Title: Spontaneous symmetry breaking from anyon condensation

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Abstract: In the context of quantum spin liquids, it is long known that the condensation of fractionalized excitations will inevitably break certain physical symmetries sometimes. For example, condensing spinons will usually break spin rotation and time reversal symmetries. We generalize these phenomena to generic continuous quantum phase transitions between symmetry enriched topological orders driven by anyon condensation. We provide a generic rule to determine whether a symmetry is enforced to break across an anyon condensation transition. Using a dimensional reduction scheme, we establish a mapping between these symmetry-breaking anyon-condensation transitions in two spatial dimensions, and deconfined quantum criticality in one spatial dimension.

Spontaneous symmetry breaking from anyon condensation

Yuan-Ming Lu

Perimeter Institute, 4/24/2019

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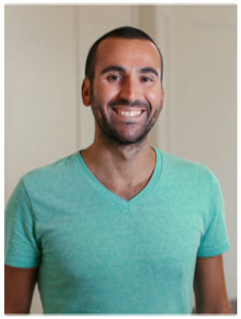
w/ **Corey Jones** & **Dave Penneys** (OSU), **Marcel Bischoff** (Ohio Univ.)

Ongoing work w/
Alex Rasmussen (OSU)

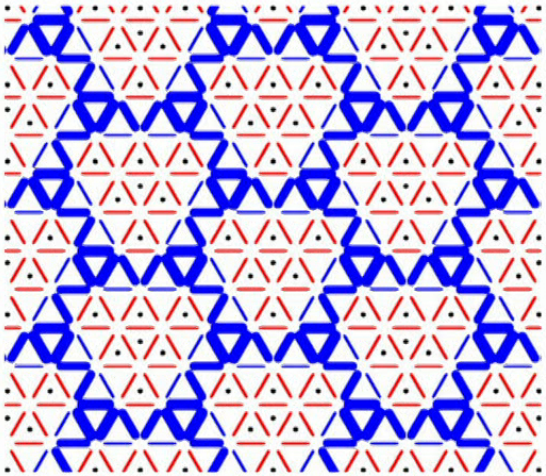


THE OHIO STATE UNIVERSITY

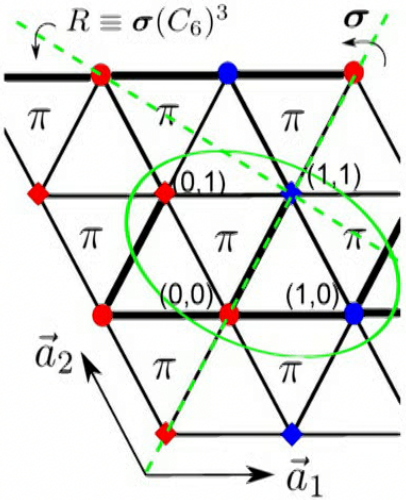
Outline

- Motivation
- Introduction to Symmetry Enriched Topological Orders (SETOs) & Anyon Condensation
- Toric code as a simplest example
- Enforced spontaneous symmetry breaking driven by anyon condensation: two sufficient conditions & more examples
- Relation to deconfined quantum criticality

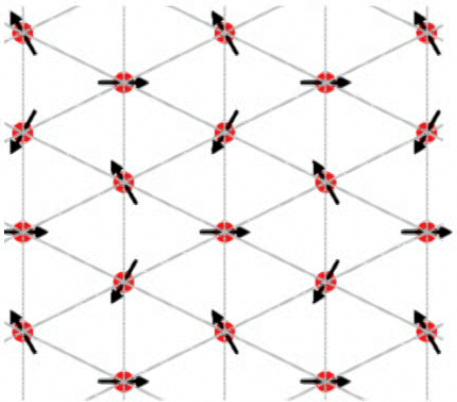
Neighboring phases of a gapped Z2 spin liquid



Valence bond crystal ($\sqrt{12} \times \sqrt{12}$) breaks *crystal symmetries* (*Moessner & Sondhi, 2000 Misguich & Mila, 2008 (see figure) Slagle & Xu, 2014*)



π-flux Dirac U(1) spin liquid + pairings
→ **symmetric Z2 spin liquid**
(#20 state in *Lu, 2016)*



Neel order breaks *spin rotation* and *time reversal symmetries* (*Sachdev 1992, Wang & Vishwanath 2006, Song et al, 2019*)

How to identify neighboring phases of a gapped topological order in 2d?

- May not have tractable parton construction
- May not have a Dirac limit to compute monopole quantum #
- May not know the long-wavelength field theory description

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(irrespective of energetics)

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E Algebraic theory of anyons

E.1 Fusion theory . . . . . .
E.2 Particle-antiparticle duality . . . . . .
E.3 Braiding and topological spin . . . . . .
E.4 Verlinde algebra and topological S-matrix
E.5 Braiding nondegeneracy = modularity . .
E.6 Gauge freedom and Ocneanu rigidity . . .
E.7 Categorical formalism (aside) . . . . . .

Kitaev 2008

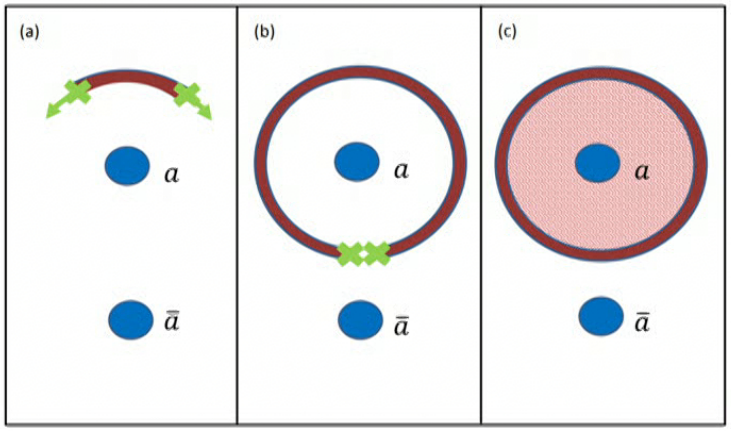
How do symmetries act on a topological order?

- 2d Topological Orders: anyons equipped w/ fusion and braidings, mathematically a unitary modular tensor category (UMTC)
- *Symmetry Enriched Topological Orders* (**SETO**s): *G*-crossed braided tensor category (for symmetry group *G*)

(1) group action ρ → *anyon permutation*

(2) 2nd group cohomology $H^2_{[\rho]}(G, \mathcal{A})$ → "*symmetry fractionalization*"

