Title: Bridging partons and coupled-wire approaches to strongly entangled quantum matter

Speakers: David Mross

Collection: Quantum Matter: Emergence & Entanglement 3

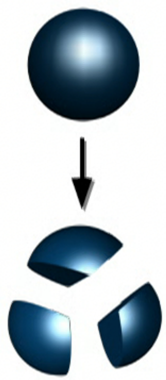
Date: April 24, 2019 - 11:30 AM

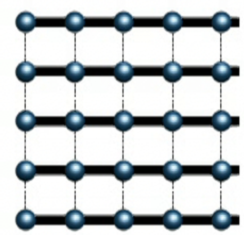
URL: http://pirsa.org/19040107

Abstract: The Hallmark of strongly entangled quantum phases is an intrinsic impossibility to describe them locally in terms of microscopic degrees of freedom. Two popular methods that have been developed to analytically describe these exotic states are known as (1) â€˜parton constructionâ€™ and (2) â€˜coupled-wire approachâ€™. The former provides a constructive route for determining which non-trivial phases may arise, in principle, for a given set of constituent degrees of freedom and symmetries. This capability comes at the expense of having very little predictive power what phases do arise, in practice, in any particular system. The latter technique, by contrast, yields explicit expressions of ground states, excitations as well as parent Hamiltonians in terms of microscopic degrees of freedom. The price to pay is a lack of flexibility, and each phase needs to be analyzed on a laborious case-by-case basis. I will show how recent understanding of two-dimensional dualities provides a natural link between the two approaches. Specifically, I will show how a wide range of parton mean-field states can be easily translated into explicit coupled-wire models, and how their universal properties can be obtained in a transparent manner.

Bridging partons and coupled-wire approaches to strongly entangled quantum matter

David F. Mross
Weizmann Institute of Science

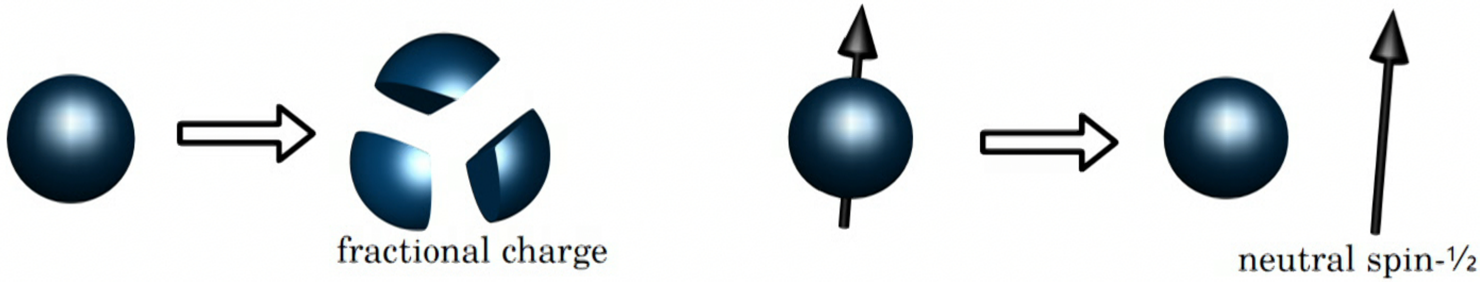




ISRAEL SCIENCE FOUNDATION
BSF United States – Israel Binational Science Foundation
Minerva Stiftung

‘Strongly entangled’ quantum matter

Phases with excitations whose quantum numbers appear to be at odds with microscopic constituents

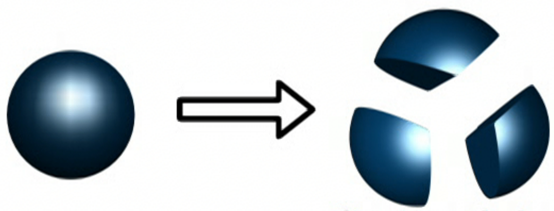


Cannot form by locally combining microscopic degrees of freedom

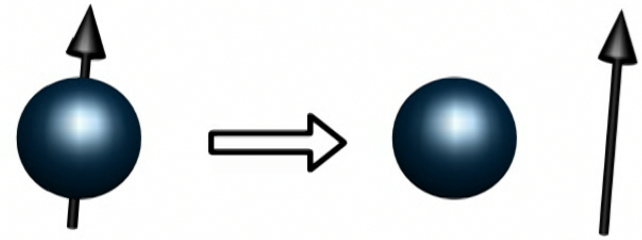

Fractional charge
Cannot form by locally combining

‘Strongly entangled’ quantum matter

Phases with excitations whose quantum numbers
appear to be at odds with microscopic constituents

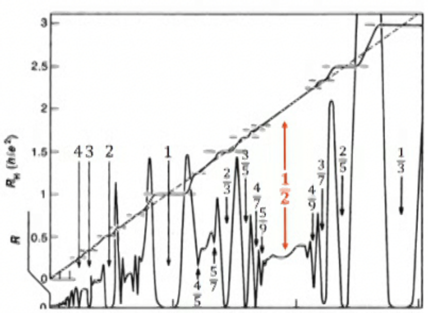


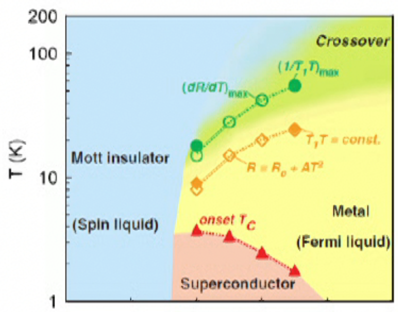
fractional charge

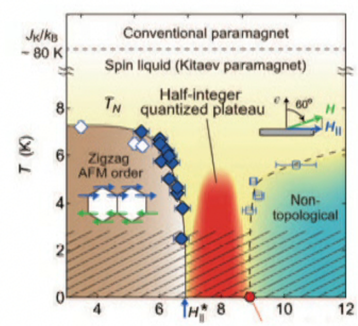


neutral spin-½

Cannot form by locally combining microscopic degrees of freedom







Strong experimental evidence of their existence

Analytical tools 1: Partons

Step 1: Dream up a state
(prototypical wave function)

$$\Psi_{\frac{1}{2m+1}} = \prod_{i<j}(z_i - z_j)^{2m+1} e^{-\sum_i \frac{|z_i|^2}{4\ell^2}} \qquad \Psi_{\mathrm{PFS}} = P\left[\det|e^{i\vec{k}_a\vec{r}_{b,\uparrow}}|\det|e^{i\vec{k}_a\vec{r}_{b,\downarrow}}|\right]$$

Step 2: Reinterpret as product of
weakly entangled wave functions

$$\Psi_{\frac{1}{2m+1}} \propto (\Psi_1)^{2m+1} \qquad \Psi_{\mathrm{PFS}} = \Psi_{\mathrm{FS}}(\vec{r}_{a,\uparrow})\Psi_{\mathrm{FS}}(\vec{r}_{b,\downarrow})\Psi_{\mathrm{Mott}}^{\Delta=\infty}(\vec{r})$$

Analytical tools 1: Partons

Step 1: Dream up a state (prototypical wave function)

$$\Psi_{\frac{1}{2m+1}} = \prod_{i<j}(z_i - z_j)^{2m+1} e^{-\sum_i \frac{|z_i|^2}{4\ell^2}}$$

$$\Psi_{\mathrm{PFS}} = P\left[\det|e^{i\vec{k}_a\vec{r}_{b,\uparrow}}|\det|e^{i\vec{k}_a\vec{r}_{b,\downarrow}}|\right]$$

Step 2: Reinterpret as product of weakly entangled wave functions

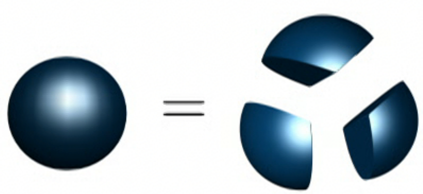
$$\Psi_{\frac{1}{2m+1}} \propto (\Psi_1)^{2m+1}$$

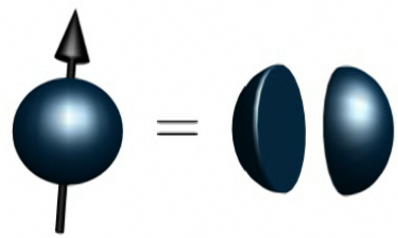
$$\Psi_{\mathrm{PFS}} = \Psi_{\mathrm{FS}}(\vec{r}_{a,\uparrow})\Psi_{\mathrm{FS}}(\vec{r}_{b,\downarrow})\Psi_{\mathrm{Mott}}^{\Delta=\infty}(\vec{r})$$

Step 3: Derive effective field theory

$$c^\dagger = \prod_{i=1}^{2m+1} f_i^\dagger$$

$$S^+ = f_\uparrow^\dagger f_\downarrow$$





Analytical tools 1: Partons

Gaussian part of Hamiltonian strictly zero

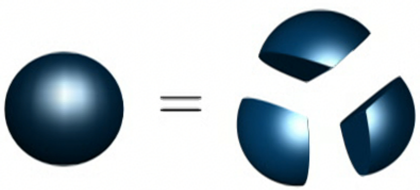
Ψ

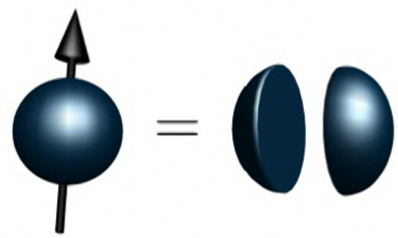
$\cdot)$

Step 3: Derive effective field theory

$$c^\dagger = \prod_{i=1}^{2m+1} f_i^\dagger$$

$$S^+ = f_\uparrow^\dagger f_\downarrow$$

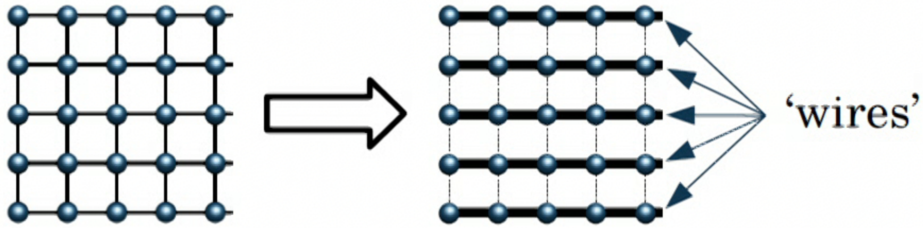




Analytical tools 2: Coupled wires

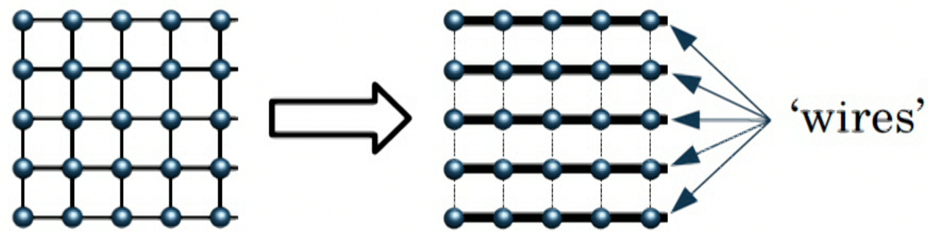
Analytical tools 2: Coupled wires

Step 1: Take Hamiltonian in anisotropic limit



Analytical tools 2: Coupled wires

Step 1: Take Hamiltonian in anisotropic limit

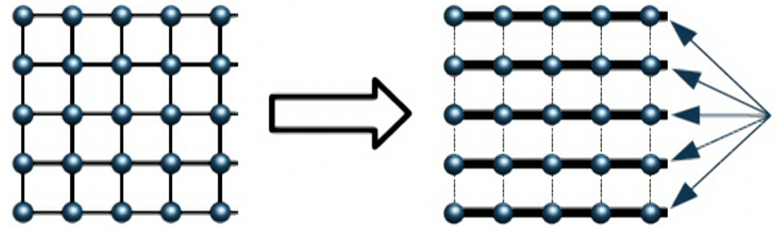


Step 2: Take long-wavelength limit of 1d theories

$$H = J\sum_i \vec{S}_i \cdot \vec{S}_{i+1} \quad \Rightarrow \quad H = \int dx[(\partial_x\theta)^2 + (\partial_x\varphi)^2]$$

