Title: Onset of Random Matrix Statistics in Scrambling Systems

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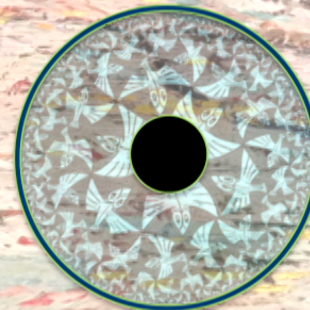
Abstract: The fine grained energy spectrum of quantum chaotic systems, which are widely believed to be characterized by random matrix statistics. A basic scale in these systems is the energy range over which this behavior persists. We defined the corresponding time scale by the time at which the linearly growing ramp region in the spectral form factor begins. We dubbed this ramp time. It is also referred to as the ergodic or Thouless time in the condensed matter physics community. The purpose of my talk is to understand this scale in many-body quantum systems that display strong chaos (such as SYK and spin chain), sometimes referred to as scrambling systems. Using numerical results and analytic estimates for random quantum circuits, I will provide summary of results on scaling of ramp time with system size in the presence/absence of conservation laws.

Onset of Random Matrix Behavior in Scrambling Systems



By H. Gharibyan, M. Hanada, S. Shenker, M. Tezuka [1803.08050]

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Outline

1. Overview of RMT
- Spectrum
- Eigenvalue repulsion
- Spectral form factor

2. Many-Body Chaotic Systems
- Sachdev-Ye-Kitaev
- Chaotic spin chain
- Numerics

3. Random Q. Circuits
- Analog of SFF
- 1D (local) and all-to-all

4. Random Q. Circuit with Conservation Law
- XXZ Random Circuit
- Diffusion and SFF
- Hamiltonian hypothesis

1. Random Matrix Theory:
spectrum and nearest-neighbor distribution

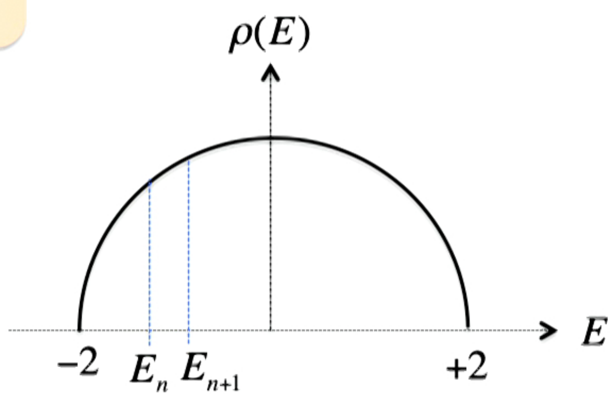
- The **partition function** is (b=1, 2, 4 for GOE/GUE/GSE)

$$\mathcal{Z}_{\mathrm{RMT}} = \int \Big(\prod_{i,j} dH_{ij}\Big)\, e^{-\frac{bL}{2}\mathrm{Tr}(H^2)}$$

H is L-by-L Hermitian matrix

- Spectrum: **Wigner semicircle**

$$\rho_{sc}(E) = \frac{1}{2\pi}\sqrt{4-E^2},$$

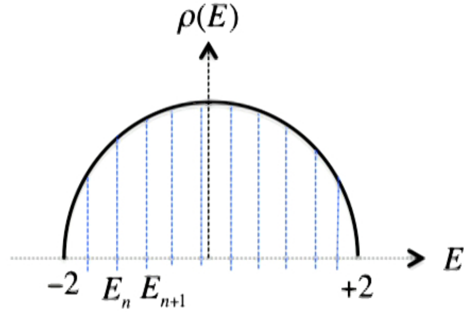


[Wigner, Mehta ,Dyson]

