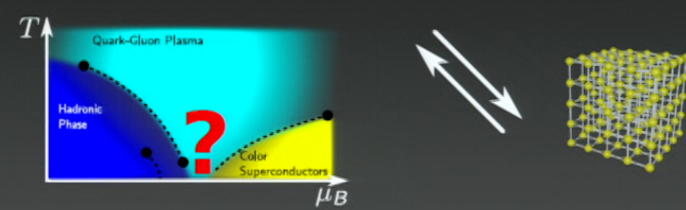

Classical simulation

- Monte Carlo methods in euclidean space-time
  - No real-time dynamics
  - Sign problem

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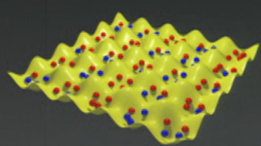

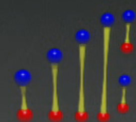

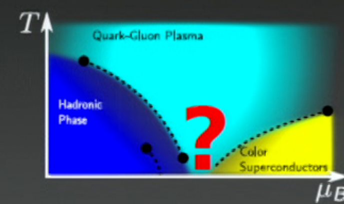

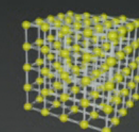

Quantum computing

- Many promising experimental platforms
- Free from purely numerical limitations

Classical simulation

- Monte Carlo methods in euclidean space-time
  - No real-time dynamics
  - Sign problem

Outline

1. Motivation
2. Tensor Network States and application to Lattice Field Theory
3. Phase structure of the O(3) nonlinear sigma model
4. Schwinger model in the presence of a topological θ-term
5. Summary & Outlook

Introduction to Tensor Network States (TNS)

What are Tensor Network States?

- Wave function for an interacting N-body system

$$|\psi\rangle = \sum_{i_1,i_2,\dots,i_N=1}^{d} c_{i_1,i_2,\dots,i_N} |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

$$= \sum_{i_1,i_2,\dots,i_N=1}^{d} \boxed{c_{i_1,i_1,\dots,i_N}} \; |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

$\Rightarrow$ Tensor $\boxed{c_{i_1,i_1,\dots,i_N}}$ with d^N entries, **exponential** number of parameters

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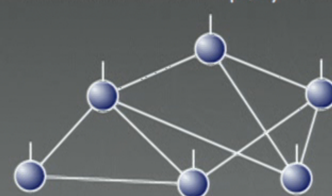
- Tensor Network State: ansatz for $|\psi\rangle$ built from smaller pieces



$\Rightarrow$ Only a **polynomial** number poly(N) of parameters

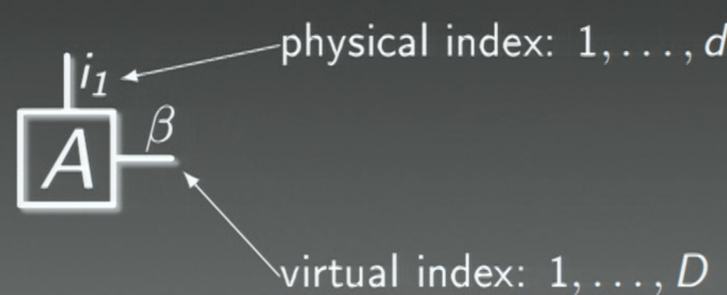
Introduction to Tensor Network States (TNS)

Matrix Product State (MPS) ansatz

- MPS ansatz with open boundary conditions (OBC) for system with N sites

$$|\psi\rangle = \sum_{i_1, i_2, \ldots, i_N}^{d} A_1^{i_1} A_2^{i_2} \ldots A_N^{i_N} |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

- Tensor $A_1^{i_1} \in \mathbb{C}^{1\times D}$



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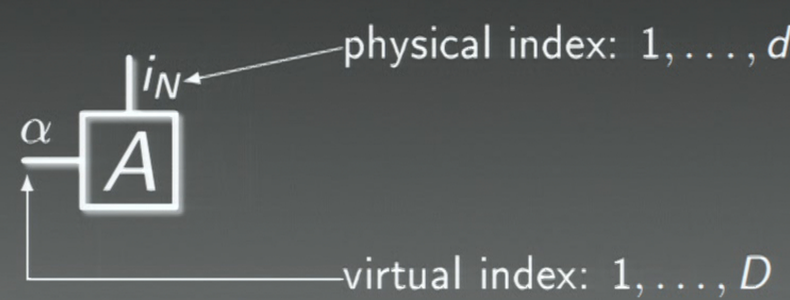
- Tensor $A_N^{i_N} \in \mathbb{C}^{D \times 1}$



- D: **Bond dimension** of the MPS

Introduction to Tensor Network States (TNS)

Matrix Product State (MPS) ansatz

- Number of parameters in the MPS with OBC

$$(N-2)D^2 d + 2Dd = \mathcal{O}(ND^2 d)$$

M. B. Hastings, J. Stat. Mech. 2007 (2007)

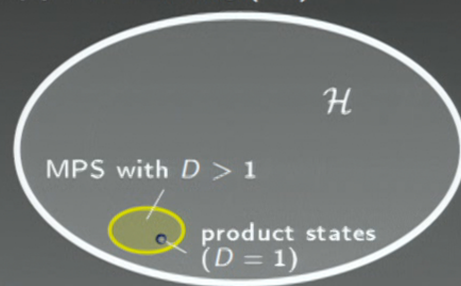
Introduction to Tensor Network States (TNS)

Matrix Product State (MPS) ansatz

- Number of parameters in the MPS with OBC

$$(N-2)D^2 d + 2Dd = \mathcal{O}(ND^2 d)$$

- Quantum information: Physically relevant states $D \ll d^{\lfloor \frac{N}{2} \rfloor}$
- Entanglement entropy: $S \leq \log(D)$



$\Rightarrow$ MPS efficiently parametrize the physically relevant subspace of slightly entangled states

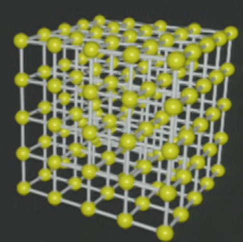
M. B. Hastings, J. Stat. Mech. 2007 (2007)

Introduction to Tensor Network States (TNS)

Why Tensor Network States?

Monte Carlo

- Action $S = \int d^4x \, \mathcal{L}$
- ✓ Expectation values
- ✗ Access to the ground state wave function
- ✗ Sign problem free



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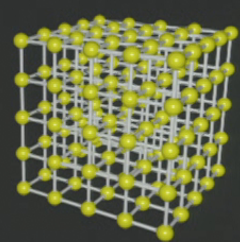
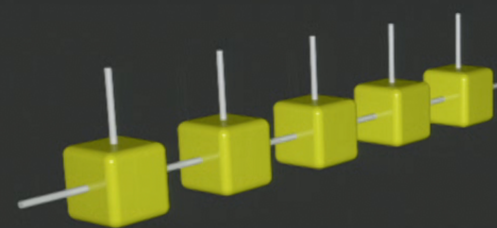


Matrix Product States

- Hamiltonian
- ✓ Ground states / Low-lying excitations
- ○ Time evolution
- ✓ **Sign problem free**



G. Vidal Phys. Rev. Lett. 93, 040502 (2004)
F. Verstraete, D. Porras, J.I. Cirac Phys. Rev. Lett. 93, 227205 (2004)

3.

