Title: When quantum-information scrambling met quasiprobabilities

Speakers: Nicole Yunger Halpern

Collection: Quantum Matter: Emergence & Entanglement 3

Date: April 23, 2019 - 4:30 PM

URL: http://pirsa.org/19040100

Abstract: Two topics have been gaining momentum in different fields of physics: At the intersection of condensed matter and high-energy physics lies the out-of-time-ordered correlator (OTOC). The OTOC reflects quantum many-body equilibration; chaos; and scrambling, the spread of quantum information through many-body entanglement. In quantum optics and quantum foundations, quasiprobabilities resemble probabilities but can become negative and nonreal. Such nonclassical values can signal nonclassical physics, such as the capacity for superclassical computation. I unite these two topics, showing that the OTOC equals an average over a quasiprobability distribution. The distribution, a set of numbers, contains more information than the OTOC, one number that follows from coarse-graining over the distribution. Aside from providing insight into the OTOCâ€™s fundamental nature, the OTOC quasiprobability has several applications: Theoretically, the quasiprobability interrelates scrambling with uncertainty relations, nonequilibrium statistical mechanics, and chaos. Experimentally, the quasiprobability points to a scheme for measuring the OTOC (via weak measurements, which refrain from disturbing the measured system much). The quasiprobability also signals false positives in attempts to measure scrambling of open systems. Finally, the quasiprobability underlies a quantum advantage in metrology.

References

â€¢ NYH, Phys. Rev. A 95, 012120 (2017). https://journals.aps.org/pra/abstract/10.1103/PhysRevA.95.012120
â€¢ NYH, Swingle, and Dressel, Phys. Rev. A 97, 042105 (2018). https://journals.aps.org/pra/abstract/10.1103/PhysRevA.97.042105
â€¢ NYH, Bartolotta, and Pollack, accepted by Comms. Phys. (in press). https://arxiv.org/abs/1806.04147
â€¢ GonzÃ lez Alonso, NYH, and Dressel, Phys. Rev. Lett. 122, 040404 (2019). https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.122.040404
â€¢ Swingle and NYH, Phys. Rev. A 97, 062113 (2018). https://journals.aps.org/pra/abstract/10.1103/PhysRevA.97.062113
â€¢ Dressel, GonzÃ lez Alonso, Waegell, and NYH, Phys. Rev. A 98, 012132 (2018). https://journals.aps.org/pra/abstract/10.1103/PhysRevA.98.012132
â€¢ Arvidsson-Shukur, NYH, Lepage, Lasek, Barnes, and Lloyd, arXiv:1903.02563 (2019). https://arxiv.org/abs/1903.02563

WHEN ~~HARRY~~ quantum-information scrambling MET ~~SALLY...~~ quasiprobabilities

Nicole Yunger Halpern

Harvard-Smithsonian ITAMP
(Institute for Theoretical Atomic, Molecular, and Optical Physics)
Harvard University Department of Physics

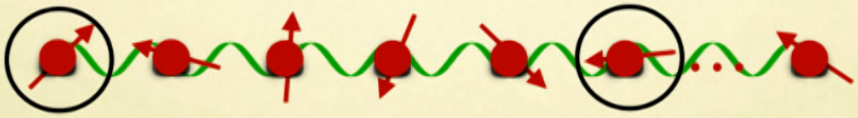
QUANTUM MATTER 3, PERIMETER INSTITUTE, 4/23/19

WHEN quantum-information scrambling ~~HARRY~~ MET quasiprobabilities ~~SALLY~~...

- 2 concepts, favored by different physics communities and seemingly disjoint
- Combination offers experimental advantages and theoretical insights

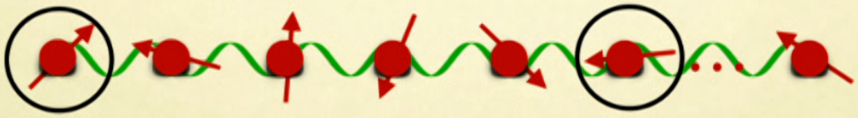
(1) Quantum-information scrambling

- Fields: condensed matter, high-energy physics, quantum information
- Setting: quantum many-body system
- Information initially encoded locally
- Interactions → especially nonintegrable, nonlocal
  information spreads across the system through many-body entanglement
  can't be recovered by any local probe
  "scrambled"
- The literature:



(1) Quantum-information scrambling

- Fields: condensed matter, high-energy physics, quantum information
- Setting: quantum many-body system
- Information initially encoded locally
- Interactions → especially nonintegrable, nonlocal
  information spreads across the system through many-body entanglement
  can't be recovered by any local probe
  "scrambled"
- The literature: 



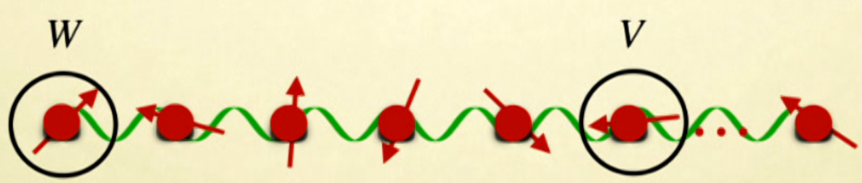
(1) Quantum-information scrambling

Out-of-time-ordered correlator (OTOC)

- Signal of scrambling
- $\langle W^{\dagger}(t) V^{\dagger} W(t) V \rangle$

Interaction evolution

Local Hermitian and/or unitary operators
(e.g., 1-qubit Pauli operators)