1. Random Matrix Theory:
spectrum and nearest-neighbor distribution

- Distance between E_{n+1} and E_n levels is

$$s_n = E_{n+1} - E_n$$

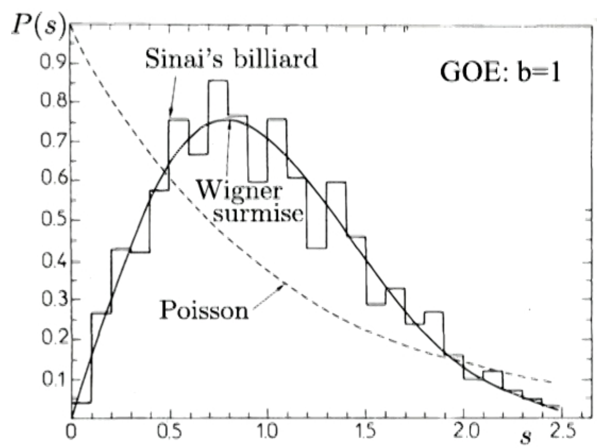


- Nearest - Neighbor: **Wigner surmise**
  - **Average NN distance $\langle s \rangle = 4/L$**
  - **The distribution of NN distance**

$$p(s) \approx c \times s^b e^{-s^2}$$

  - **This is standard test of chaos**

[Wigner, Mehta ,Dyson]



1. Random Matrix Theory: beyond Wigner surmise

- Wigner surmise has to do with **eigenvalue repulsion**.
- To illustrate this one can re-write the GUE action in terms of energy density $\mathcal{Z}_{GUE} = \int D\rho e^{-S[\rho]}$, where

$$S[\rho] = -\frac{L^2}{2}\int dE\rho(E)E^2 + L^2\int dE_1 dE_2\rho(E_1)\rho(E_2)\log|E_1 - E_2|$$

- There is a logarithmic repulsion (also know as Dyson gas)
- Wigner surmise tests this repulsive behavior only in short range, we need a new quantity to test it for further eigenvalues.

1. Random Matrix Theory: spectral form factor

- Denote the Lorentzian evolved partition function by (H is the GUE Hamiltonian)

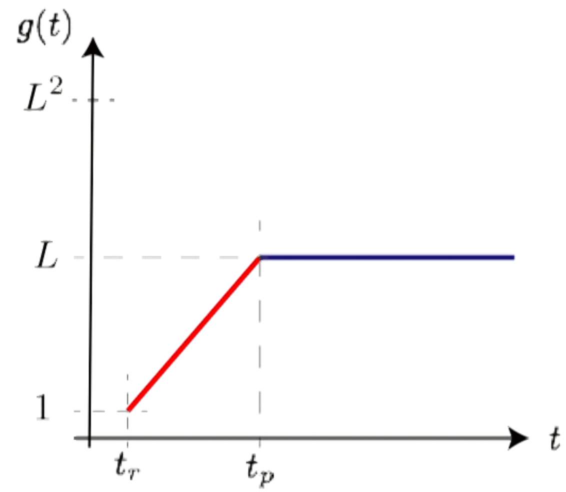
$$Z(\beta,t) = Tr\left(e^{-\beta H - iHt}\right) = \sum_n e^{-\beta E_n - iE_n t}$$

- The spectral form factor (SFF) is

$$\left\langle Z(\beta,t)Z^*(\beta,t)\right\rangle_{RMT}$$

In the rest of the talk we will focus on β=0 case.

$$g(t) = \left\langle Z(0,t)Z^*(0,t)\right\rangle_{RMT}$$



1. Random Matrix Theory: spectral form factor

- In infinite temperature limit (β=0) we can express

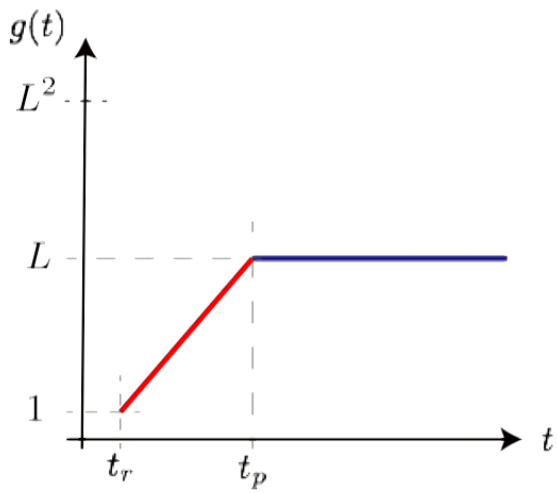
$$g(t) = \left\langle Z(0,t) Z^*(0,t) \right\rangle_{RMT} = \sum_{m,n} e^{-i(E_m - E_n)t}$$

- For the large dimension L, we can express it as continues integral of energies

$$g(t) = \int dE_1\, dE_2 L^2 \;\; R(E_1, E_2)\; e^{-i(E_1 - E_2)t}$$

- $R(E_1,E_2)$ is the two-point eigenvalue correlator for RMT

1. Random Matrix Theory: spectral form factor



[Dyson, Gaudin, Mehta]