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3. **Phase structure of the O(3) nonlinear sigma model**
4. Schwinger model in the presence of a topological θ-term
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The O(3) rotor model

Continuum formulation

- Euclidean time Lagrangian of the model

$$\mathcal{L}_{O(3)} = \frac{1}{2g_0^2}(\partial_\nu \mathbf{n})^2, \qquad \mathbf{n} \in \mathbb{R}^3, \qquad \mathbf{n} \cdot \mathbf{n} = 1$$

$\Rightarrow$ Nonlinearity because of the constraint

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⇒ Nonlinearity because of the constraint

- O(3) is at the same time the simplest nontrivial CP^{N-1} model

$$\mathcal{L}_{CP^{N-1}} = \frac{1}{\bar{g}_0^2}(D_\nu \mathbf{z})^\dagger (D_\nu \mathbf{z}), \qquad \mathbf{z} \in \mathbb{C}^N, \qquad \mathbf{z}^\dagger \cdot \mathbf{z} = 1$$

$D_\nu = \partial_\nu + iA_\nu$: covariant derivative with gauge field A_ν

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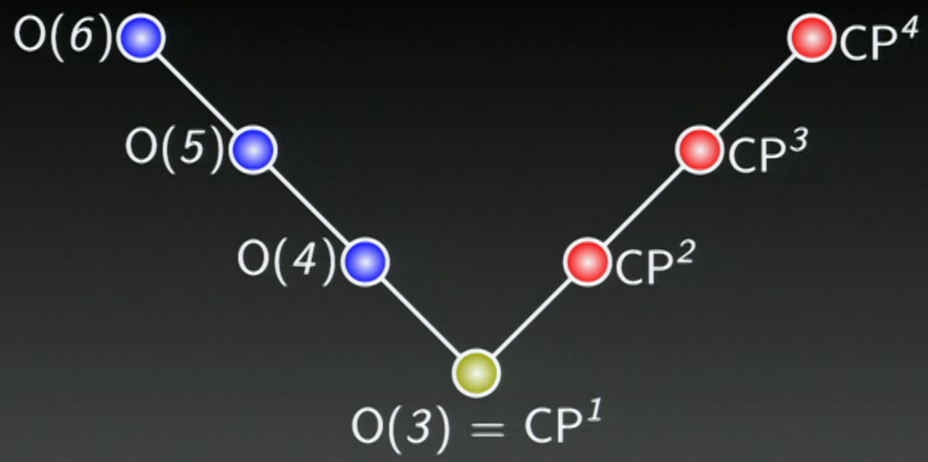
$D_\nu = \partial_\nu + iA_\nu$: covariant derivative with gauge field A_ν

- Gauge field is not dynamical, "no $F_{\mu\nu}F^{\mu\nu}$-term"
- Can be eliminated at the expense of a nonlinearity
- Relation between CP^1 and O(3): $n_a = z_\alpha^\dagger \sigma^a_{\alpha\beta} z_\beta$

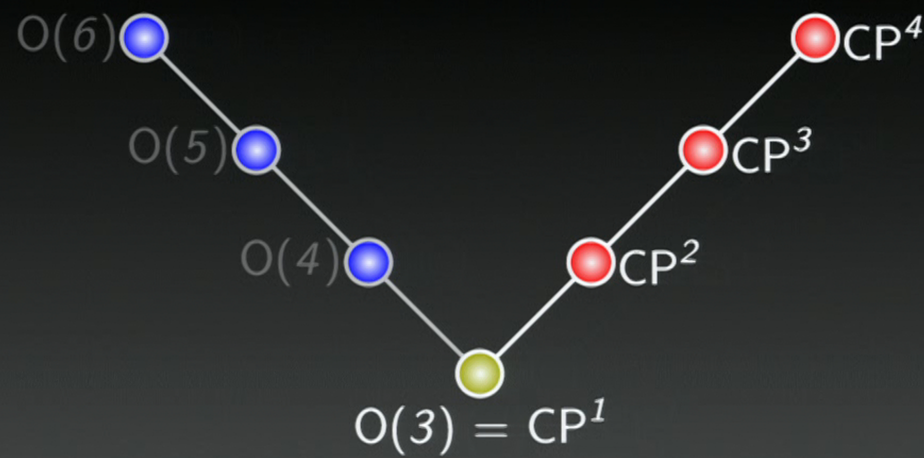
The O(3) rotor model

Relation between rotor and CP^{N-1} models



The O(3) rotor model

Relation between rotor and CP^{N-1} models



- Focus on two (1+1) dimension: $\nu = 0, 1$
  - Asymptotic freedom
  - Dynamically generated mass gap
  - Nontrivial topology and instantons

"The CP^{N-1} system does its best to imitate QCD."

– A. Actor, Fortschr. Phys., 33: 333-374 (1985)

The O(3) rotor model

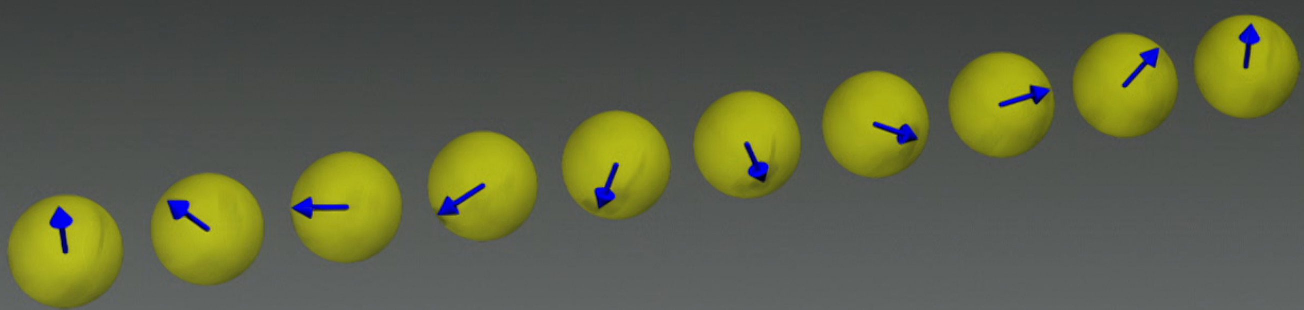
Lattice discretization

- Dimensionless Hamiltonian on a lattice with spacing a

$$aH = \frac{1}{2\beta}\sum_{k=1}^{N}\mathbf{L}_k^2 - \beta\sum_{k=1}^{N-1}\mathbf{n}_k\mathbf{n}_{k+1}$$

$$[L^\alpha, L^\beta] = i\varepsilon^{\alpha\beta\gamma}L^\gamma, \qquad [L^\alpha, n^\beta] = i\varepsilon^{\alpha\beta\gamma}n^\gamma, \qquad [n^\alpha, n^\beta] = 0$$

- Hamiltonian describes chain of coupled quantum rotors



- Continuum limit: $\beta \to \infty$ (thanks to asymptotic freedom)

Spectral properties

F. Bruckmann, K. Jansen, SK, Phys. Rev. D **99**, 074501 (2019)

Spectral properties

Numerical approach

- Suitable basis: angular momentum eigenstates $\otimes|l_k m_k\rangle_{k=1}^N$
- Angular momentum of each rotor is unbounded