Analytical tools 2: Coupled wires

Step 1: Take Hamiltonian in anisotropic limit



Step 2: Take long-wavelength limit of 1d theories

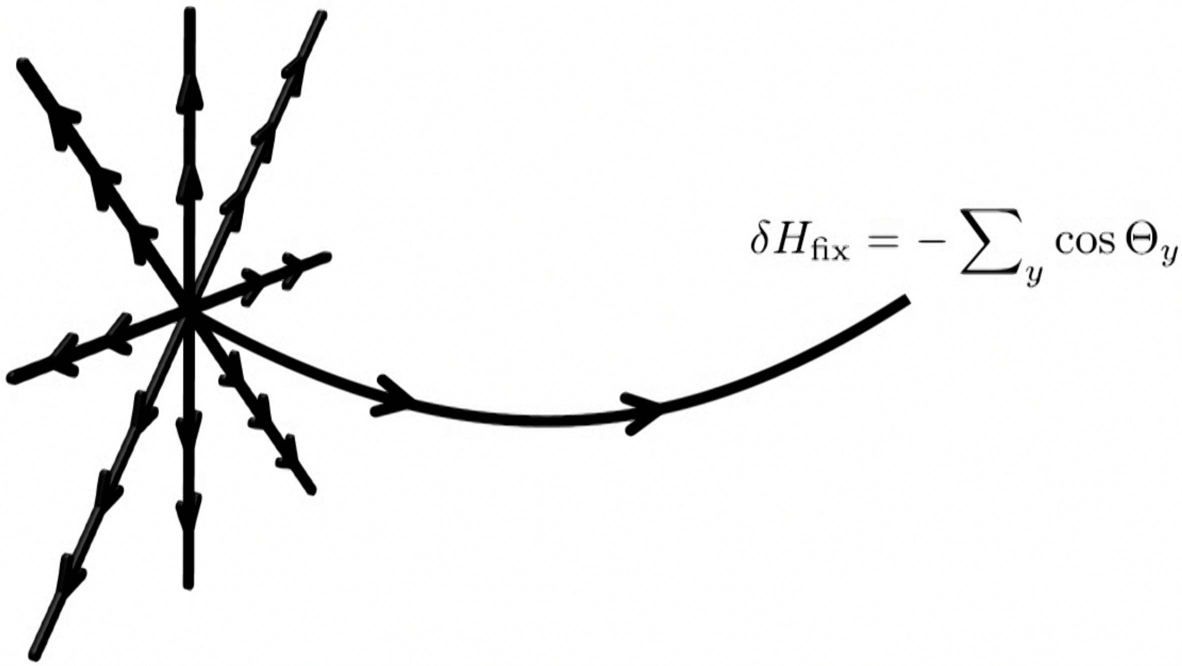
$$H = J\sum_i \vec{S}_i \cdot \vec{S}_{i+1} \quad \Rightarrow \quad H = \int dx[(\partial_x\theta)^2 + (\partial_x\varphi)^2]$$

Step 3: Couple wires with solvable 'fixed point Hamiltonian'

Find conjugate variables Θ_y, Φ_y such that:

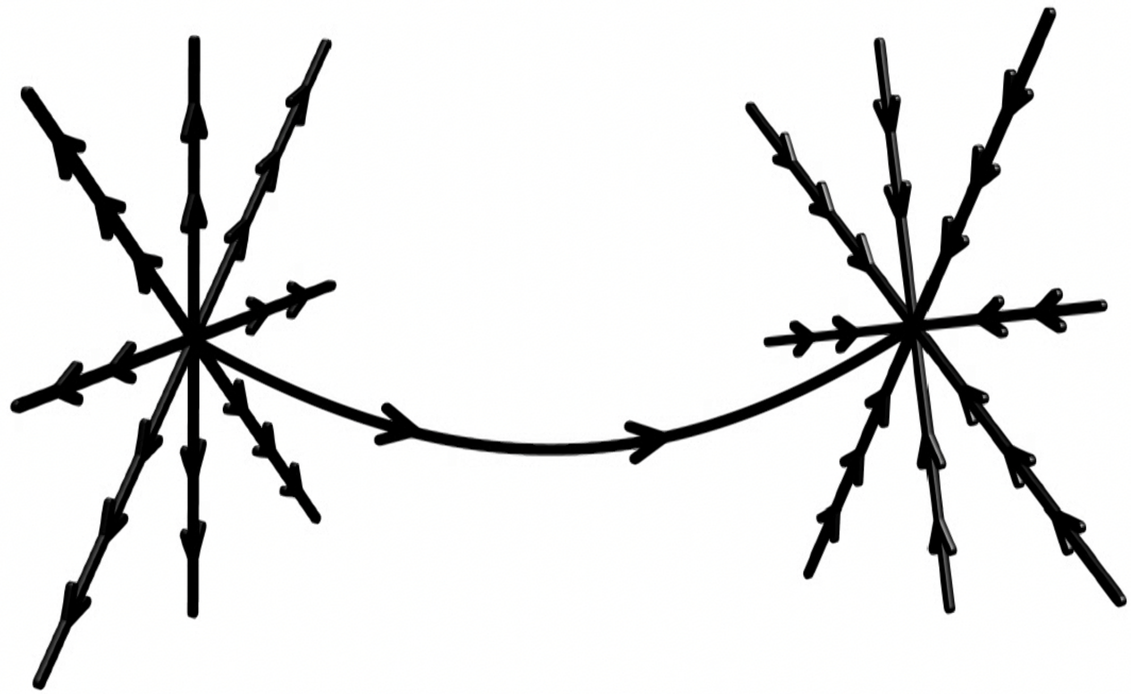
1. $\delta H = -\sum_y \cos\Theta_y$
2. $\cos\Theta_y$ are local operators
3. $[\Theta_y(x), \Theta_{y'}(x')] = 0$
4. $\mathcal{U}_{\text{symm}}|\{\Theta_y\}\rangle = |\{\Theta_y\}\rangle$

Analytical tools 2: Coupled wires



Many relevant directions at
decoupled-wired fixed point

Analytical tools 2: Coupled wires



Many relevant directions at decoupled-wired fixed point

Stable phase described by
$\delta H_{\mathrm{fix}} = -\sum_y \cos\Theta_y$

Partons vs. coupled wires

- \+ Determine 'what is possible'
- \+ Any symmetries and degrees of freedom
- − No analytic control over energetics

- − ~~Cumbersome piece-by-piece approach~~
- − Spatial anisotropy unavoidable
- \+ Generate fixed-point Hamiltonians
- \+ Analytic control (where applicable)

Partons vs. coupled wires

+ Determine 'what is possible'
+ Any symmetries and degrees of freedom

− No analytic control over energetics

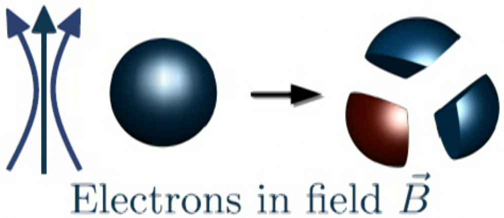
− Cumbersome piece-by-piece approach
− Spatial anisotropy unavoidable

+ Generate fixed-point Hamiltonians
+ Analytic control (where applicable)

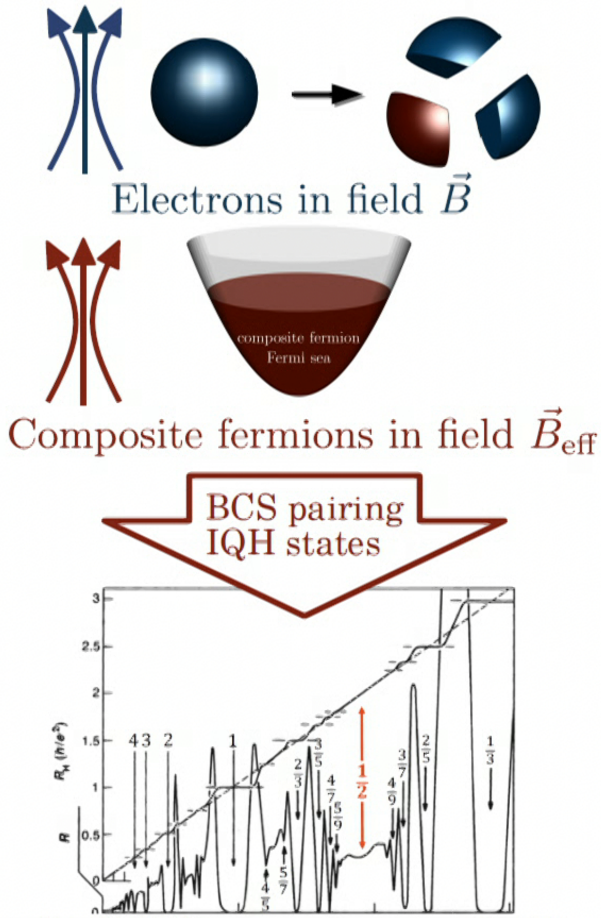
Example 1:
Quantum Hall Effect

DFM, Alicea, Motrunich, PRL 117, 016802 (2016); PRX 7, 041016 (2017)

Composite fermions

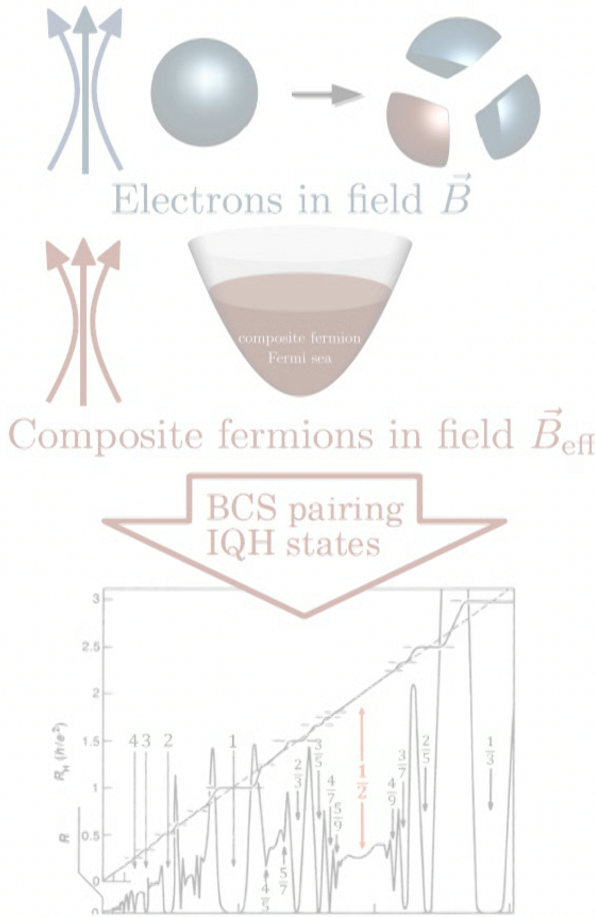


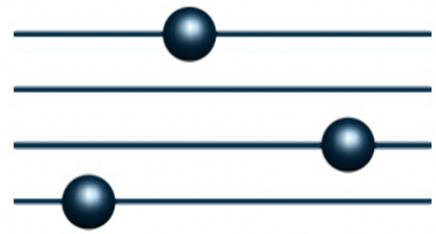
Composite fermions



Coupled wires

my convention:
$\psi \sim \exp[i\varphi \pm i\theta]$

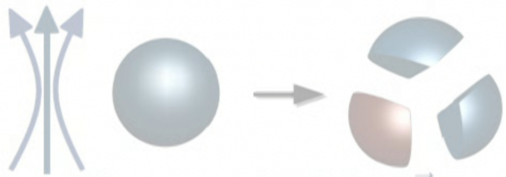




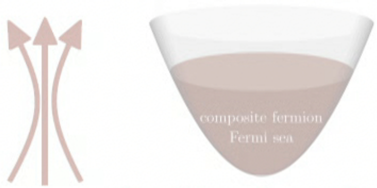
$$H_{\text{wire}} = (\partial_x\varphi - A_x)^2 + (\partial_x\theta)^2 - iA_\tau \frac{\partial_x\theta}{\pi}$$

Coupled wires

my convention:
$\psi \sim \exp[i\varphi \pm i\theta]$

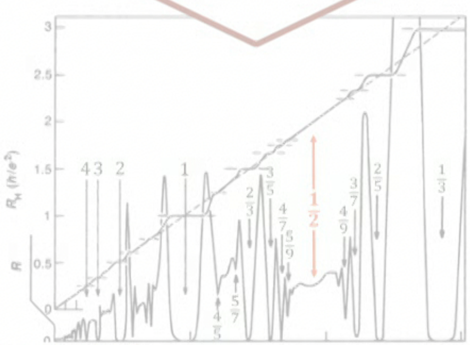


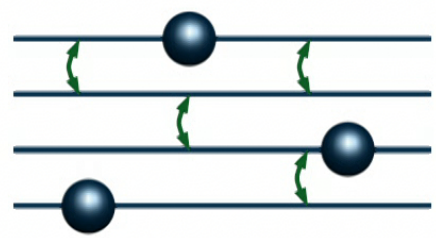
Electrons in field $\vec{B}$



Composite fermions in field $\vec{B}_{\text{eff}}$

BCS pairing
IQH states





$$H_{\text{wire}} = (\partial_x\varphi - A_x)^2 + (\partial_x\theta)^2 - iA_\tau \frac{\partial_x\theta}{\pi}$$

Couple wires with δH

Laughlin states:

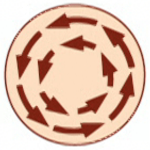
$$\delta H = \psi^\dagger_{y+1,L}\psi_{y,R}e^{2in(\theta_y+\theta_{y+1})} + \text{H.c.}$$

$\delta H_{\text{Abelian FQH}}$
Kane, Mukhopadhyay, Lubensky (2002)

$\delta H_{\text{non-Abelian FQH}}$
Teo, Kane (2014)

