Title: From cold to lukewarm to hot electrons

Speakers: Andres Schlief

Collection: Quantum Matter: Emergence & Entanglement 3

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Abstract: I will present a study of the single-particle properties of hot, lukewarm and cold electrons that coexist in the two-dimensional antiferromagnetic quantum critical metal within a unified theory. I will show how to generalize the theory that describes the interaction of critical spin-density wave fluctuations and electrons near the hot spots on the Fermi surface (hot electrons) by including electrons far away from the hot spots (lukewarm and cold electrons). Through an analytically tractable functional renormalization group scheme it will be shown that low-energy electrons are characterized by a universal momentum-dependent quasi-particle weight that decays to zero as the hot spots are approached along the Fermi surface, owing to the coexistence of quasiparticle and non-quasiparticle excitations within the same metallic state. This approach allows to characterize how the global shape of the Fermi surface is renormalized due to the strong interaction between the electrons and the critical spin fluctuations. I will finalize by commenting on the scope of this approach to study properties that are sensitive to the entirety of the Fermi surface, paying special attention to some preliminary results on the superconducting instability of this metallic state.

From Cold To Lukewarm to Hot Electrons

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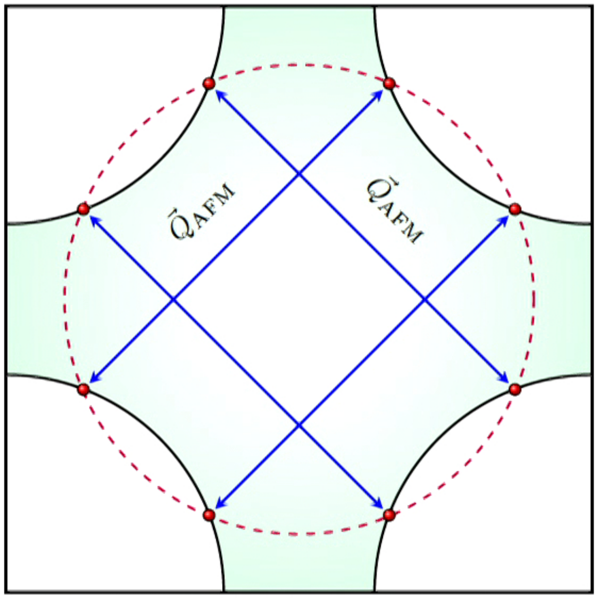
Quantum Matter: Emergence & Entanglement 3
Perimeter Institute for Theoretical Physics

The AFM Quantum Critical Metal in $2d$

McMaster University

@ QCP:

Spontaneous appearance of a spatially modulated electronic spin polarization: Spin Density Waves.



- Electronic spin polarization

$$\vec{S}(\vec{r}_j) = \vec{\phi}(\vec{r}_j) e^{i\vec{Q}_{\rm AFM}\cdot\vec{r}_j}$$

- $\vec{Q}_{\rm AFM} = (\pi, \pi)$, commensurate.
- $\vec{\phi}(\vec{r}_j)$- SU(2) order parameter:
  - $\langle\vec{\phi}(\vec{r}_j)\rangle \neq 0$: AFM phase
  - $\langle\vec{\phi}(\vec{r}_j)\rangle = 0$: Paramagnetic phase.

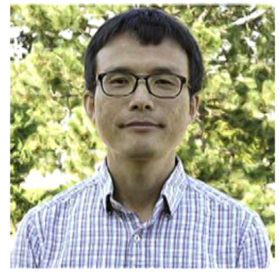
At QCP, the Fermi surface (FS) develops *hot spots*.

Outline: AFM Quantum Critical Metal in $2d$

First Part

The Theory of Hot Spot Electrons

A. S., P. Lunts and S.-S. Lee. Phys. Rev. X. **7**, 021010 (2017)
P. Lunts, **A. S.** and S.-S. Lee. Phys. Rev. B. **95**, 245109 (2017)
A. S., P. Lunts and S.-S. Lee. Phys. Rev. B. **98**, 075140 (2018)



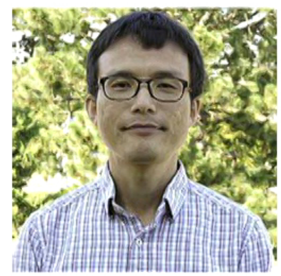
Sung-Sik Lee

Peter Lunts

Second Part

Beyond The Theory of Hot Spot Electrons:
Single-particle properties of the AFM Quantum Critical Metal

A. S. and S.-S. Lee. In preparation



Sung-Sik Lee

Take Away Messages

McMaster University

Message I

The normal state AFM Quantum Critical Metal supports *both* Fermi-liquid-like and non-Fermi-liquid-like electronic excitations.

Message II

Despite the imminent onset of superconducting order, Non-Fermi-liquid signatures survive in the normal state of the AFM Quantum Critical Metal.

