Title: Twisted foliated fracton order

Speakers: Xie Chen

Collection: Quantum Matter: Emergence & Entanglement 3

Date: April 22, 2019 - 10:45 AM

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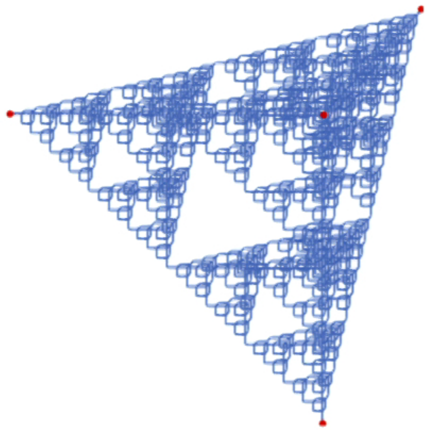
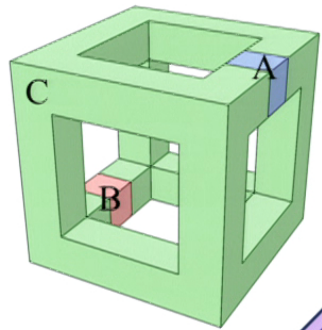
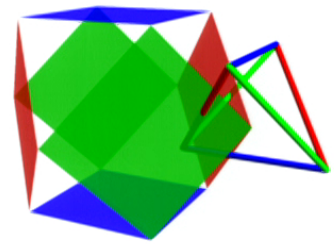
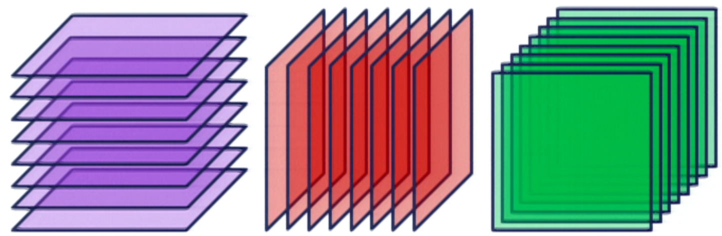
Abstract: In the study of three-dimensional gapped models, two-dimensional gapped states can be considered as a free resource. This is the basic idea underlying our proposal of the notion of `foliated fracton order'. Using this idea, we have found that many of the known type-I fracton models, like the X-cube model and the checkerboard model, have the same foliated fracton order. In this talk, I will present three-dimensional fracton models with a different kind of foliated fracton order. The previously known foliated fracton order corresponds to the gauge theory of a simple paramagnet with subsystem planar symmetry. The new order corresponds to a twisted version of the gauge theory where the system before gauging has nontrivial order protected by the subsystem planar symmetries. I will discuss a way to identify the nontrivial order by compactifying the system in the z direction and analyzing the resulting two dimensional order.





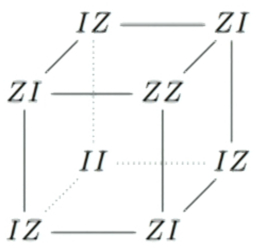


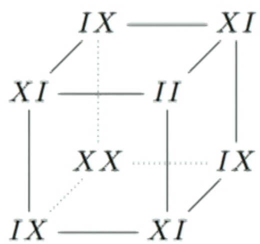


Twisted Foliated Fracton Phases

XIE CHEN, CALTECH
PERIMETER INSTITUTE
APR. 2019

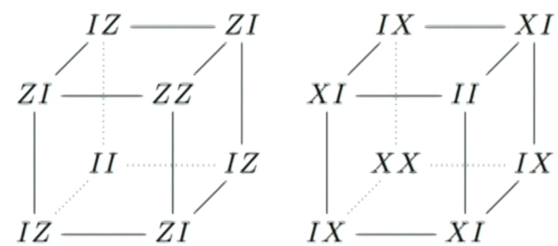
Fractal cubic code





Haah; Yoshida

Fractal cubic code



Gapped

Ground State Degeneracy 2^k

$$\frac{k+2}{4} = \deg_x \ \gcd \begin{bmatrix} 1+(1+x)^L, \\ 1+(1+\omega x)^L, \\ 1+(1+\omega^2 x)^L \end{bmatrix}_{\mathbb{F}_4}$$

$$= \begin{cases} 1 & \text{if } L = 2^p + 1 \ (p \geq 1), \\ L & \text{if } L = 2^p \ (p \geq 1) \end{cases}$$

$$\mathbb{F}_4 = \{0, 1, \omega, \omega^2\}$$

Haah; Yoshida

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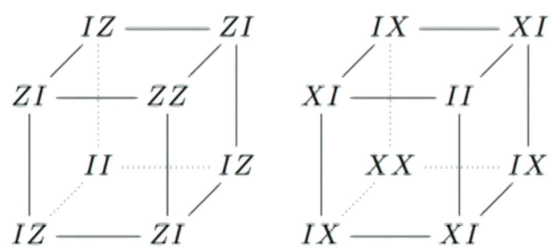

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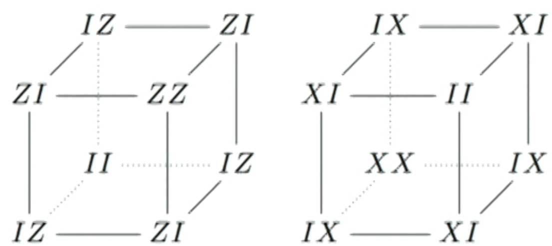

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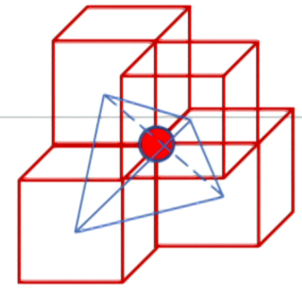

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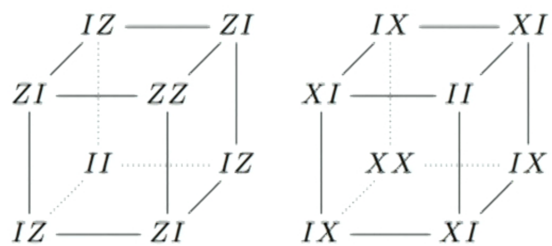
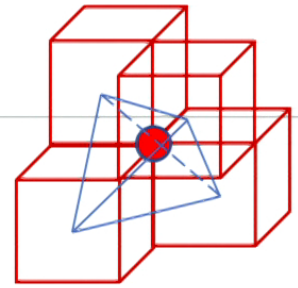


Gapped



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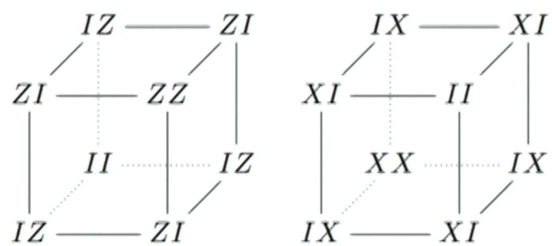
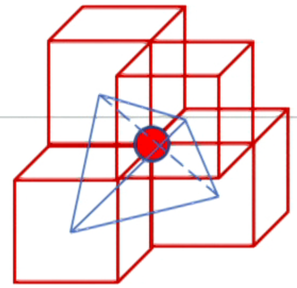


Gapped



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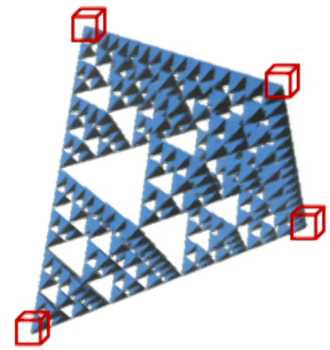

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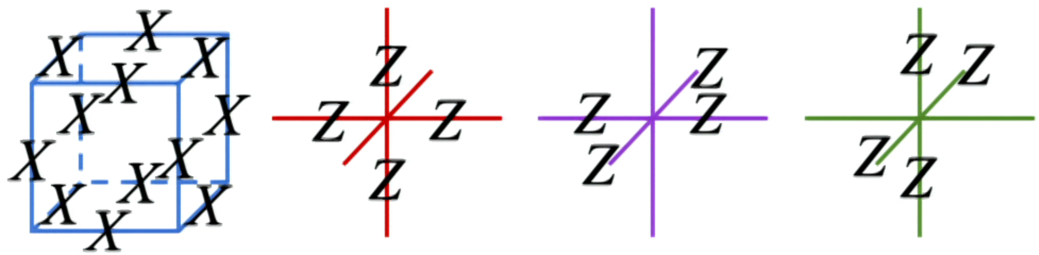
Haah; Yoshida

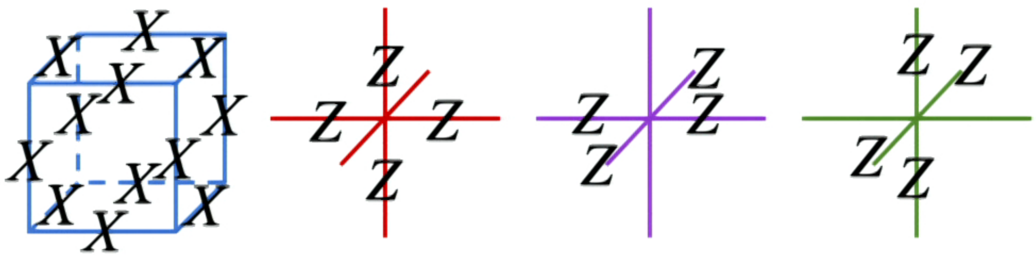
Type I Models (also gapped)

