

Title: QED and quantum magnetism in (2+1)d

Speakers: Chong Wang

Collection: Quantum Matter: Emergence & Entanglement 3

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Abstract: The interplay of symmetry and topology has been at the forefront of recent progress in quantum matter. In this talk I will discuss an unexpected connection between band topology and competing orders in a quantum magnet. The key player is the two-dimensional Dirac spin liquid (DSL), which at low energies is described by an emergent Quantum Electrodynamics (QED) with massless Dirac fermions (a.k.a. spinons) coupled to a U(1) gauge field. A long-standing open question concerns the symmetry properties of the magnetic monopoles, an important class of critical degrees of freedom. I will show that the monopole properties can be determined from the topology of the underlying spinon band structure. In particular, the lattice momentum and angular momentum of monopoles can be determined from the charge (or Wannier) centers of the corresponding spinon insulators. I will then discuss the consequences of the monopole properties, such as the stability of the DSL on different lattices, universal (experimental and numerical) signatures of DSL, and competing symmetry-breaking phases near the DSL state.

QED: towards a unified view on 2D quantum magnetism

Chong Wang

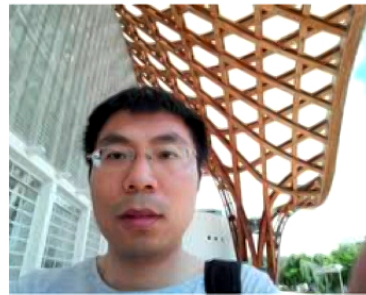
Perimeter Institute

QUANTUM MATTER: EMERGENCE & ENTANGLEMENT 3

Perimeter Institute, April 22, 2019



Xue-Yang Song
(Harvard)



Yin-Chen He
(Perimeter)

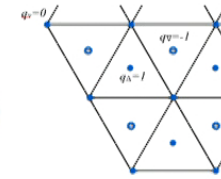


Ashvin Vishwanath
(Harvard)

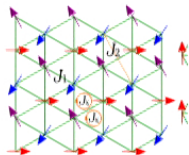
arXiv: 1811.11182, 1811.11186



Band topology (SPT):
Topological insulator, Wannier center,
Polarization...



Instantons (monopoles) in QED₃



Massive phase

Competing orders:
Anti-Ferromagnet, Valence-bond solid...

Critical phase (CFT)

$$\sum_{\alpha,s} \bar{\psi}_{\alpha,s} \gamma^\mu (\partial_\mu - i a_\mu) \psi_{\alpha,s}$$

U(1) Dirac spin liquid:
consequences in experiments and numerics

Quantum magnetism

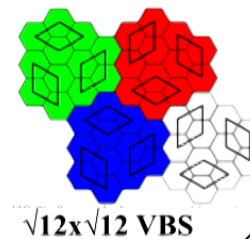
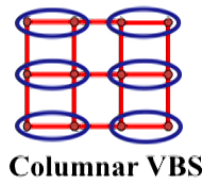
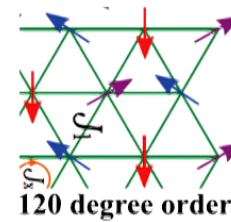
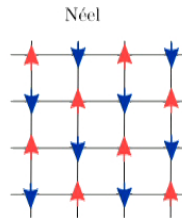
$$H = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \dots$$

**Square and
Honeycomb**

**Deconfined
criticality**

???

**Triangular and
Kagome**



Dirac spin liquid: a mother state?

$$\mathcal{L}_{QED3} = \sum_{\alpha,s} \bar{\psi}_{\alpha,s} \gamma^{\mu} (\partial_{\mu} - i a_{\mu}) \psi_{\alpha,s}$$

Algebraic spin liquid as the mother of many competing orders

Michael Hermele,¹ T. Senthil,^{2,3} and Matthew P. A. Fisher⁴

Deconfined Quantum Critical Point on the Triangular Lattice

Chao-Ming Jian,^{1,2} Alex Thomson,^{3,2} Alex Rasmussen,⁴ Zhen Bi,⁵ and Cenke Xu⁴