The Theory of Hot Spot Electrons

McMaster University

Low-energy effective field theory:

$$S = S_\psi + S_\phi + S_{\psi^\dagger \Phi \psi} + S_{\Phi^4},$$

$$S_\psi = \sum_{N=1}^{8} \sum_{\sigma=\uparrow,\downarrow} \int \mathrm{d}k \; \psi^\dagger_{N,\sigma}(k) \left[ik_0 + ie_N(\vec{k}; \boldsymbol{v}) \right] \psi_{N,\sigma}(k)$$

$$S_\Phi = \frac{1}{4} \int \mathrm{d}q \; \left[q_0^2 + \boldsymbol{c}^2 |\vec{q}|^2 \right] \mathrm{Tr}\left[\Phi(q)\Phi(-q) \right],$$

$$S_{\psi^\dagger\Phi\psi} = \boldsymbol{g} \sum_{N=1}^{8} \sum_{\sigma,\sigma'=\uparrow,\downarrow} \int \mathrm{d}k \int \mathrm{d}q \; \psi^\dagger_{\bar{N},\sigma}(k+q) \Phi_{\sigma\sigma'}(q) \psi_{N,\sigma'}(k),$$

$$S_{\Phi^4} = \boldsymbol{u} \int \mathrm{d}q_1 \int \mathrm{d}q_2 \int \mathrm{d}q_3 \; \mathrm{Tr}\left[\Phi(q_1+q_2)\Phi(q_3-q_2) \right] \mathrm{Tr}\left[\Phi(-q_1)\Phi(-q_3) \right]$$

A. Abanov, E. Abrahams, E. Berg, V. de Carvalho, L. Classen, A. Chubukov, H. Freire, M. Gerlach, S. Hartnoll, D. F. Hofman, D.-H. Lee, J. Lee, Z.-X. Li, S. A. Maier, Z. Y. Meng, M. Metlitski, A. Patel, S. Sachdev, Y. Schattner, J. Schmalian, P. Strack, S. Sur, S. Trebst, A. Tsvelik, F. Wang, S. Whitsitt, P. Wölfe, H. Yao, amongst many others...

Electrons close to the Hot Spots

- Electron close to hot spot N:

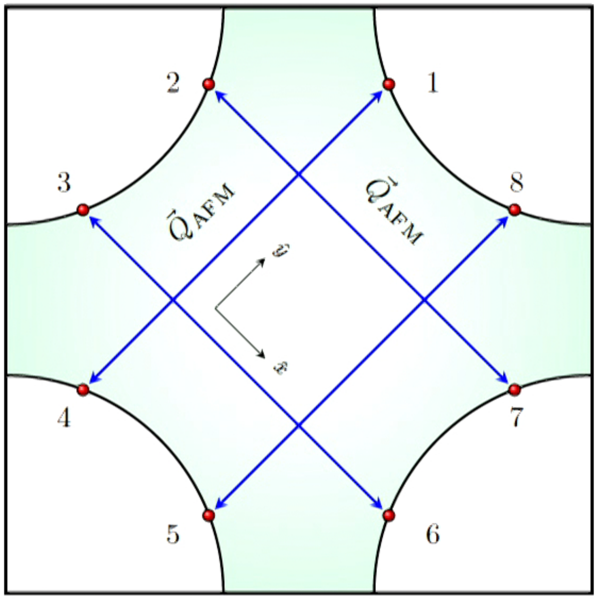
$$\psi_{N,\sigma}(k) = \psi_{N,\sigma}(k_0, \vec{k})$$

- Dispersion @ $N = 1$

$$e_1(\vec{k}, v) = vk_x + k_y.$$

- Electronic spectrum

$$\omega = \pm|e_N(\vec{k}; v)|.$$



S_ψ

$$S_\psi = \sum_{N=1}^{8} \sum_{\sigma=\uparrow,\downarrow} \int \mathrm{d}k \; \psi^\dagger_{N,\sigma}(k) \left[ik_0 + ie_N(\vec{k}; v) \right] \psi_{N,\sigma}(k).$$

Electrons close to the Hot Spots

- Electron close to hot spot N:

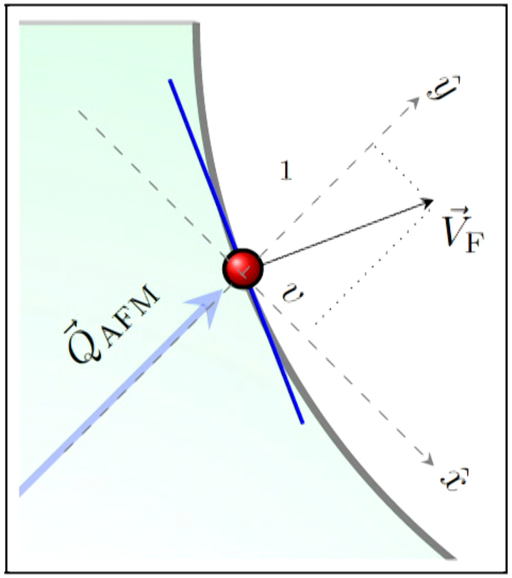
$$\psi_{N,\sigma}(k) = \psi_{N,\sigma}(k_0, \vec{k})$$

- Dispersion @ $N = 1$

$$e_1(\vec{k}, v) = \boldsymbol{v} k_x + k_y.$$

- Electronic spectrum

$$\omega = \pm |e_N(\vec{k}; \boldsymbol{v})|.$$



S_ψ

$$S_\psi = \sum_{N=1}^{8} \sum_{\sigma=\uparrow,\downarrow} \int \mathrm{d}k \; \psi^\dagger_{N,\sigma}(k) \left[ik_0 + ie_N(\vec{k}; \boldsymbol{v}) \right] \psi_{N,\sigma}(k).$$