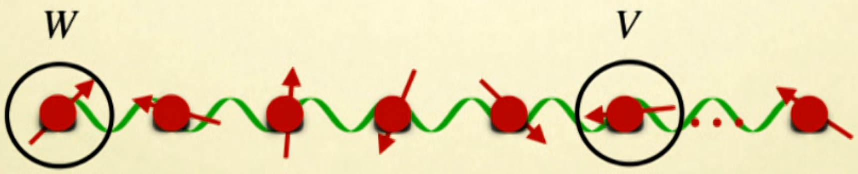
(1) Quantum-information scrambling

Out-of-time-ordered correlator (OTOC)

- Signal of scrambling
- $\langle W^\dagger(t) V^\dagger W(t) V \rangle$ ← $\mathrm{Tr}(\ldots\rho)$

Interaction evolution   Local operators

- Applications to quantum chaos, phases of matter, black-hole information paradox, quantum advantage/supremacy, …



(2) Quasiprobability distributions

- From quantum optics, as well as quantum foundations and quantum computation
- Like probability distributions
- But can behave nonclassically
  - Assume negative and nonreal values
  - Can signal nonclassical physics

Main point

- **OTOC = average over quasiprobability distribution**

  coarse-grained   more detailed

- **The OTOC quasiprobability offers several experimental advantages and theoretical insights.**

Where we're headed

- Review: out-of-time-ordered correlator
- OTOC = average over quasiprobability distribution
- Quasiprobability background
- Weak-measurement scheme
- Symmetry breaking and chaos-like bifurcation
- Theoretical and experimental applications of the OTOC quasiprobability
- Outlook

References and collaborators

NYH, Phys. Rev. A **95**, 012120 (2017).

NYH, Swingle, and Dressel, Phys. Rev. A **97**, 042105 (2018).

González Alonso, NYH, and Dressel, Phys. Rev. Lett. **122**, 040404 (2019).

Swingle and NYH, Phys. Rev. A **97**, 062113 (2018).

Dressel, Gonzàlez Alonso, Waegell, and NYH, Phys. Rev. A **98**, 012132 (2018).

NYH, Bartolotta, and Pollack, accepted by Comms. Phys. (in press).

Arvidsson-Shukur, NYH, Lepage, Lasek, Barnes, and Lloyd, arXiv:1903.02563 (2019).

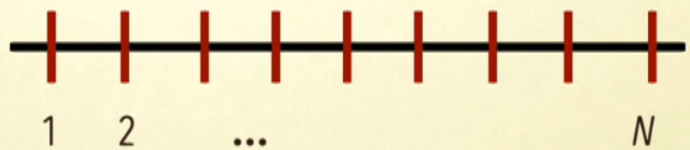
Out-of-time-ordered correlator

OTOC: Set-up

- Quantum many-body system

(1) Boundary dual of a gravitational theory

(2) Chain of spins



Motivating the OTOC's definition with chaos

- Chaos $\longleftrightarrow$ sensitivity to initial conditions
- Compare 2 protocols that differ by an initial perturbation.

(1) $|\psi\rangle$

Motivating the OTOC's definition with chaos

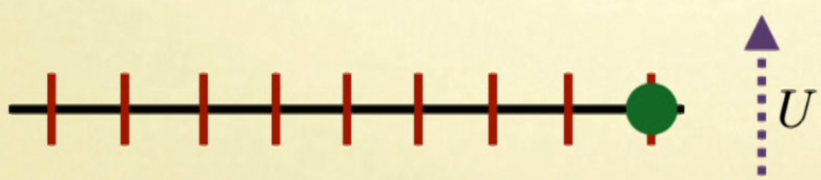
- Chaos $\longleftrightarrow$ sensitivity to initial conditions
- Compare 2 protocols that differ by an initial perturbation.

(1) $|\psi\rangle \quad \mapsto \qquad\qquad V\,|\psi\rangle$

Motivating the OTOC's definition with chaos

- Chaos $\longleftrightarrow$ sensitivity to initial conditions
- Compare 2 protocols that differ by an initial perturbation.

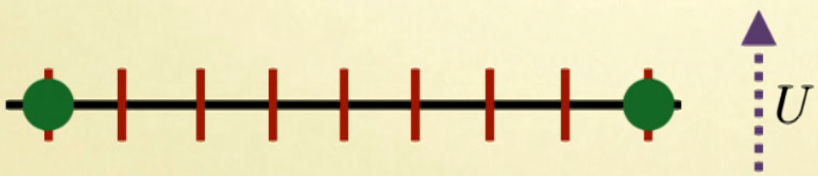
(1) $|\psi\rangle \quad \mapsto \qquad\qquad U \; V \; |\psi\rangle$



Motivating the OTOC's definition with chaos

- Chaos $\longleftrightarrow$ sensitivity to initial conditions
- Compare 2 protocols that differ by an initial perturbation.

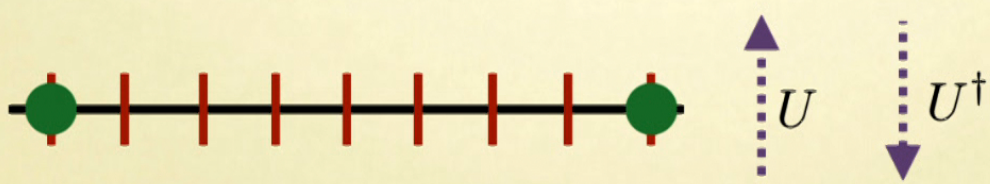
(1) $|\psi\rangle \mapsto \quad \mathcal{W}\ U\ V\ |\psi\rangle$



Motivating the OTOC's definition with chaos

- Chaos $\longleftrightarrow$ sensitivity to initial conditions
- Compare 2 protocols that differ by an initial perturbation.