Abelian anyons



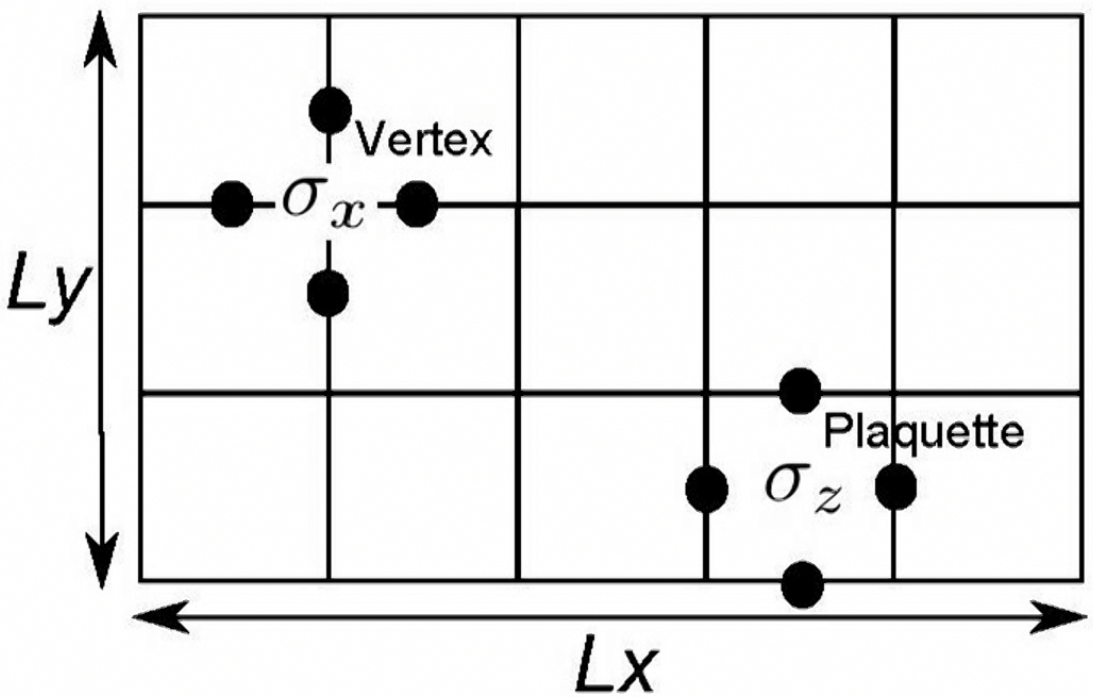
$$U_g^{\mathrm{loc}} U_h^{\mathrm{loc}} = \omega_a(g,h) U_{gh}^{\mathrm{loc}}$$

$$\omega_a(g,h) = S_{\omega(g,h),a}$$

Essin&Hermele, Mesaros&Ran, Hung&Wen, Lu&Vishwanath.....

Barkeshli, Bonderson, Cheng, Wang, 2014; Tarantino, Lindner, Fidkowski, 2016

Toric code as an SET (Kitaev 1997)



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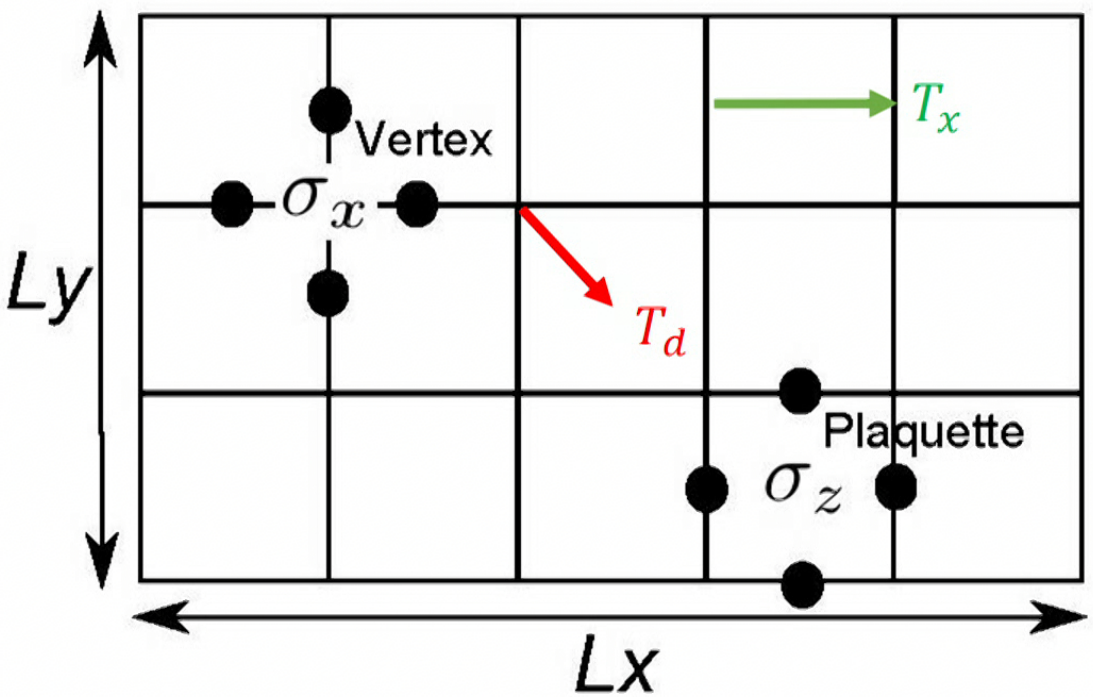

Diagonal translation T_d: $e \overset{T_d}{\leftrightarrow} m$

(Wen, 2003)

Horizontal translation T_x

+ Z2 spin flip $X \equiv \prod_l \sigma_x(l)$:

$(T_x X T_x^{-1} X^{-1})_e = -1$

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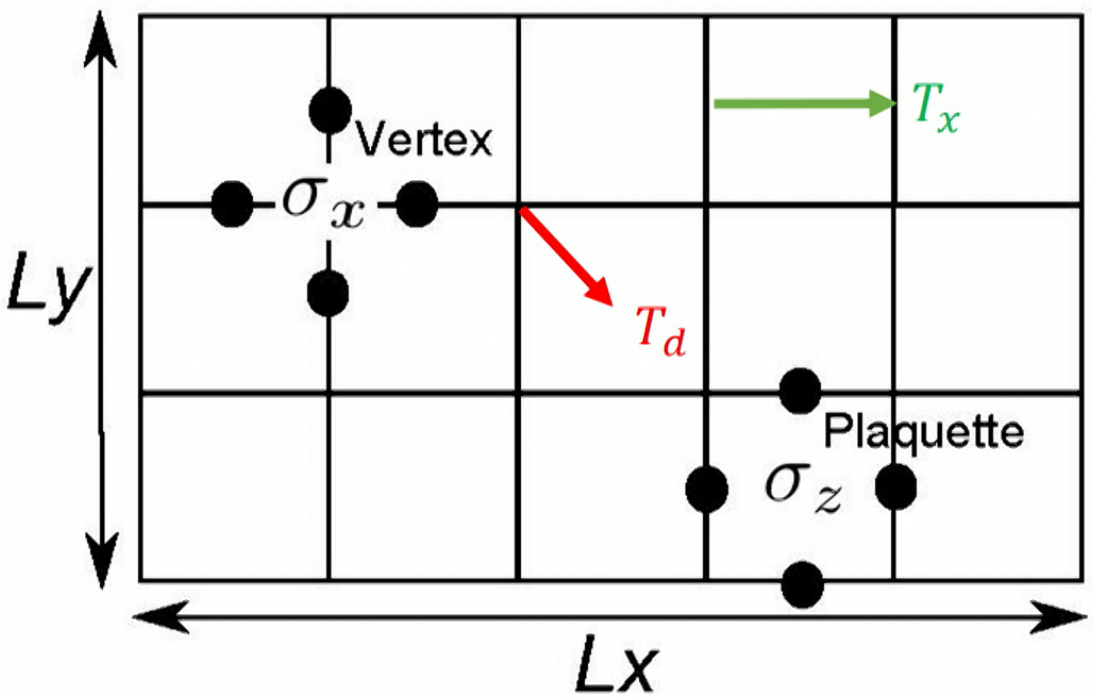

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2nd group cohomology invariant:

$$\frac{\omega(T_x, X)}{\omega(X, T_x)} = m$$

(Jiang, Vishwanath, Lu, unpublished)

What is the mechanism for transitions b/w topological orders?