Composite fermions on wires

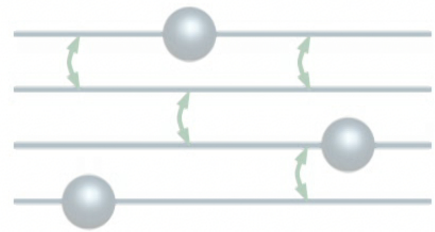
my convention:
$\psi \sim \exp[i\varphi \pm i\theta]$



$$V_{\vec{R}} = \exp\left[i \sum_{\vec{r}} \alpha(\vec{r} - \vec{R}) \hat{n}_{\vec{r}}\right]$$

$$\oint d\vec{s} \cdot \vec{\nabla}\alpha = 2\pi$$

Fradkin (1989)



$$H_{\text{wire}} = (\partial_x \varphi - A_x)^2 + (\partial_x \theta)^2 - iA_\tau \frac{\partial_x \theta}{\pi}$$

Couple wires with δH

Laughlin states:

$$\delta H = \psi^\dagger_{y+1,L} \psi_{y,R} e^{2in(\theta_y + \theta_{y+1})} + \text{H.c.}$$

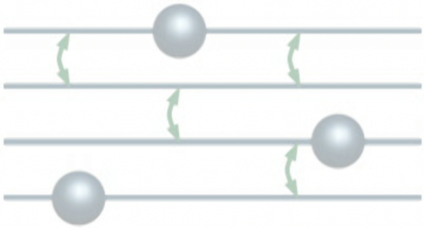
$\delta H_{\text{Abelian FQH}}$
Kane, Mukhopadhyay, Lubensky (2002)

$\delta H_{\text{non-Abelian FQH}}$
Teo, Kane (2014)

Composite fermions on wires

my convention:
$\psi \sim \exp[i\varphi \pm i\theta]$

$V_{\vec{R}} = \exp\left[i \sum_y \operatorname{sgn}(y - Y)\theta\right]$



$$H_{\text{wire}} = (\partial_x \varphi - A_x)^2 + (\partial_x \theta)^2 - i A_\tau \frac{\partial_x \theta}{\pi}$$

Couple wires with δH

Laughlin states:

$\delta H = \psi^\dagger_{y+1,L} \psi_{y,R} e^{2in(\theta_y + \theta_{y+1})} + \text{H.c.}$

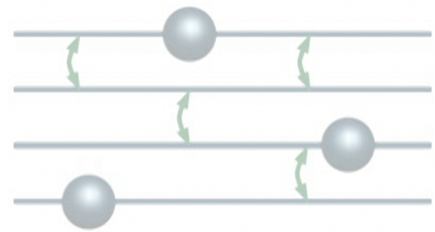
$\delta H_{\text{Abelian FQH}}$
Kane, Mukhopadhyay, Lubensky (2002)

$\delta H_{\text{non-Abelian FQH}}$
Teo, Kane (2014)

Composite fermions on wires

my convention:
$\psi \sim \exp[i\varphi \pm i\theta]$

$$V_{\vec{R}} = \exp\left[i \sum_y \mathrm{sgn}(y - Y)\theta\right]$$



$$H_{\mathrm{wire}} = (\partial_x\varphi - A_x)^2 + (\partial_x\theta)^2 - iA_\tau \frac{\partial_x\theta}{\pi}$$

Couple wires with δH

Laughlin states:

$$\delta H = \psi^\dagger_{y+1,L}\psi_{y,R}e^{2in(\theta_y+\theta_{y+1})} + \mathrm{H.c.}$$

$\delta H_{\mathrm{Abelian\ FQH}}$

Kane, Mukhopadhyay, Lubensky (2002)

$\delta H_{\mathrm{non\text{-}Abelian\ FQH}}$

Teo, Kane (2014)

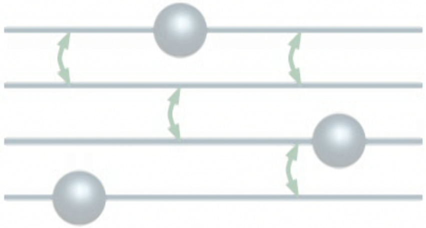
Composite fermions on wires

my convention:
$\psi \sim \exp[i\varphi \pm i\theta]$

$$V_{\vec{R}} = \exp\left[i \sum_y \mathrm{sgn}(y - Y)\theta\right]$$

Vortex hopping is a local process

$$V^\dagger_{y-1/2} V_{y+1/2} = e^{2i\theta_y}$$



$$H_{\rm wire} = (\partial_x \varphi - A_x)^2 + (\partial_x \theta)^2 - i A_\tau \frac{\partial_x \theta}{\pi}$$

Couple wires with δH

Laughlin states:

$$\delta H = \psi^\dagger_{y+1,L} \psi_{y,R} e^{2in(\theta_y + \theta_{y+1})} + \text{H.c.}$$

$\delta H_{\text{Abelian FQH}}$

Kane, Mukhopadhyay, Lubensky (2002)

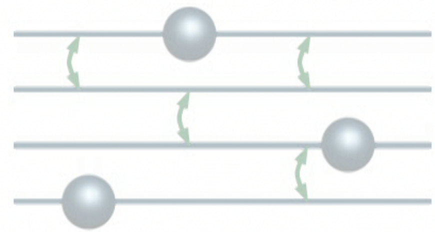
$\delta H_{\text{non-Abelian FQH}}$

Teo, Kane (2014)

Composite fermions on wires

my convention:
$\psi \sim \exp[i\varphi \pm i\theta]$

$V_{\vec{R}} = \exp\left[i \sum_y \mathrm{sgn}(y - Y)\theta\right]$



$H_{\mathrm{wire}} = (\partial_x\varphi - A_x)^2 + (\partial_x\theta)^2 - iA_\tau \frac{\partial_x\theta}{\pi}$

$\delta H = (\psi^\dagger_{y+1,L} V^2_{y+1/2})(\psi_{y,R} V^{2\dagger}_{y-1/2}) + \mathrm{H.c.}$

Couple wires with δH

Laughlin states:

$\delta H = \psi^\dagger_{y+1,L}\psi_{y,R} e^{2in(\theta_y + \theta_{y+1})} + \mathrm{H.c.}$

$\delta H_{\text{Abelian FQH}}$

Kane, Mukhopadhyay, Lubensky (2002)

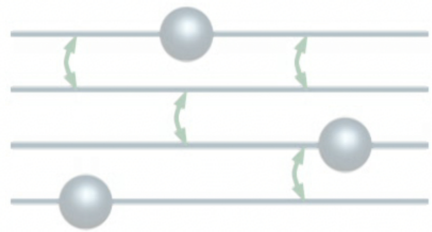
$\delta H_{\text{non-Abelian FQH}}$

Teo, Kane (2014)

Composite fermions on wires

my convention:
$\psi \sim \exp[i\varphi \pm i\theta]$

$$V_{\vec{R}} = \exp\left[i \sum_y \mathrm{sgn}(y - Y)\theta\right]$$



$$H_{\text{wire}} = (\partial_x \varphi - A_x)^2 + (\partial_x \theta)^2 - iA_\tau \frac{\partial_x \theta}{\pi}$$

$$\delta H = (\psi^\dagger_{y+1,L} V^2_{y+1/2})(\psi_{y,R} V^{2\dagger}_{y-1/2}) + \text{H.c.}$$

Composite fermions

$$\psi_{\text{CF}}(\vec{r}) = \psi_{\text{el}}(\vec{r}) V^{2\dagger}_{\vec{r}}$$

Couple wires with δH

Laughlin states:

$$\delta H = \psi^\dagger_{y+1,L} \psi_{y,R} e^{2in(\theta_y + \theta_{y+1})} + \text{H.c.}$$

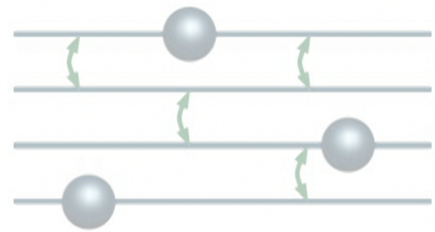
$\delta H_{\text{Abelian FQH}}$
Kane, Mukhopadhyay, Lubensky (2002)

$\delta H_{\text{non-Abelian FQH}}$
Teo, Kane (2014)

Fixed point Hamiltonians

my convention:
$\psi \sim \exp[i\varphi \pm i\theta]$

$V_{\vec{R}} = \exp\left[i \sum_y \mathrm{sgn}(y - Y)\theta\right]$



$$H_{\text{wire}} = (\partial_x \varphi - A_x)^2 + (\partial_x \theta)^2 - i A_\tau \frac{\partial_x \theta}{\pi}$$

Couple wires with δH

Laughlin states:

$$\delta H = \psi^\dagger_{y+1,L} \psi_{y,R} e^{2in(\theta_y + \theta_{y+1})} + \text{H.c.}$$

$\delta H_{\text{Abelian FQH}}$
Kane, Mukhopadhyay, Lubensky (2002)

$\delta H_{\text{non-Abelian FQH}}$
Teo, Kane (2014)

Composite fermions

$$\psi_{\text{CF}}(\vec{r}) = \psi_{\text{el}}(\vec{r}) V^{2\dagger}_{\vec{r}}$$

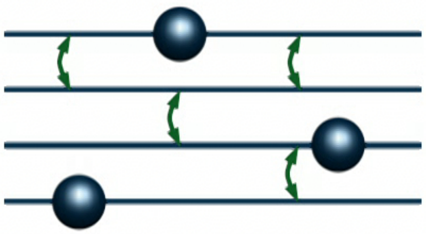
$\delta H_{\text{MF}}[\ \to\ ]$

David F. Mross — Bridging partons and coupled wires — PI 04/2019

Fixed point Hamiltonians

my convention:
$\psi \sim \exp[i\varphi \pm i\theta]$

$V_{\vec{R}} = \exp\left[i \sum_y \mathrm{sgn}(y - Y)\theta\right]$



$$H_{\mathrm{wire}} = (\partial_x \varphi - A_x)^2 + (\partial_x \theta)^2 - i A_\tau \frac{\partial_x \theta}{\pi}$$

Composite fermions

$\psi_{\mathrm{CF}}(\vec{r}) = \psi_{\mathrm{el}}(\vec{r}) V_{\vec{r}}^{2\dagger}$

$\delta H_{\mathrm{MF}}[\;\to\;] = \delta H_{\mathrm{FQH}}[\;]$

Couple wires with δH

Laughlin states:

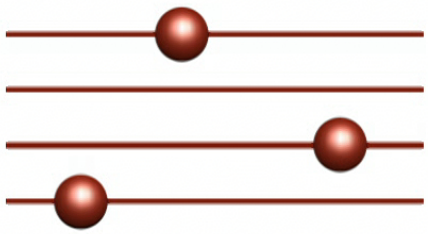
$\delta H = \psi^\dagger_{y+1,L} \psi_{y,R} e^{2in(\theta_y + \theta_{y+1})} + \mathrm{H.c.}$

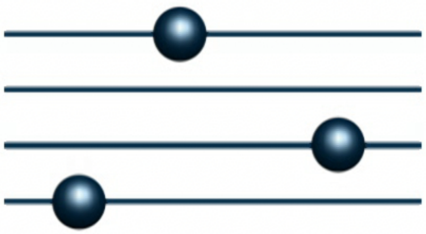
$\delta H_{\mathrm{Abelian\ FQH}}$
Kane, Mukhopadhyay, Lubensky (2002)

$\delta H_{\mathrm{non\text{-}Abelian\ FQH}}$
Teo, Kane (2014)

Emergent gauge fields

my convention:
$\psi \sim \exp[i\varphi \pm i\theta]$

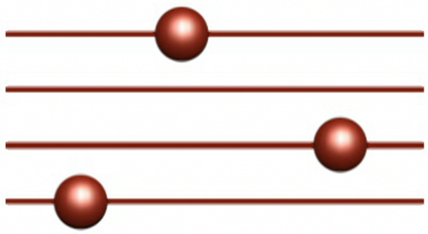




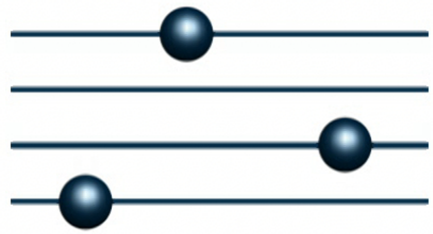
$$H_{\text{wire}} = (\partial_x \varphi - A_x)^2 + (\partial_x \theta)^2 - i A_\tau \frac{\partial_x \theta}{\pi}$$

Emergent gauge fields

my convention:
$\psi \sim \exp[i\varphi \pm i\theta]$



$$H_{\text{wire}}[\varphi, \theta] = H_{\text{non-local}}[\varphi_{\text{CF}}, \theta_{\text{CF}}]$$



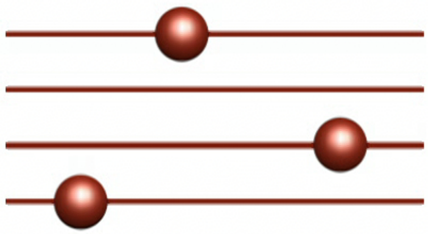
$$H_{\text{wire}} = (\partial_x \varphi - A_x)^2 + (\partial_x \theta)^2 - i A_\tau \frac{\partial_x \theta}{\pi}$$