Key open issue: monopoles (instantons)

A crash course on Dirac spin liquid

Parton construction using fermionic spinons:

$$\mathbf{S}_i = \frac{1}{2} f_{i,\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{i,\beta}$$

Mean field ansatz:

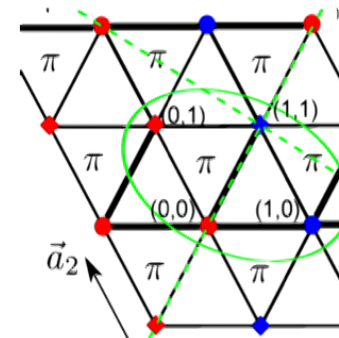
$$H_{MF} = - \sum_{ij} f_i^\dagger t_{ij} f_j$$

A handy recipe to access possible interesting states of matter

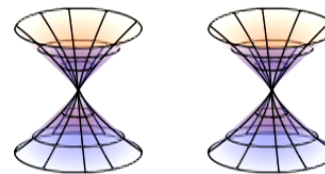
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Arrange hoppings to have 2x2 Dirac
cones at low energy

(Historical motivation:
RVB, cuprates...)



Picture from
Lu, 2016



U(1) gauge redundancy* on spinon fermions:

$$f \rightarrow e^{i\alpha_i} f$$

=> emergent U(1) gauge field coupled to spinons

Gauge charge per site:

$$q_i = \sum_{\alpha} f_{i,\alpha}^{\dagger} f_{i,\alpha} - 1$$

Gauge invariance: $\langle q \rangle = 0$

Low-energy continuum theory: QED₃ with $N_f=4$

$$\mathcal{L}_{QED3} = \sum_{\alpha,s} \bar{\psi}_{\alpha,s} \gamma^{\mu} (\partial_{\mu} - i a_{\mu}) \psi_{\alpha,s}$$

A few words on QED₃

- Strongly coupled in IR
- Interesting scenario: a critical state (CFT)
 - Emergent SU(4) spin-valley symmetry ([Hermele, et. al, 2004](#))
 - Emergent U(1) flux conservation symmetry:
$$\dot{j}_\mu = \frac{1}{2\pi} \epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda$$
 - Supported by recent numerics ([Karthik, Narayanan, PRD, 2016](#)) when monopoles are ignored (non-compact U(1) gauge field)

Monopoles in QED₃

- A 2π -flux insertion operator $\mathcal{M}$
— charged under $U(1)_{\text{flux}}$
- In QED₃: one zero mode per Dirac fermion
- Gauge invariance: half-fill the zero modes

$$\Phi \sim f_i^\dagger f_j^\dagger \mathcal{M}_{bare}$$

- $\{\Phi_1, \dots, \Phi_6\}$: vector under $SO(6)=SU(4)/Z_2$
— “fundamental order parameter”
- 3 spin singlets $\{\Phi_{1,2,3}\}$ + 3 spin triplets $\{\Phi_{4,5,6}\}$

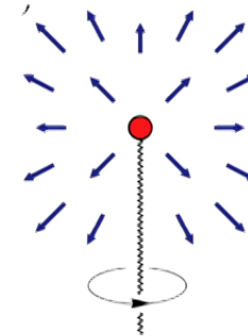
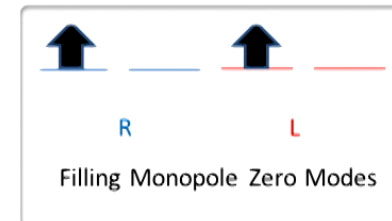


Image from Zyga, phys.org



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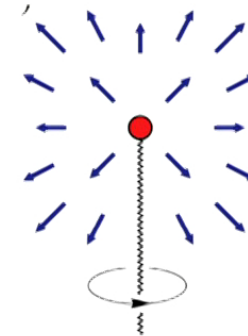
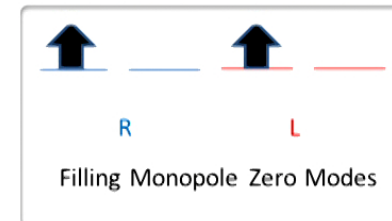