X-Cube



Chamon, 05; Vijay, Haah, Fu, 15, 16

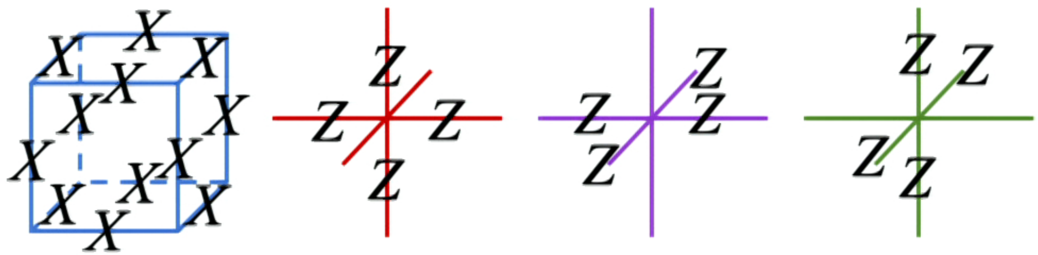
X-cube



Log(Ground state degeneracy) = 6L-3

Vijay, Haah, Fu, 16

X-cube



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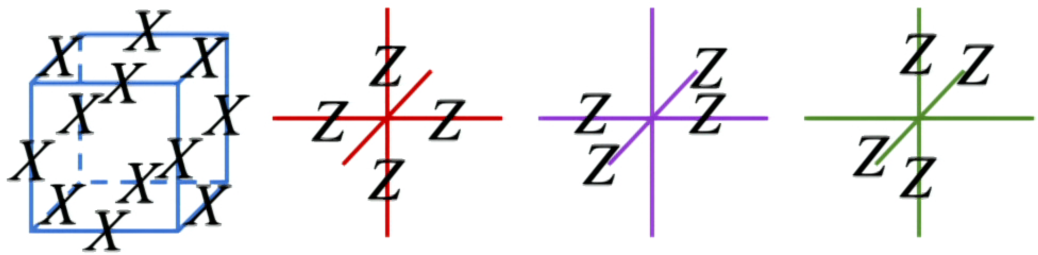
Lineon excitations:

Vijay, Haah, Fu, 16

X-cube



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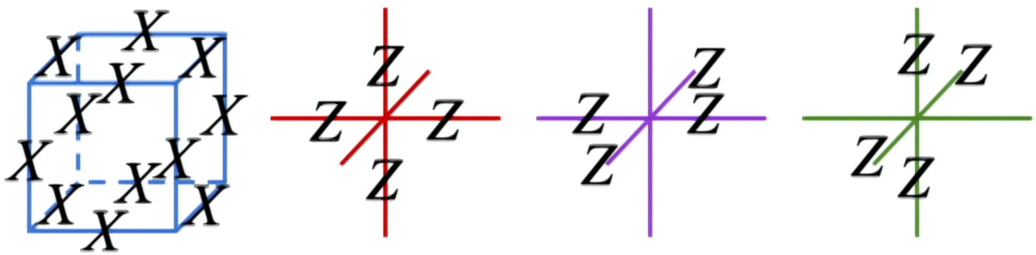

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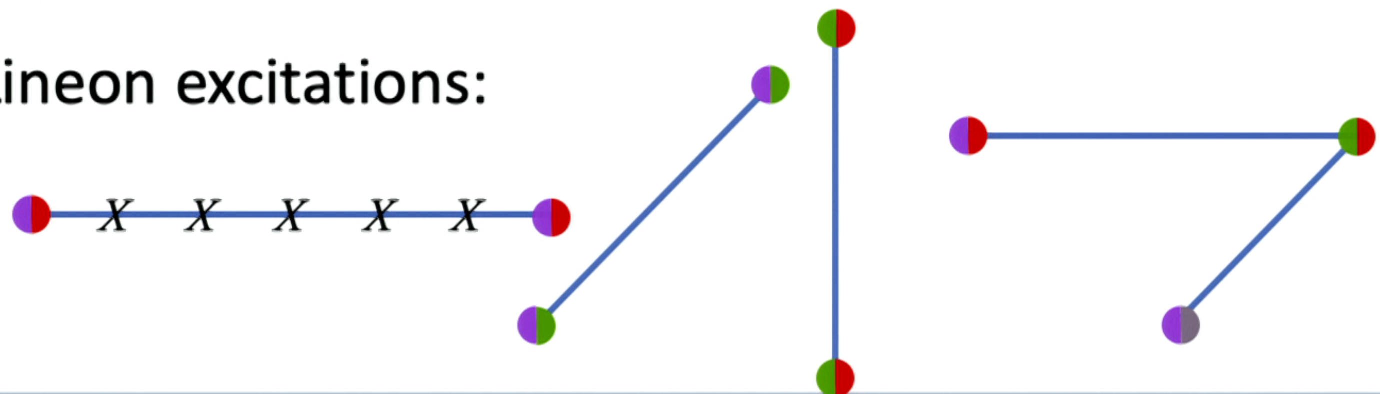

Vijay, Haah, Fu, 16

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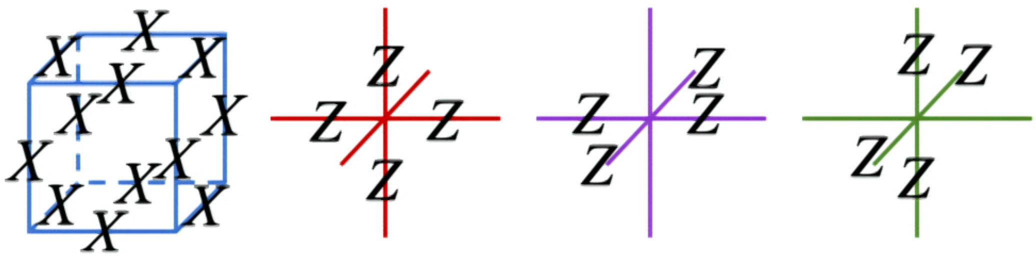
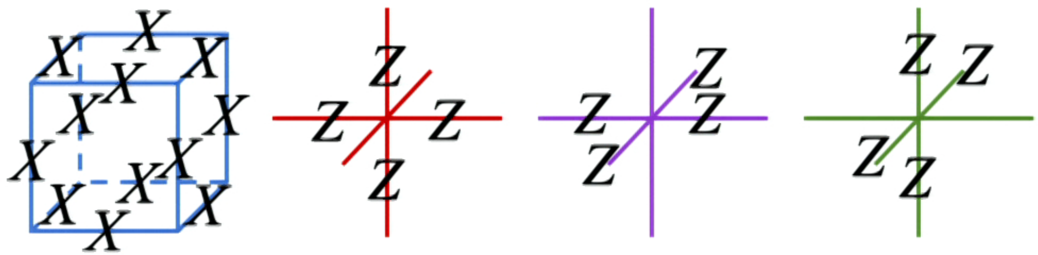


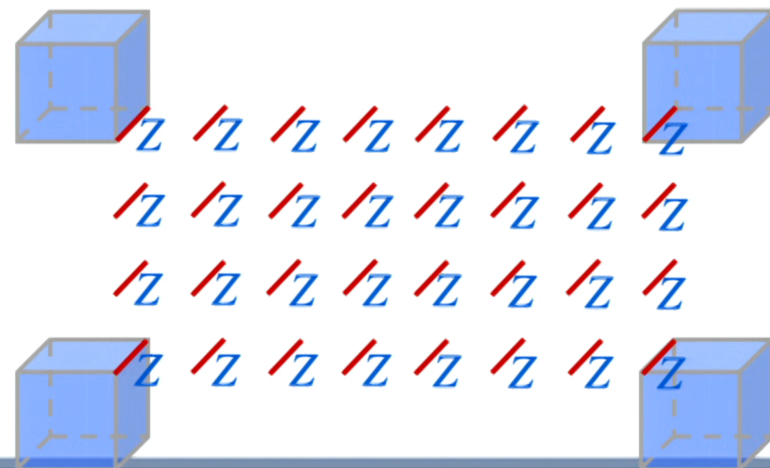
Fracton Excitations

Vijay, Haah, Fu, 16

X-cube



Fracton Excitations



Vijay, Haah, Fu, 16

Fracton models

- Point excitation with restricted motion
- Slow dynamics
- Localization in the absence of disorder
- Unusual entanglement scaling
- Exponential ground state degeneracy

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How to build a self-correcting quantum memory (quantum hard drive)?

Fracton models

- Point excitation with restricted motion
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How to build a self-correcting quantum memory (quantum hard drive)? → **How to understand the new order in these models?**

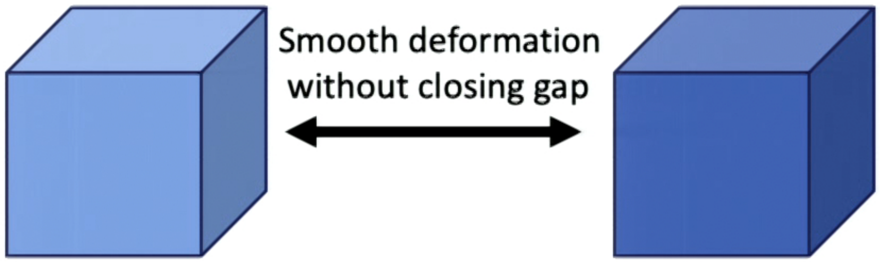
Fracton models

- Point excitation with restricted motion
- Slow dynamics
- Localization in the absence of disoı
- Unusual entanglement scaling
- Exponential ground state degenerɛ

How to build a self-correcting quantum memory (quantum hard drive)?