- "Anyon condensation"

(1) Trivial topological spin (bosons) → "commutative"

(2) Closed form: $A = 1 + a + a^2 + \cdots$ → "algebra"

(3) Unique vacuum in algebra A → "connected"

(4) Decomposable into simple objects → "separable" (or Frobenius)

"Connected étale (commutative & separable) algebra A" (*Kong 2014*)

See *Burnell, 2017* and references therein
Kong, 2014 (category theory of anyon condensation)

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- Condensed phase is a topological order $D = C_A^{loc}$ ("local A-modules")

(i) anyons non-local w.r.t. condensate A are confined

(ii) two anyons related by the condensate are identified

(iii) new anyons can emerge (from splitting of old anyons)

See *Burnell, 2017* and references therein
Kong, 2014 (category theory of anyon condensation)

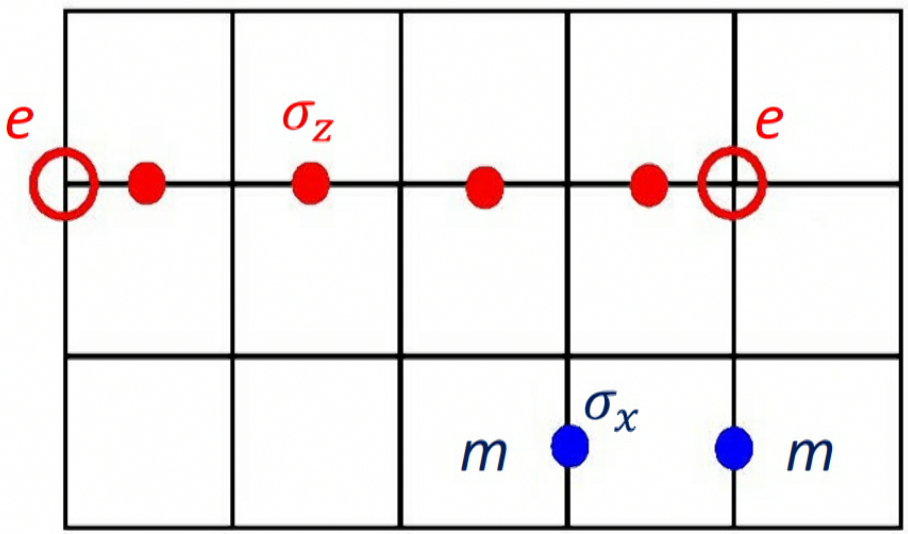
Anyon condensation in toric code

- Condensing electric charge: $A = 1 + e$ (similarly $A = 1 + m$)

- $H = H_{T.C.} - h_z \sum_l \sigma_z(l)$

(Vidal, Dusuel, Schmidt,.....)

- $\lim_{h_z \to \infty} |g.s.\rangle = \prod_l (\frac{1+\sigma_z(l)}{2}) |T.C.\rangle$



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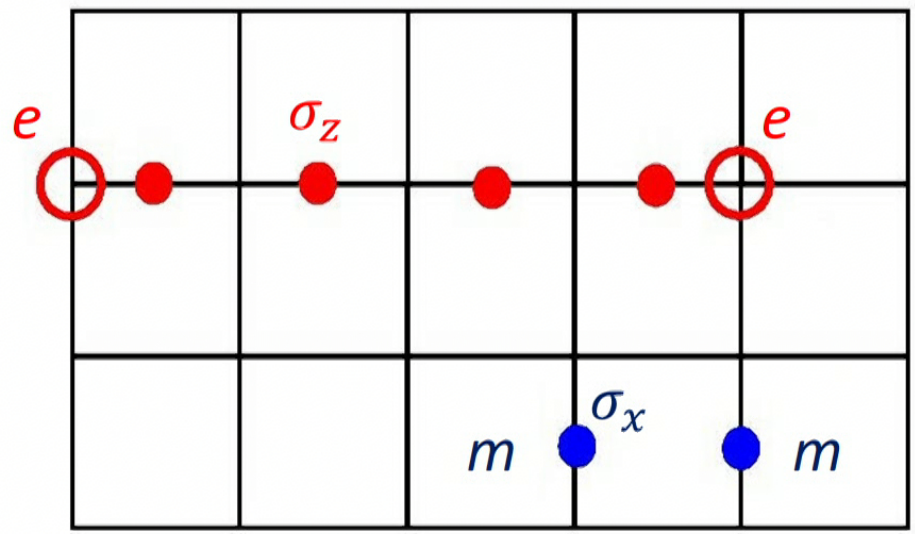
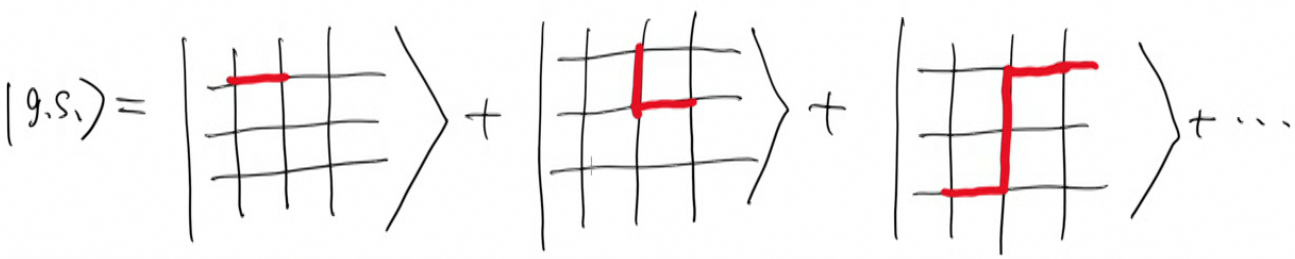
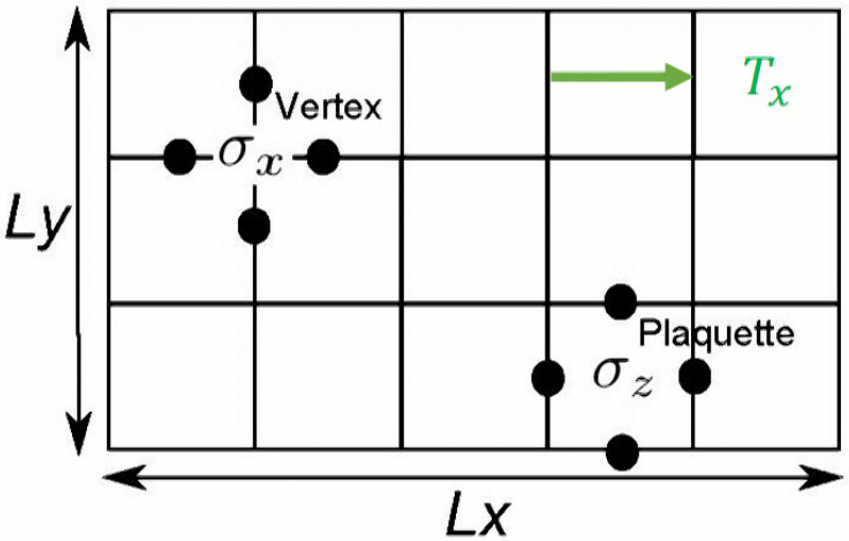