Gaussian integral over auxiliary variables

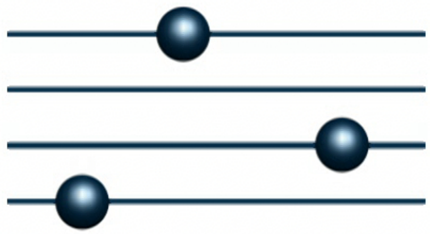
$$e^{-H_{\text{non-local}}[\varphi_{\text{CF}}, \theta_{\text{CF}}]} \propto \int \mathcal{D}\vec{a}\, e^{-H^{\text{CF}}_{\text{wire}}[\varphi_{\text{CF}}, \theta_{\text{CF}}, \vec{a}]}$$

Emergent gauge fields

my convention:
$\psi \sim \exp[i\varphi \pm i\theta]$



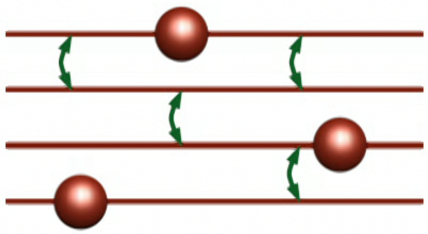
$$H^{\mathrm{CF}}_{\mathrm{wire}} = \vec{J}^{\mathrm{CF}}_{\tau,x} \cdot \vec{a} + \frac{i}{8\pi}(a - A)d(a - A)$$



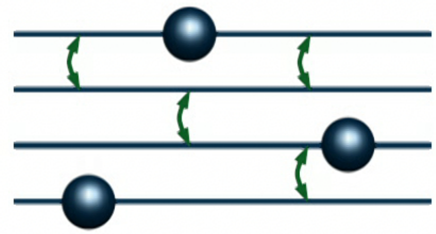
$$H_{\mathrm{wire}} = \vec{J}_{\tau,x} \cdot \vec{A}$$

Emergent gauge fields

my convention:
$\psi \sim \exp[i\varphi \pm i\theta]$

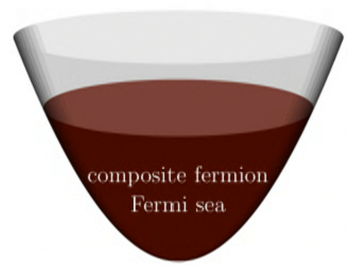


$$H_{\text{wire}}^{\text{CF}} = \vec{J}_{\tau,x}^{\text{CF}} \cdot \vec{a} + \frac{i}{8\pi}(a-A)d(a-A)$$



$$H_{\text{wire}} = \vec{J}_{\tau,x} \cdot \vec{A}$$

Non-trivial compressible state:
(Dirac) Composite Fermi liquid



Example 2:
Quantum Magnetism

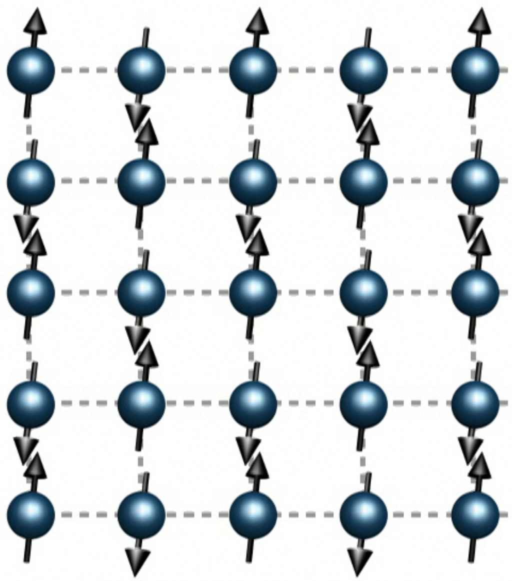
Spin-½ with conserved S_z / Bosons at half-filling

Eyal Leviatan

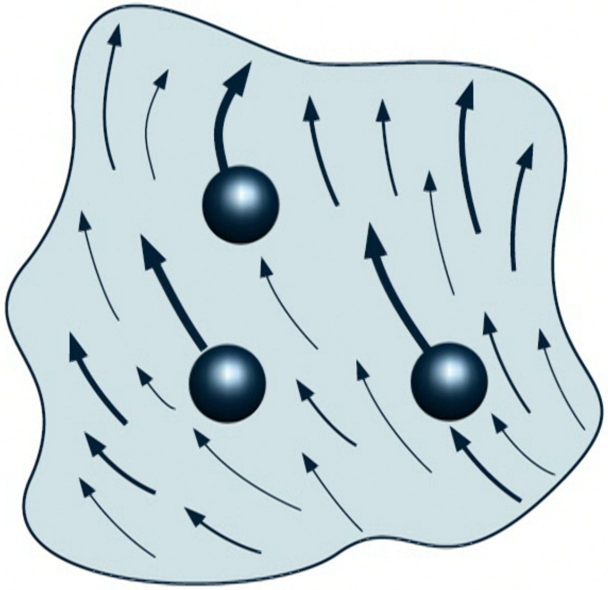
Many connections to deconfined criticality

Senthil, Vishwanath, Balents, Sachdev, Fisher (2004)

Trivial (ordered) phases

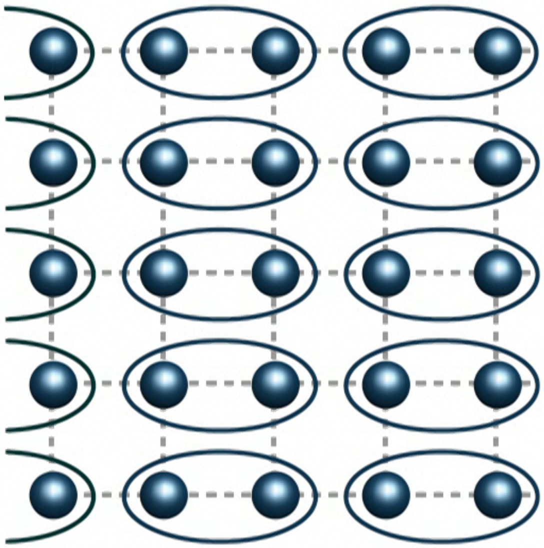


Néel state

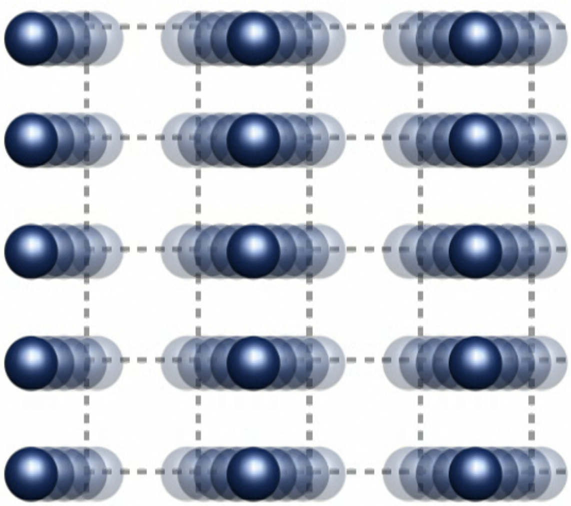


Superfluid

Trivial (ordered) phases



Valence bond solid



Bond density wave

Trivial phases on wires

my convention:
$B \sim \exp[i\sqrt{2}\varphi]$

Decoupled wires / spin chains

$$H_{\text{wire}} = (\partial_x \varphi)^2 + (\partial_x \theta)^2 \qquad H'_{\text{wire}} = \cos 2\sqrt{2}\theta$$

Leading coupling terms

$$\delta H_{\text{SF}} = \cos[\sqrt{2}\varphi_y - \sqrt{2}\varphi_{y+1}] \qquad \delta H_{\text{BDW}} = \cos[\sqrt{2}\theta_y + \sqrt{2}\theta_{y+1}]$$

Trivial phases on wires

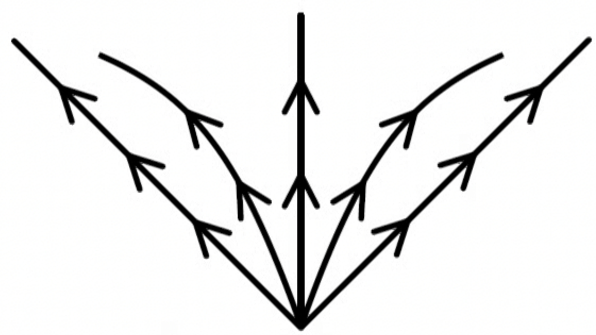
my convention:
$B \sim \exp[i\sqrt{2}\varphi]$

Decoupled wires / spin chains

$$H_{\text{wire}} = (\partial_x \varphi)^2 + (\partial_x \theta)^2 \qquad H'_{\text{wire}} = \cos 2\sqrt{2}\theta$$

Leading coupling terms

$$\delta H_{\text{SF}} = \cos[\sqrt{2}\varphi_y - \sqrt{2}\varphi_{y+1}] \qquad \delta H_{\text{BDW}} = \cos[\sqrt{2}\theta_y + \sqrt{2}\theta_{y+1}]$$



decoupled wires

Trivial phases on wires

my convention:
$B \sim \exp[i\sqrt{2}\varphi]$

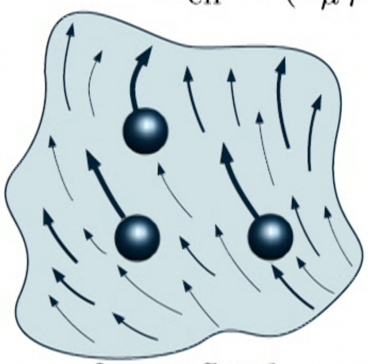
Decoupled wires / spin chains

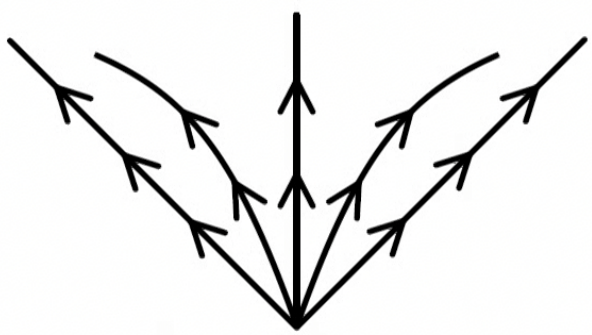
$$H_{\text{wire}} = (\partial_x\varphi)^2 + (\partial_x\theta)^2 \qquad H'_{\text{wire}} = \cos 2\sqrt{2}\theta$$

Leading coupling terms

$$\delta H_{\text{SF}} = \cos[\sqrt{2}\varphi_y - \sqrt{2}\varphi_{y+1}] \qquad \delta H_{\text{BDW}} = \cos[\sqrt{2}\theta_y + \sqrt{2}\theta_{y+1}]$$

$$\mathcal{L}_{\text{eff}} = (\partial_\mu\varphi)^2$$


Superfluid


decoupled wires

Trivial phases on wires

my convention:
$B \sim \exp[i\sqrt{2}\varphi]$

Decoupled wires / spin chains

$$H_{\text{wire}} = (\partial_x \varphi)^2 + (\partial_x \theta)^2 \qquad H'_{\text{wire}} = \cos 2\sqrt{2}\theta$$

Leading coupling terms

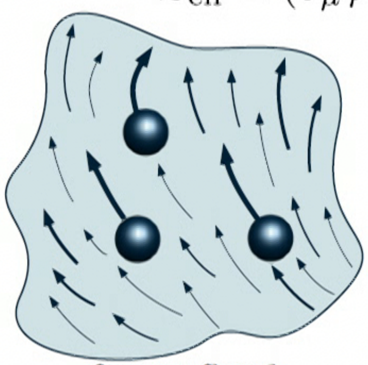
$$\delta H_{\text{SF}} = \cos[\sqrt{2}\varphi_y - \sqrt{2}\varphi_{y+1}] \qquad \delta H_{\text{BDW}} = \cos[\sqrt{2}\theta_y + \sqrt{2}\theta_{y+1}]$$

$$\delta H_{\text{BDW}} \to (\partial_y \theta')^2 \qquad \theta' = (-1)^y \theta$$