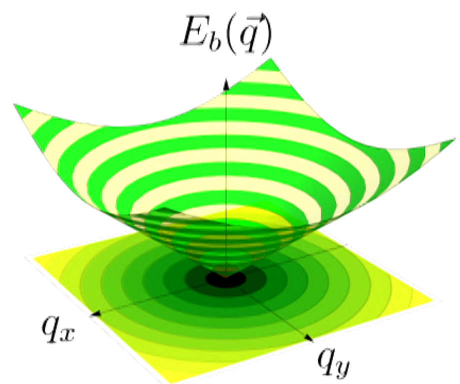
Bosonic Collective mode

- $\vec{q}$ measured with respect to $\vec{Q}_{\mathrm{AFM}}$
- SU(2)-matrix bosonic field:

$$\Phi_{\sigma\sigma'}(q) = \sum_{\alpha=1}^{3} (\tau^a)_{\sigma\sigma'} \phi^a(q), \qquad \tau^a \in \mathfrak{su}(2).$$

- Bosonic spectrum

$$E_b(\vec{q}) = \mathbf{c}|\vec{q}|.$$



$S_\Phi + S_{\Phi^4}$

$$S_\Phi = \frac{1}{4}\int \mathrm{d}q \; \left[q_0^2 + \mathbf{c}^2|\vec{q}|^2\right] \mathrm{Tr}\left[\Phi(q)\Phi(-q)\right]$$

$$S_{\Phi^4} = \boldsymbol{u}\int \mathrm{d}q_1 \int \mathrm{d}q_2 \int \mathrm{d}q_3 \; \mathrm{Tr}\left[\Phi(q_1+q_2)\Phi(q_3-q_2)\right] \mathrm{Tr}\left[\Phi(-q_1)\Phi(-q_3)\right]$$

The Theory of Hot Spot Electrons

McMaster University

Low-energy effective field theory:

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$$S_{\Phi^4} = \boldsymbol{u} \int \mathrm{d}q_1 \int \mathrm{d}q_2 \int \mathrm{d}q_3 \; \mathrm{Tr}\left[\Phi(q_1+q_2)\Phi(q_3-q_2) \right] \mathrm{Tr}\left[\Phi(-q_1)\Phi(-q_3) \right]$$

Strongly-coupled field theory that cannot be studied perturbatively in controlled way!

Low-Energy Theory of Hot Spots Electrons

McMaster University

Characterized by exact critical exponents:

Interaction-Driven Scaling

$$z = 1, \qquad [\psi] = -2, \qquad \& \qquad [\Phi] = -2.$$

Low-energy properties encoded in Minimal Local Action:

Minimal Local Action

$$S = \sum_{N=1}^{8} \sum_{\sigma=\uparrow,\downarrow} \int \mathrm{d}k \; \psi^{\dagger}_{N,\sigma}(k) \left[i k_0 + i e_N(\vec{k}; \boldsymbol{v}) \right] \psi_{N,\sigma}(k)$$

$$+ \boldsymbol{g} \sum_{N=1}^{8} \sum_{\sigma,\sigma'=\uparrow,\downarrow} \int \mathrm{d}k \int \mathrm{d}q \; \psi^{\dagger}_{\bar{N},\sigma}(k+q) \Phi_{\sigma\sigma'}(q) \psi_{N,\sigma'}(k),$$

Bosonic action is *irrelevant* $\Rightarrow$ Freedom to fix $\frac{g^2}{v} \sim \mathcal{O}(1)$ (marginal).

Low-Energy Theory of Hot Spots Electrons

McMaster University

Minimal Local Action

$$S = \sum_{N=1}^{8} \sum_{\sigma=\uparrow,\downarrow} \int \mathrm{d}k \; \psi^{\dagger}_{N,\sigma}(k) \left[ik_0 + ie_N(\vec{k}; \boldsymbol{v}) \right] \psi_{N,\sigma}(k)$$

$$+ \sqrt{\frac{\pi \boldsymbol{v}}{2}} \sum_{N=1}^{8} \sum_{\sigma,\sigma'=\uparrow,\downarrow} \int \mathrm{d}k \int \mathrm{d}q \; \psi^{\dagger}_{\bar{N},\sigma}(k+q) \Phi_{\sigma\sigma'}(q) \psi_{N,\sigma'}(k),$$

Emergent nesting of the FS @ hot spots:

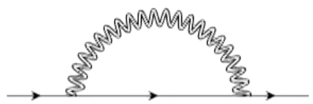
If $\quad v_0 \ll 1, \quad v \to 0 \qquad$ Stable low-energy fixed-point.