Pictorial representation:
superposition of all open string configurations



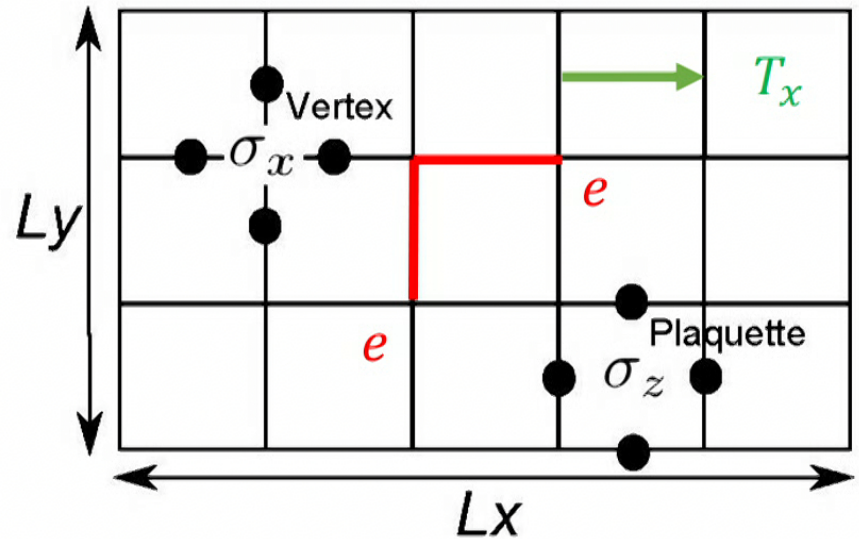
Spontaneous symmetry breaking by e-condensation in toric code



Spontaneous symmetry breaking by e-condensation in toric code

A symmetric Hamiltonian that condenses e (w.r.t. T_x and X):

$$H_{sym} = H_{T.C.} - n_z \sum_{\langle i,j \rangle} \sigma_z(i)\sigma_z(j)$$



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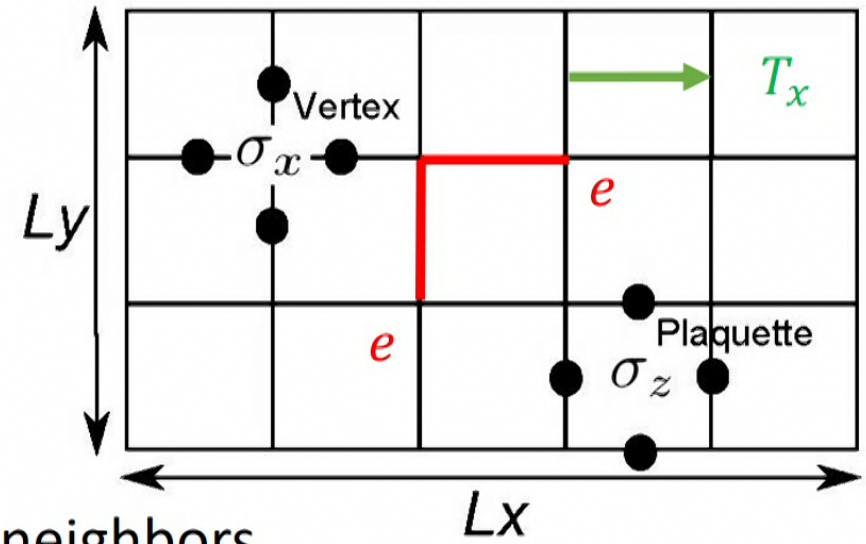


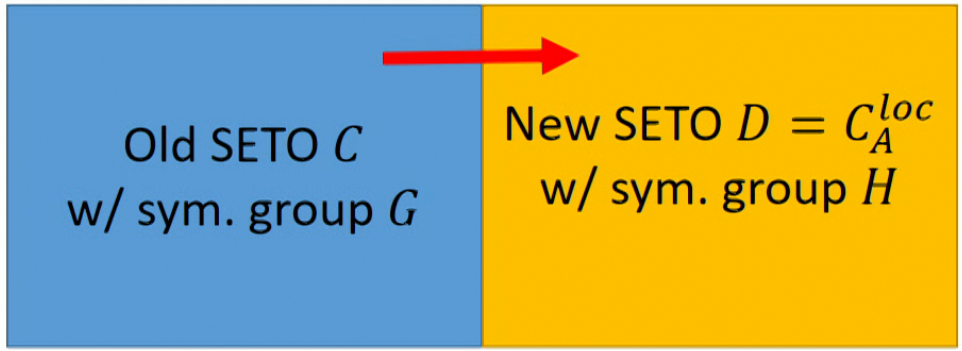
Hopping of e particles b/w next nearest neighbors (within the same sublattice)

→ two dispersion minima for e particles

Anyon condensation in SETOs

Continuous quantum phase transitions

driven by condensing $A = 1 + a + \cdots$



Physical argument: bulk wavefunctions

Consider local symmetry action (within the green region) on the ground state of condensed phase

$$U_g^{\mathrm{loc}} U_h^{\mathrm{loc}} = \omega_a(g,h) U_{gh}^{\mathrm{loc}} \qquad \omega_a(g,h) = S_{\omega(g,h),a}$$

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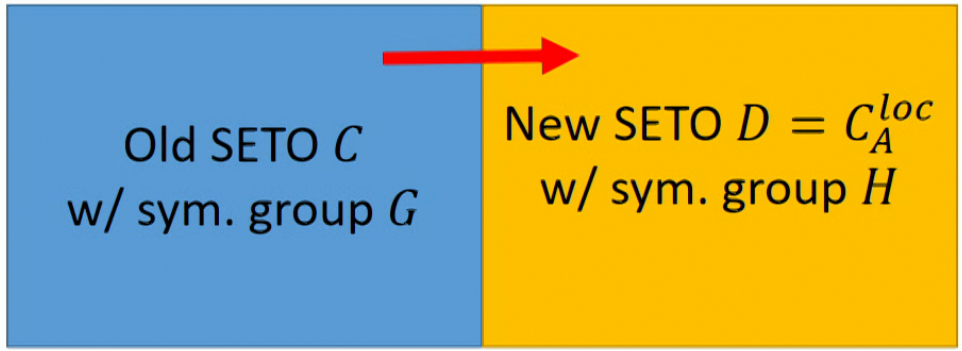

- *Can* $H = G$*?*
- If not, what symmetries must be broken? *Which subgroup* $H \leq G$ *is compatible with the new T.O.?*