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The problem

How do monopoles transform under discrete physical symmetries?

$$g : \Phi_i \rightarrow e^{i\theta_g} O_{ij} \Phi_j, \quad O \in SO(6)$$

Berry phase from Dirac sea

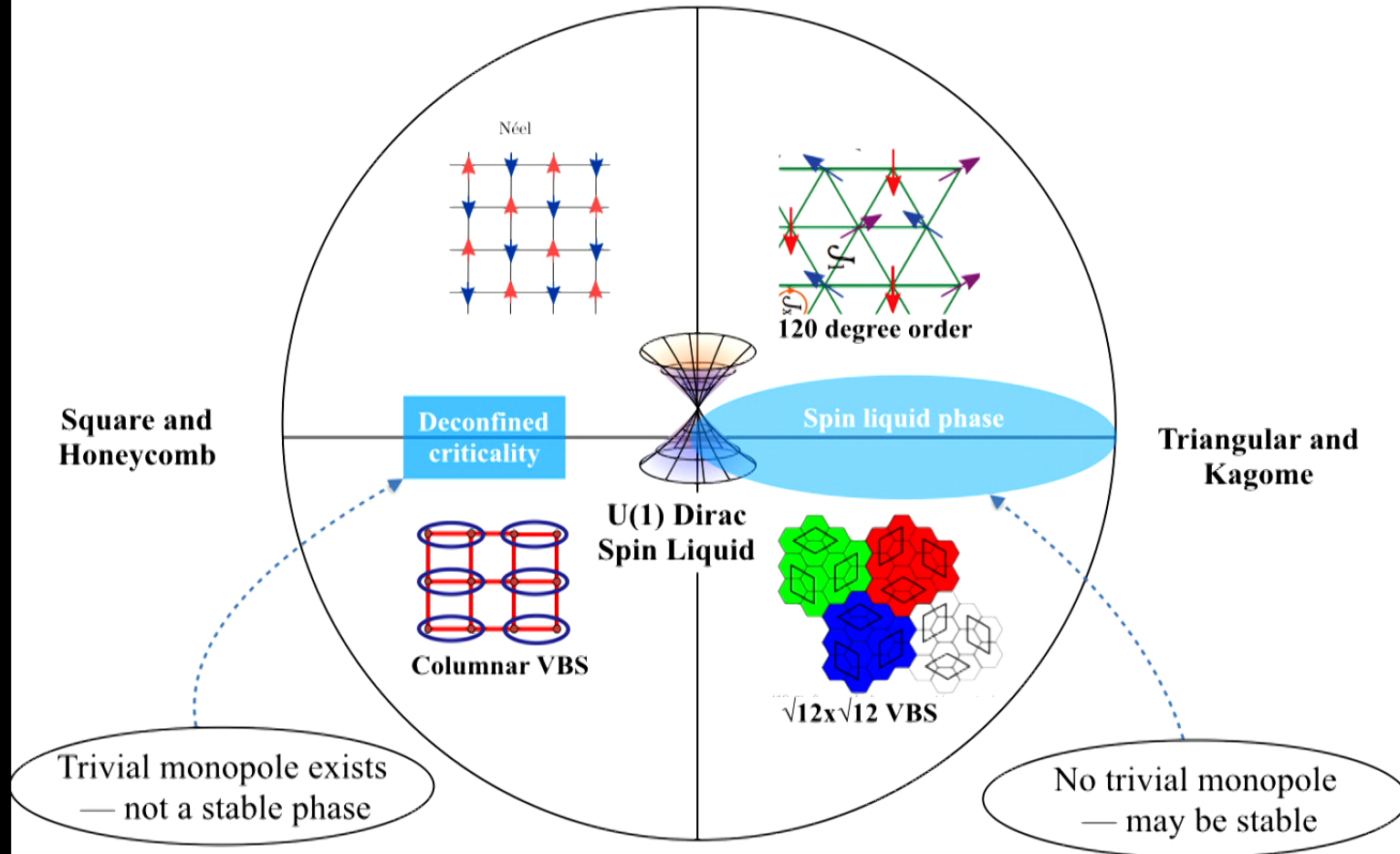
Determined by transformation of Dirac fermions — relatively easy