- What are the universal properties of these phases?
- What is the relation between different models?
- How to define a fracton phase?

Definition of gapped phases



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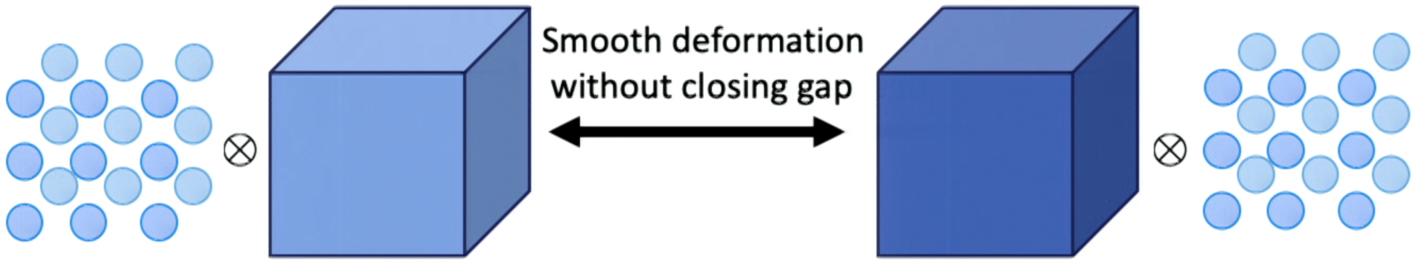
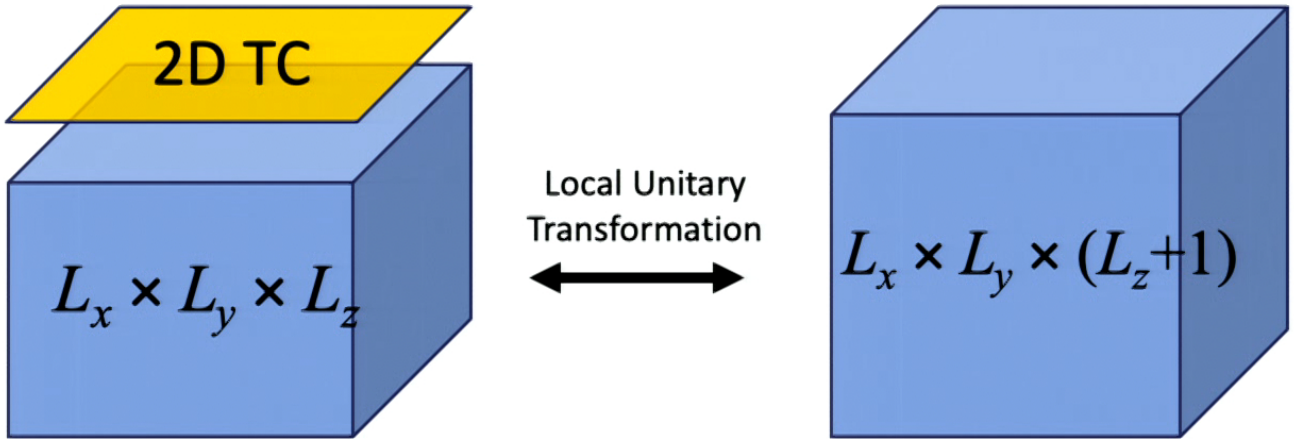
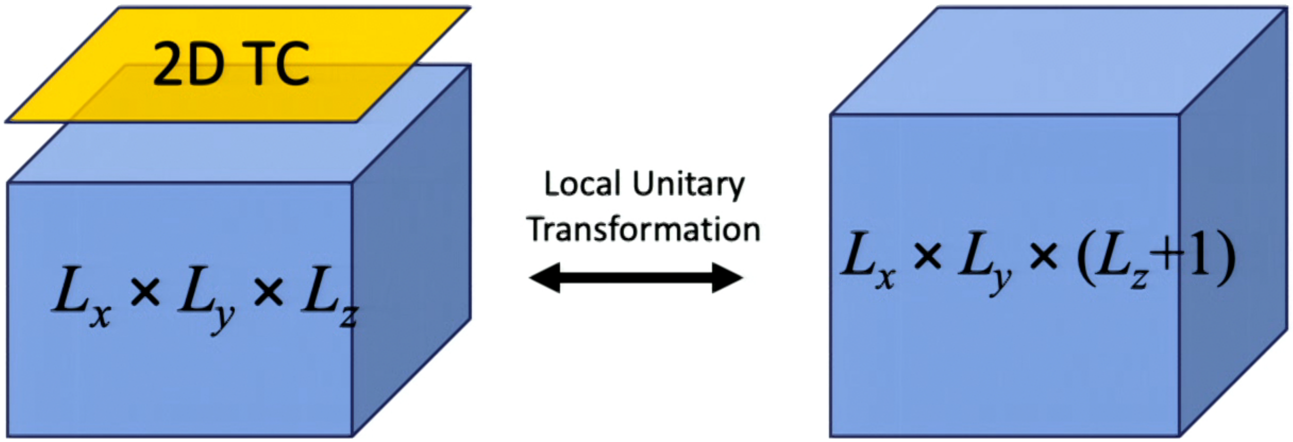


X-cube RG



- Fracton Models on General Three-Dimensional Manifolds, *Phys. Rev. X* 8, 031051 (2018)

X-cube RG

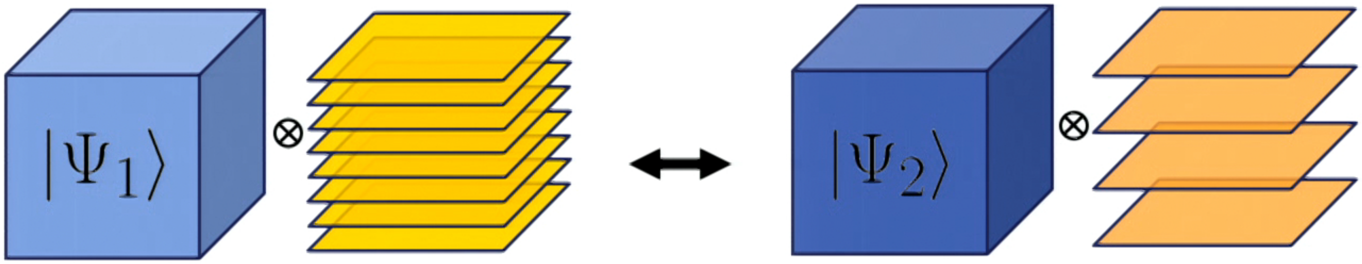


- Define model on different manifolds
- Define phase up to addition / removal of gapped 2D layers (and smooth deformation): foliated fracton phase

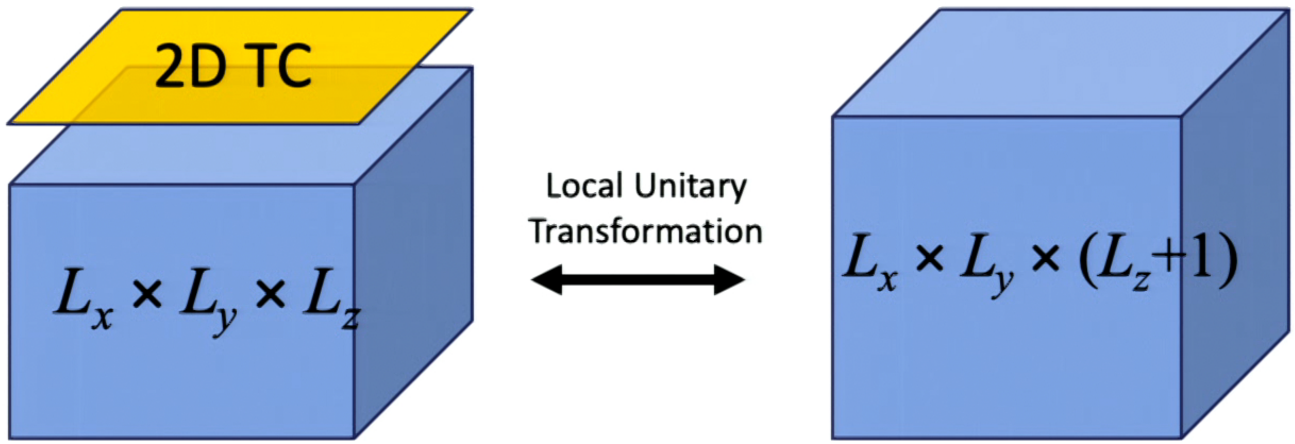
- Fracton Models on General Three-Dimensional Manifolds, *Phys. Rev. X* 8, 031051 (2018)

Foliated Fracton Phase

Definition. Two Hamiltonians H_1 and H_2 belong to the same ***foliated fracton phase*** if their ground states can be mapped into each other through local unitary transformation up to addition or removal of 2D topological orders