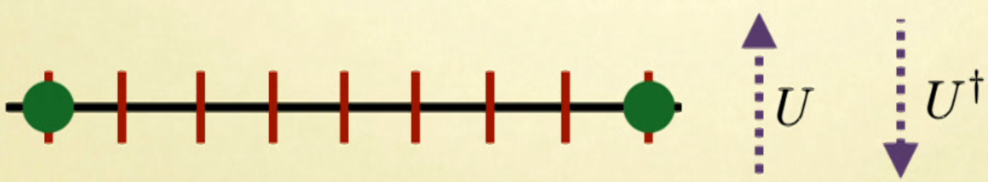
(1) $|\psi\rangle \;\mapsto\; U^\dagger \, \mathcal{W} \, U \, V \, |\psi\rangle$



Motivating the OTOC's definition with chaos

- Chaos $\longleftrightarrow$ sensitivity to initial conditions
- Compare 2 protocols that differ by an initial perturbation.

(1) $|\psi\rangle \mapsto U^\dagger \mathcal{W} \, U \, V \, |\psi\rangle =: |\psi'\rangle$



Motivating the OTOC's definition with chaos

- Chaos $\longleftrightarrow$ sensitivity to initial conditions
- Compare 2 protocols that differ by an initial perturbation.

(1) $|\psi\rangle \;\mapsto\; U^\dagger \, \mathcal{W} \; U \; V \, |\psi\rangle =: |\psi'\rangle$

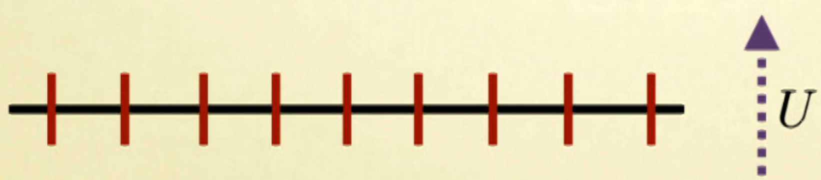
(2) $|\psi\rangle$

Motivating the OTOC's definition with chaos

- Chaos $\longleftrightarrow$ sensitivity to initial conditions
- Compare 2 protocols that differ by an initial perturbation.

(1) $|\psi\rangle \mapsto U^\dagger \, \mathcal{W} \, U \, V \, |\psi\rangle =: |\psi'\rangle$

(2) $|\psi\rangle \mapsto U \, |\psi\rangle$

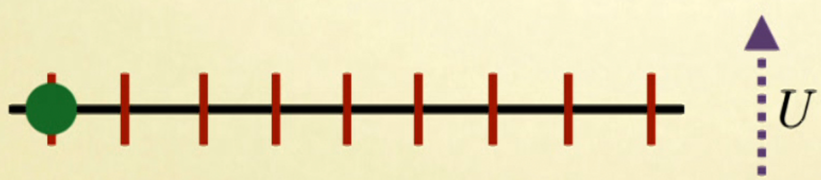


Motivating the OTOC's definition with chaos

- Chaos $\longleftrightarrow$ sensitivity to initial conditions
- Compare 2 protocols that differ by an initial perturbation.

(1) $|\psi\rangle \;\mapsto\; U^\dagger \, \mathcal{W} \; U \; V \, |\psi\rangle =: |\psi'\rangle$

(2) $|\psi\rangle \;\mapsto\; \mathcal{W} \; U \, |\psi\rangle$

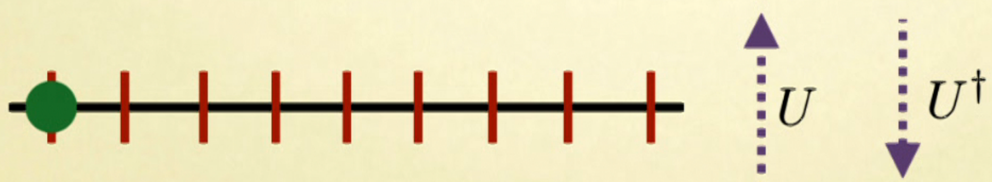


Motivating the OTOC's definition with chaos

- Chaos $\longleftrightarrow$ sensitivity to initial conditions
- Compare 2 protocols that differ by an initial perturbation.

(1) $|\psi\rangle \;\mapsto\; U^\dagger \, \mathcal{W} \; U \; V \, |\psi\rangle =: |\psi'\rangle$

(2) $|\psi\rangle \;\mapsto\; U^\dagger \, \mathcal{W} \; U \, |\psi\rangle$

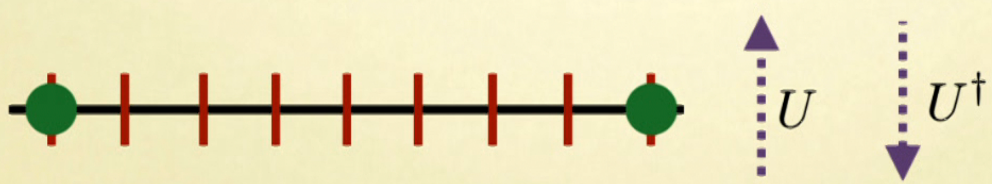


Motivating the OTOC's definition with chaos

- Chaos $\longleftrightarrow$ sensitivity to initial conditions
- Compare 2 protocols that differ by an initial perturbation.

(1) $|\psi\rangle \mapsto U^\dagger \mathcal{W} \; U \; V \, |\psi\rangle =: |\psi'\rangle$

(2) $|\psi\rangle \mapsto V \; U^\dagger \mathcal{W} \; U \, |\psi\rangle$



Motivating the OTOC's definition with chaos

- Chaos $\longleftrightarrow$ sensitivity to initial conditions
- Compare 2 protocols that differ by an initial perturbation.