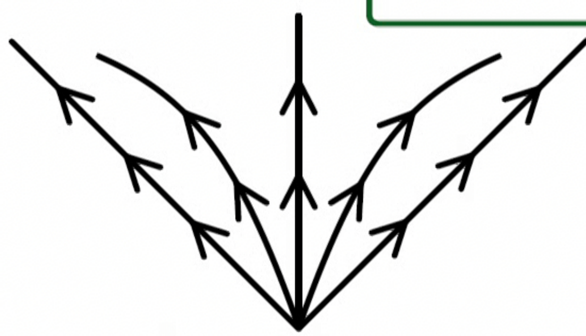
$$\mathcal{L}_{\text{wire}} = (\partial_x \theta')^2 + (\partial_\tau \theta')^2$$

$$\Rightarrow \quad \mathcal{L}_{\text{eff}} = (\partial_\mu \theta')^2$$

$$\mathcal{L}_{\text{eff}} = (\partial_\mu \varphi)^2$$



Superfluid



decoupled wires

Trivial phases on wires

my convention: $B \sim \exp[i\sqrt{2}\varphi]$

Decoupled wires / spin chains

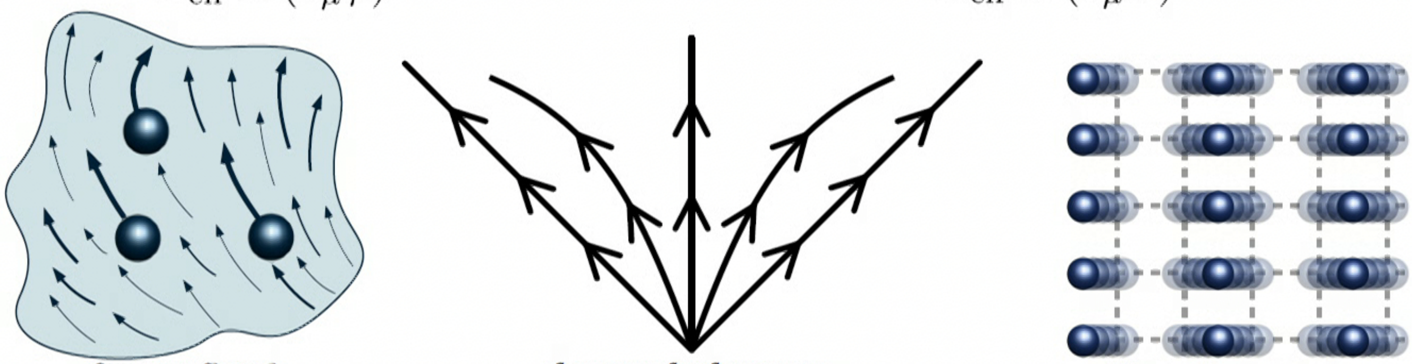
$$H_{\text{wire}} = (\partial_x \varphi)^2 + (\partial_x \theta)^2 \qquad H'_{\text{wire}} = \cos 2\sqrt{2}\theta$$

Leading coupling terms

$$\delta H_{\text{SF}} = \cos[\sqrt{2}\varphi_y - \sqrt{2}\varphi_{y+1}] \qquad \delta H_{\text{BDW}} = \cos[\sqrt{2}\theta_y + \sqrt{2}\theta_{y+1}]$$

Phonon pinned by H'_{wire}

$\mathcal{L}_{\text{eff}} \sim (\partial_\mu \varphi)^2$ $\qquad$ $\mathcal{L}_{\text{eff}} \sim (\partial_\mu \theta')^2$



Superfluid $\qquad$ decoupled wires $\qquad$ Density wave

Topological defects 1: Vortices

my convention:
$B \sim \exp[i\sqrt{2}\varphi]$

$$V_{\vec{R}} = \exp\left[i\frac{1}{\sqrt{2}}\sum_y \mathrm{sgn}(y-Y)\theta\right]$$

Topological defects 1: Vortices

my convention:
$B \sim \exp[i\sqrt{2}\varphi]$

$$V_{\vec{R}} = \exp\left[i\frac{1}{\sqrt{2}}\sum\nolimits_y \mathrm{sgn}(y-Y)\theta\right]$$

Properties:

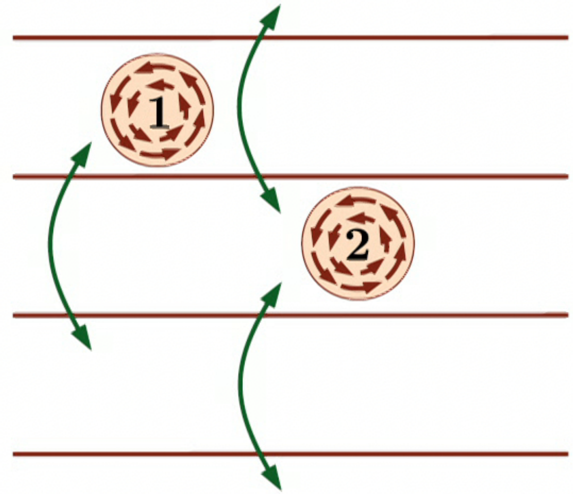
1. Coupled to Maxwell-photon

$$H_{\text{wire}} = \vec{J}^{\text{vortex}}_{\tau,x} \cdot \vec{a}$$

Topological defects 1: Vortices

my convention:
$B \sim \exp[i\sqrt{2}\varphi]$

$$V_{\vec{R}} = \exp\left[i\frac{1}{\sqrt{2}}\sum_y \mathrm{sgn}(y-Y)\theta\right]$$



Properties:

1. Coupled to Maxwell-photon

$$H_{\mathrm{wire}} = \vec{J}^{\mathrm{vortex}}_{\tau,x} \cdot \vec{a}$$

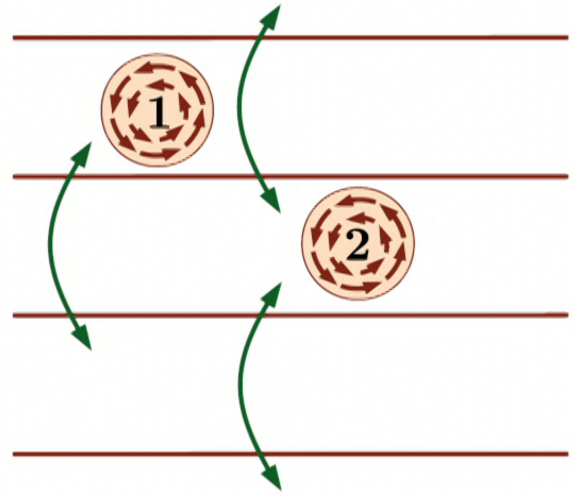
2. Can only hop in steps of two

$$\cos[\sqrt{2}\theta_y + \sqrt{2}\theta_{y+1}] = V^\dagger_{y-1/2} V_{y+3/2} + \mathrm{H.c.}$$

Topological defects 1: Vortices

my convention:
$B \sim \exp[i\sqrt{2}\varphi]$

$$V_{\vec{R}} = \exp\left[i\frac{1}{\sqrt{2}}\sum\nolimits_y \mathrm{sgn}(y-Y)\theta\right]$$



Properties:

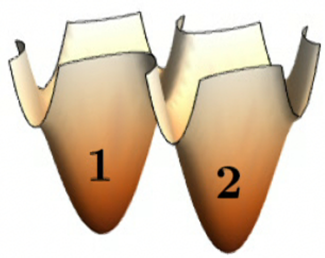
1. Coupled to Maxwell-photon

$$H_{\mathrm{wire}} = \vec{J}^{\mathrm{vortex}}_{\tau,x} \cdot \vec{a}$$

2. Can only hop in steps of two

$$\cos[\sqrt{2}\theta_y + \sqrt{2}\theta_{y+1}] = V^\dagger_{y-1/2}V_{y+3/2} + \mathrm{H.c.}$$

at half-filling, π-flux



Trivial phases on wires

my convention:
$B \sim \exp[i\sqrt{2}\varphi]$

Decoupled wires / spin chains

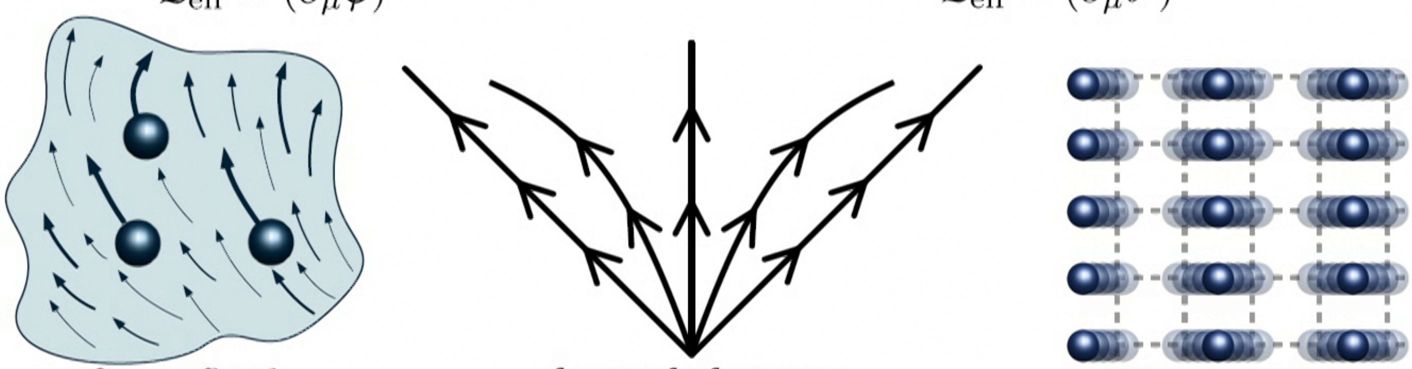
$$H_{\text{wire}} = (\partial_x\varphi)^2 + (\partial_x\theta)^2 \qquad H'_{\text{wire}} = \cos 2\sqrt{2}\theta$$

Leading coupling terms

$$\delta H_{\text{SF}} = \cos[\sqrt{2}\varphi_y - \sqrt{2}\varphi_{y+1}] \qquad \delta H_{\text{BDW}} = \cos[\sqrt{2}\theta_y + \sqrt{2}\theta_{y+1}]$$

Phonon pinned by H'_{wire}

$\mathcal{L}_{\text{eff}} \sim (\partial_\mu\varphi)^2$ $\qquad$ $\mathcal{L}_{\text{eff}} \sim (\partial_\mu\theta')^2$



Superfluid $\qquad$ decoupled wires $\qquad$ Density wave

Topological defects 1: Vortices

my convention:
$B \sim \exp[i\sqrt{2}\varphi]$

$$V_{\vec{R}} = \exp\left[i\frac{1}{\sqrt{2}}\sum_y \mathrm{sgn}(y-Y)\theta\right]$$

Topological defects 1: Vortices

my convention:
$B \sim \exp[i\sqrt{2}\varphi]$

$$V_{\vec{R}} = \exp\left[i\frac{1}{\sqrt{2}}\sum_y \mathrm{sgn}(y-Y)\theta\right]$$

Properties:

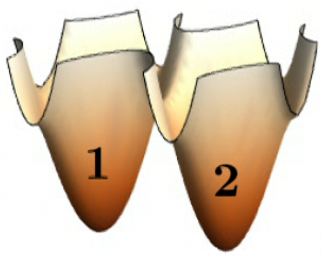
1. Coupled to Maxwell-photon

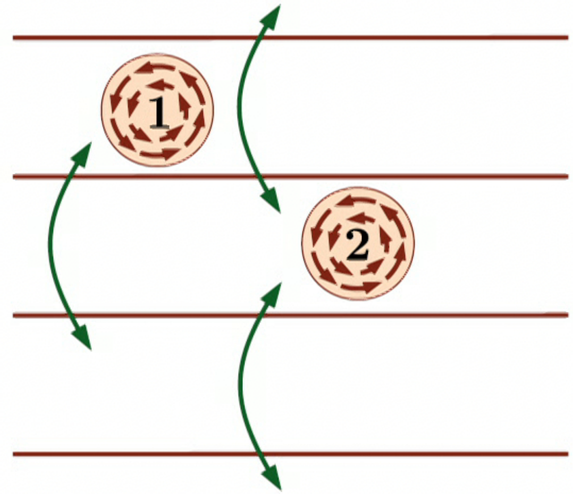
$$H_{\mathrm{wire}} = \vec{J}^{\mathrm{vortex}}_{\tau,x}\cdot\vec{a}$$

2. Can only hop in steps of two

$$\cos[\sqrt{2}\theta_y + \sqrt{2}\theta_{y+1}] = V^{\dagger}_{y-1/2}V_{y+3/2} + \mathrm{H.c.}$$

at half-filling, π-flux



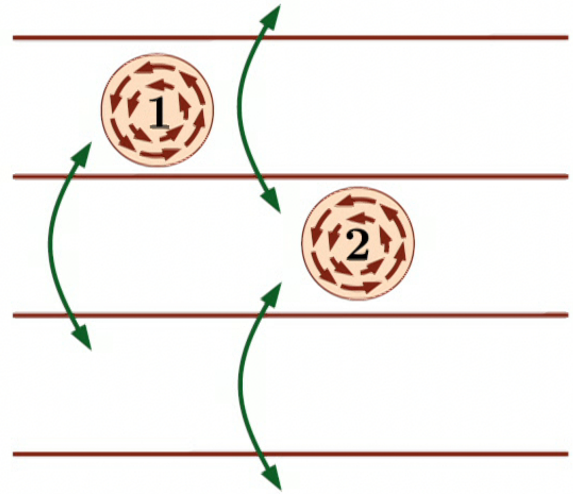


$$H_{\mathrm{wire}} = \left(\vec{J}^{\mathrm{vortex},1}_{\tau,x} + \vec{J}^{\mathrm{vortex},2}_{\tau,x}\right)\cdot\vec{a}$$

Topological defects 1: Vortices

my convention:
$B \sim \exp[i\sqrt{2}\varphi]$

$$V_{\vec{R}} = \exp\left[i\frac{1}{\sqrt{2}}\sum\nolimits_y \mathrm{sgn}(y-Y)\theta\right]$$



$$H_{\mathrm{wire}} = \left(\vec{J}^{\mathrm{vortex},1}_{\tau,x} + \vec{J}^{\mathrm{vortex},2}_{\tau,x}\right)\cdot\vec{a}$$

$$\delta H_{\mathrm{BDW}} = \sum_{i=1,2}\left[V^{\dagger}_{i,y-1/2}V_{i,y+3/2} + \mathrm{H.c.}\right]$$

$$\delta H_{\mathrm{SF}} = \cos 2\theta_{\mathrm{vortex},i}$$

Topological defects 2: Dislocations

my convention:
$B \sim \exp[i\sqrt{2}\varphi]$

$$b_{\vec{R}} = \exp\left[i \frac{1}{\sqrt{2}} \sum_y \mathrm{sgn}(y - Y)(-1)^{y+Y} \varphi \right]$$