$\Rightarrow$ Local Hilbert spaces are infinite dimensional

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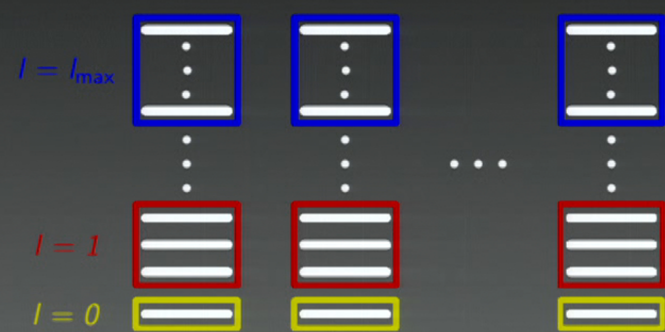
$\Rightarrow$ Local Hilbert spaces are infinite dimensional

- Truncate maximum angular momentum at each site

$$l \leq l_{\max} \qquad \forall k$$



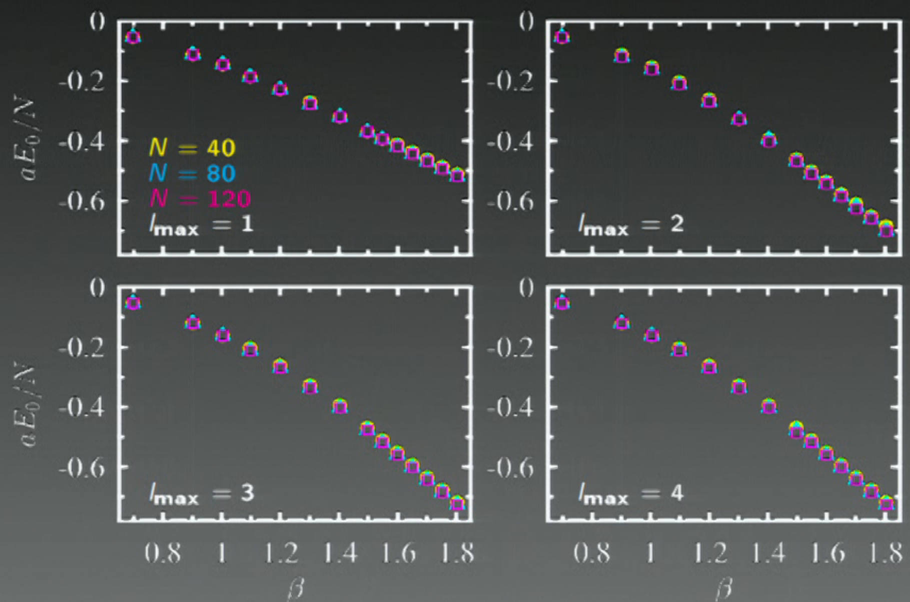
$\Rightarrow$ Local Hilbert spaces of dimension $d = (l_{\max} + 1)^2$

- Use MPS with open boundary conditions to solve the model

Spectral properties

Ground-state energy density

- For each combination of $(\beta, l_{\max}, N)$ bond dimensions $D \in [80, 180]$
- Linear extrapolation in $1/D$



$\Rightarrow$ Hardly any finite-size effects and almost no dependence on $l_{\max}$

Spectral properties

Mass gap

- Asymptotic scaling

$$am = \frac{8}{e}\, a\Lambda_{\overline{\mathrm{MS}}} = 64 a\Lambda_L = 128\pi\beta \exp(-2\pi\beta)$$

Hasenfratz, Maggiore, Niedermayer, Phys. Lett. B, 245 522 (1990)
Shigemitsu, Kogut, Nucl. Phys. B, 190, 365 (1981)

Spectral properties

Mass gap

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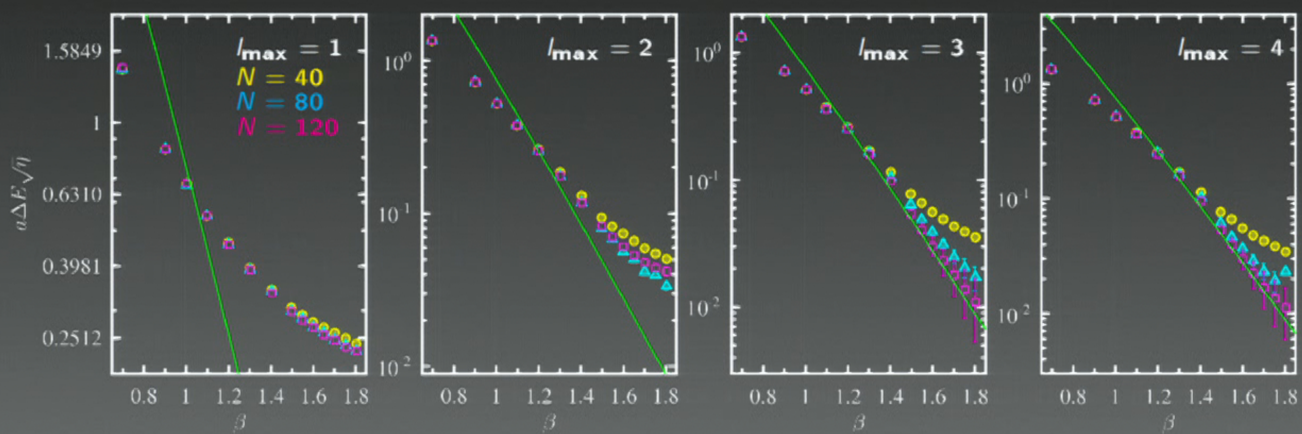
- Numerical data



⇒ Good agreement with the continuum prediction

Hasenfratz, Maggiore, Niedermayer, Phys. Lett. B, 245 522 (1990)
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Numerical results

Entanglement entropy in the ground state

- As we approach $\beta \to \infty$ the mass gap closes
- The correlation length in lattice units $\xi/a = 1/am$ diverges

$$S = \frac{c}{6} \log \frac{\xi}{a} + \bar{k} = \frac{c}{6} (2\pi\beta - \log \beta) + k$$

Calabrese, Cardy, J. Stat. Mech., 2004, P06002 (2004)