- **The Ramp** (red) & **Plateau** (blue)

$$g(t) = \begin{cases} \frac{t}{2}, & t < t_p = 2L \\ L, & t \geq t_p = 2L \end{cases}$$

Example: GUE ensemble

- Important timescales
  - Ramp time: time scale t_r that universal linear growth starts. [*** $t_r \sim O(1)$ ***]
  - Plateau time: time scale t_p when ramp saturates. [*** $t_p \sim L$ ***]

2. Many-Body Chaos: overview

One-body chaos is a relatively old field of study. Spectral form factor, ramp and plateau are well understood in a single particle random hoping problem.

- SFF has a linear ramp and a plateau
- Ramp starts at the time t_{diff} that is the time for a particle to diffuse across the system. Thus, $t_r \sim N^2$ for system of size N.
- The corresponding energy is known as **Thouless energy**.

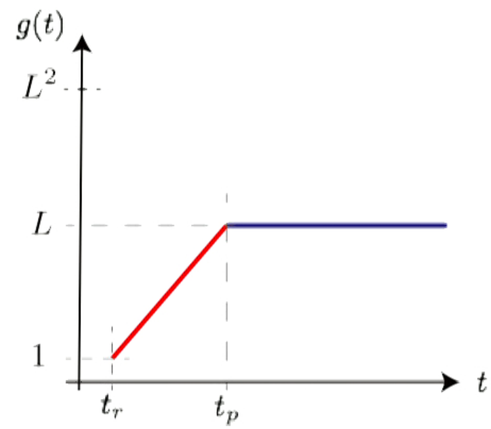
[Altshuler, Shklovskii]

2. Many-Body Chaos: overview

Many-body chaos on the other hand is a relatively young and rapidly growing field of research. Recent results can be categorized into two themes,

A. Short time chaos
- OTOC / Scrambling
- Bound on quantum chaos

B. Long time chaos
- Nearest-neighbor distributions
- Spectral form factor (SFF)



2. Many-Body Chaos: SYK model

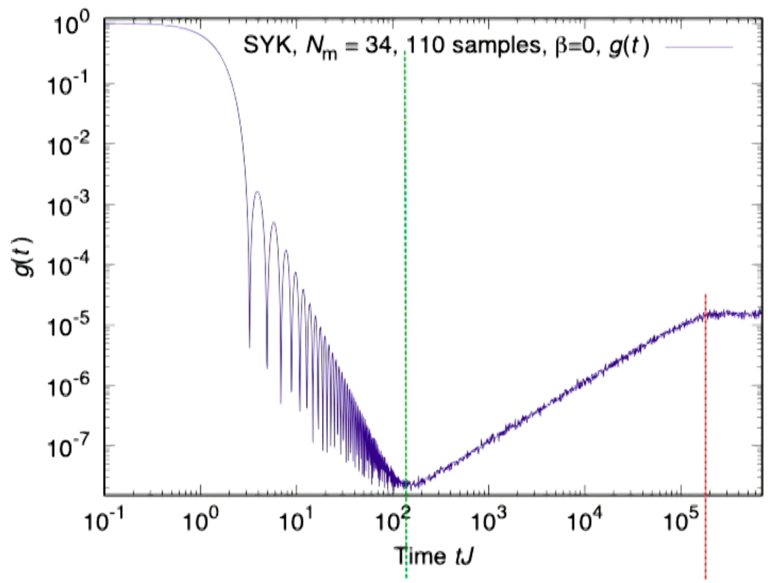
- Sachdev-Ye-Kitaev (SYK) is a theory of N Majorana fermions

$$H = \frac{1}{4!} \sum_{i,j,k,l=1}^{N} J_{ijkl} \; \psi_i \psi_j \psi_k \psi_l$$

[Kitaev, Sachdev-Ye, Maldacena -Stanford, Polchinski et al.]

- Here ψ_i denote Majorana fermions $\{\psi_i, \psi_j\} = \delta_{ij}$ and $\psi_i^+ = \psi_i$.
- J's are independent Gaussian random numbers with mean 0 and width $\sqrt{6/N^3}J$
- The Hilbert space of SYK model had dimension $L=2^{N/2}$.