X-cube RG



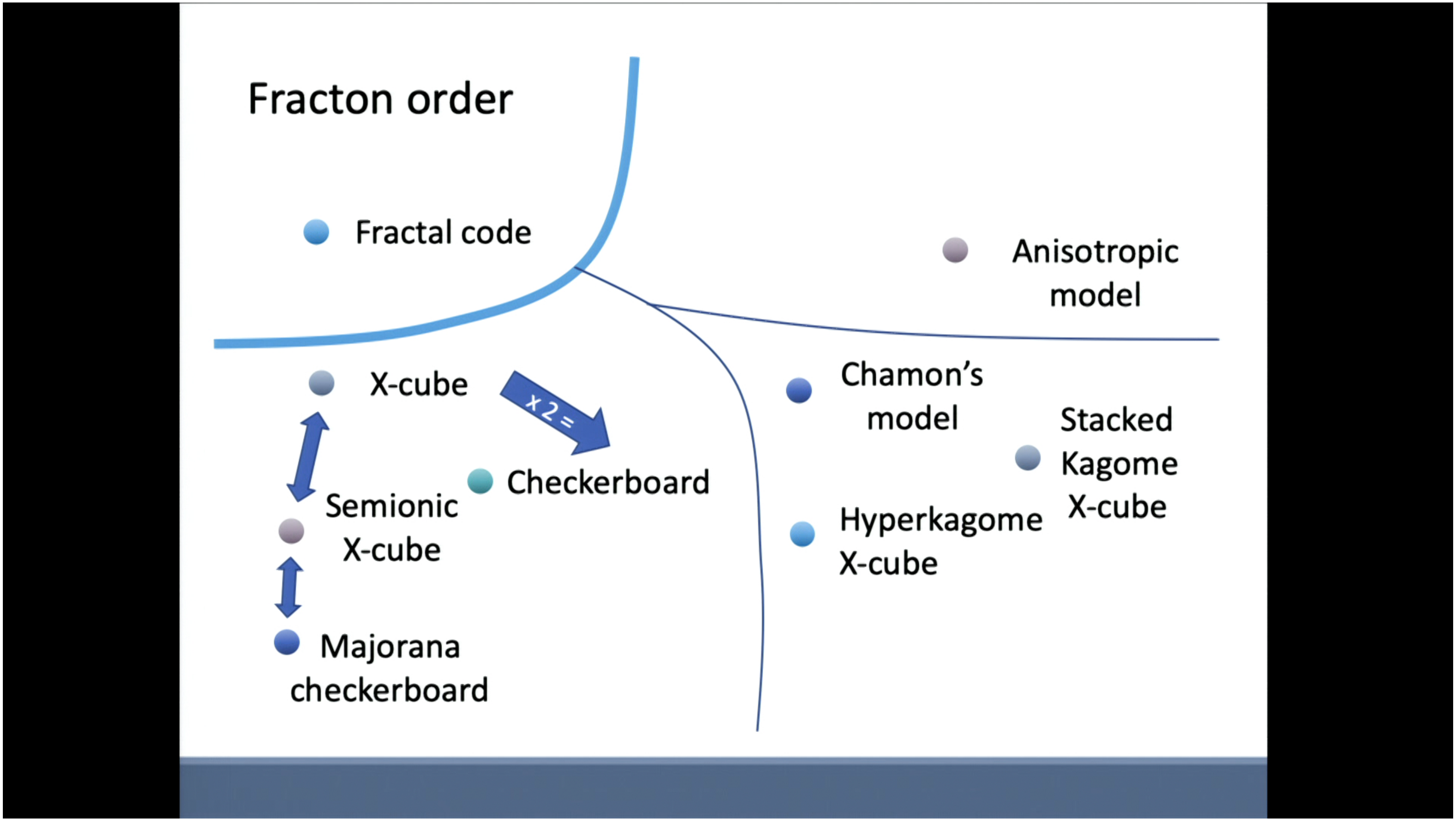
- Define model on different manifolds
- Define phase up to addition / removal of gapped 2D layers (and smooth deformation): foliated fracton phase
- Characterize universal properties by removing contribution from 2D layers

- Fracton Models on General Three-Dimensional Manifolds, *Phys. Rev. X* 8, 031051 (2018)

Fracton order

- Fractal code
- X-cube
- Checkerboard
- Semionic X-cube
- Majorana checkerboard
- Chamon's model
- Stacked Kagome X-cube
- Hyperkagome X-cube

Vijay, Haah, Fu, 15; Slagle, Kim, 17; Ma, Lake, Chen, Hermele, 17


Fracton order
Fractal code
Anisotropic model
X-cube
x 2 =
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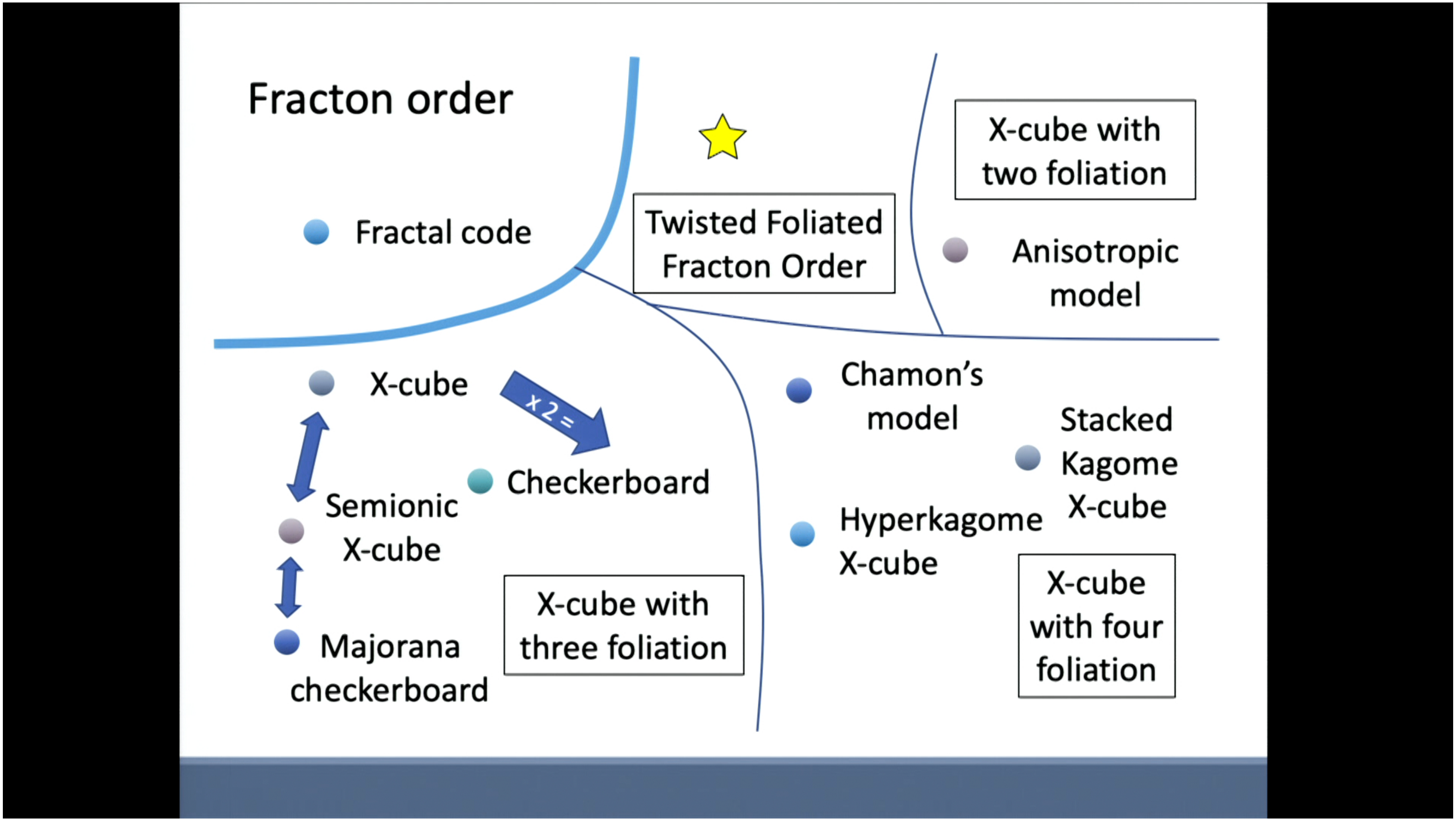


Fracton order
Fractal code
Twisted Foliated Fracton Order
X-cube with two foliation
Anisotropic model
X-cube
x 2 =
Checkerboard
Semionic X-cube
Majorana checkerboard
X-cube with three foliation
Chamon's model
Stacked Kagome X-cube
Hyperkagome X-cube
X-cube with four foliation

Topological order as gauge theory

- Toric Code
- Double Semion

Levin, Gu, 12

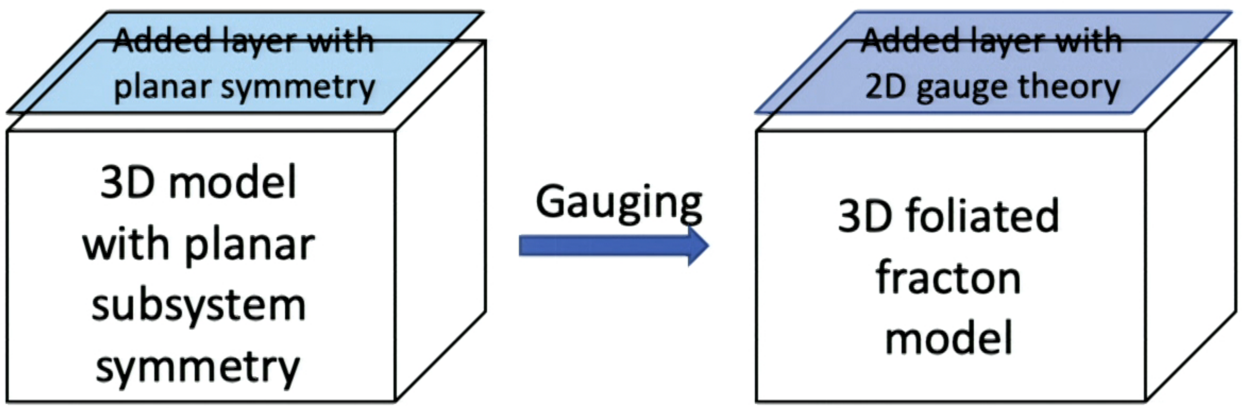
Gauging subsystem symmetry

- X-cube, Checkerboard, Semionic X-cube, Anisotropic model, Hypercube X-cube… models from gauging 3D paramagnets with planar symmetry