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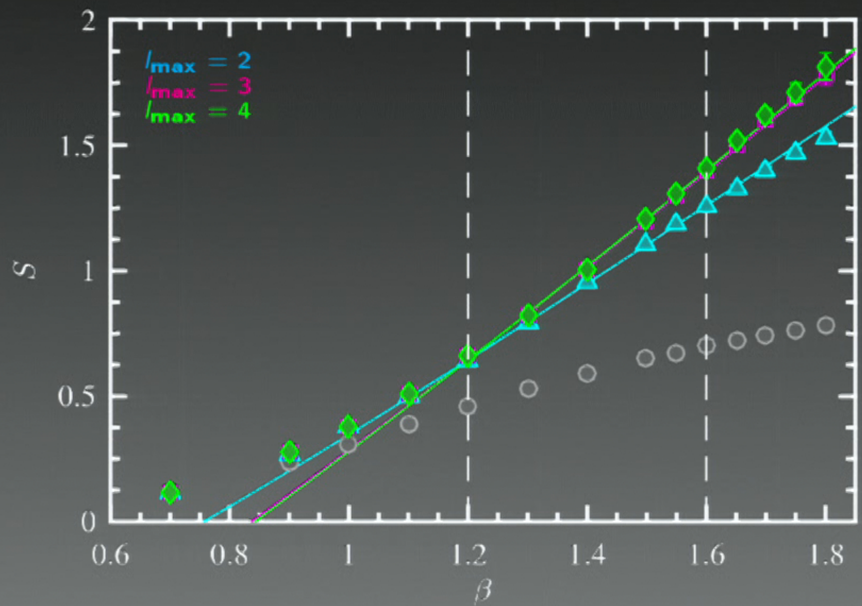

l_{max}	c
2	1.66 ± 0.19
3	2.01 ± 0.12
4	2.04 ± 0.14

Calabrese, Cardy, J. Stat. Mech., 2004, P06002 (2004)

Phase structure at nonvanishing chemical potential

F. Bruckmann, K. Jansen, SK, Phys. Rev. D **99**, 074501 (2019)

Phase structure at nonvanishing chemical potential

Adding a chemical potential

- Conventional Monte Carlo approach: sign problem
- Lattice Hamiltonian with chemical potential

$$aH = \frac{1}{2\beta}\sum_{k=1}^{N}\mathbf{L}_k^2 - \beta\sum_{k=1}^{N-1}\mathbf{n}_k\mathbf{n}_{k+1} - a\mu Q$$

$$Q = \sum_{k=1}^{N} L_k^z, \qquad [H, Q] = 0$$

Bruckmann, Gattringer, Kloiber, Sulejmanpasic, Phys. Rev. D, 94, 114503 (2016)

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$$Q = \sum_{k=1}^{N} L_k^z, \qquad [H, Q] = 0$$

- Hamiltonian is block diagonal, inside a block with charge eigenvalue q

$$aH|_q = -a\mu q \mathbb{1} + aW_{\text{aux}}|_q.$$

- Ground-state energy inside a block with charge q

$$aE_{0,q}(\mu) = -a\mu q + aE_0[aW_{\text{aux}}|_q]$$

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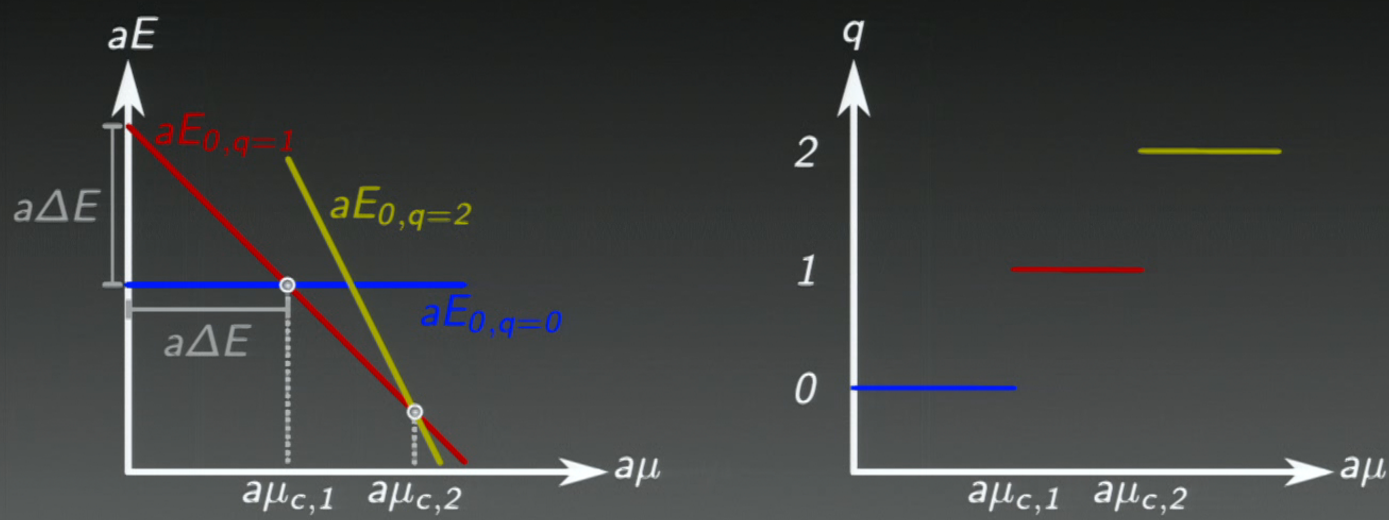
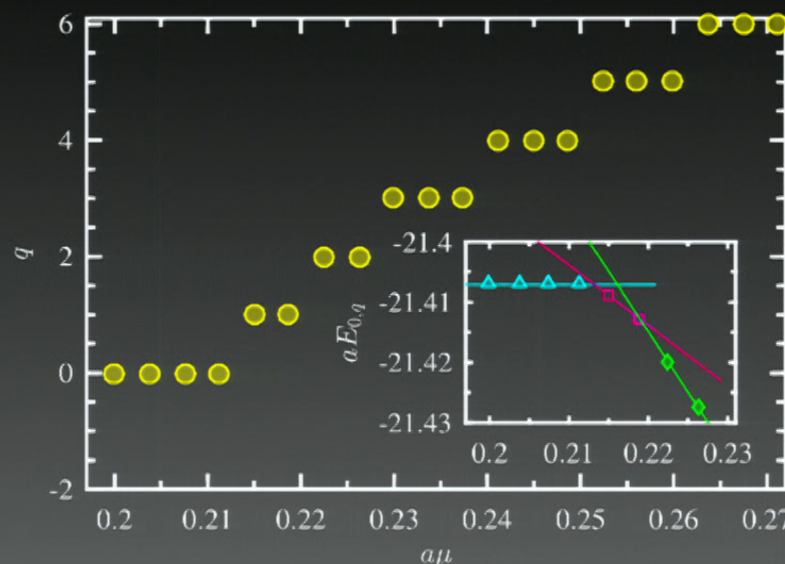


- For $a\mu = 0$:
  - ▶ Ground state is in sector $q = 0$
  - ▶ First excited states: massive particles form a triplet with $q = 0, \pm 1$

Phase structure at nonvanishing chemical potential

Phase structure at nonvanishing chemical potential

- Numerical data for $l_{\max} = 4$, $\beta = 1.2$, $N = 80$ and $D = 200$



$\Rightarrow$ Excellent agreement with the analytical prediction

- Intersection of the energy levels allows for determining the transition points precisely with small resolution in $a\mu$

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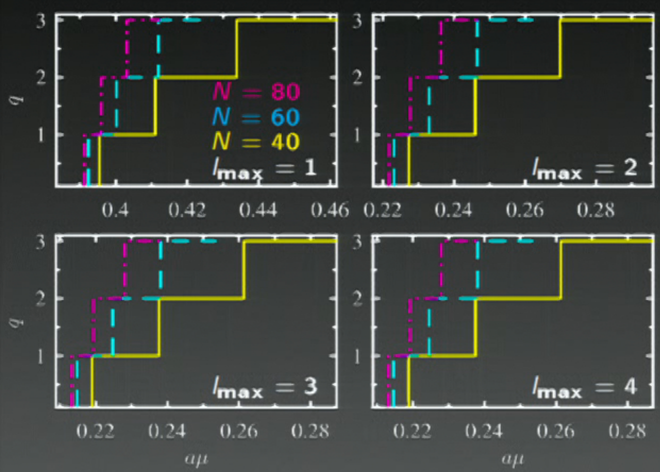