(1) $|\psi\rangle \;\mapsto\; U^\dagger \, \mathcal{W} \; U \; V \, |\psi\rangle \; =: \; |\psi'\rangle$

(2) $|\psi\rangle \;\mapsto\; V \; U^\dagger \, \mathcal{W} \; U \, |\psi\rangle \; =: \; |\psi''\rangle$

- How much does an initial perturbation change the final state?
  - Overlap: $|\langle \psi'' | \psi' \rangle| = \boxed{\left| \langle \mathcal{W}^\dagger(t) V^\dagger \mathcal{W}(t) V \rangle \right|} =: |F(t)|$

$$\sim 1 - \frac{1}{N} \, e^{\lambda_{\mathrm{L}} t}$$

Lyapunov-type exponent

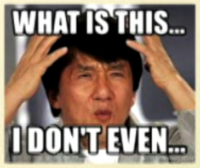
Motivation for the definition of the OTOC quasiprobability

$$F(t) = \mathrm{Tr}\left(\mathcal{W}^\dagger(t)\, V^\dagger\, \mathcal{W}(t)\, V\, \rho\right)$$

$$\sum_{w_2} w_2^*\, \Pi_{w_2}^{\mathcal{W}(t)} \qquad \sum_{v_2} v_2^*\, \Pi_{v_2}^{V} \qquad \sum_{w_1} w_1\, \Pi_{w_1}^{\mathcal{W}(t)} \qquad \sum_{v_1} v_1\, \Pi_{v_1}^{V}$$

$v_1 \longleftarrow \pm 1$

$$= \sum_{v_1, w_1, v_2, w_2} w_2^* v_2^* w_1 v_1\ \underbrace{\mathrm{Tr}\left(\Pi_{w_2}^{\mathcal{W}(t)} \Pi_{v_2}^{V} \Pi_{w_1}^{\mathcal{W}(t)} \Pi_{v_1}^{V}\, \rho\right)}_{=:\ \tilde{A}_\rho(v_1, w_1, v_2, w_2)}$$



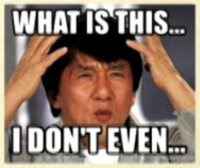
$\tilde{A}_\rho(v_1, w_1, v_2, w_2)$

Quasiprobability behind the out-of-time-ordered correlator

Motivating the definition of the OTOC quasiprobability

- Is it a probability? No, it has too little symmetry.
- But… it's normalized to one, like a probability distribution.

$$F(t) = \sum_{v_1, w_1, v_2, w_2} w_2^* v_2^* w_1 v_1 \ \underbrace{\mathrm{Tr}\left(\Pi_{w_2}^{\mathcal{W}(t)} \Pi_{v_2}^{V} \Pi_{w_1}^{\mathcal{W}(t)} \Pi_{v_1}^{V} \rho \right)}_{\stackrel{..}{=}\ \tilde{A}_\rho(v_1, w_1, v_2, w_2)}$$

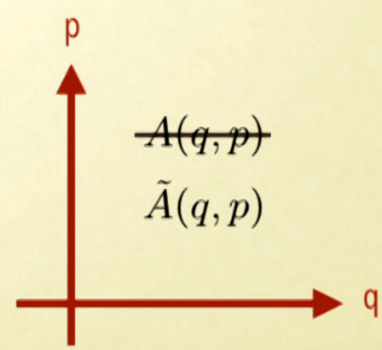


Quasiprobability behind the out-of-time-ordered correlator

Quasiprobability distributions

Review: quasiprobability distributions

- Tools used in quantum optics, quantum computation, and quantum foundations
- Similar: phase-space distribution in classical statistical mechanics
- Function that represents a quantum state ρ
  - Can use to calculate probabilities, expectation values, …
- Like a probability, but violates $\geq$1 axiom of probability theory
- Nonclassical behaviors (e.g., negativity) can signal nonclassical physics.



Review: quasiprobability distributions

- Examples: Wigner function, Glauber-Sudarshan distribution, Husimi Q-function, **Kirkwood-Dirac distribution**

Kirkwood, Phys. Rev. **44**, 31 (1933).

Dirac, Rev. Mod. Phys. **17**, 195 (1945).

- Offers alternative route to quantum state tomography
- Lies behind fundamental theory (e.g., Leggett-Garg inequalities)
- Well-defined for discrete systems, even qubits

- OTOC quasiprobability: $\tilde{A}_\rho(v_1, w_1, v_2, w_2) = \text{Tr}\left(\cancel{\Pi_{w_2}^{\mathcal{W}(t)}}\cancel{\Pi_{v_2}^{V}}\,\Pi_{w_1}^{\mathcal{W}(t)}\Pi_{v_1}^{V}\rho\right)$

1D, and $\langle w_1(t) | v_1 \rangle \neq 0$

- "Extended" Kirkwood-Dirac quasiprobability

QI scrambling $\longleftrightarrow$ Q. optics

- Kirkwood-Dirac quasiprobability: $\tilde{A}_\rho(v_1, w_1) = \text{Tr}\left(\Pi_{w_1}^{\mathcal{W}(t)}\Pi_{v_1}^{V}\rho\right)$

BASIC PROPERTIES

$$\tilde{A}_\rho(v_1, w_1, v_2, w_2) = \mathrm{Tr}\left(\Pi^{\mathcal{W}(t)}_{w_2} \Pi^{V}_{v_2} \Pi^{\mathcal{W}(t)}_{w_1} \Pi^{V}_{v_1} \rho\right)$$

- **Can become negative**
  - Significance: *V* and *W(t)* can't both have well-defined values.
  - **Negativity empirically distinguishes scrambling from decoherence.**
    - González Alonso, NYH, and Dressel, Phys. Rev. Lett. **122**, 040404 (2019).
- **Can become nonreal**
  - Unlike the Wigner function
  - Significance: Measuring *V* disturbs *W(t)*.
- **Can be measured experimentally** ⟶

Weak-measurement scheme

- The story and formalism behind weak measurements ⟶ Ask during the Q&A!
- (1) NYH, Phys. Rev. A **95**, 012120 (2017).