Physical argument: bulk wavefunctions

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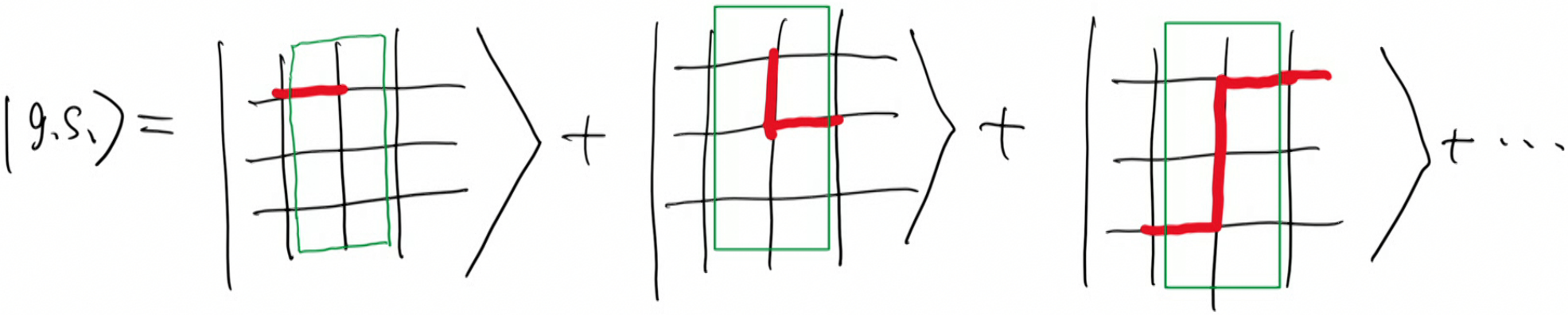

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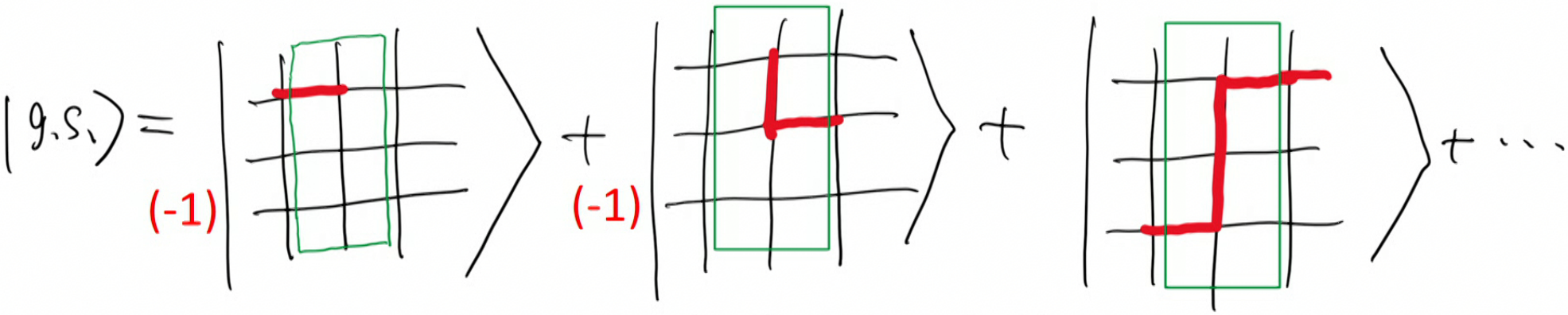

Consider e.g. toric code, $A = 1 + e, \omega(g,h) = m,\ \omega_e(g,h) = S_{m,e} = -1$

Condensed anyons cannot have nontrivial symmetry fractionalization!

Main results: two necessary conditions to preserve symmetry group G

- #1: condensation algebra $A = 1 + a + \cdots$ must be preserved by action of symmetry group G

Bischoff, Jones, Lu, Penneys, 2018

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Main results: two necessary conditions to preserve symmetry group G

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- #2: no symmetry fractionalization within condensed anyons

  $\Leftrightarrow$ splitting of the short exact sequence:

$$1 \longrightarrow \mathrm{Aut}_{\mathcal{C}}(A) \xrightarrow{\iota} \mathrm{Aut}_{\mathcal{C}^G}(I(A)) \xrightarrow{\pi} G \longrightarrow 1$$

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Generalization of "Projective Symmetry Group" (PSG) in spin liquids (*Wen, 2002*)

- Extension of sym. group G by "invariant gauge group" (*IGG*)

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- 1 → *IGG* → *PSG* → G → 1

(1) Roughly speaking, $Aut_{\mathcal{C}}(A)$ is the *IGG* of condensed anyons in $A = 1 + a + \cdots$
(2) $Aut_{\mathcal{C}}(A)$ can be Abelian or non-Abelian
(3) Include the cases where symmetry G can permute anyons

Bischoff, Jones, Lu, Penneys, 2018

Splitting of the exact sequence

$$1 \longrightarrow \mathrm{Aut}_{\mathcal{C}}(A) \xrightarrow{\iota} \mathrm{Aut}_{\mathcal{C}^G}(I(A)) \underset{\mu}{\overset{\pi}{\rightleftarrows}} G \longrightarrow 1$$

- Splitting == existence of a map μ such that $\pi * \mu = id_G$

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$$1 \to N \to M \to G \to 1 \text{ splits} \Longleftrightarrow M = G \rtimes N$$

- The exact sequence splits ~ no symmetry fractionalization on the condensed anyons

Bischoff, Jones, Lu, Penneys, 2018

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- The exact sequence splits ~ no symmetry fractionalization on the condensed anyons
- Splitting → choice of G sym. defects consistent with the condensate
- *Inequivalent splittings (up to "gauge choice") can lead to distinct SETOs after the anyon condensation.*

Bischoff, Jones, Lu, Penneys, 2018

Exp. #0: Landau theory

- Start from a trivial phase $C = Vec$ (no anyons)
- $A = 1$ → condensing local bosons
- End up with the condensed phase $D = C = Vec$ (no anyons)
- $1 \to Aut_C(A) = 1 \to G \to G \to 1$ clearly splits

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- No enforced symmetry breaking: can preserve full sym. G
- Can also breaks down to any subgroup $H \leq G$ since the sequence splits for any H
- *QPTs b/w SPT phases is not within the anyon-condensation framework*

Exp #1.1 Toric code w/ $G = Z_2 = \{1, g\}$ which permutes anyons

- $C_{T.C.} = \{1, e, m, \epsilon = e \times m\}$
- Sym. generator permutes anyons: $e \overset{g}{\leftrightarrow} m$
- Again consider condensing e: $A = 1 + e$

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- $A = 1 + e$ is not preserved by g sym. action → $G = Z_2$ must break
- Enforced SSB from anyon condensation, based on condition #1

Exp #1.2 Toric code w/ $G = Z_2 = \{1, g\}$ where sym. fractionalized on e

- $C_{T.C.} = \{1, e, m, \epsilon = e \times m\}$
- $[\omega] \in H^2_\rho(G, \{1, e, m, \epsilon\})$ given by $\omega(g, g) = m$
- Consider condensing e: $A = 1 + e$
- $Aut_C(A) = Z_2 \cong \{1, m\}$: whether e obtains a (-1) sign or not
- $1 \to Z_2 \cong \{1, m\} \to Z_4 = \{1, \frac{m}{2}, m, \frac{3m}{2}\} \to Z_2 = \{1, \alpha_g \cong \frac{m}{2}\} \to 1$

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Exercise 1.3.4: *Show that there is a short exact sequence*

Group rep. theory lecture by D. Bump

$$1 \longrightarrow Z_n \longrightarrow Z_{nm} \longrightarrow Z_m \longrightarrow 1,$$

and that this short exact sequence is split if and only if n and m are coprime.