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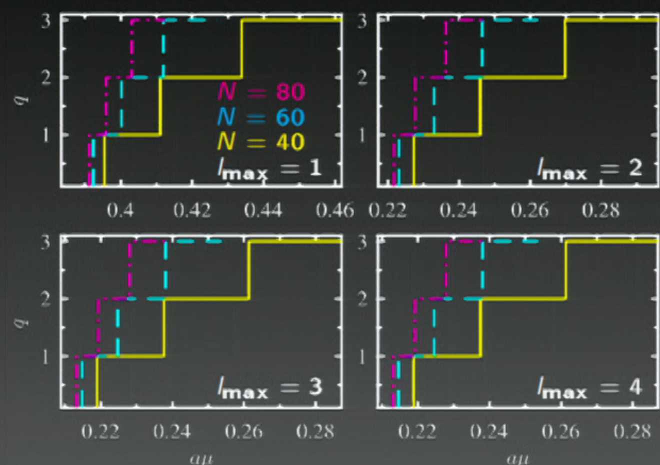

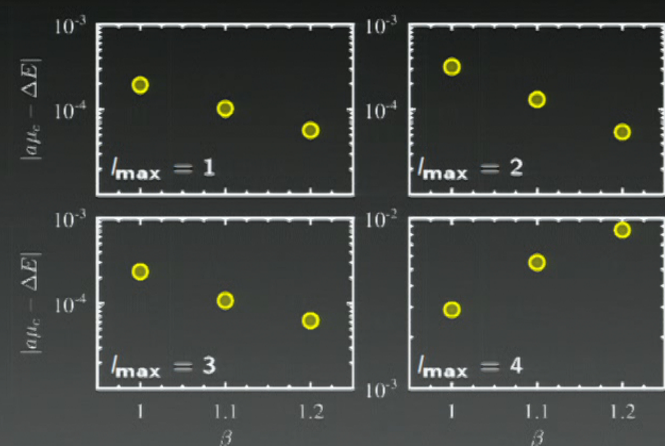

- Noticeable truncation effects only for $l_{\text{max}} = 1$
- $a\mu_c$ at first transition is in excellent agreement with the gap

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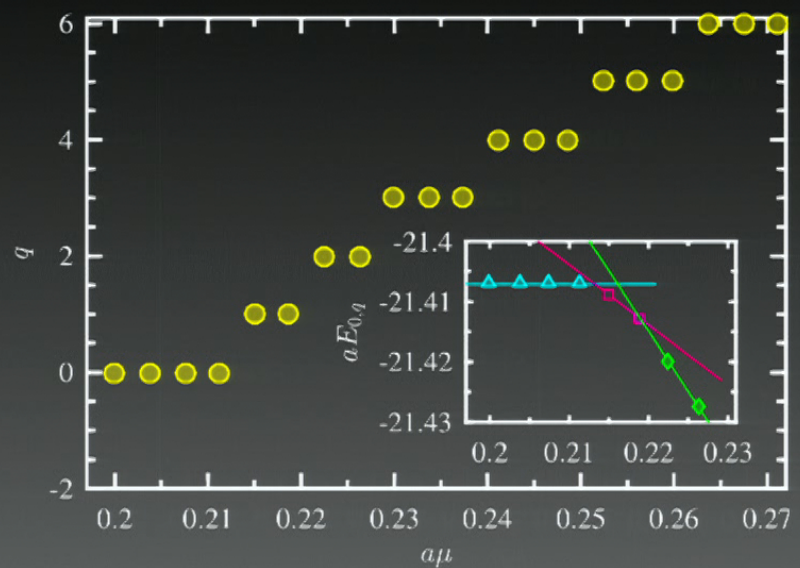

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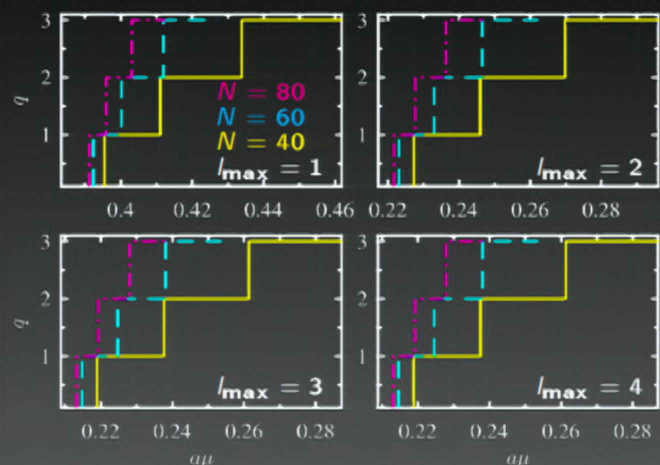

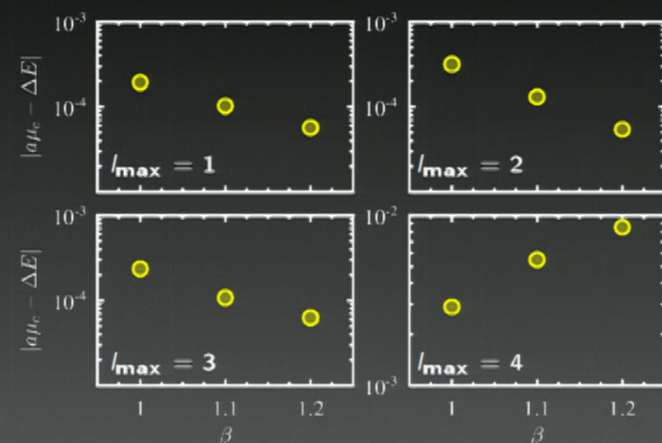

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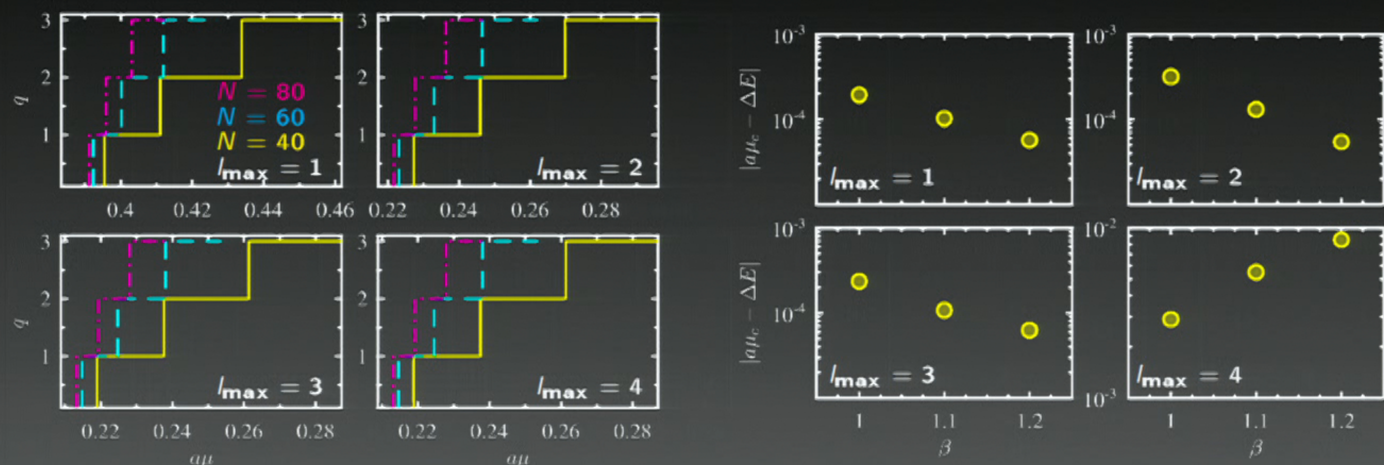

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$\Rightarrow$ Example of overcoming the sign problem

4.