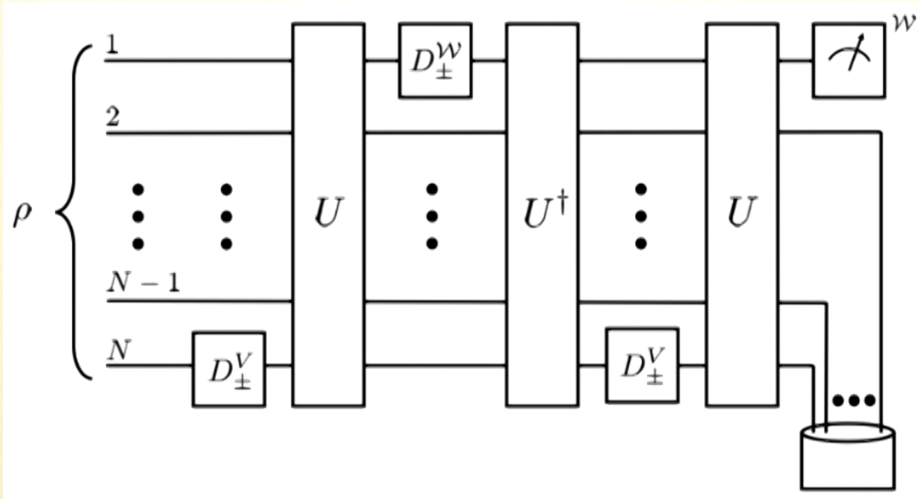
  (2) NYH, Swingle, and Dressel, Phys. Rev. A **97**, 042105 (2018).

  (3) Dressel, Gonzàlez Alonso, Waegell, and NYH, Phys. Rev. A **98**, 012132 (2018).

  (4) González Alonso, NYH, and Dressel, Phys. Rev. Lett. **122**, 040404 (2019).

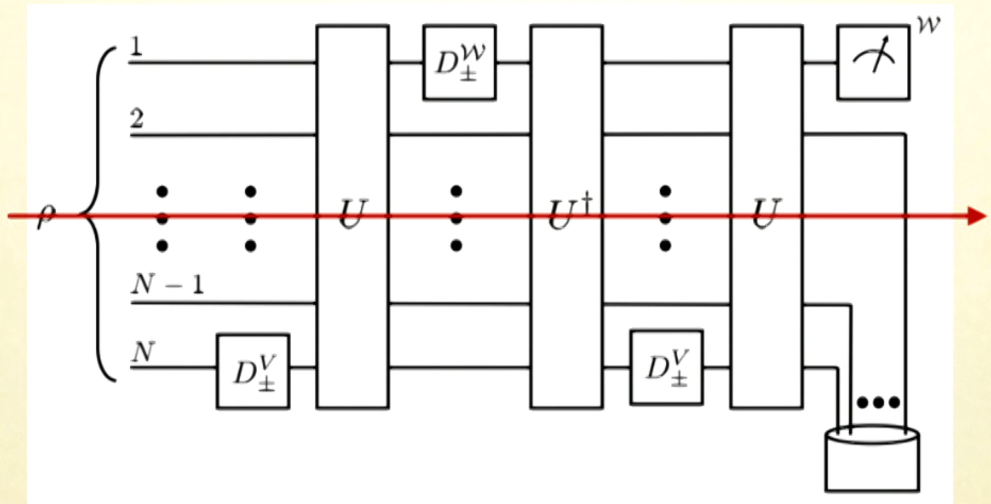
  (5) Swingle and NYH, Phys. Rev. A **97**, 062113 (2018).
- Platforms: superconducting qubits, trapped ions, quantum dots, ultracold atoms, …

Weak-measurement circuit



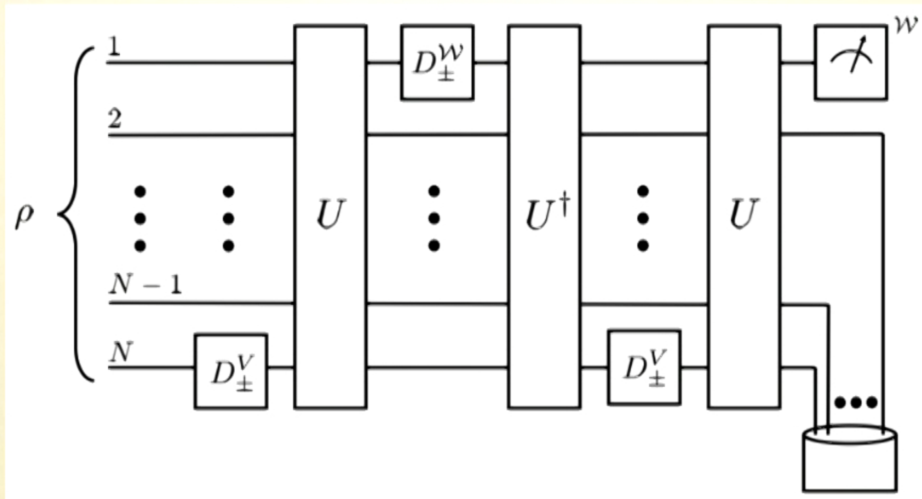
$$\tilde{\mathcal{A}}_\rho(v_1, w_1, v_2, w_2) = \mathrm{Tr}\left(U^\dagger \Pi^{\mathcal{W}}_{w_2} U \Pi^V_{v_2} U^\dagger \Pi^{\mathcal{W}}_{w_1} U \Pi^V_{v_1} \rho\right)$$