Exp #1.3 Toric code w/ $G = Z_2 = \{1, g\}$ where sym. is fractionalized on m

- $C_{T.C.} = \{1, e, m, \epsilon = e \times m\}$
- $[\omega] \in H^2_\rho(G, \{1, e, m, \epsilon\})$ given by $\omega(g, g) = e$
- Consider condensing e: $A = 1 + e$
- $Aut_C(A) = Z_2 \cong \{1, m\}$: whether e obtains a (-1) sign or not
- $1 \to Z_2 \cong \{1, m\} \to Z_2 \times Z_2 = \{1, \frac{e}{2}, m, \frac{e}{2} \times m\} \to Z_2 = \{1, \alpha_g \cong \frac{e}{2}\} \to 1$

Exp #1.3 Toric code w/ $G = Z_2 = \{1, g\}$ where sym. is fractionalized on m

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- Consider condensing e: $A = 1 + e$
- $Aut_C(A) = Z_2 \cong \{1, m\}$: whether e obtains a (-1) sign or not
- $1 \to Z_2 \cong \{1, m\} \to Z_2 \times Z_2 = \{1, \frac{e}{2}, m, \frac{e}{2} \times m\} \to Z_2 = \{1, \alpha_g \cong \frac{e}{2}\} \to 1$

- Two inequivalent splittings $\alpha_g = \frac{e}{2}$ or $\alpha_g = \frac{e}{2} \times m$ (semion! *Levin, Gu 2012*)
- Two distinct condensed phases: trivial phase vs. Z2-SPT phase *(Jiang, Ran 2016)*

Exp. #2 No enforced SSB for any group G

- $C_{SU(2)_{10}} = \left\{[0], \left[\frac{1}{2}\right], [1], \dots [5]\right\}$
- $A = [0] + [3]$
- $C_A^{loc} = SO(5)_1 = \{1, \psi, \sigma\},\ T = \mathrm{diag}\left(1, -1, e^{\frac{i5\pi}{8}}\right)$ *(Eliens, Romers, Bais, 2014)*
- $Aut_C(A) = Z_1$
- $1 \to 1 \to G \to G \to 1$

Exp #3: a universal example for any exact sequence

- C = Quantum double of group N *(Kitaev 1997)*
- Condensing all gauge charges: $A = A(Rep(N))$
- $Aut_C(A) = N$
- Can realize any exact sequence $1 \to N \to M \to G \to 1$ by choosing a proper G action on the quantum double

- Lattice models exist for the SET phases (Heinrich et al 2016, Cheng et al 2017, Tarantino et al 2016)

Take home messages

- Two checks to determine whether symmetry G can be preserved:

(1) Symmetry G preserves the condensed anyons $A = 1 + a + \cdots$

(2) Short exact sequence $1 \to Aut_C(A) \to Aut_{C^G}(I(A)) \to G \to 1$ splits

- Different splittings lead to distinct condensed phases

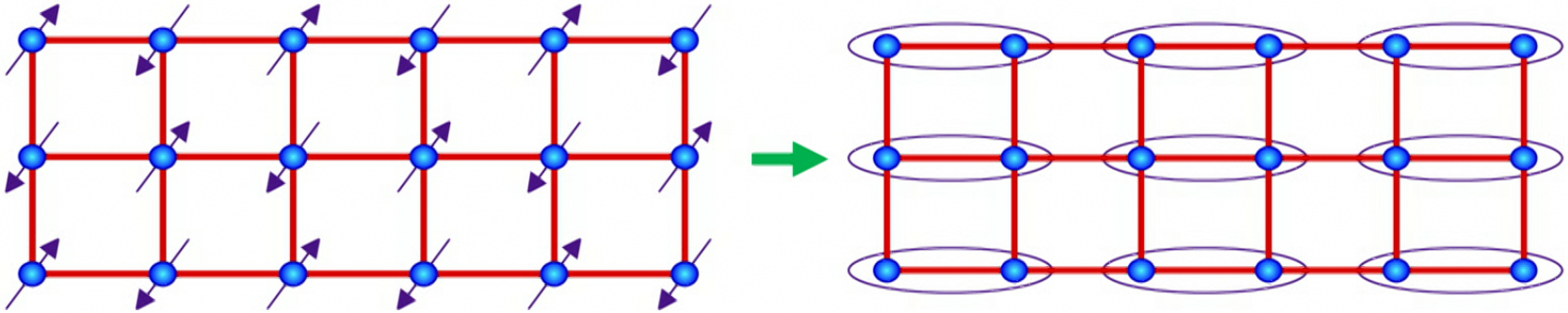
- If G cannot be preserved, the same checking for its subgroup $H \subseteq G$

Same conditions apply to existence of gapped boundaries b/w two SETOs
(Kong&Kitaev, Kong,……)

Bischoff, Jones, Lu, Penneys, 2018

Relation to deconfined quantum critical points (DQCPs)

A simple version: $G = C_4 \times Z_2^T$



Senthil, Vishwanath, Balents, Sachdev, Fisher 2003

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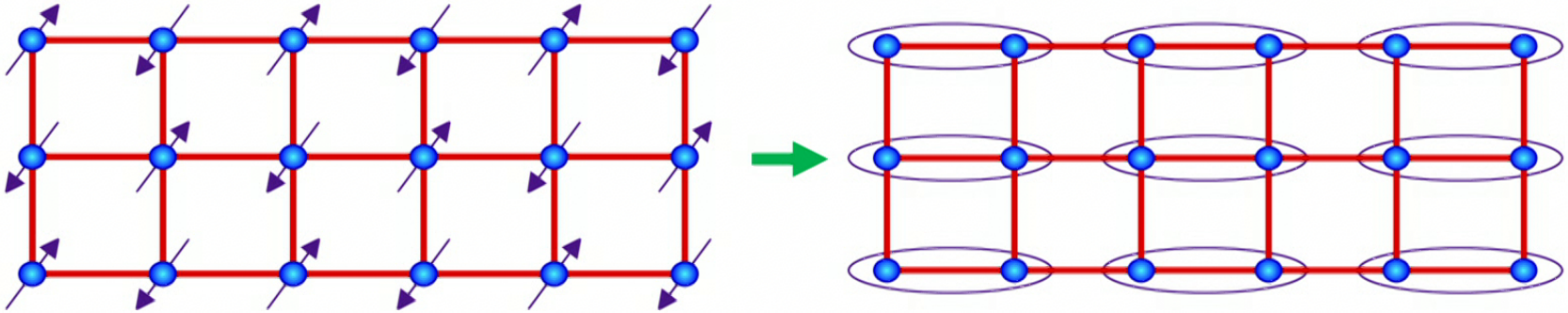
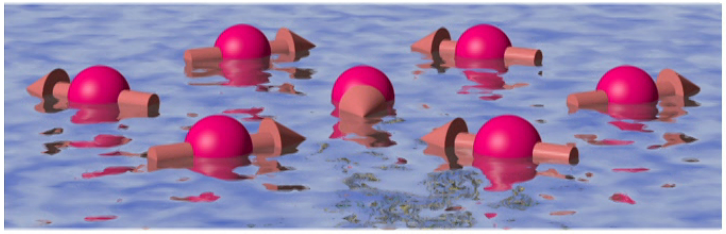
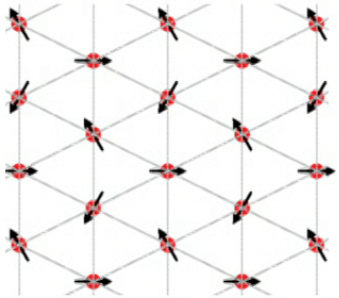