2. Many-Body Chaos: numerical SFF in SYK model



$$t_r \propto ???$$

$$t_p \propto 2^{N/2}$$

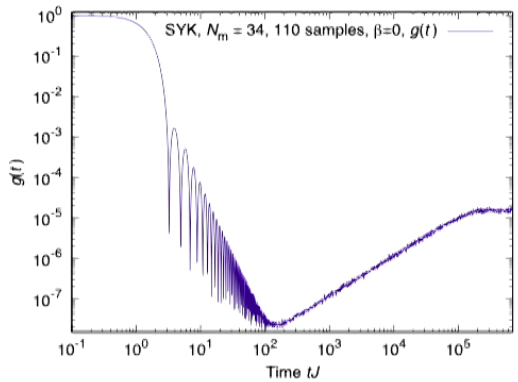
[CGHPSSSST]

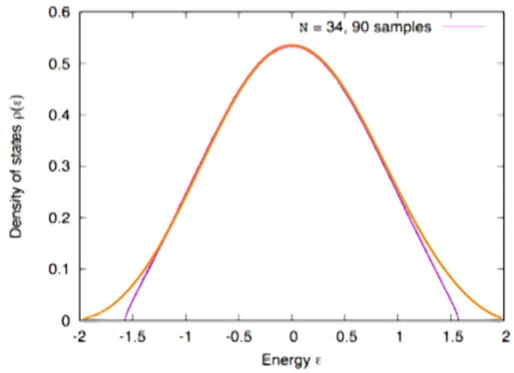
- The **Slope** is non-universal and comes from the sharp edge

$$g(t) = ZZ^*(t) \propto \frac{1}{t^3}$$

- The **Ramp** and **Plateau** are believed to be universal in quantum chaotic systems

2. Many-Body Chaos: <YY*> instead of <ZZ*>





- How early does the ramp start?
- The place slope & ramp intersect is dubbed dip time
- To really study the start time of the universal ramp one can use alternative quantity to SFF [Stanford]

$$Y(\alpha,t)Y^*(\alpha,t) = \sum_{m,n} e^{-\alpha(E_m^2+E_n^2)} e^{-it(E_m-E_n)}$$

3. Random Circuit Model: overview

- At this point, there is no systematic way to compute spectral form factor analytically in many-body chaotic systems.

- In the case of SYK, there has been some recent progress using large N properties, but the question is still not settled. [Shenker et.al, Altland, Bagrets]

- To understand the physical mechanism behind ramp and plateau, specifically the ramp time. We instead turn to random quantum circuit models that can be studied analytically.

3. Random Circuit Model: 1D Circuit

- Random quantum circuit in **1D (brickwork)**
  - N qubits on 1D lattice
  - We have two qubit Haar random gate

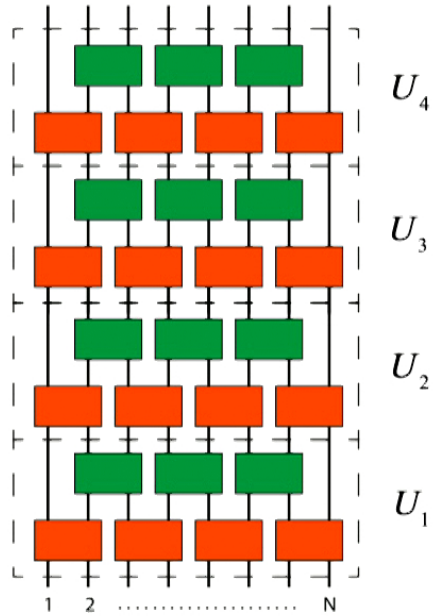
$$g_{ij} \subset U(4)$$

  - i-th time step is a unitary

$$U_i = g_{23} g_{45} \cdots g_{(N-2)(N-1)} g_{12} g_{34} \cdots g_{(N-1)N}$$

  - Full unitary at time k

$$U(t) = U_t U_{t-1} \dots U_2 U_1$$



3. Random Circuit Model: spectral properties

- We are interested to compute at what time **t** this quantity gets very close to it's Haar value.