Weak-measurement circuit



$$\tilde{\mathcal{A}}_\rho(v_1, w_1, v_2, w_2) = \mathrm{Tr}\left(U^\dagger \Pi^{\mathcal{W}}_{w_2} U \Pi^{V}_{v_2} U^\dagger \Pi^{\mathcal{W}}_{w_1} U \Pi^{V}_{v_1} \rho\right)$$

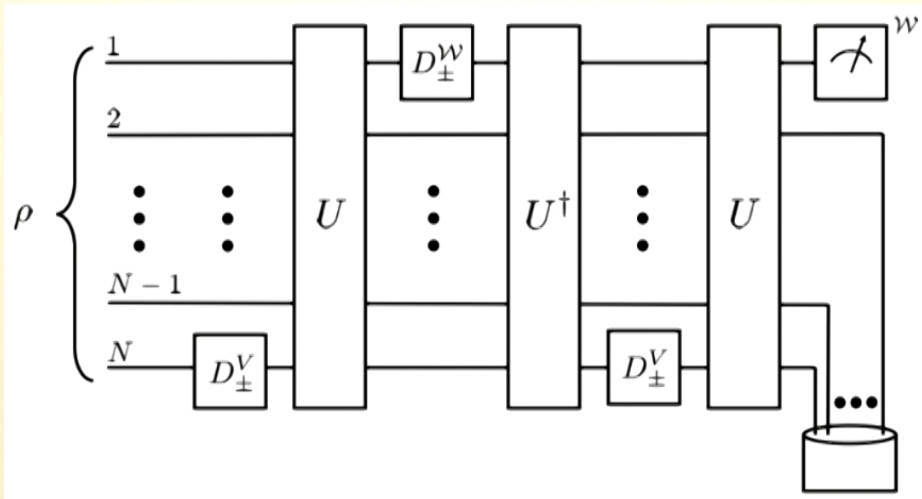
Weak-measurement circuit



$$\tilde{\mathcal{A}}_\rho(v_1, w_1, v_2, w_2) = \mathrm{Tr}\left(U^\dagger \Pi^{\mathcal{W}}_{w_2} U \Pi^{V}_{v_2} U^\dagger \Pi^{\mathcal{W}}_{w_1} U \Pi^{V}_{v_1} \rho\right)$$

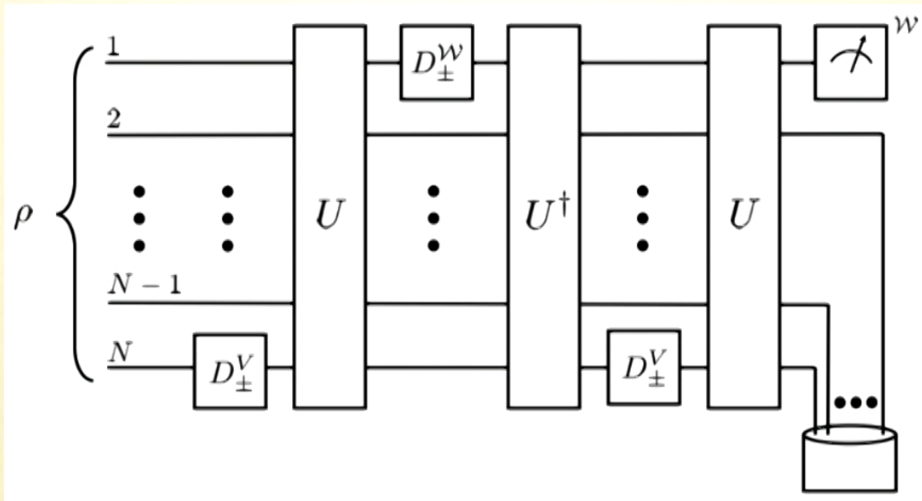
Asymmetric about $\rho \Rightarrow$
weak measurement

Weak-measurement circuit



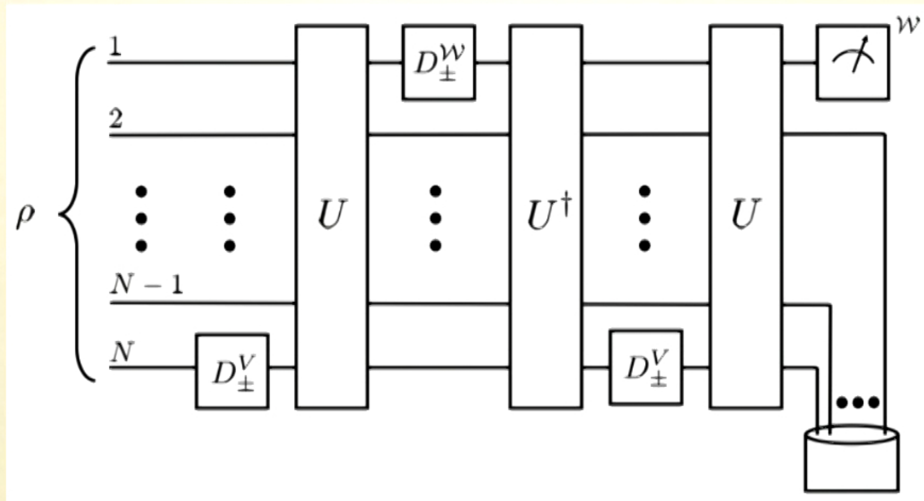
$$\tilde{\mathcal{A}}_\rho(v_1, w_1, v_2, w_2) = \mathrm{Tr}\left(U^\dagger \Pi^{\mathcal{W}}_{w_2} U \Pi^{V}_{v_2} U^\dagger \Pi^{\mathcal{W}}_{w_1} U \Pi^{V}_{v_1} \rho\right)$$