Topological defects 2: Dislocations

my convention:
$B \sim \exp[i\sqrt{2}\varphi]$

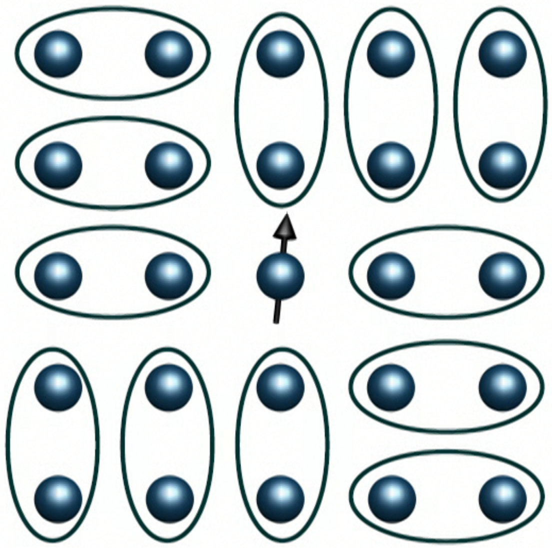
$$b_{\vec{R}} = \exp\left[i \frac{1}{\sqrt{2}} \sum_y \mathrm{sgn}(y - Y)(-1)^{y+Y} \varphi \right]$$

Properties:

1. $b_{y+1/2} b_{y-1/2} = B_y$

Topological defects 2: Dislocations

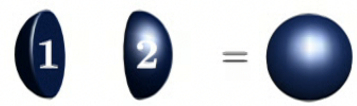
my convention:
$B \sim \exp[i\sqrt{2}\varphi]$



$$b_{\vec{R}} = \exp\left[i\frac{1}{\sqrt{2}}\sum_y \mathrm{sgn}(y-Y)(-1)^{y+Y}\varphi\right]$$

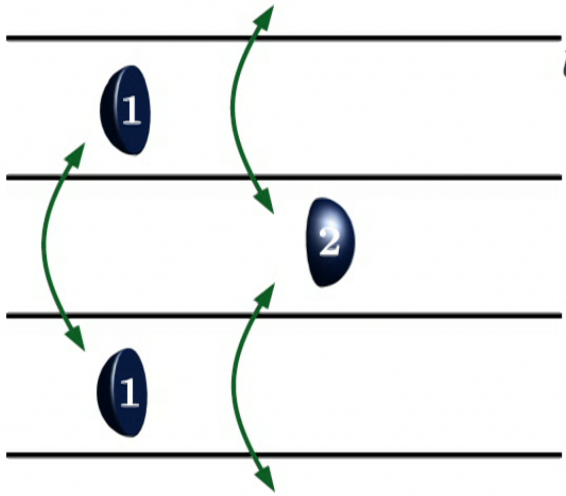
Properties:

1. $b_{y+1/2}b_{y-1/2} = B_y$



Topological defects 2: Dislocations

my convention:
$B \sim \exp[i\sqrt{2}\varphi]$



$$b_{\vec{R}} = \exp\left[i\frac{1}{\sqrt{2}}\sum_y \mathrm{sgn}(y-Y)(-1)^{y+Y}\varphi\right]$$

Properties:

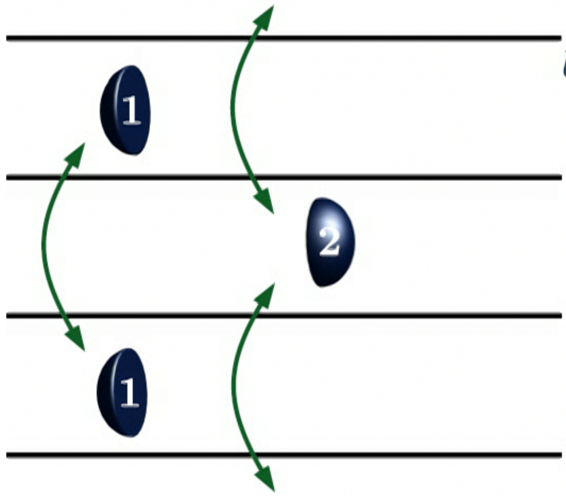
1. $b_{y+1/2}b_{y-1/2} = B_y$

2. two species

$$b^\dagger_{y-1/2}b_{y+3/2} = B^\dagger_y B_{y+1}$$

Topological defects 2: Dislocations

my convention:
$B \sim \exp[i\sqrt{2}\varphi]$



$$b_{\vec{R}} = \exp\left[i\frac{1}{\sqrt{2}}\sum_y \mathrm{sgn}(y-Y)(-1)^{y+Y}\varphi\right]$$

Properties:

1. $b_{y+1/2}b_{y-1/2} = B_y$

2. two species

$$b^\dagger_{y-1/2}b_{y+3/2} = B^\dagger_y B_{y+1}$$

3. b_i is vortex in $V_i = e^{i\varphi_{\mathrm{vortex},i}}$

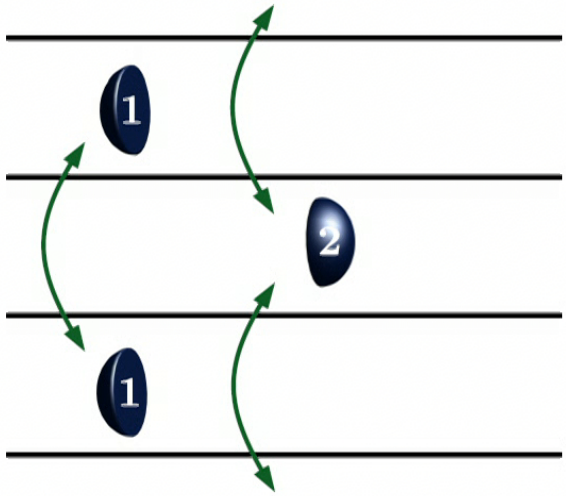
$$b_{i,\vec{R}} = \exp\left[i\sum_y \mathrm{sgn}(y-Y)\varphi_{\mathrm{vortex},i}\right]$$

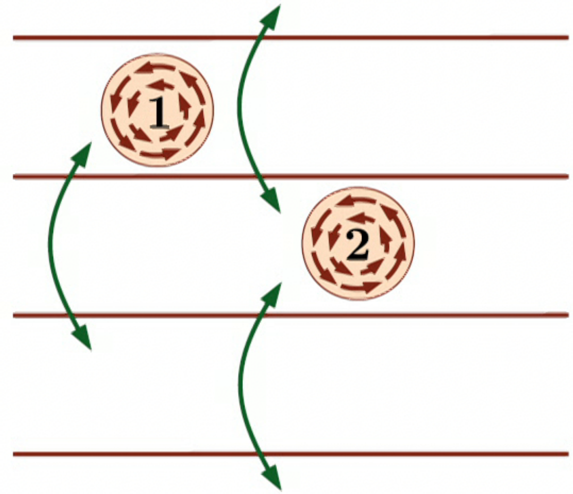
$$H_{\mathrm{wire}} = \left(\vec{J}^{\mathrm{parton},1}_{\tau,x} + \vec{J}^{\mathrm{parton},2}_{\tau,x}\right)\cdot\vec{a}$$

$$\delta H_{\mathrm{SF}} = \sum_{i=1,2}\left[b^\dagger_{i,y-1/2}b_{i,y+3/2} + \mathrm{H.c.}\right]$$

Emergent gauge fields

my convention:
$B \sim \exp[i\sqrt{2}\varphi]$





$$H_{\text{wire}} = \left(\vec{J}^{\text{parton},1}_{\tau,x} + \vec{J}^{\text{parton},2}_{\tau,x}\right) \cdot \vec{a}$$

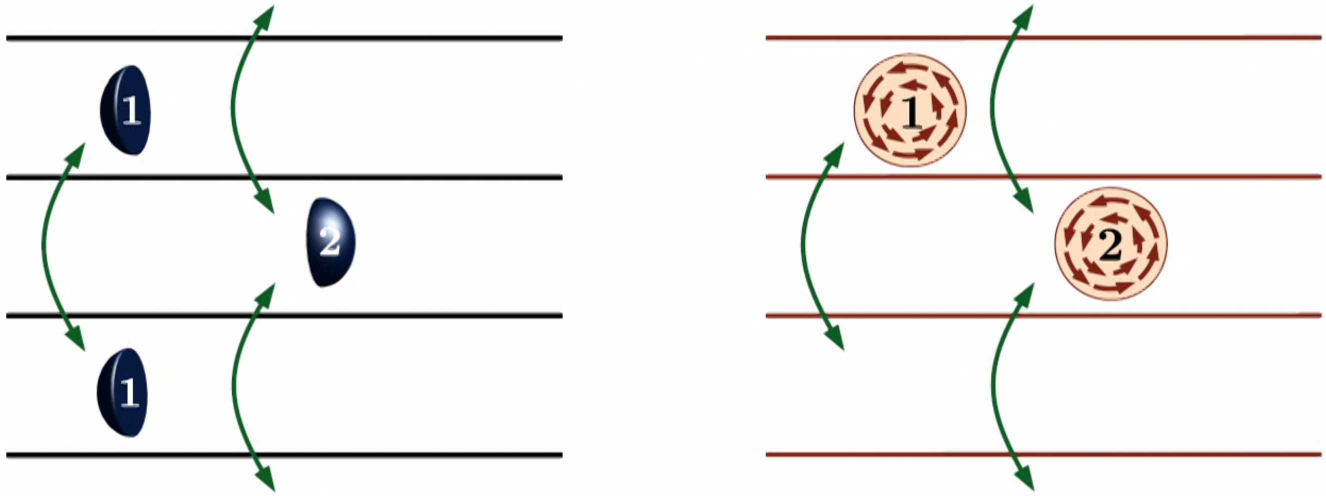
$$H_{\text{wire}} = \left(\vec{J}^{\text{vortex},1}_{\tau,x} + \vec{J}^{\text{vortex},2}_{\tau,x}\right) \cdot \vec{a}$$

$$\delta H_{\text{SF}} = \sum_{i=1,2} \left[b^\dagger_{i,y-1/2} b_{i,y+3/2} + \text{H.c.}\right]$$

$$\delta H_{\text{BDW}} = \sum_{i=1,2} \left[V^\dagger_{i,y-1/2} V_{i,y+3/2} + \text{H.c.}\right]$$

Emergent gauge fields

my convention: $B \sim \exp[i\sqrt{2}\varphi]$



$$H_{\text{wire}} = \left(\vec{J}^{\text{parton},1}_{\tau,x} + \vec{J}^{\text{parton},2}_{\tau,x}\right) \cdot \vec{a}$$

$$H_{\text{wire}} = \left(\vec{J}^{\text{vortex},1}_{\tau,x} + \vec{J}^{\text{vortex},2}_{\tau,x}\right) \cdot \vec{a}$$

$$\delta H_{\text{SF}} = \sum_{i=1,2} \left[b^\dagger_{i,y-1/2} b_{i,y+3/2} + \text{H.c.}\right]$$

$$\delta H_{\text{BDW}} = \sum_{i=1,2} \left[V^\dagger_{i,y-1/2} V_{i,y+3/2} + \text{H.c.}\right]$$

$H'_{\text{wire}} = $ flux insertion (monopole)

$$H'_{\text{wire}} = V_1^{2\dagger} V_2^2 + \text{H.c.}$$

Phases of matter / fixed point Hamiltonians

Expect:
I. Trivial phases
II. Chiral phases
III. Higgs phases
IV. Gapless phases

Emergent gauge fields

my convention:
$B \sim \exp[i\sqrt{2}\varphi]$

$$(\vec{\Delta} \times \vec{a})^2 + \text{monopoles} \quad \leftrightarrow \quad \cos(\vec{\Delta} \times \vec{a})$$

(real gauge field) (compact gauge field)

$$(\vec{\Delta}\varphi)^2 + \cos\theta \quad \leftrightarrow \quad \cos(\vec{\Delta}\varphi)$$

$$H_{\text{wire}} = \left(\vec{J}^{\text{parton},1}_{\tau,x} + \vec{J}^{\text{parton},2}_{\tau,x}\right) \cdot \vec{a}$$

$$H_{\text{wire}} = \left(\vec{J}^{\text{vortex},1}_{\tau,x} + \vec{J}^{\text{vortex},2}_{\tau,x}\right) \cdot \vec{a}$$

$$\delta H_{\text{SF}} = \sum_{i=1,2} \left[b^\dagger_{i,y-1/2} b_{i,y+3/2} + \text{H.c.}\right]$$

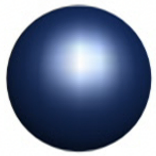
$$\delta H_{\text{BDW}} = \sum_{i=1,2} \left[V^\dagger_{i,y-1/2} V_{i,y+3/2} + \text{H.c.}\right]$$