Gapped Topo. Order w/ symmetry G



Confined phase with SSB

Senthil, Vishwanath, Balents, Sachdev, Fisher 2003

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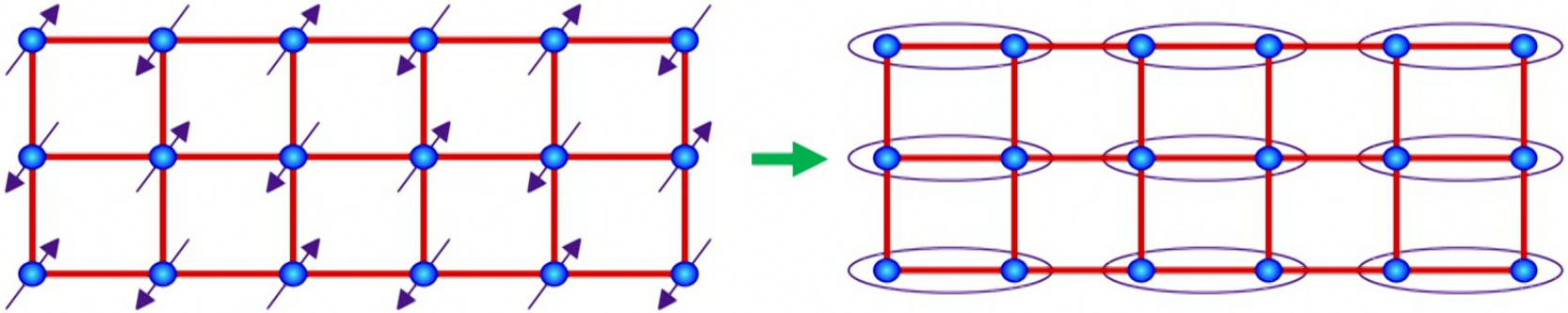

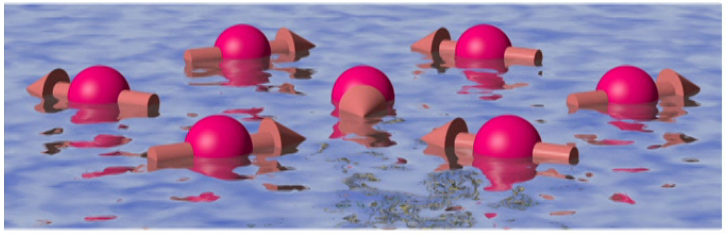

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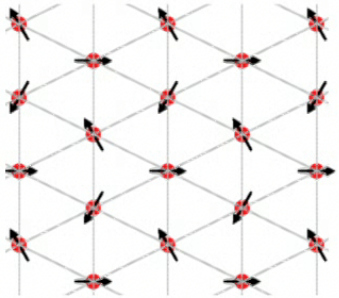

Confined phase with SSB

Can they possibly be related?

Senthil, Vishwanath, Balents, Sachdev, Fisher 2003

Yes, via a dimensional reduction trick

- 2d topological orders on a thin cylinder → 1d SSB (long-range entangled)

(Tao&Thouless 1983, Seidel&Lee 2006, Huang, Chen, Pollmann 2013, Zaletel, Lu, Vishwanath, 2015, Qi, Fu, 2015……)

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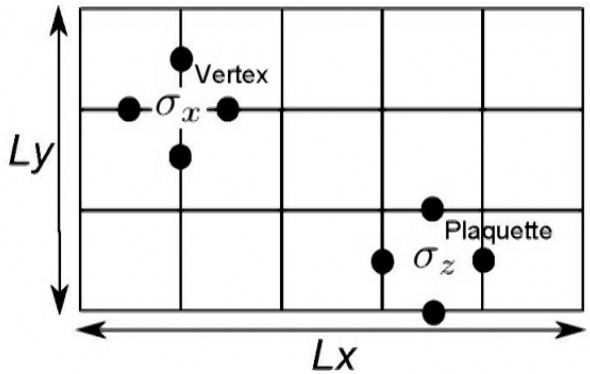

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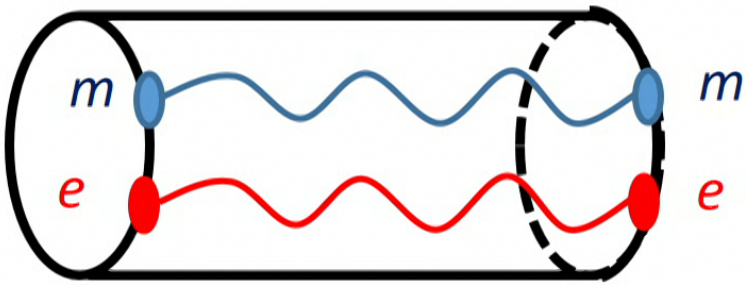
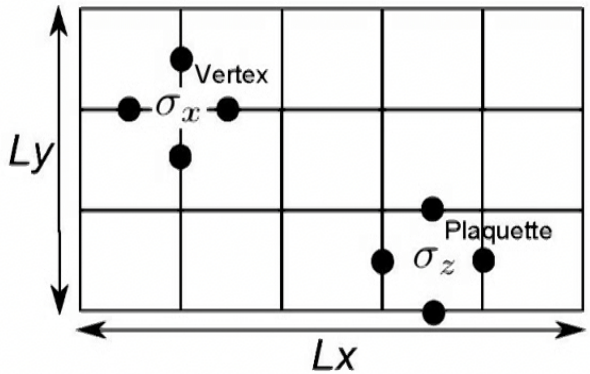
Toric code → 1d SSB w/ emergent $Z_2^e \times Z_2^m$ sym.





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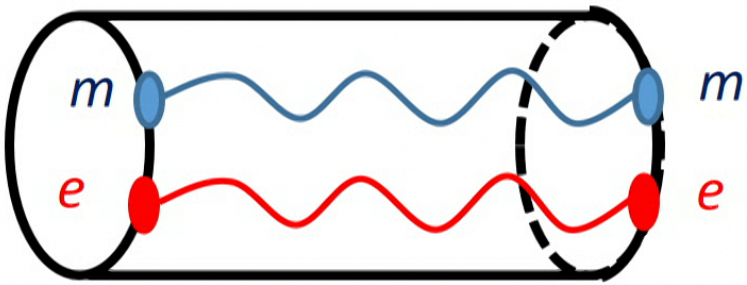
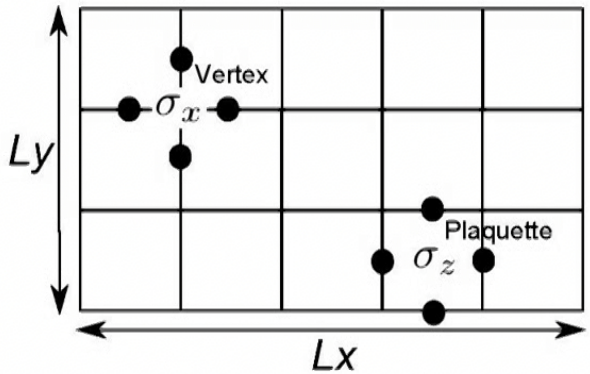