WEAK-MEASUREMENT CIRCUIT



$$\tilde{A}_\rho(v_1, w_1, v_2, w_2) = \mathrm{Tr}\left(U^\dagger \Pi^{\mathcal{W}}_{w_2} U \Pi^{V}_{v_2} U^\dagger \Pi^{\mathcal{W}}_{w_1} U \Pi^{V}_{v_1} \rho\right)$$

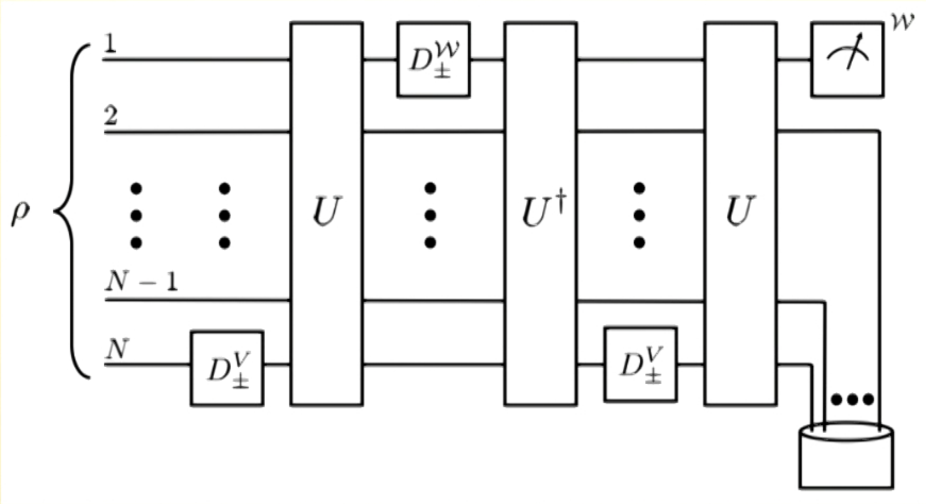
Weak-measurement circuit



$$\tilde{\mathcal{A}}_\rho(v_1, w_1, v_2, w_2) = \mathrm{Tr}\left(\cancel{U}^\dagger \Pi^{\mathcal{W}}_{w_2} U \Pi^{V}_{v_2} U^\dagger \Pi^{\mathcal{W}}_{w_1} U \Pi^{V}_{v_1} \rho\right)$$

$$U^\dagger \Pi^{\mathcal{W}}_{w_2} \cancel{U}$$

Weak-measurement circuit



$$\tilde{A}_\rho(v_1, w_1, v_2, w_2) = \mathrm{Tr}\left(\cancel{U^\dagger}\Pi^{\mathcal{W}}_{w_2} U \Pi^{V}_{v_2} U^\dagger \Pi^{\mathcal{W}}_{w_1} U \Pi^{V}_{v_1} \rho\right)$$

$U\Pi^{\mathcal{W}}_{w_2}$

Symmetry breaking and chaos-like bifurcation in the OTOC quasiprobability

NYH, Swingle, and Dressel, Phys. Rev. A **97**, 042105 (2018).

Symmetry breaking and chaos-like Bifurcation

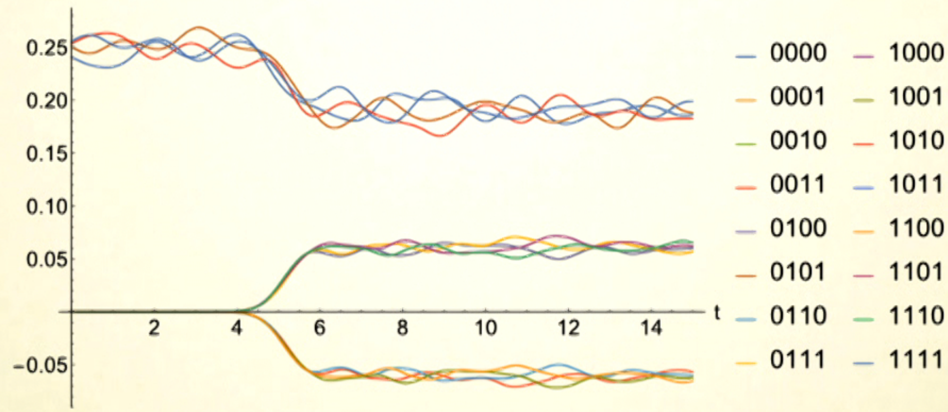
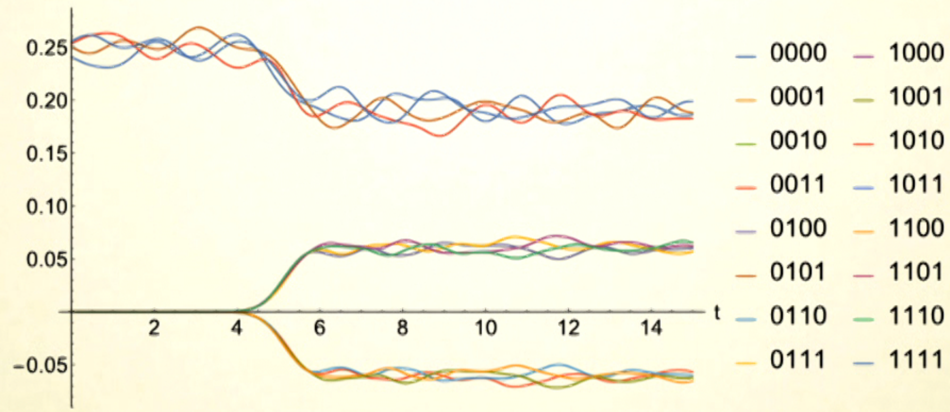