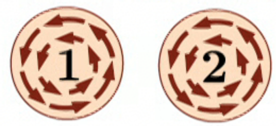
$H'_{\text{wire}} = $ flux insertion (monopole)

$$H'_{\text{wire}} = V_1^{2\dagger} V_2^2 + \text{H.c.}$$

Phases of matter / fixed point Hamiltonians

Expect: I. Trivial phases
II. Chiral phases
III. Higgs phases
IV. Gapless phases

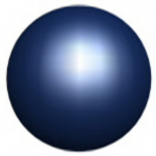


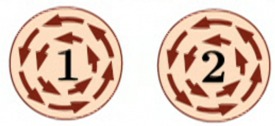


Microscopic bosons / Spins	Vortices (+ real photon)	Partons (+ compact photon)
Superfluid / Neel	Mott (Photon)	Superfluid (Higgs)
Density wave / VBS	Superfluid (Higgs)	Mott + confinement
Quantum Hall / Chiral SL	Quantum Hall	Quantum Hall

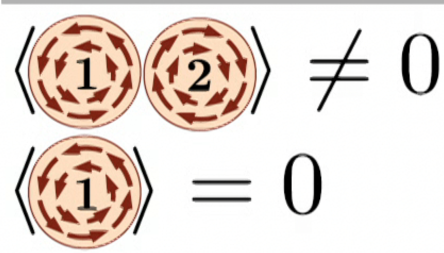
Phases of matter / fixed point Hamiltonians

Expect:
I. Trivial phases
II. Chiral phases
III. Higgs phases
IV. Gapless phases



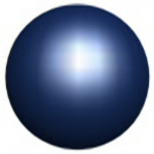


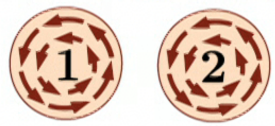
Microscopic bosons / Spins	Vortices (+ real photon)	Partons (+ compact photon)
Superfluid / Neel	Mott (Photon)	Superfluid (Higgs)
Density wave / VBS	Superfluid (Higgs)	Mott + confinement
Quantum Hall / Chiral SL	Quantum Hall	Quantum Hall
$\mathbb{Z}_2$ Spin liquid	Exciton condensate	



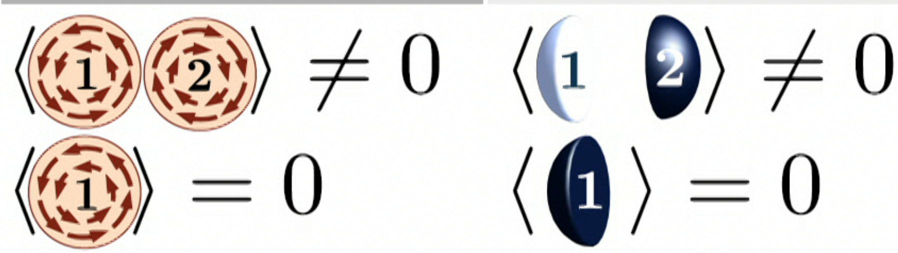
Phases of matter / fixed point Hamiltonians

Expect:
I. Trivial phases
II. Chiral phases
III. Higgs phases
IV. Gapless phases





Microscopic bosons / Spins	Vortices (+ real photon)	Partons (+ compact photon)
Superfluid / Neel	Mott (Photon)	Superfluid (Higgs)
Density wave / VBS	Superfluid (Higgs)	Mott + confinement
Quantum Hall / Chiral SL	Quantum Hall	Quantum Hall
$\mathbb{Z}_2$ Spin liquid	Exciton condensate	Exciton condensate



Identification of phases

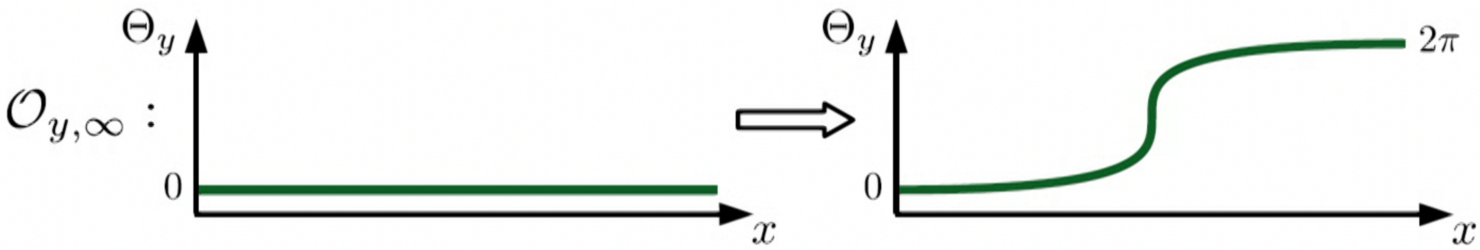
I. Simple phases: $\delta H = -\sum_y \cos\Theta_y$ already known ✓

Identification of phases

I. Simple phases: $\delta H = -\sum_y \cos\Theta_y$ already known ✓

II. Chiral phases: edge states

III. Quantum numbers and statistics of excitations



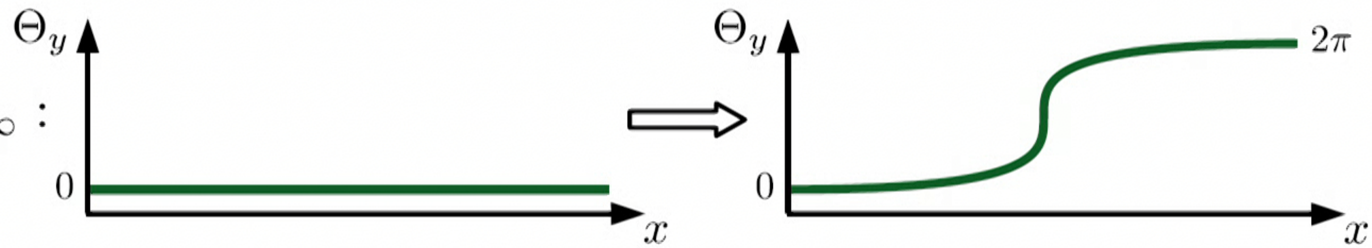
Identification of phases

I. Simple phases: $\delta H = -\sum_y \cos\Theta_y$ already known ✓

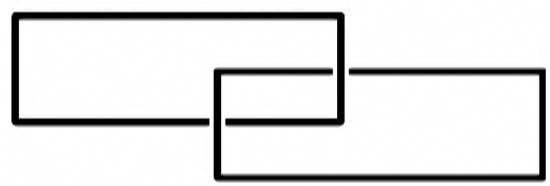
II. Chiral phases: edge states

III. Quantum numbers and statistics of excitations

$\mathcal{O}_{y,\infty}$:



$$\mathcal{O}_1(\vec{r}_1,\vec{r}_1')\mathcal{O}_2(\vec{r}_2,\vec{r}_2')\mathcal{O}_1^\dagger(\vec{r}_1,\vec{r}_1')\mathcal{O}_2^\dagger(\vec{r}_2,\vec{r}_2')$$

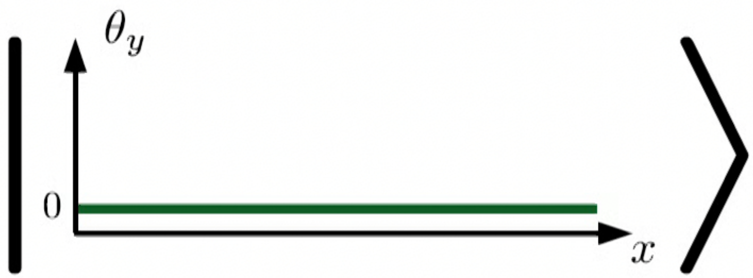


Deconfinement of partons

$$b_{\vec{R}} = \exp\left[i\frac{1}{\sqrt{2}}\sum\nolimits_y \mathrm{sgn}(y-Y)(-1)^{y+Y}\varphi\right]$$

1. Density wave state $\delta H = -\cos 2\sqrt{2}\theta_y - \cos(\sqrt{2}\theta_y + \sqrt{2}\theta_{y+1})$

$|\mathrm{DW}\rangle =$



Deconfinement of partons

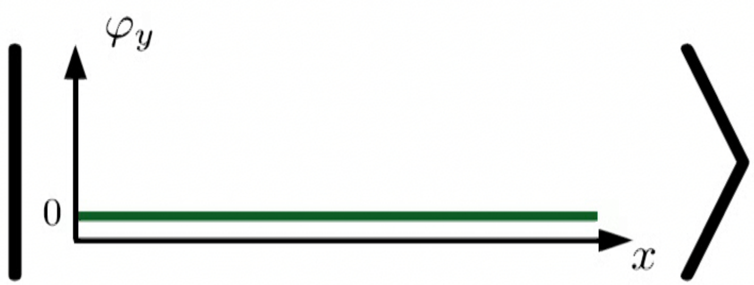
$$b_{\vec{R}} = \exp\left[i\frac{1}{\sqrt{2}}\sum_y \mathrm{sgn}(y-Y)(-1)^{y+Y}\varphi\right]$$

2. Superfluids

$$\delta H = -\cos(\sqrt{2}\varphi_y - \sqrt{2}\varphi_{y+1})$$

$|\mathrm{SF}\rangle =$

$b^\dagger|\mathrm{SF}\rangle =$



Deconfinement of partons

$$b_{\vec{R}} = \exp\left[i \frac{1}{\sqrt{2}} \sum_y \mathrm{sgn}(y - Y)(-1)^{y+Y} \varphi \right]$$

3. Spin liquid

$$\delta H = -\cos(\sqrt{2}\theta_{2y-1} + 2\sqrt{2}\theta_{2y} + \sqrt{2}\varphi_{2y+1}) - \cos(\sqrt{2}\varphi_{2y} - 2\sqrt{2}\varphi_{2y+1} + \sqrt{2}\varphi_{2y+2})$$

$$b_{\vec{R}} = \exp \begin{array}{l} \vdots \\ -i/\sqrt{2}\varphi_{2y+3} \\ +i/\sqrt{2}\varphi_{2y+2} \\ -i/\sqrt{2}\varphi_{2y+1} \\ +i/\sqrt{2}\varphi_{2y} \\ +i/\sqrt{2}\varphi_{2y-1} \\ -i/\sqrt{2}\varphi_{2y-2} \\ +i/\sqrt{2}\varphi_{2y-3} \\ -i/\sqrt{2}\varphi_{2y-4} \\ \vdots \end{array}$$

Deconfinement of partons

$$b_{\vec{R}} = \exp\left[i\frac{1}{\sqrt{2}}\sum_y \mathrm{sgn}(y-Y)(-1)^{y+Y}\varphi\right]$$

3. Spin liquid

$$\delta H = -\cos(\sqrt{2}\theta_{2y-1} + 2\sqrt{2}\theta_{2y} + \sqrt{2}\varphi_{2y+1}) - \cos(\underbrace{\sqrt{2}\varphi_{2y} - 2\sqrt{2}\varphi_{2y+1} + \sqrt{2}\varphi_{2y+2}}_{4\Phi_{2y+1}})$$

$$b_{\vec{R}} = \exp \begin{matrix} \vdots \\ -i/\sqrt{2}\varphi_{2y+3} \\ +i/\sqrt{2}\varphi_{2y+2} \\ -i/\sqrt{2}\varphi_{2y+1} \\ +i/\sqrt{2}\varphi_{2y} \\ +i/\sqrt{2}\varphi_{2y-1} \\ -i/\sqrt{2}\varphi_{2y-2} \\ +i/\sqrt{2}\varphi_{2y-3} \\ -i/\sqrt{2}\varphi_{2y-4} \\ \vdots \end{matrix} = \exp \begin{matrix} \vdots \\ +i\Phi_{2y+3} \\ \\ +i\Phi_{2y+1} \\ +i/\sqrt{2}\varphi_{2y} \\ -i\Phi_{2y-1} \\ \\ -i\Phi_{2y-3} \\ \vdots \end{matrix}$$

Deconfinement of partons

$$b_{\vec{R}} = \exp\left[i\frac{1}{\sqrt{2}}\sum_y \mathrm{sgn}(y-Y)(-1)^{y+Y}\varphi\right]$$

3. Spin liquid

$$\delta H = -\cos(\sqrt{2}\theta_{2y-1} + 2\sqrt{2}\theta_{2y} + \sqrt{2}\varphi_{2y+1}) - \cos(\underbrace{\sqrt{2}\varphi_{2y} - 2\sqrt{2}\varphi_{2y+1} + \sqrt{2}\varphi_{2y+2}}_{4\Phi_{2y+1}})$$

$$b_{\vec{R}} = \exp \begin{matrix} \vdots \\ -i/\sqrt{2}\varphi_{2y+3} \\ +i/\sqrt{2}\varphi_{2y+2} \\ -i/\sqrt{2}\varphi_{2y+1} \\ +i/\sqrt{2}\varphi_{2y} \\ +i/\sqrt{2}\varphi_{2y-1} \\ -i/\sqrt{2}\varphi_{2y-2} \\ +i/\sqrt{2}\varphi_{2y-3} \\ -i/\sqrt{2}\varphi_{2y-4} \\ \vdots \end{matrix} = \exp \begin{matrix} \vdots \\ +i\Phi_{2y+3} \\ +i\Phi_{2y+1} \\ +i/\sqrt{2}\varphi_{2y} \\ -i\Phi_{2y-1} \\ -i\Phi_{2y-3} \\ \vdots \end{matrix} \Longrightarrow b_{\vec{R}}|\mathrm{SL}\rangle = e^{i/\sqrt{2}\varphi_{2y}}|\mathrm{SL}\rangle$$