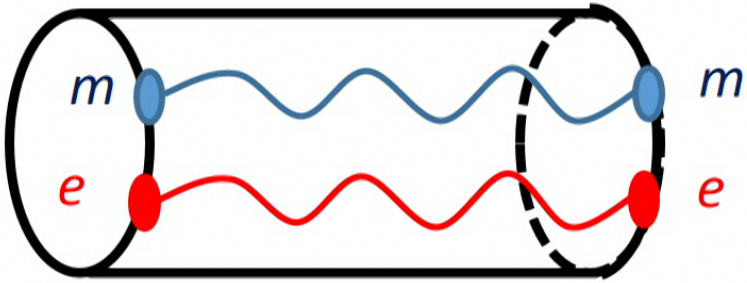


Anyon strings along cylinder length: emergent *global (0-form) symmetries*

Anyon strings around cylinder circumference: order parameter of SSB

Anyon condensation on a thin cylinder

- $H = H_{T.C.} - h_z \sum_l \sigma_z(l)$



Rasmussen, Lu
In preparation

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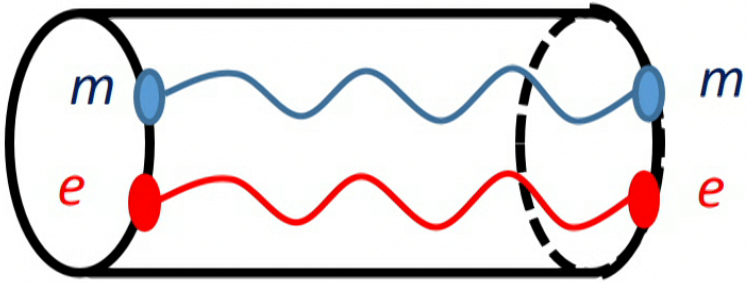
  → explicitly breaking of Z_2^m



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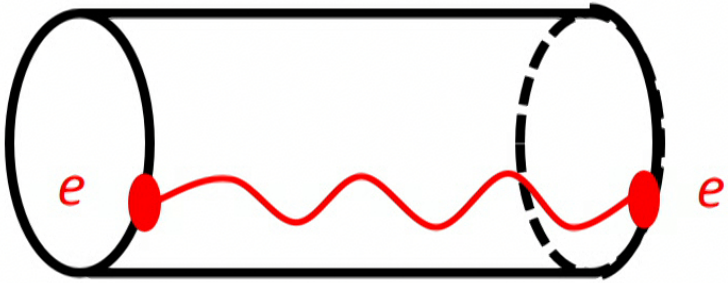

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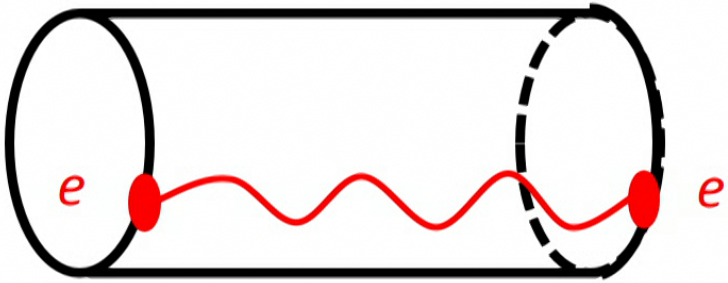

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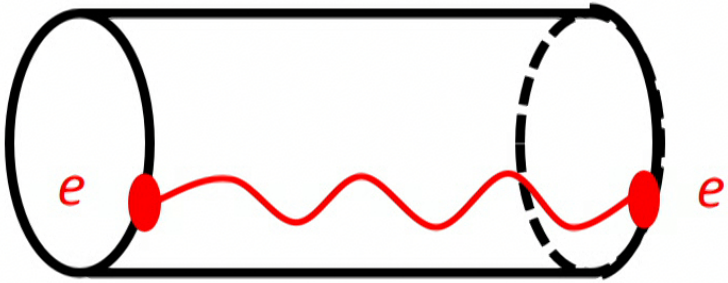

Toric code w/ sym. G Trivial phase w/ SSB of G

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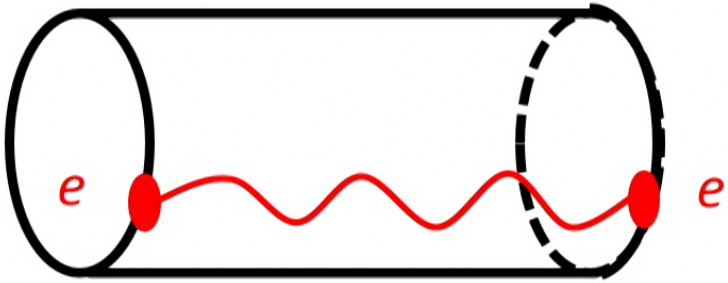

Toric code w/ sym. G | Trivial phase w/ SSB of G

hz

Dimensional reduction to a thin cylinder/torus
→ 1d spin chain w/ $G \times Z_2^e$ symmetry

Rasmussen, Lu
In preparation

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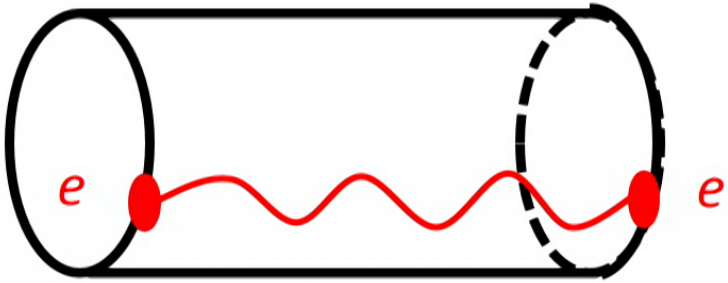

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Preserving G
but SSB of Z_2^e

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Rasmussen, Lu
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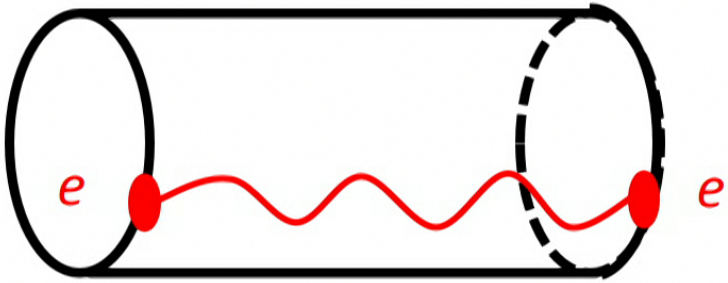

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Preserving G but SSB of Z_2^e

1d DQCP

Preserving Z_2^e but SSB of G

Dimensional reduction to a thin cylinder/torus
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In preparation

Algebraic connection to DQCP

- Consider no anyon permutation by symmetry
- Consider $IGG = Aut_C\ (A) = N$ to be *Abelian* s.t. $H^1\big(N, U(1)\big) = N$
- $1 \to N \to M \to G \to 1 \cong H^2(G, N) = H^2(G, H^1\big(N, U(1)\big))$ does not split (sym. G must be broken by anyon condensation)
- $H^2(G, H^1\big(N, U(1)\big)) \subseteq H^3(G \times N, U(1))$ w/ mixed anomaly of sym. G and sym. N → 2+1-D SPT w. $G \times N$ sym. → DQCP on 1+1-D edge!

(Vishwanath&Senthil 2012,……)

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