FIG. 13: Real part of $\tilde{\mathscr{A}}_\rho$ as a function of time. Random pure state. Nonintegrable parameters, $N = 10$, $\mathcal{W} = \sigma_1^z$, $V = \sigma_N^z$.

$$\tilde{A}_\rho(v_1, w_1, v_2, w_2) = \mathrm{Tr}\left(\Pi_{w_2}^{\mathcal{W}(t)} \Pi_{v_2}^{V} \Pi_{w_1}^{\mathcal{W}(t)} \Pi_{v_1}^{V} \rho\right)$$

Symmetry breaking and chaos-like Bifurcation



- At early times, $\tilde{\mathcal{A}}_\rho(v_1, w_1, v_2, w_2) \approx (\ldots)^2 - (\ldots)^2 + \ldots$

  Quasiprobability invariant under negation of this

- Symmetry breaking → phase transition?

Applications of the OTOC quasiprobability to theory and experiment

(1) Fundamental understanding of the OTOC

- NYH, Swingle, and Dressel, Phys. Rev. A **97**, 042105 (2018).
- OTOC = average over quasiprobability

(2) Scheme for measuring the OTOC and its quasiprobability

- NYH, Phys. Rev. A **95**, 012120 (2017).
  NYH, Swingle, and Dressel, Phys. Rev. A **97**, 042105 (2018).
  Dressel, Gonzàlez Alonso, Waegell, and NYH, Phys. Rev. A **98**, 012132 (2018).
  González Alonso, NYH, and Dressel, Phys. Rev. Lett. **122**, 040404 (2019).
  Swingle and NYH, Phys. Rev. A **97**, 062113 (2018).

(3) Flags false positives in attempts to detect scrambling in open systems

- González Alonso, NYH, and Dressel, Phys. Rev. Lett. **122**, 040404 (2019).

Applications of the OTOC quasiprobability to theory and experiment

(4) Motivates a generalization of quasiprobability theory

- NYH, Swingle, and Dressel, Phys. Rev. A **97**, 042105 (2018).

(5) Strengthens link between the OTOC and chaos

- NYH, Swingle, and Dressel, Phys. Rev. A **97**, 042105 (2018).

Applications of the OTOC quasiprobability to theory and experiment

(6) Lies behind an fluctuation-like relation for the OTOC

- NYH, Phys. Rev. A **95**, 012120 (2017).

Strengthening of the second law of thermodynamics

- Summed quasiprobability = generating function for OTOC

(7) Lies behind an entropic uncertainty relation for scrambling

- NYH, Bartolotta, and Pollack, accepted by Comms. Phys. (in press) arXiv:1806.04147.

(8) Flags a nonclassical advantage in metrology

- Arvidsson-Shukur, NYH, Lepage, Lasek, Barnes, and Lloyd, arXiv:1903.02563 (2019).

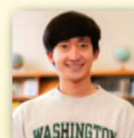
Outlook

Outlook

- **Measure the OTOC and its quasiprobability with weak measurements.**

  - OTOC experiments have begun, but a thorough study, and measuring the quasiprobability distribution, remain opportunities.
  - Platforms: superconducting qubits, trapped ions, ultracold atoms, quantum dots, …
  - Entropic uncertainty relations for weak and strong measurements
    - NYH, Bartolotta, and Pollack, accepted by Comms. Phys. (in press) arXiv:1806.04147.
    - **Washington U. in St. Louis:**

Kater Murch



Taeho Lee

Jonathan Monroe

OUTLOOK

- **Does the OTOC quasiprobability flag a new type of phase transition at the scrambling time?**
  - NYH, Swingle, and Dressel, Phys. Rev. A **97**, 042105 (2018).
- **How does the OTOC quasiprobability in holographic, high-energy systems?**
- **Applications of the quasiprobability's imaginary part**
  - NYH, Swingle, and Dressel, Phys. Rev. A **97**, 042105 (2018).
- **Leggett-Garg inequalities for scrambling**
  - Leggett and Garg, PRL **54**, 857 (1985).
    Emary, Lambert, and Nori, Rep. Prog. Phys. **77**, 1 (2014).
  - Like Bell inequalities
  - But reflect correlations across time, rather than across space
  - Underlaid by OTOC-like quasiprobabilities

Recap

- **OTOC = average over quasiprobability distribution**

  coarse-grained more detailed

- **The OTOC quasiprobability offers experimental advantages and theoretical insights.**

QI SCRAMBLING

QUASIPROBABILITIES

THANKS FOR YOUR TIME!

NYH, Phys. Rev. A **95**, 012120 (2017).

NYH, Swingle, and Dressel, Phys. Rev. A **97**, 042105 (2018).

González Alonso, NYH, and Dressel, Phys. Rev. Lett. **122**, 040404 (2019).

Swingle and NYH, Phys. Rev. A **97**, 062113 (2018).

Dressel, Gonzàlez Alonso, Waegell, and NYH, Phys. Rev. A **98**, 012132 (2018).

NYH, Bartolotta, and Pollack, accepted by Comms. Phys. (in press) arXiv:1806.04147.

Arvidsson-Shukur, NYH, Lepage, Lasek, Barnes, and Lloyd, arXiv:1903.02563 (2019).