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The Schwinger model/QED in 1+1 dimensions

Continuum formulation

- Eucledian time Lagrangian of the model

$$\mathcal{L} = \overline{\psi}\gamma_\mu D^\mu \psi + m\overline{\psi}\psi + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

kinetic energy + coupling to the gauge field — mass term — dynamics of gauge field

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \; D_\mu = \partial_\mu + igA_\mu, \; F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

J. Schwinger, Phys. Rev. 128 2425 (1962)

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- Simplest nontrivial gauge theory with matter
- Many similarities with QCD
  - Confinement
  - Chiral symmetry breaking

J. Schwinger, Phys. Rev. 128 2425 (1962)

The Schwinger model/QED in 1+1 dimensions

Continuum formulation

- Eucledian time Lagrangian of the model

$$\mathcal{L} = \overline{\psi}\gamma_\mu D^\mu \psi + m\overline{\psi}\psi + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\frac{g}{2}\frac{\theta}{2\pi}\varepsilon_{\mu\nu}F_{\mu\nu}$$

kinetic energy + coupling to the gauge field — mass term — dynamics of gauge field — topological θ-term

$$\psi = \begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix},\ D_\mu = \partial_\mu + igA_\mu,\ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,\ \theta \in [0, 2\pi)$$

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  - Confinement
  - Chiral symmetry breaking
  - Theta vacuua, strong CP problem

$\Rightarrow$ Action is no longer real if $\theta \neq 0$, sign problem

J. Schwinger, Phys. Rev. 128 2425 (1962)

The Schwinger model

Continuum predictions for the behavior

- For small masses m/g the θ-dependence can be computed with mass perturbation theory
  - Energy density in units of the coupling

$$\frac{\varepsilon(m,\theta)}{g^2} = c_1 \frac{m}{g} \cos(\theta) + \frac{\pi c_1^2}{4} \left(\frac{m}{g}\right)^2 (c_2 \cos(2\theta) + c_3)$$

C. Adam, Ann. Phys. 259, 1 (1997)

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  - Electric field in units of the coupling

$$\frac{F(m,\theta)}{g} = 2\pi \times \frac{\partial}{\partial\theta}\frac{\varepsilon(\theta,m)}{g^2}$$

  - Topological susceptibility

$$\frac{\chi_{\text{top}}(m,\theta)}{g} = -\frac{\partial^2}{\partial\theta^2}\frac{\varepsilon(\theta,m)}{g^2} = -\frac{1}{2\pi}\frac{\partial}{\partial\theta}\frac{F(\theta,m)}{g}$$

  - Chiral condensate

$$\frac{\Sigma(m,\theta)}{g} = \frac{\langle\bar{\psi}(x)\psi(x)\rangle}{g} = g\frac{\partial}{\partial m}\frac{\varepsilon(m,\theta)}{g^2}$$

- For $m/g = 0$ physics is independent of θ C. Adam, Ann. Phys. 259, 1 (1997)

The Schwinger model

Lattice Hamiltonian formulation

- Kogut-Susskind staggered fermions in temporal gauge $A^0 = 0$

$$H = -\frac{i}{2a}\sum_{n=1}^{N-1}\left(\phi_n^\dagger \, e^{i\alpha_n} \, \phi_{n+1} - \text{h.c.}\right) + \sum_{n=1}^{N} (-1)^n m \, \phi_n^\dagger \phi_n + \frac{ag^2}{2}\sum_{n=1}^{N-1} \left(L_n + \frac{\theta}{2\pi}\right)^2$$

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ϕ_n: single-component fermionic field $\qquad [L_n, \alpha_m] = i\delta_{nm}$

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$$L_n - L_{n-1} = Q_n = \phi_n^\dagger\phi_n - \tfrac{1}{2}\left(1 - (-1)^n\right)$$

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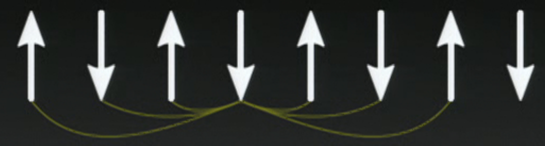
- Use OBC and Gauss Law to integrate out the gauge field

The Schwinger model

Numerical simulation with Matrix Product States

- Map fermions to spins with a Jordan-Wigner transformation



- Use MPS with open boundary conditions to solve the model

M. C. Bañuls, K. Cichy, J. I. Cirac, K. Jansen, JHEP 2013, 158 (2013)
M. C. Bañuls, K. Cichy, K. Jansen, J. I. Cirac, SK, Phy. Rev. Lett. 118, 071601 (2017)

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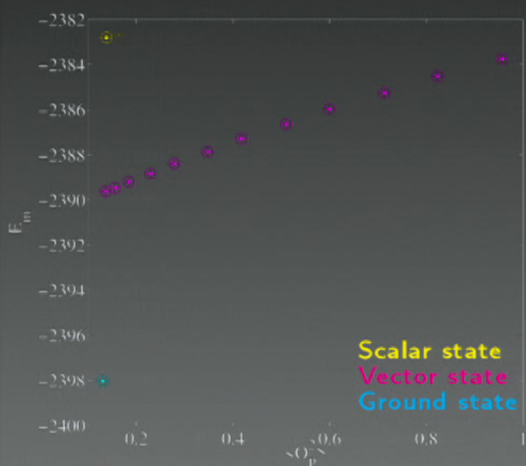

Vector mass

m/g	DMRG	MPS
0	0.5641859	0.56414(26)
0.125	0.53950(7)	0.53946(20)
0.25	0.51918(5)	0.51915(14)
0.5	0.48747(2)	0.48748(6)

Scalar mass

m/g	SCE	MPS
0	1.128379	1.1283(10)
0.125	1.22(2)	1.221(2)
0.25	1.24(3)	1.239(6)
0.5	1.20(3)	1.231(5)

M. C. Bañuls, K. Cichy, J. I. Cirac, K. Jansen, JHEP 2013, 158 (2013)
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The Schwinger model

Numerical simulation with Matrix Product States

- Measure the lattice equivalents of ε/g^2, F/g and Σ/g

The Schwinger model

Numerical simulation with Matrix Product States

- Measure the lattice equivalents of ε/g^2, F/g and Σ/g
- Extrapolate to the continuum similar to lattice calculations