- Lattice-scale origin of the phase factor: Berry phase accumulated by monopole when moving in charge background
(Haldane; Read & Sachdev)
- Partial success in the past, rely on numerics
(Alicea; Hermele, Ran, Lee, Wen; Ran, Vishwanath, Lee)

Why do we care?

- Conceptually: necessary information to characterize the state
- Practically: 1) stability of U(1) Dirac spin liquids:
if monopole perturbation symmetry-allowed, likely relevant and
drives confinement (Borokhov, Kapustin, Wu; Pufu; Polyakov)
- 3) measurable consequences (e.g. in spectroscopy)
- 2) determine nearby symmetry-breaking orders
— monopoles are the natural order parameters

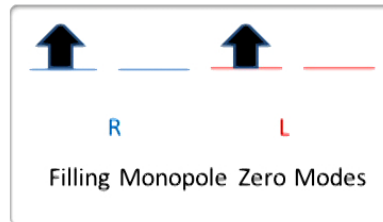
Summary of results



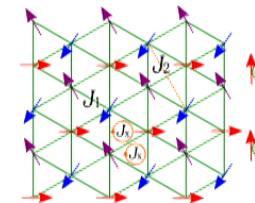
Example: Triangular magnetic order

- Mass term - (Quantum Spin Hall) $-\bar{\psi}\sigma^z\psi$
 - Breaks spin rotation but not lattice symmetries

- *Monopole picked out*



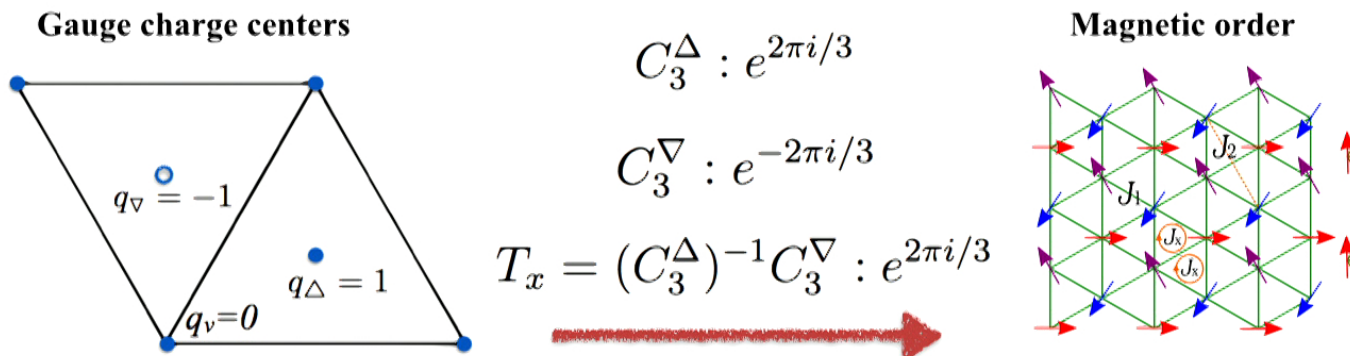
- This monopole behaves like $S_x + iS_y$
 - non-collinear magnetic order (Senthil, Fisher; Ran, Vishwanath, Lee; Jian, Thomson, Rasmussen, Bi, Xu)



- What about lattice symmetries (translation & rotation)?