Energy cost: $\Delta E = O(1)$

Deconfinement of partons

$$b_{\vec{R}} = \exp\left[i\frac{1}{\sqrt{2}}\sum_y \mathrm{sgn}(y-Y)(-1)^{y+Y}\varphi\right]$$

3. Spin liquid

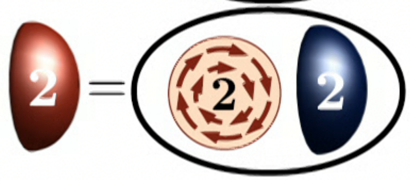
$$\delta H = -\cos(\sqrt{2}\theta_{2y-1} + 2\sqrt{2}\theta_{2y} + \sqrt{2}\varphi_{2y+1}) - \cos(\sqrt{2}\varphi_{2y} - 2\sqrt{2}\varphi_{2y+1} + \sqrt{2}\varphi_{2y+2})$$

Fermionic partons

Combine different types of defects

$f_1 = V_1 b_1$

$f_2 = V_2^\dagger b_2$

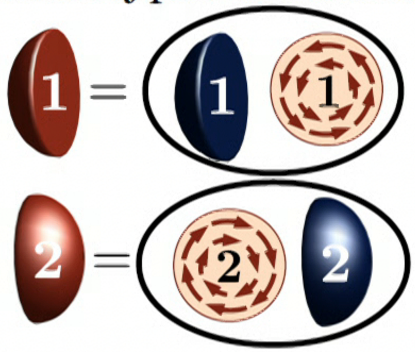


Fermionic partons

Combine different types of defects

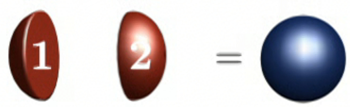
$$f_1 = V_1 b_1$$

$$f_2 = V_2^\dagger b_2$$



Properties:

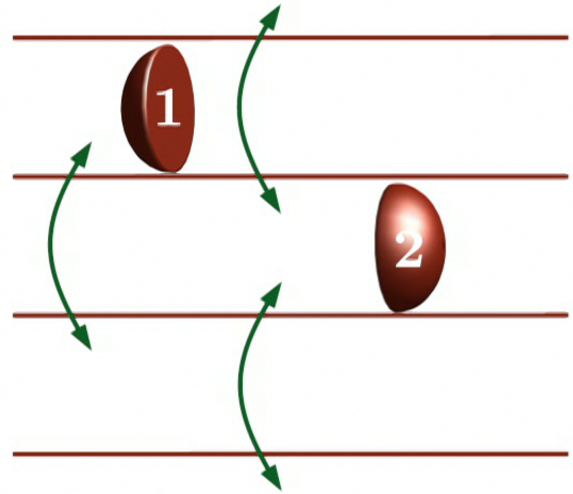
1. $f_1 f_2 \sim e^{i\sqrt{2}\varphi \pm i\sqrt{2}\theta} \sim B$



2. two species

$$\delta H_{\mathrm{SF}} = f^\dagger_{1,R,y+1} f_{1,L,y} + f^\dagger_{2,R,y} f_{2,L,y+1} + \mathrm{H.c.}$$

$$\delta H_{\mathrm{BDW}} = f^\dagger_{2,R,y} f_{2,L,y} + f^\dagger_{1,R,y} f_{1,L,y} + \mathrm{H.c.}$$

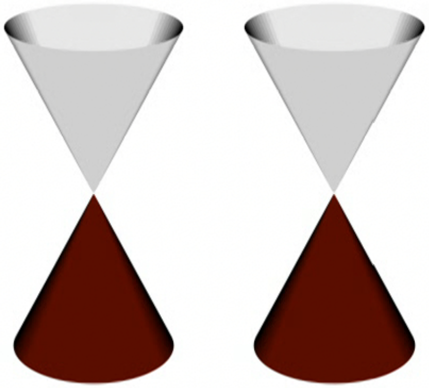


$$H_{\mathrm{wire}} = \left(\vec{J}^{\,\mathrm{fermion},1}_{\tau,x} + \vec{J}^{\,\mathrm{fermion},2}_{\tau,x}\right) \cdot \vec{a}$$

$\delta H_{\mathrm{SF}} + \delta H_{\mathrm{BDW}} =$ fermion hopping

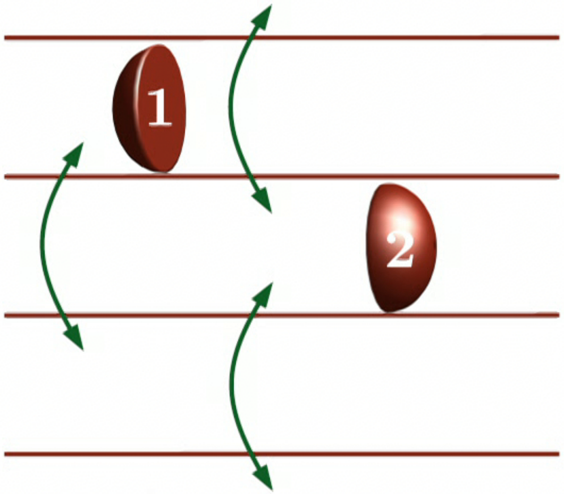
Phases / fixed point Hamiltonians

$\delta H_{\rm SF} + \delta H_{\rm BDW}$ with equal coefficients



$H'_{\rm wire}$ = flux insertion (monopole)

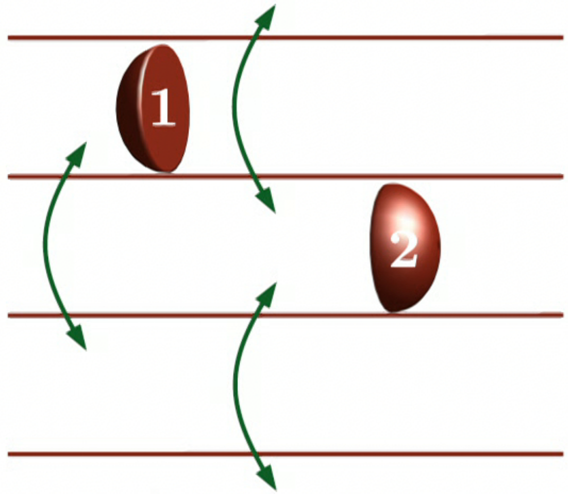
(compact QED3 at transition)



$H_{\rm wire} = \left(\vec{J}^{\rm fermion,1}_{\tau,x} + \vec{J}^{\rm fermion,2}_{\tau,x} \right) \cdot \vec{a}$

Phases / fixed point Hamiltonians

Gapped band structure: ν_1 and ν_2



$$H_{\text{wire}} = \left(\vec{J}^{\text{fermion},1}_{\tau,x} + \vec{J}^{\text{fermion},2}_{\tau,x}\right) \cdot \vec{a}$$

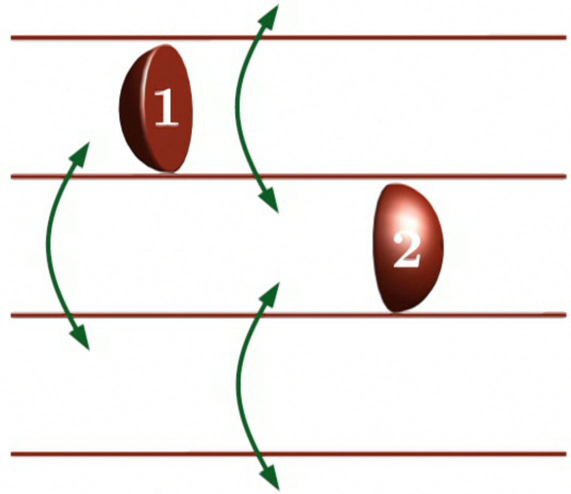
Phases / fixed point Hamiltonians

Gapped band structure: ν_1 and ν_2

$$\delta H_{0,\,0}\left[\quad\right] = \delta H_{\mathrm{BDW}}\left[\quad\right]$$

$$\delta H_{1,\,-1}\left[\quad\right] = \delta H_{\mathrm{SF}}\left[\quad\right]$$

$$\delta H_{\nu_1 \neq -\nu_2}\left[\quad\right] = \delta H_{\mathrm{CSL}}\left[\quad\right]$$



$$H_{\mathrm{wire}} = \left(\vec{J}^{\,\mathrm{fermion},1}_{\tau,x} + \vec{J}^{\,\mathrm{fermion},2}_{\tau,x}\right) \cdot \vec{a}$$

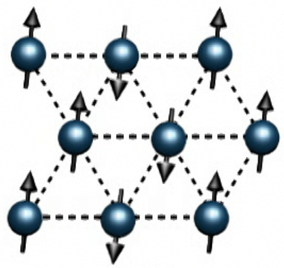
Mean-field phases:

$$\delta H_{\mathrm{BCS}}\left[\ 1\ \ 2\ \right] = \delta H_{\mathbb{Z}_2\ \text{Spin liquid}}\left[\quad\right]$$

Outlook

Outlook

Lattice symmetries and geometric frustration



δH_{SF} and δH_{BDW} absent

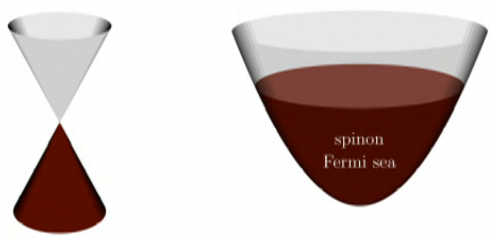
Isotropic limit and lattice Hamiltonians

- Translate to lattice spins

$$\cos[\ldots] \to S_1 S_2 S_3 S_4$$

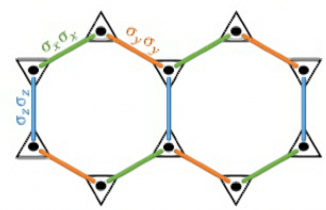
- Deform to simpler, isotropic model (numerics)

Gapless spin liquids



- Dirac SL and spinon Fermi surface

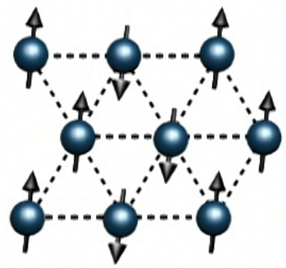
Internal symmetries



- Broken S_z
- Full $SU(2)$

Outlook

Lattice symmetries and geometric frustration



δH_{SF} and δH_{BDW} absent

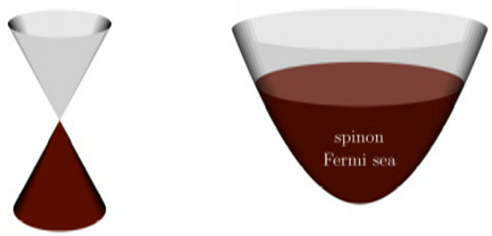
Isotropic limit and lattice Hamiltonians

- Translate to lattice spins

$$\cos[\ldots] \to S_1 S_2 S_3 S_4$$

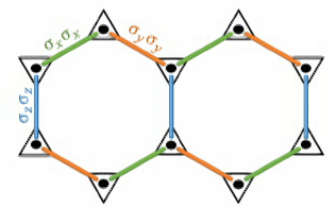
- Deform to simpler, isotropic model (numerics)

Gapless spin liquids



- Dirac SL and spinon Fermi surface

Internal symmetries

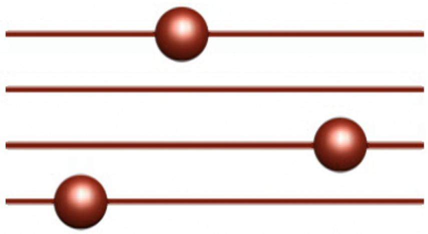


- Broken S_z
- Full $SU(2)$

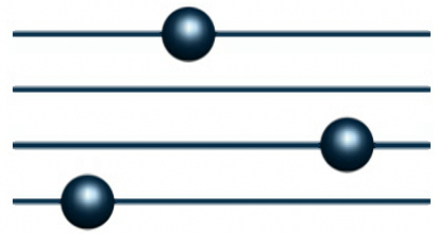
Thank you!

Emergent gauge fields

my convention:
$\psi \sim \exp[i\varphi \pm i\theta]$



$$H_{\text{wire}}[\varphi,\theta] = H_{\text{non-local}}[\varphi_{\text{CF}},\theta_{\text{CF}}]$$



$$H_{\text{wire}} = (\partial_x\varphi - A_x)^2 + (\partial_x\theta)^2 - iA_\tau\frac{\partial_x\theta}{\pi}$$

Gaussian integral over auxiliary variables

$$e^{-H_{\text{non-local}}[\varphi_{\text{CF}},\theta_{\text{CF}}]} \propto \int \mathcal{D}\vec{a}\, e^{-H^{\text{CF}}_{\text{wire}}[\varphi_{\text{CF}},\theta_{\text{CF}},\vec{a}]}$$