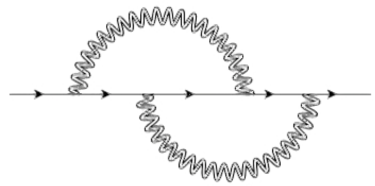
Dynamics of the collective mode damped by particle-hole excitations:

$$D(q)^{-1} = |q_0| + c(v)(|q_x| + |q_y|), \qquad c(v) \sim \sqrt{v \log\left(\frac{1}{v}\right)}$$

Low-Energy Hot Spot Electrons

Low-energy spectral function of hot spot electrons dominated by:





Higher-loops suppressed:

$$\Sigma_{\mathrm{F}}(L, L_f) \sim \left(\frac{v}{c(v)}\right)^{L-L_f},$$

with $v/c(v) \sim \sqrt{v} \ll 1.$

Close the fixed-point,

$$z = 1 + \mathcal{O}(\sqrt{v}), \qquad [\Psi] = -2 + \mathcal{O}(\sqrt{v}).$$

Low-energy Hot Spot Spectral Function:

$$\mathcal{A}_N(\vec{0}, \omega) \sim \frac{e^{-\frac{2\sqrt{3}\sqrt{\log(\Lambda_f/\omega)}}{\log\log(\Lambda_f/\omega)}}}{\omega\sqrt{\log(\Lambda_f/\omega)}\,\log\log(\Lambda_f/\omega)} \frac{1}{[\log(\Lambda_f/\omega)]^{\frac{3}{8}}}$$

@ Hot spots: Highly incoherent electronic excitations $\rightarrow$ Non-Fermi-liquid-like.

Outline: AFM Quantum Critical Metal in $2d$

McMaster University

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P. Lunts, **A. S.** and S.-S. Lee. Phys. Rev. B. **95**, 245109 (2017)
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Beyond The Theory of Hot Spot Electrons:
Single-particle properties of the AFM Quantum Critical Metal

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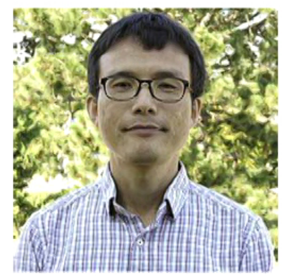

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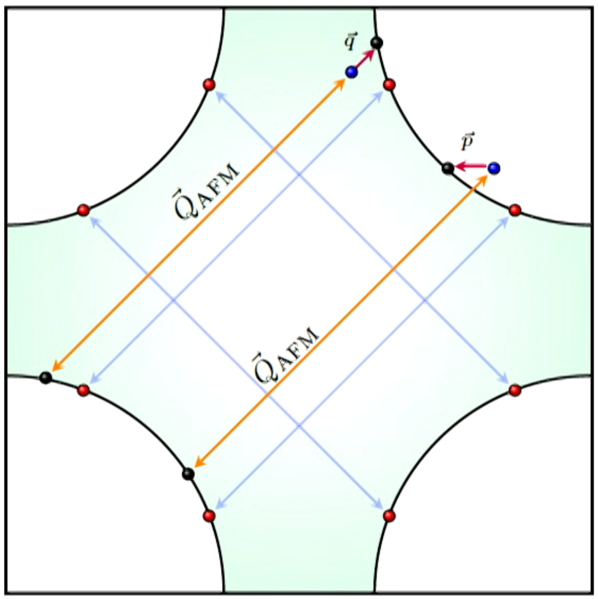
Beyond The Theory of Hot Spot Electrons

Theory of hot spot electrons is not a complete low-energy theory for the AFM quantum critical metal!



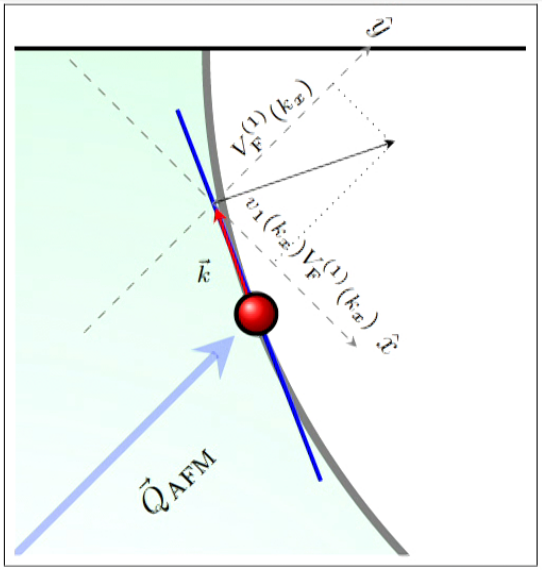
- Hot spot electrons are only a fraction of all gapless degrees of freedom along the FS.
- Fixed-point: Stable up to *superconducting instabilities.*

Need to incorporate electrons away from the hot spots in the low-energy description.

Beyond The Theory of Hots Spot Electrons

Generalized Minimal Local Action

$$S = \sum_{N=1}^{8} \sum_{\sigma=\uparrow,\downarrow} \int \mathrm{d}k\, \psi^{\dagger}_{N,\sigma}(k) \left[ik_0 + iV_{\mathrm{F}}^{(N)}(k_N) e_N[\vec{k}; v_N(k_N)]\right] \psi_{N,\sigma}(k)$$
$$+ \sum_{N=1}^{8} \sum_{\sigma,\sigma'=\uparrow,\downarrow} \int \mathrm{d}k \int \mathrm{d}k'\, g_N(k'_N, k_N) \psi^{\dagger}_{N,\sigma}(k+q) \Phi_{\sigma\sigma'}(q) \psi_{N,\sigma'}(k).$$



- k_N: Parametrizes FS locally. $k_1 = k_x, k_2 = -k_y, \cdots$.
- At zero momentum,

$$v_N(0) = v,$$
$$V_{\mathrm{F}}^{(N)}(0) = 1,$$
$$g_N(0,0) = \sqrt{\frac{\pi}{2} v}.$$

Low-Energy Theory: Functional RG Analysis

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Functional RG Analysis

What is the momentum profile of the coupling functions in the low-energy limit, i.e., their functional form at the low-energy fixed point?