Charge center: a recipe

- Deform gapped fermions to “atomic limit”
- Focus on rotations: use $L = e m$
- Translation generated by successive rotations



Meaning of negative charge: “Fragile Topology”
Po, Watanabe, Vishwanath

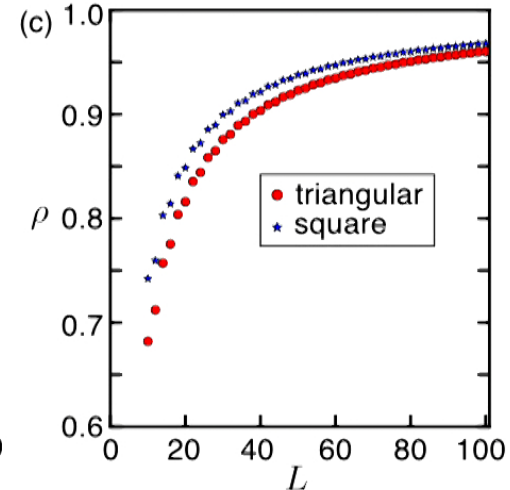
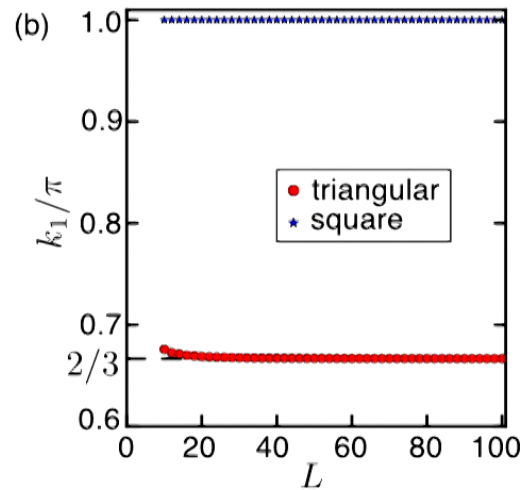
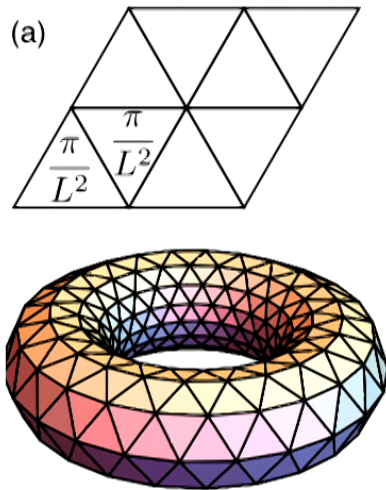


(After breaking spin-rotation and time-reversal symmetries)

Calculational recipe from Po, Vishwanath, Watanabe:
Symmetry-based indicators

sym.	$\Gamma_{k/M}^{\Delta}$	$\Gamma_{k/M}^{\nabla}$	$\Gamma_{k/M}$	Γ_k^{PSG} (2-fold particle, 2-fold hole)	Γ_M^{PSG} (4-fold deg.)
C_3^{site}	$\omega^L \times [\omega^2/1, 1, \omega, \omega^2]$	$\omega^L \times [\omega/1, 1, \omega, \omega^2]$	$[1/1, 1, \omega, \omega^2]$	$[\omega^2, 1], [\omega, 1]$	$[1, 1, \omega, \omega^2]$
C_3^{up-tri}	$\omega^L \times [1/1, 1, \omega, \omega^2]$	$\omega^L \times [\omega^2/1, 1, \omega, \omega^2]$	$[\omega/1, 1, \omega, \omega^2]$	$[\omega, 1], [\omega, \omega^2]$	$[1, 1, \omega, \omega^2]$
$C_3^{down-tri}$	$\omega^L \times [\omega/1, 1, \omega, \omega^2]$	$\omega^L \times [1/1, 1, \omega, \omega^2]$	$[\omega^2/1, 1, \omega, \omega^2]$	$[\omega^2, \omega], [\omega^2, 1]$	$[1, 1, \omega, \omega^2]$
$\Gamma^{particle} = \Gamma_{L=0} + \Gamma_{L=0}^{\Delta} - \Gamma_{L=0}^{\nabla} \rightarrow \text{fragile topology}$ $\Gamma^{hole} = \Gamma_{L=0} + \Gamma_{L=0}^{\nabla} - \Gamma_{L=0}^{\Delta} \rightarrow \text{fragile topology}$					