Vijay, Haah, Fu, 16; Williamson, 16; Kubica, Yoshida, 18; You, Devakul, Burnell, Sondhi, 18; Song, Prem, Huang, Martin-Delgado, 18

- arXiv:1806.08679, Foliated fracton order from gauging subsystem symmetries

Gauging subsystem symmetry



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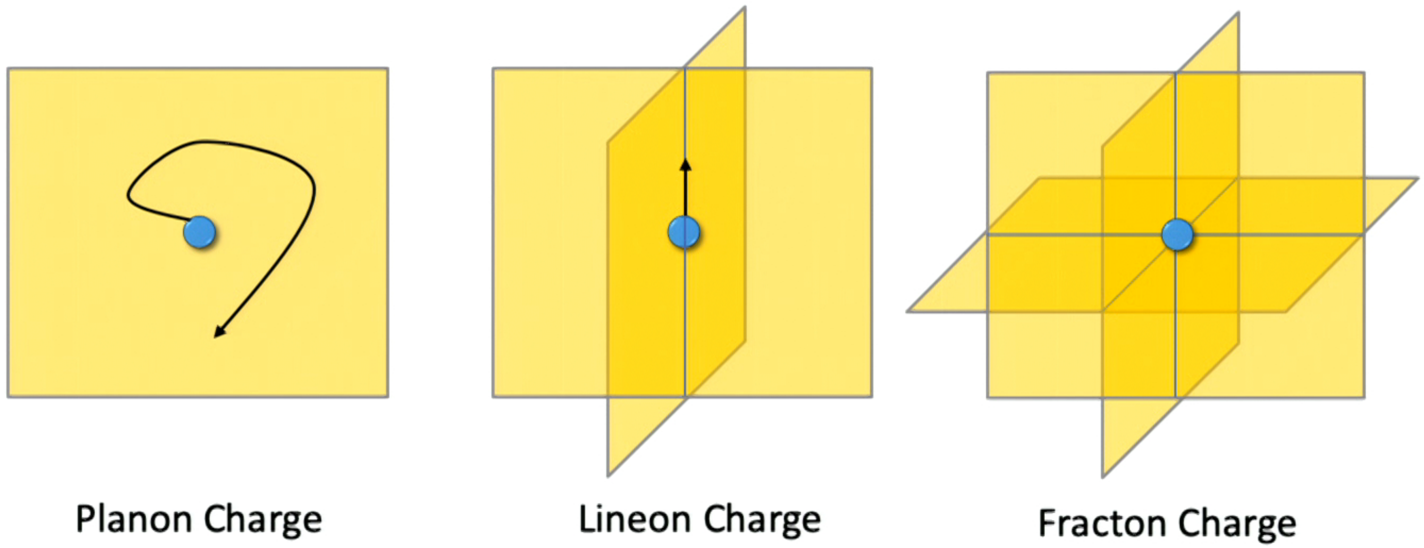

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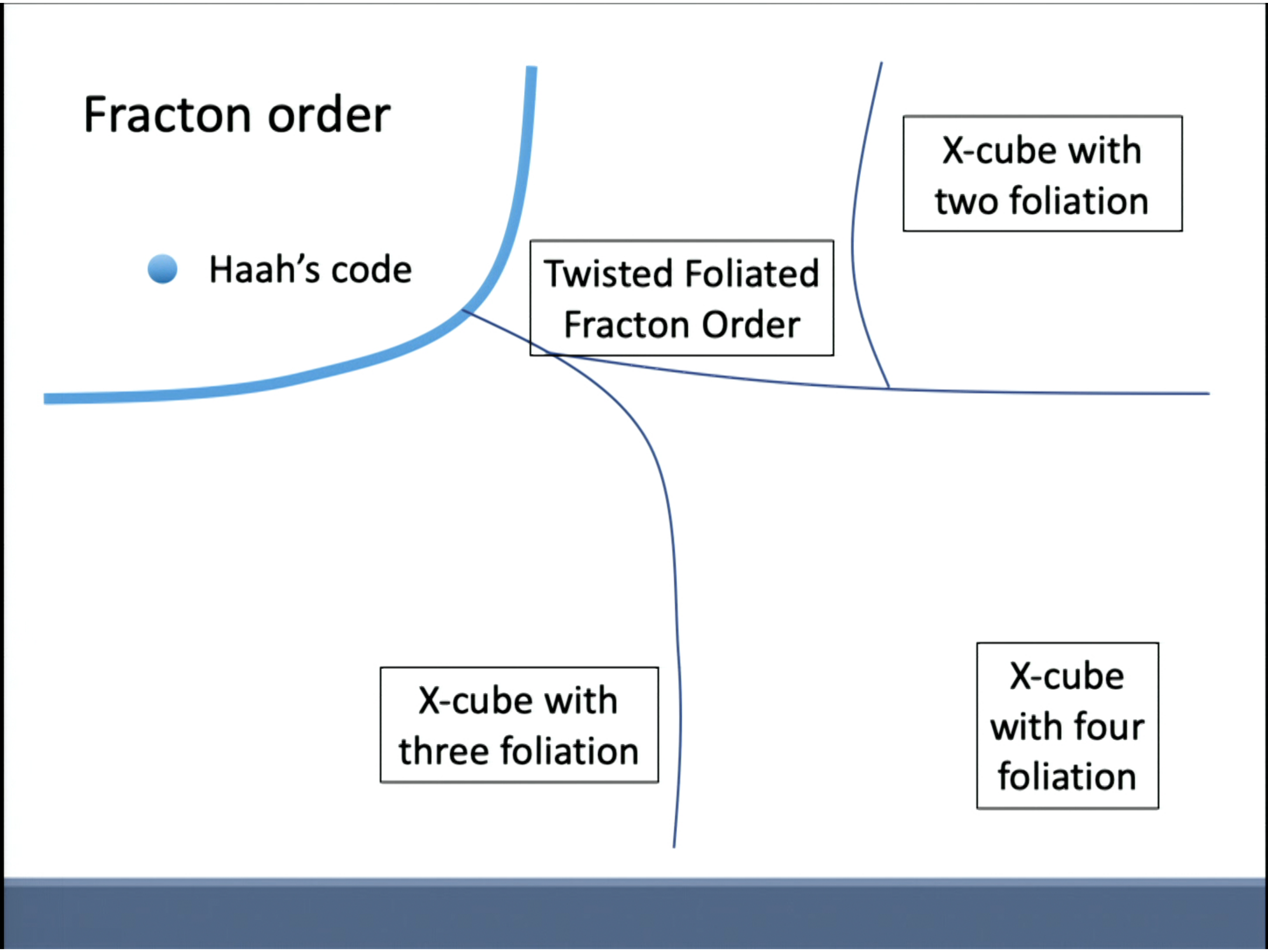
Fracton order
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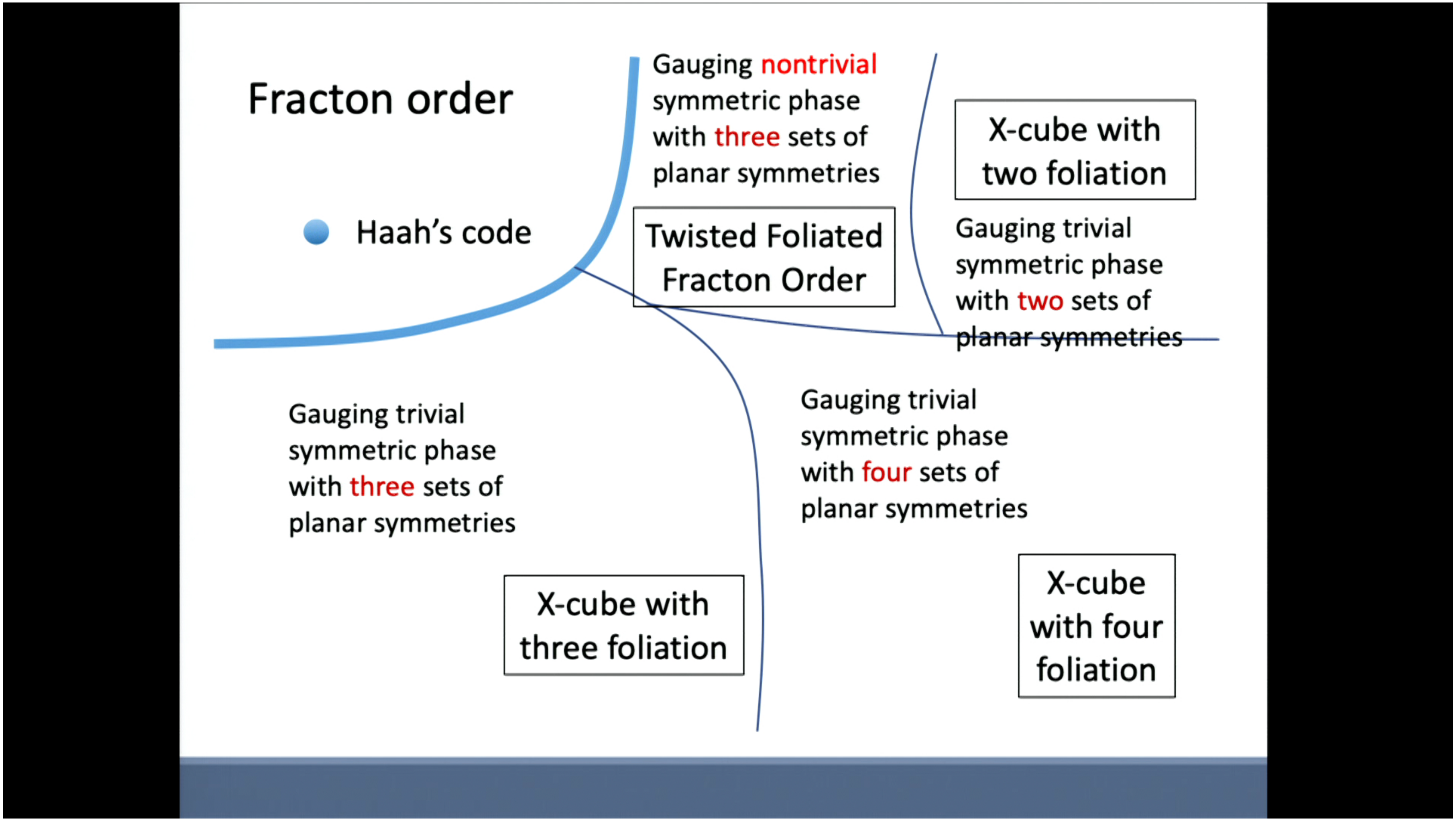
X-cube with two foliation
X-cube with three foliation
X-cube with four foliation