Very hard to answer for arbitrary momentum dependence of the coupling functions!

The Weak Momentum Dependence Limit (WMDL)

Functional RG is *analytically tractable* in the case that

$$v_N(0) = v \ll 1 \qquad \text{(Control in theory of hot spot electrons)},$$

$$\frac{\partial \log J_N(k_N)}{\partial \log k_N} \ll 1, \qquad \text{(Slow variation along the FS)},$$

with $J_N(k_N) = v_N(k_N), V_{\mathrm{F}}^{(N)}$ and $g_N(k'_N, k_N)$.

The WMDL provides a window in which the emergent momentum profiles can be understood in a controlled way.

Weak-Momentum Dependence Limit

Start with the UV theory ($\mu \sim \Lambda_f$):

$$\mathsf{v}_N(k_N) = v_0 \ll 1, \qquad \mathsf{V}_{\mathrm{F}}^{(N)}(k_N) = 1, \qquad \& \qquad \mathsf{g}_N(k'_N, k_N) = \sqrt{\frac{\pi}{2} v_0}$$

In practice the WMDL means:

(i) Quantum corrections computed with momentum-independent coupling functions.

(ii) Momentum dependence arises from the IR cutoffs quantum corrections and

(iii) appears in observables through the RG flow of v.

Result: Low-energy fixed-point with

$$v_N(0) = 0, \qquad \& \qquad \frac{\partial \log J_N(k_N)}{\partial \log k_N} \sim v_0 \ll 1.$$

(i) Quantum Corrections

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In the WMDL:

- Collective mode's dynamics governed by

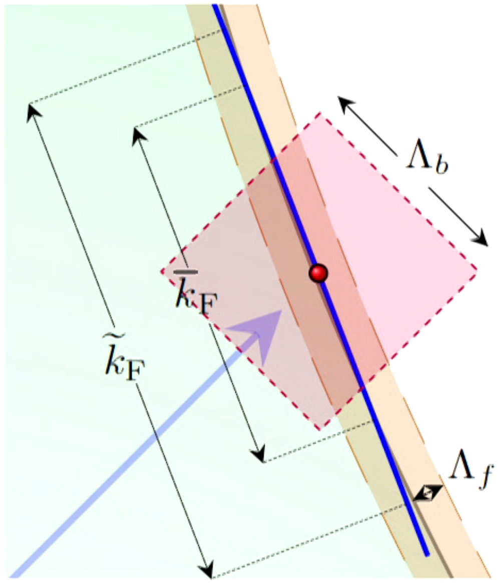
$$D(q)^{-1} = |q_0| + c(v)(|q_x| + |q_y|)$$
$$c(v) \sim \sqrt{v \log(1/v)}.$$

- Higher-loops suppressed

$$\mathcal{G}(E, L, L_f) \sim v^{\frac{(E-2)}{2}} \left(\frac{v}{c(v)}\right)^{L-L_f}$$

- Control parameter is still $v/c(v) \ll 1$!

The price to pay:

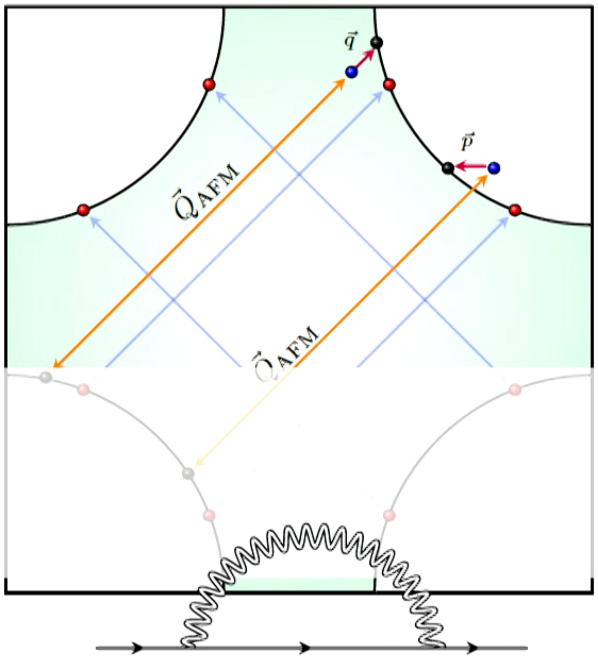


$$\overline{k}_F \sim v_0^{-3/2}\Lambda_b, \ \Lambda_f \ll \Lambda_b$$

Extended region of the FS governed by the universal low-energy theory in WMDL.

(ii) Momentum-dependent IR Scales: One-loop e.g.

Quantum corrections have IR cutoff scales depending on momentum of external electrons.



One-loop fermion self-energy:

1. Virtual fermion at FS at expense of creating bosonic excitation: $e_1(k_N) \sim vc(v)|k_N|$.
2. Virtual boson at zero energy at expense of exciting an electron: $e_2(k_N) \sim vk_N$.