Bond dimension: $D \in [20, 140]$

→ Finite size: $N \in [40, 570]$

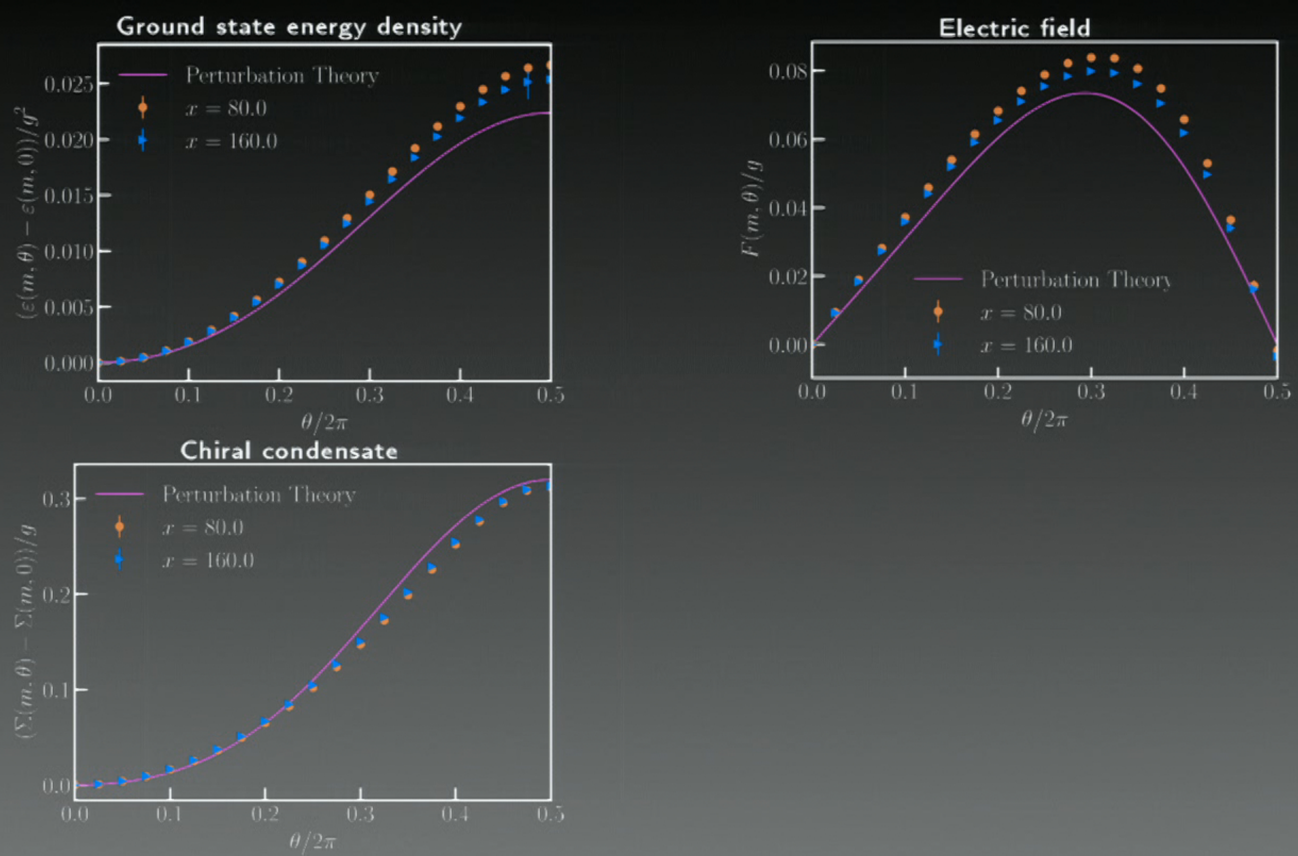
→ Continuum: $\frac{1}{(ag)^2} \in [80, 160]$

Preliminary Results

L. Funcke, K. Jansen, SK, in preparation

The Schwinger model

Results for finite lattice spacing $m/g = 0.07$



The Schwinger model

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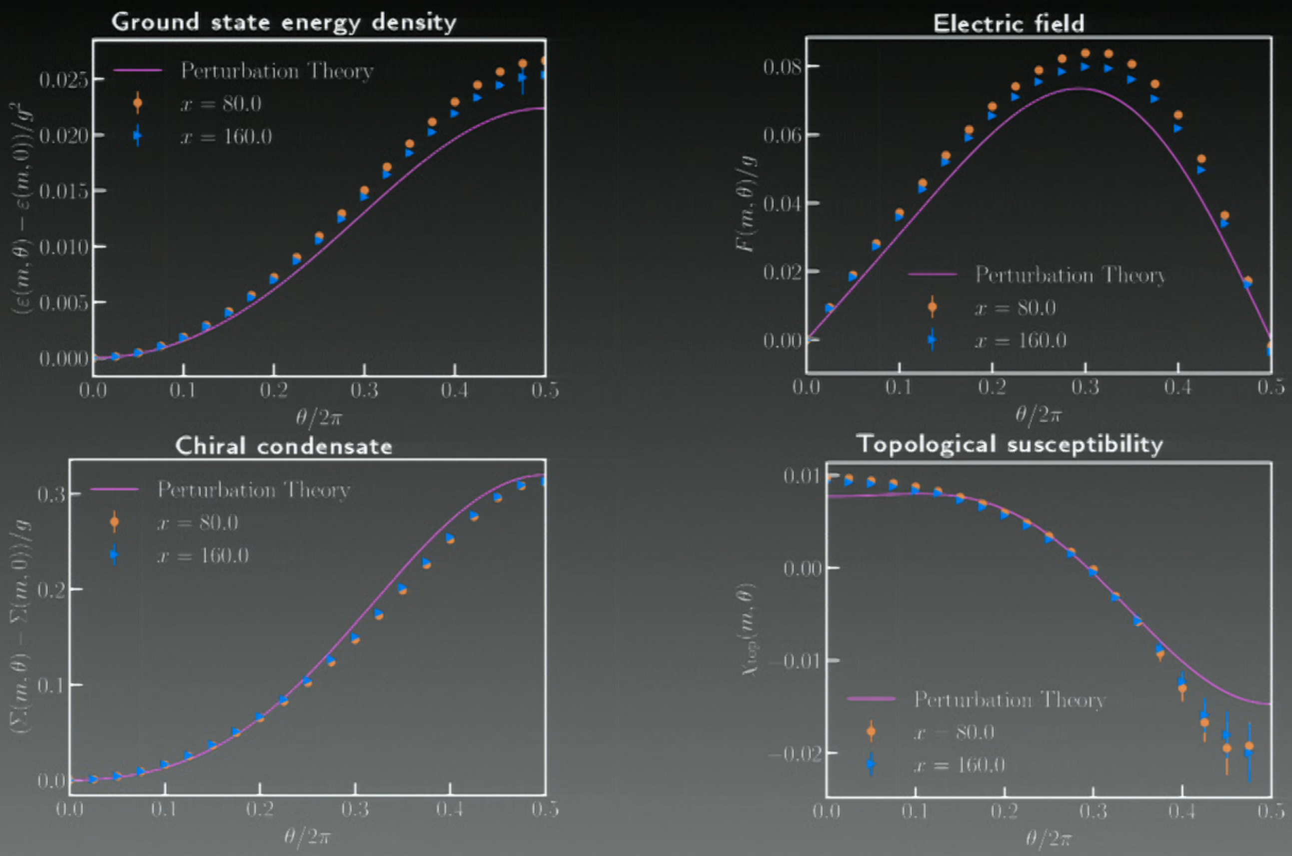
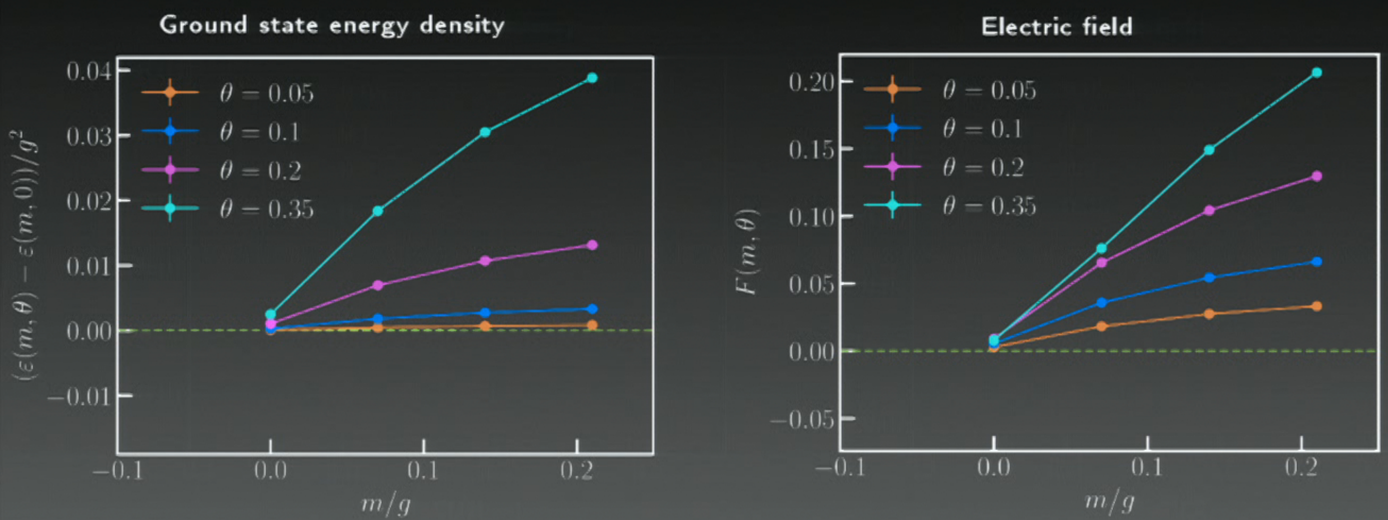