Fracton order
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Gauging nontrivial symmetric phase with three sets of planar symmetries
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Gauging trivial symmetric phase with four sets of planar symmetries
X-cube with three foliation
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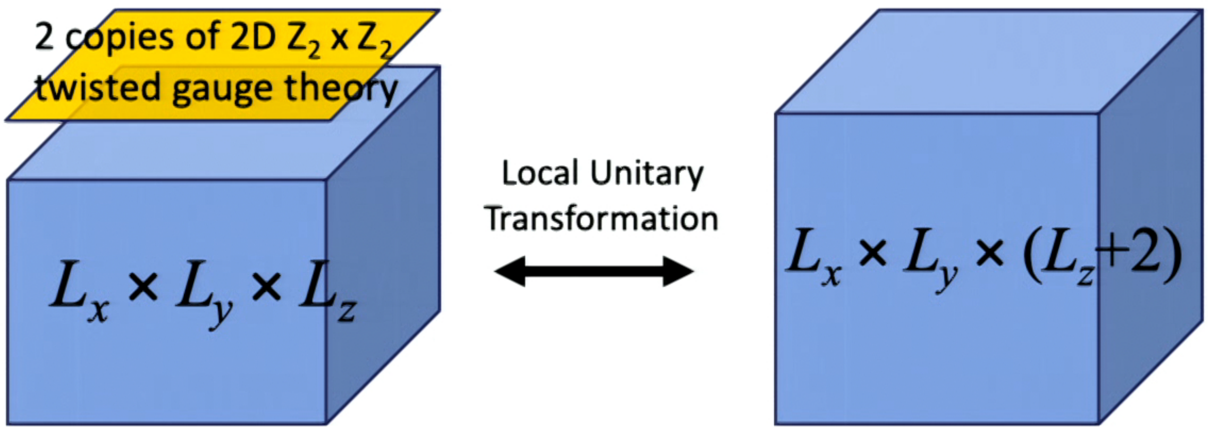
Model construction

Coupled layer construction of X-cube model

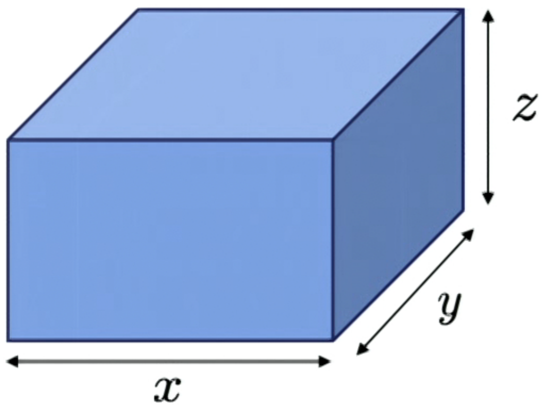
Vijay 2017; Ma, Lake, Chen, Hermele 2017

Foliated fracton order

$Z_2 \times Z_2$ twisted X-cube model



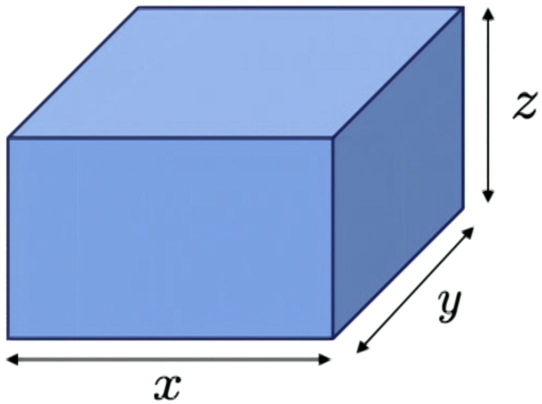
Fracton order and compactification



Finite, periodic boundary condition

Dua, Williamson, Haah, Cheng, arxiv:1903.12246

Fracton order and compactification



Finite, periodic boundary condition

- Fractons and lineons are confined
- Only xy planons remain, with z direction locality

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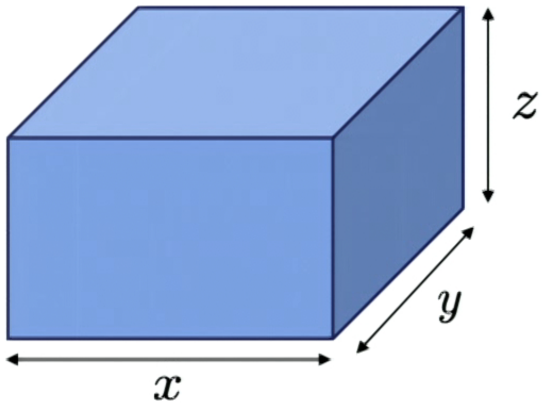


Finite, periodic boundary condition

- Fractons and lineons are confined
- Only xy planons remain, with z direction locality
- As z increases, number of planons increase

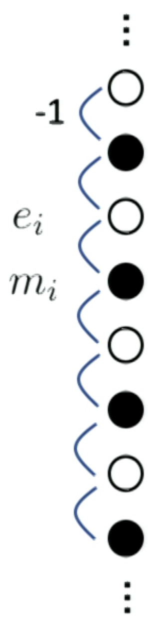
Distinguish fracton orders using the planons