IR scale: *minimum energy of virtual excitations!*

If $\mu \ll e_1(k_N)$, electrons and spin fluctuations decouple!

(iii) Momentum-dependent slope $v_N(k_N)$

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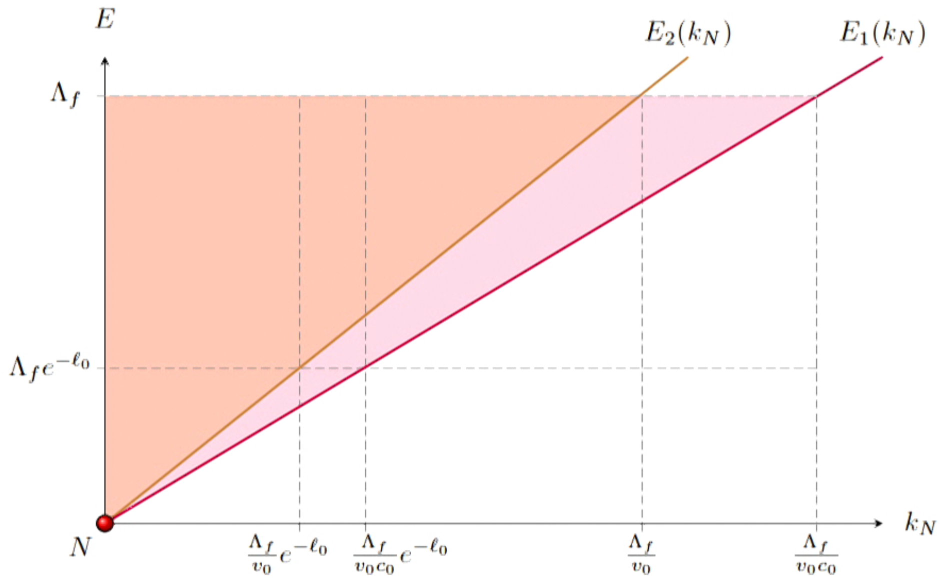
Different quantum corrections have different IR scales!



Flow of v:

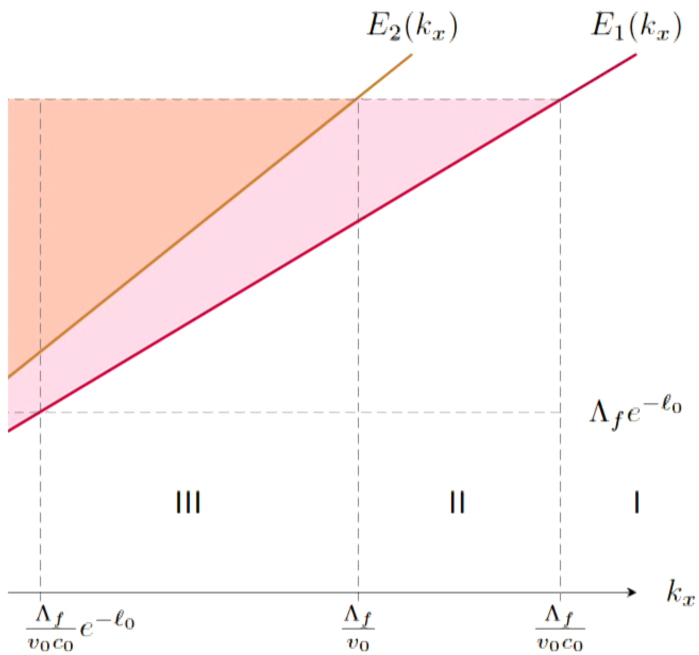
$$v(\ell) \sim \frac{1}{(\ell+\ell_0)\log(\ell+\ell_0)}$$

$$\ell_0 \sim \frac{1}{v_0 \log(1/v_0)}.$$

$$v_N(k_N; l) = v_0 \exp\left(-\frac{3}{2\pi^2}\int_0^l \mathrm{d}\ell\, v(\ell) \log\left(\frac{1}{v(\ell)}\right) \times \left[\Theta(\Lambda_f e^{-\ell} - 2v(\ell)c(\ell)|k_N|) + \Theta(\Lambda_f e^{-\ell} - 2v(\ell)|k_N|)\right]\right)$$

Ideal Case: From Cold to Lukewarm

Regions I, II and III:



Renormalized Fermi Surface

$$\text{I: } k_y = -v_0 k_x$$

$$\text{II: } k_y \sim -v_0 k_x \left(\frac{|k_x|}{\Lambda_f}\right)^{v_0}$$

$$\text{III: } k_y \sim -v_0 k_x \left(\frac{|k_x|}{\Lambda_f}\right)^{2v_0}$$

Quasiparticle Weight in I, II & III

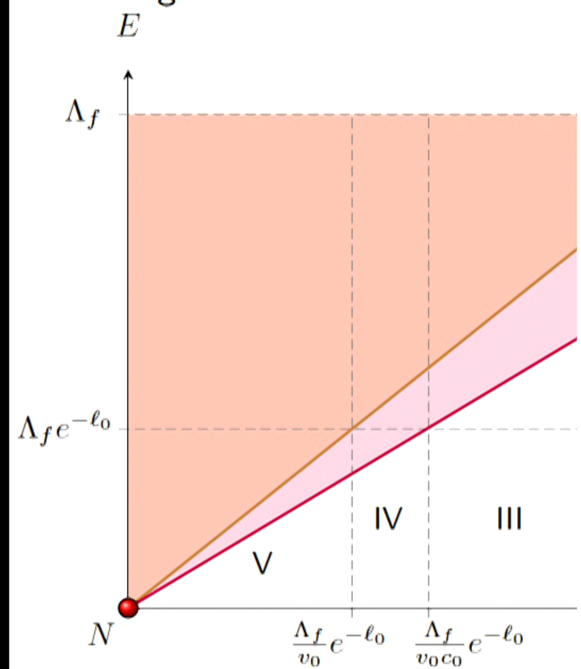
$$\text{I: } \mathcal{Z}_1(k_x) = 1 \quad \textbf{(COLD)}$$

$$\text{II \& III: } \mathcal{Z}_1(k_x) \sim \left(\frac{|k_x|}{\Lambda_f}\right)^{\sqrt{v_0}}$$

(LUKEWARM)

Ideal Case: From Lukewarm to Hot

Regions IV and V:



Renormalized Fermi Surface

$$\text{IV:}\quad k_y \sim \frac{-\sqrt{v_0}k_x}{\sqrt{\log(\Lambda_f/|k_x|)\log\log(\Lambda_f/|k_x|)}}\left(\frac{|k_x|}{\Lambda_f}\right)^{v_0}$$

$$\text{V:}\quad k_y \sim -\frac{k_x}{\log(\Lambda_f/|k_x|)\log\log(\Lambda_f/|k_x|)}$$

Quasiparticle Weight

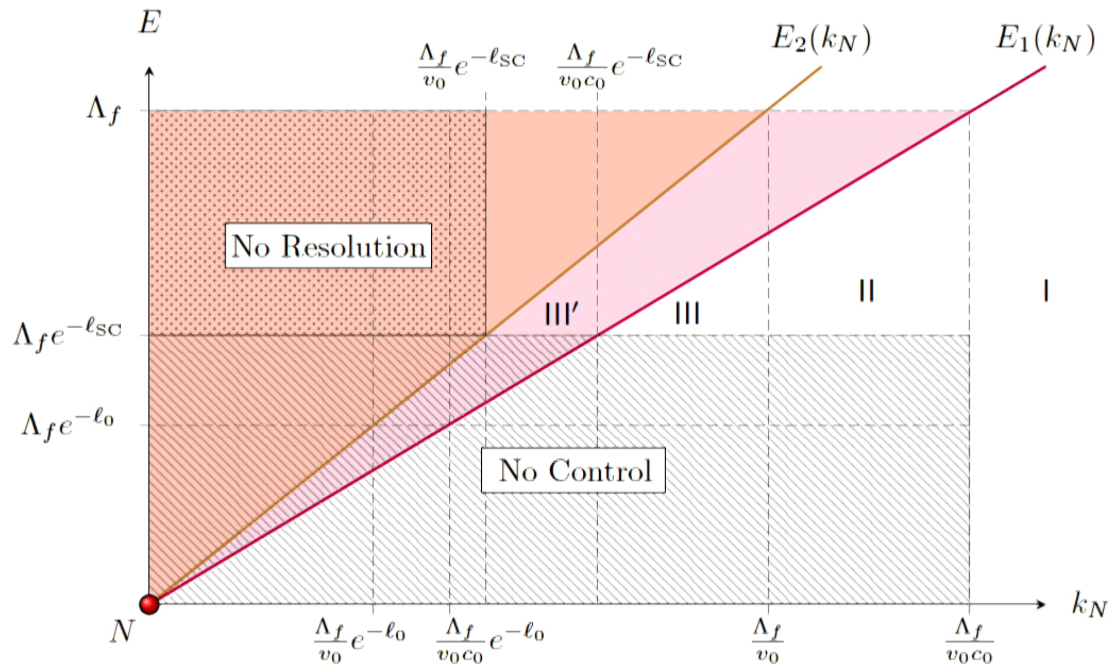
$$\mathcal{Z}_1(k_x) \sim \exp\left(-\frac{\sqrt{\log\left(\frac{\Lambda_f}{|k_x|}\right)}}{\log\log\left(\frac{\Lambda_f}{|k_x|}\right)}\right) \quad \textbf{(HOT)}$$

$Z_1(0) = 0$! Consistent with lack of quasiparticle peak of hot spot electrons!