Numerical check



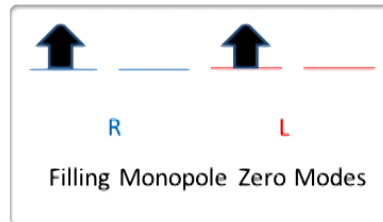
$$\langle \psi | G_{T_1} \cdot T_1 | \psi \rangle = \rho e^{i\tilde{k}_1}$$

- Dirac sea phase factor for (spin triplet) monopole.
- Square lattice (π, π) - in agreement with *Alicea*'s cylinder numerics (torus is better).

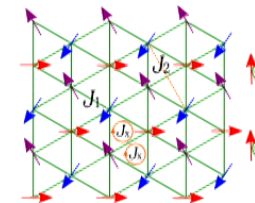
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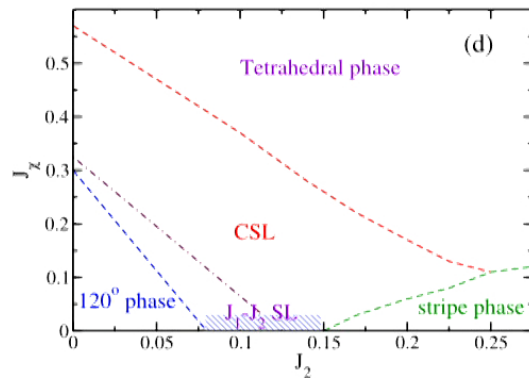
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Numerics on Triangular/Kagome

U(1) Dirac spin liquid naturally related to:



Global phase diagram and quantum spin liquids in spin-1/2 triangular antiferromagnet

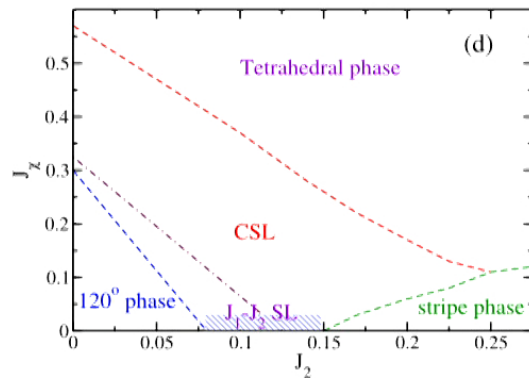
Shou-Shu Gong¹, W. Zhu², J.-X. Zhu^{3,3}, D. N. Sheng⁴, and Kun Yang⁵

- 120 magnetic order
— quantum spin Hall mass
- Chiral spin liquid
— quantum Hall mass

Similar story on Kagome
(He, et, al; Ran, et, al; Xiang, et, al)

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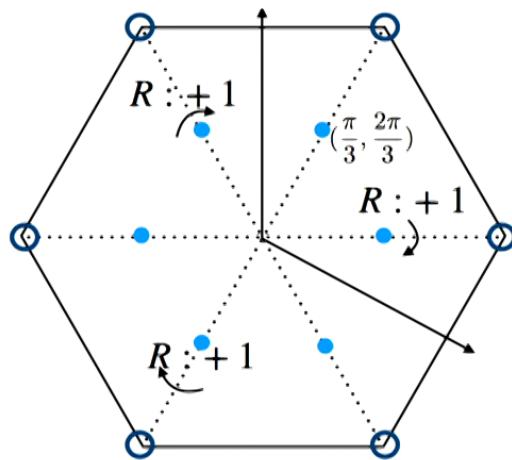
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Consequence for Dirac spin liquid

- Spin liquid phase: Dirac fermions massless, no magnetic order
- Monopoles still important as critical fluctuations
- Consequences: spectral weight at nontrivial momentum...



Recently observed numerically
Ferrari, Becca; He et. al (unpublished)