Dua, Williamson, Haah, Cheng, arxiv:1903.12246

Fracton order and compactification

X CUBE

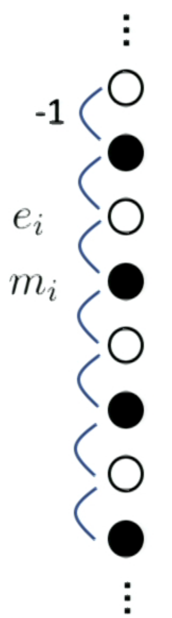
STACK OF TORIC CODE

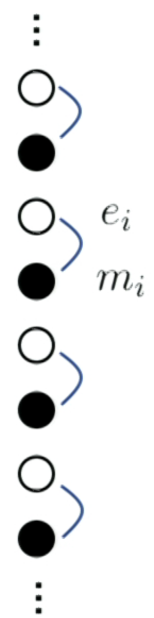


Fracton order and compactification

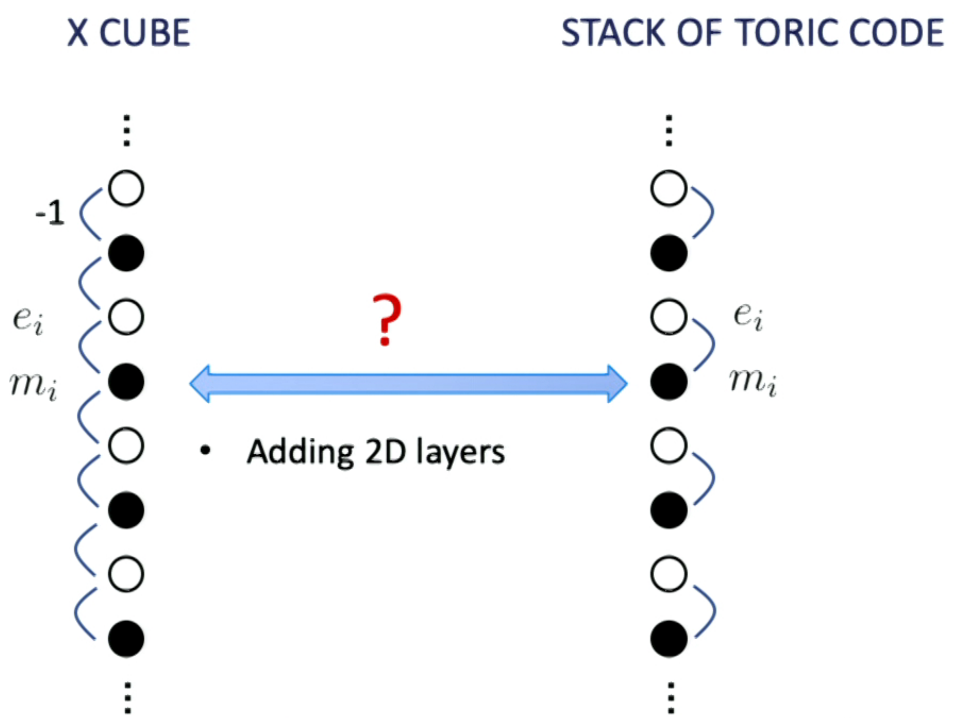
X CUBE

STACK OF TORIC CODE





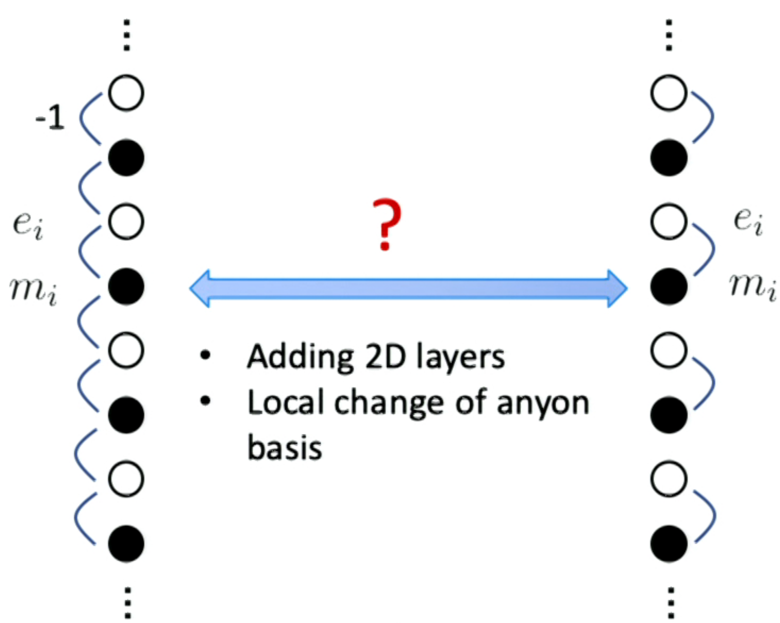
Fracton order and compactification



- Adding 2D layers

Fracton order and compactification

X CUBE — **STACK OF TORIC CODE**

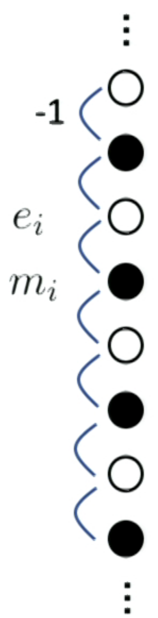


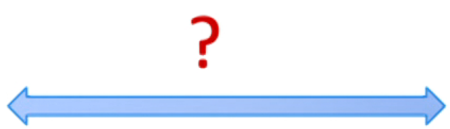
- Adding 2D layers
- Local change of anyon basis

- Locally complete generating set

Fracton order and compactification

X CUBE | **STACK OF TORIC CODE**





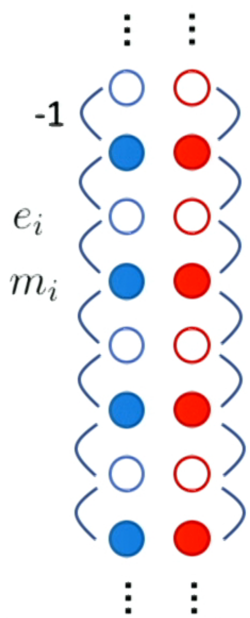
- Adding 2D layers
- Local change of anyon basis

- Locally complete generating set
- Global redundancy in X-cube model (that cannot be removed locally)

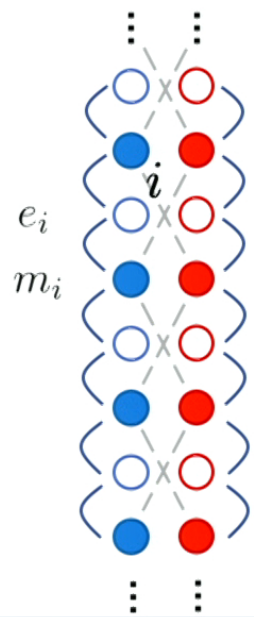
$$\prod_i e_i = \prod_i m_i = 1$$

Fracton order and compactification

2 COPIES OF X CUBE

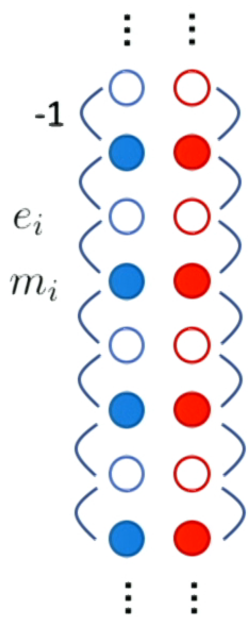


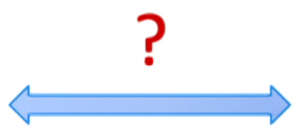
Z_2 X Z_2 TWISTED X CUBE



Fracton order and compactification

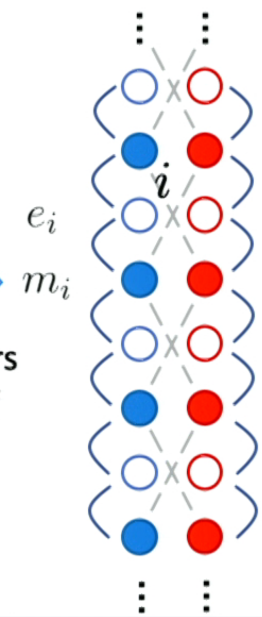
2 COPIES OF X CUBE





- Adding 2D layers
- Local change of anyon basis

Z_2 X Z_2 TWISTED X CUBE

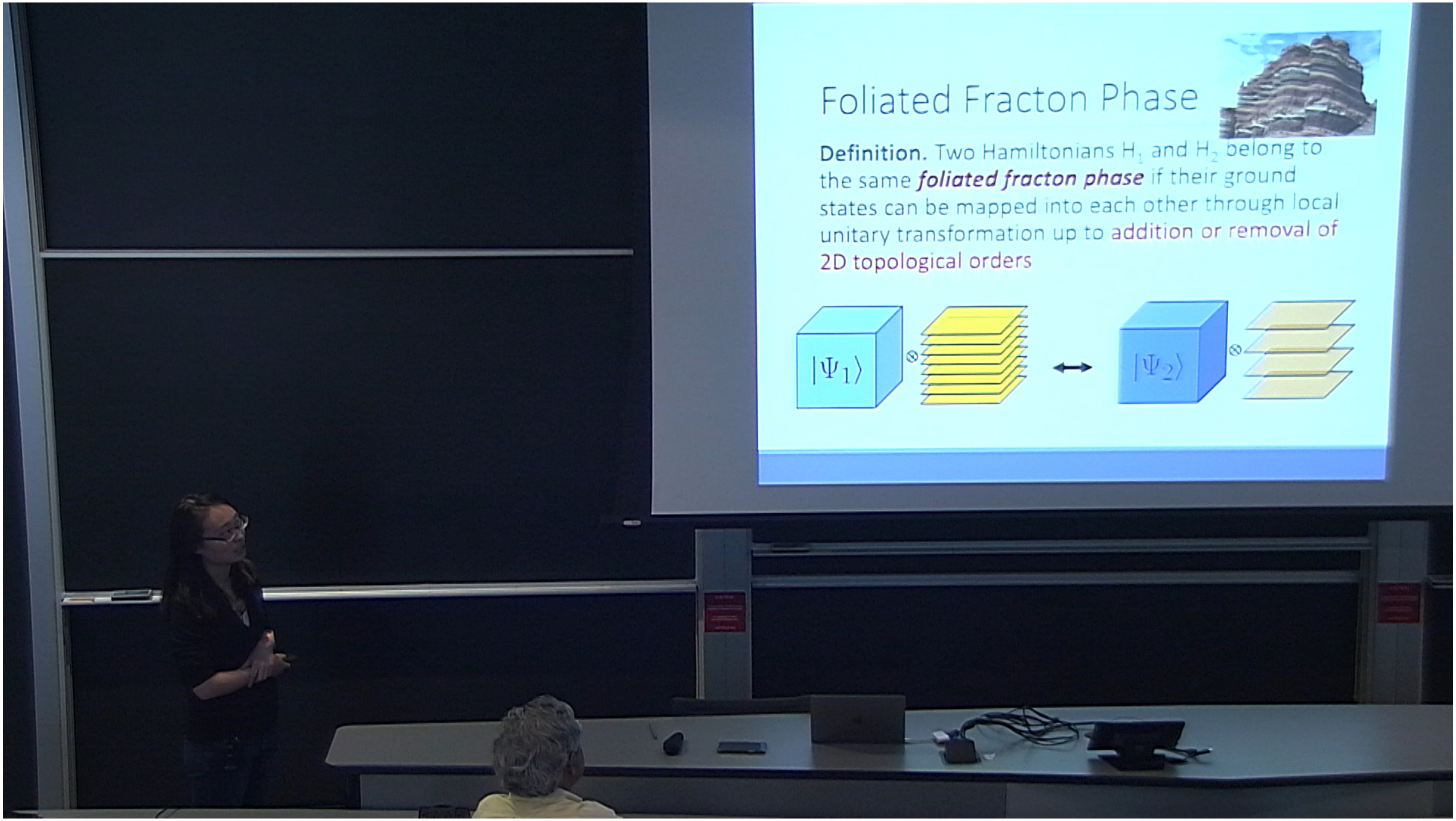


- Locally complete generating set
- Global redundancy in X-cube model (that cannot be removed locally)