The Schwinger model

Results for finite lattice spacing $x = 1/(ag)^2 = 160$



$\Rightarrow$ For $m/g = 0$ physics is independent of θ

The Schwinger model

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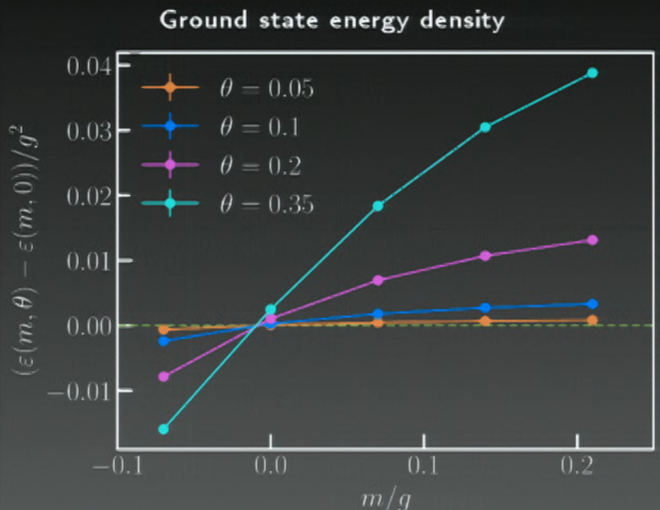

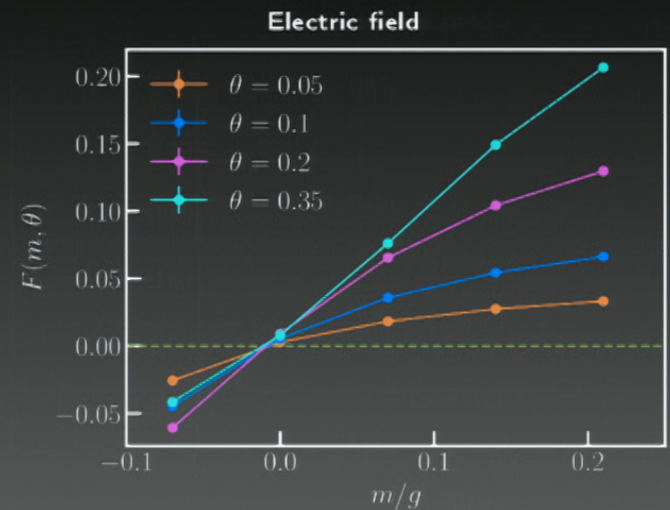

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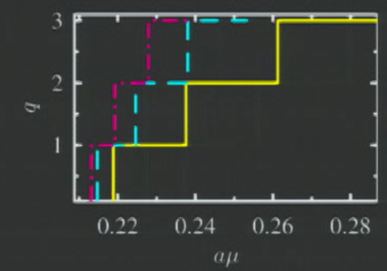
Summary

Phase structure of the O(3) nonlinear sigma model

- Good numerical precision
- We are able to enter the asymptotic scaling regime
- Reliable calculations for the phase structure



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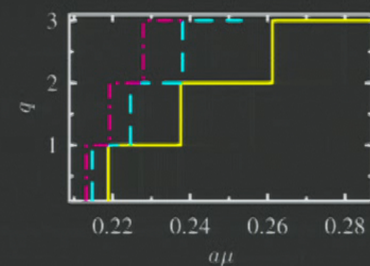
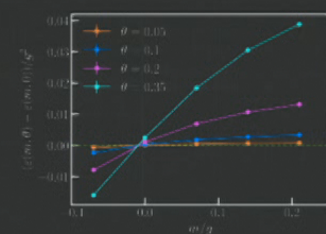


Schwinger model in the presence of a topological θ-term

- Calculations in the presence of a θ-term
- Good precision for finite lattice data
- Still have to take the continuum limit



$\Rightarrow$ Numerical methods based on TNS are promising candidates for exploring Lattice Field Theories

Outlook

Future perspectives

- Many interesting questions in (1+1) dimensions
  - Study models with topological θ-terms
  - Finite temperature
  - Real-time dynamics
  - ...
- Numerical simulations with TNS in (2+1) dimensions
- Quantum Computing

E. A. Martinez et al., Nature 534, 516 (2016)
N. Klco et al., Phys. Rev. A 98, 032331 (2018)
T. Hartung, K. Jansen, arXiv:1808.06784

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Thank you for your attention!