Realistic Scenario I: Superconducting Instability Kicks In

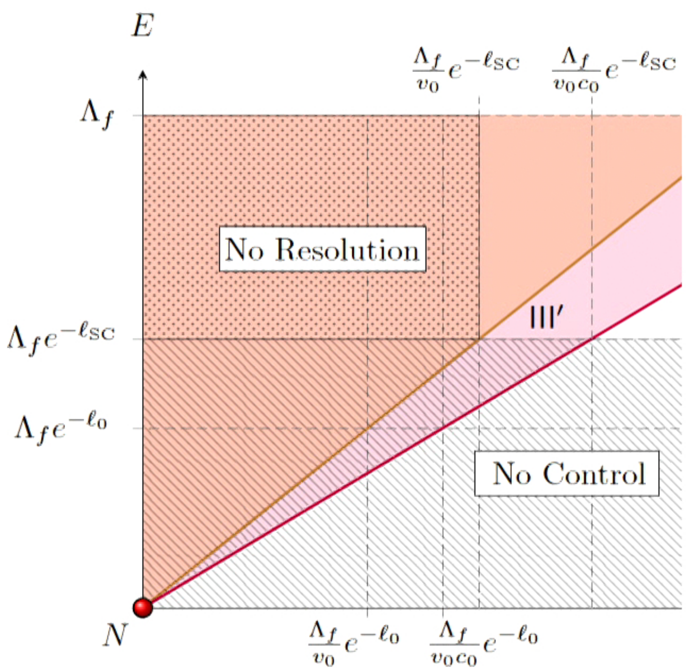
For $\omega \sim \Lambda_f e^{-\ell_{SC}} \gg \Lambda_f e^{-\ell_0}$ and $k_N \neq 0$, generically:

$$\mathcal{A}_N(\vec{k};\omega) = \frac{\mathcal{Z}_N(k_N;\omega(\vec{k}))}{\tau_N(k_N;\omega(\vec{k}))} \frac{1}{(\omega-\omega(\vec{k}))^2 + \tau(\vec{k};\omega(\vec{k}))^{-2}}$$



Realistic Scenario I: Main Difference:

Region III' and no resolution region:



Lifetime:

$$\boxed{\tau_N(k_N;\omega(\vec{k}))^{-1} \sim v_0\omega(\vec{k})}$$

Spectral weight:

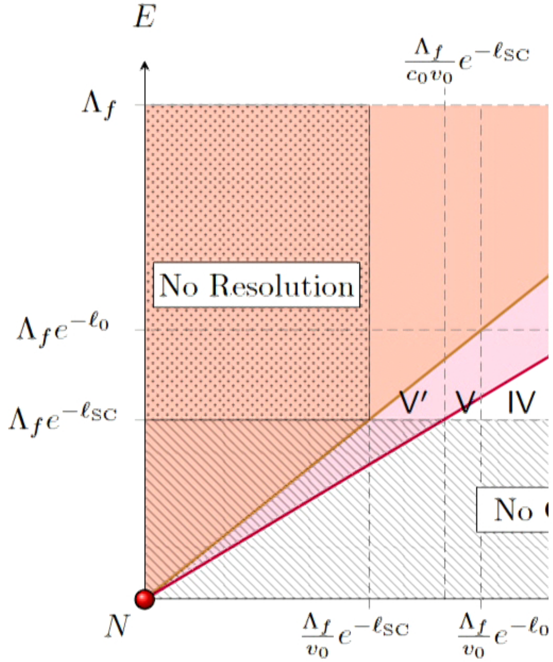
$$\mathcal{Z}_N(k_N;\omega(\vec{k})) \sim \left(\frac{\omega(\vec{k})}{\Lambda_f}\right)^{\sqrt{v_0}}$$

$\omega(\vec{k})$ receives only power-law corrections in k_N.

Scattering rate scales *linearly* in the renormalized energy: Deviation from Fermi-liquid theory! (Marginal Fermi-liquid)

Realistic Scenario II: Mani Difference

Region V' and no resolution region:



Lifetime:

$$\boxed{\tau_N(k_N;\omega(\vec{k}))^{-1} \sim \frac{\omega(\vec{k})}{\sqrt{\log(\Lambda_f/\omega(\vec{k}))}}}$$

Spectral weight:

$$\mathcal{Z}_N(k_N;\omega(\vec{k})) \sim \exp\left(-\frac{\sqrt{\log\left(\frac{\Lambda_f}{\omega(\vec{k})}\right)}}{\log\log\left(\frac{\Lambda_f}{\omega(\vec{k})}\right)}\right)$$

$\omega(\vec{k})$ receives superlogarithmic corrections in k_N.

Scattering rate deviates from Fermi-liquid theory!

Take Away Messages

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Message I

The normal state AFM Quantum Critical Metal supports *both* Fermi-liquid-like and non-Fermi-liquid-like low-energy electronic excitations:

Cold: Noninteracting: Unit quasiparticle weight
Lukewarm: Quasiparticle weight has power-law decay in momentum
Hot: Quasiparticle weight has superlogarithmic decay in momentum

At hot spots: no quasiparticles!

Message II

Despite the imminent onset of superconducting order, Non-Fermi-liquid signatures survive in the normal state of the AFM Quantum Critical Metal.

ARPES Experiments: Expect to see physics consistent with lukewarm electrons and scattering rate linear in energy!

Realistic Scenario II: Superconducting Instability

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For $\omega \sim \Lambda_f e^{-\ell_{\mathrm{SC}}} \ll \Lambda_f e^{-\ell_0}$ and $k_N \neq 0$, generically:

$$\mathcal{A}_N(\vec{k};\omega) = \frac{\mathcal{Z}_N(k_N;\omega(\vec{k}))}{\tau_N(k_N;\omega(\vec{k}))} \frac{1}{(\omega - \omega(\vec{k}))^2 + \tau(\vec{k};\omega(\vec{k}))^{-2}}$$