$$f(k,t) = \left\langle Tr[U^k(t)] \times Tr[U^{*k}(t)] \right\rangle_{RQC}$$

- Consider k=2 case. Slowest decaying monomial is: $U_{aa}U_{aa}U^*_{aa}U^*_{aa}$
- We study evolution of $U|a\rangle\langle a|U^\dagger$, where $|a\rangle = |00..00\rangle$
- We can expend operator

$$|a\rangle\langle a| = \frac{1}{2^N}(I+Z)^{\otimes N}$$

3. Random Circuit Model:
Markov process on Pauli strings

- Random circuit evolution → Markov chain on Pauli strings

$$O(t) = U(t)O(0)U^{+}(t) \rightarrow O(t) = \sum_{p} \gamma_{p}(t)\sigma_{p}$$

$$\sigma_{p} = IIZZIIXIZYIZI$$

[Harrow-Low]

- Defines a Markov process on Pauli strings: IXXZZXXIIZZ

- Markov rules:
  - Identity remains invariant: **II → II**
  - All remaining 2 qubit strings spread uniformly: **AB → 15 others**

3. Random Circuit Model:
ramp time in a quantum circuit model

- We need to equilibrate a typical string in

$$|a\rangle\langle a| = \frac{1}{2^N}(I+Z)^{\otimes N}$$

- That is of a form: $IIZZZIIZZIZIIZZIII$

- And are easy equilibrate for both **1D** and **all-to-all** random quantum circuit models. The ramp time estimate for those models

$$t_r \propto \log(N)$$

[Yoshida et. al]

This was in conflict with our numerics for Hamiltonian systems.

4. Random Circuit with Conservation Law:

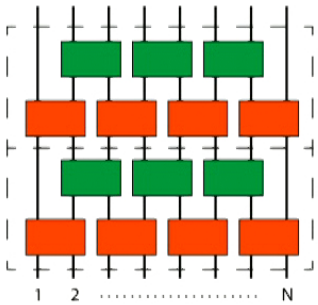
Hamiltonian versus random quantum circuit

- The key difference between Hamiltonian and random quantum circuit model is the **conservation law**.
  - Hamiltonian evolution conserve total energy of the system.

- Can we construct an analytically tractable **random circuit with a conserved charge**? (not energy that is hard)
  - XXZ random quantum circuit, which is a circuit that conserves the **total spin in Z-direction**.
  - We now study ramp time for the XXZ random q-circuit.

4. Random Circuit with Conservation Law:
random quantum circuit with conserved total spin

- Study the XXZ Random Q-Circuit, which has a **conserved charge** (total spin in Z direction) [Khemani-Vishwanath-Huse]

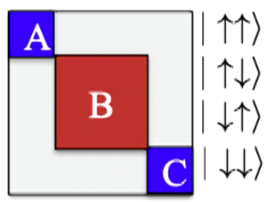


$$g_{ij} \subset \Gamma$$

$$U_i = g_{23} g_{45} \cdots g_{(N-2)(N-1)} g_{12} g_{34} \cdots g_{(N-1)N}$$

$$U(t) = U_t U_{t-1} \ldots U_2 U_1$$

- Where gateset Γ is such that

$g_{ij} =$ 

***A and C is Haar random from U(1)**
***B is Haar random from U(2)**

In the basis of spin Z

4. Random Circuit with Conservation Law:
Markov process for XXZ random circuit

- The dynamics of this circuit can also be interpreted as Markov process on the string of Pauli's, but a different one.

- Again, we compute f(k, t) for given spin sector.

- Markov rules for XXZ random quantum circuit;
  - $\{II, ZZ, (IZ+ZI)/2\}$ are invariant [1710.09835, 1803.08050]
  - $\{I\sigma^{\pm}, Z\sigma^{\pm}, \sigma^{\pm}I, \sigma^{\pm}Z\}$ are uniformly mixed
  - $\{\sigma^{+}\sigma^{-}, \sigma^{-}\sigma^{+}, (IZ-ZI)/2\}$ are uniformly mixed
  - $\{\sigma^{+}\sigma^{+}, \sigma^{-}\sigma^{-}\}$ each pick up a random phase

Summary

- **Conclusion:** In a chaotic many-body system with conserved charge, onset of the ramp is the time it takes for a local charge to diffusive in the entire system.

Evidence:

- Single particle hoping problem (conserved particle number).
- **XXZ random quantum circuits with conserved Z spin.**
- **Numerical results for chaotic Hamiltonian spin chain.**
- SYK analytic result with G and Σ variables by [Shenker et. al 1806.06840].