$$\prod_i e_i = \prod_i m_i = 1$$

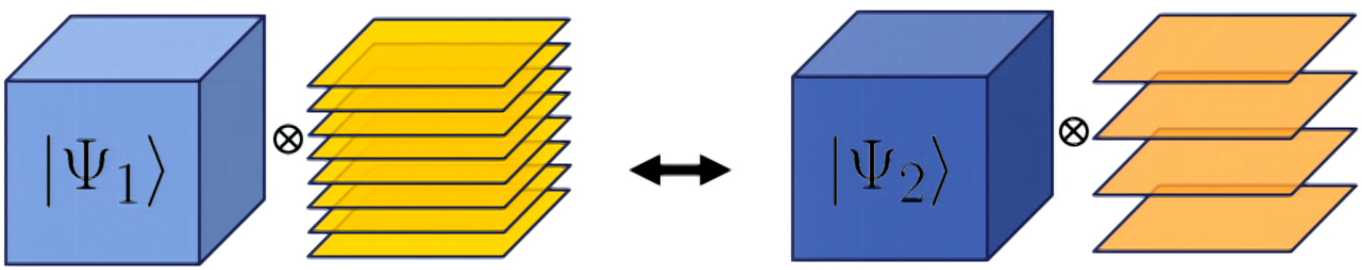
- Local redundancy in twisted model (that cannot be removed locally)

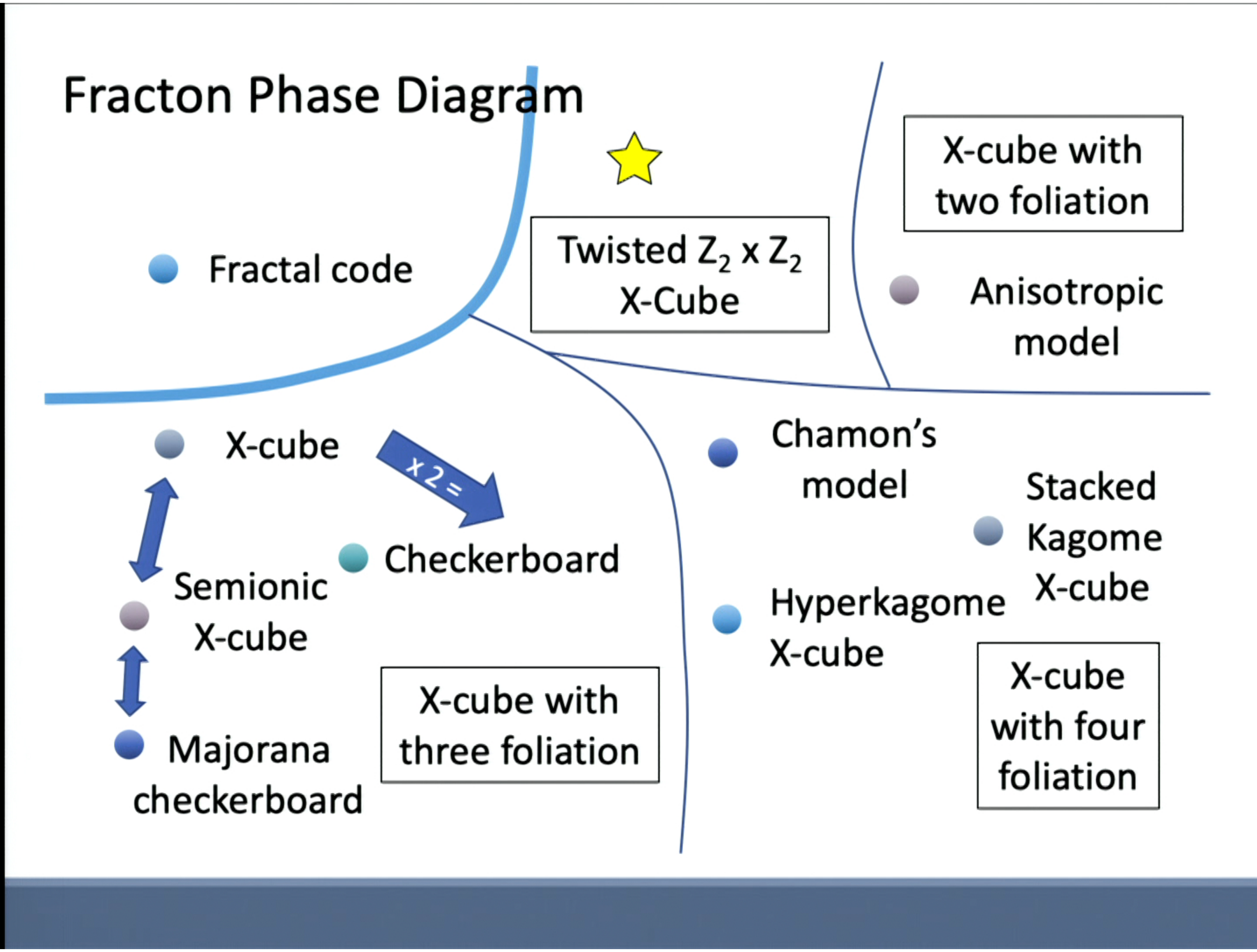
$$\left(m^A_{i,i+1}\right)^2 = e^B_i \times e^B_{i+1}$$
$$\left(m^B_{i,i+1}\right)^2 = e^A_i \times e^A_{i+1}$$


Foliated Fracton Phase
Definition. Two Hamiltonians H1 and H2 belong to the same foliated fracton phase if their ground states can be mapped into each other through local unitary transformation up to addition or removal of 2D topological orders
|Ψ1⟩
|Ψ2⟩

Foliated Fracton Phase

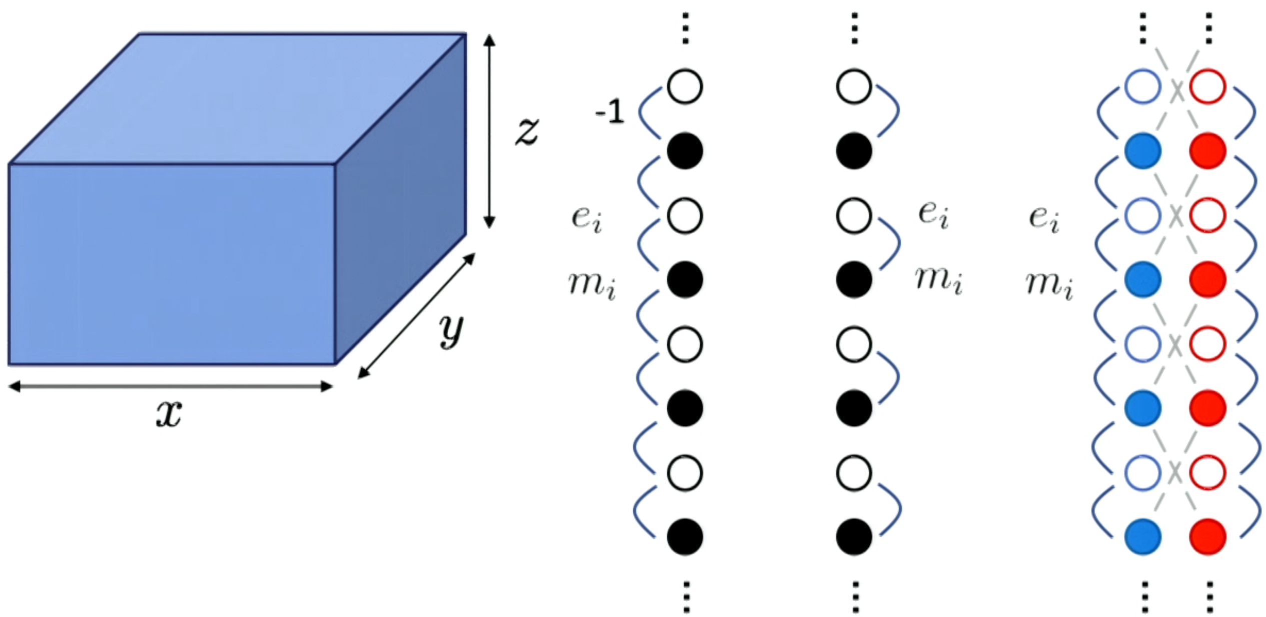
Definition. Two Hamiltonians H_1 and H_2 belong to the same ***foliated fracton phase*** if their ground states can be mapped into each other through local unitary transformation up to addition or removal of 2D topological orders





Fracton Phase Diagram
Twisted Z_2 x Z_2 X-Cube
X-cube with two foliation
Fractal code
Anisotropic model
X-cube
x 2 =
Chamon's model
Stacked Kagome X-cube
Checkerboard
Semionic X-cube
Hyperkagome X-cube
X-cube with three foliation
X-cube with four foliation
Majorana checkerboard

Fracton order and compactification



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- Universal entanglement signatures of foliated fracton phases, *SciPost Phys.* 6, 015 (2019)
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