Title: Renormalizable quantum gravity with anisotropic scaling

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Abstract: Despite intensive theoretical research for several decades, the theory of quantum gravity remains elusive. I will review the obstacles that prevent from reconciling the principles of general relativity with those of quantum mechanics. It is plausible that an eventual ultraviolet completion of general relativity will require sacrificing some of these principles. I will then focus on the class of theories where the abandoned property is local Lorentz invariance, replaced by an approximate anisotropic scaling symmetry in deep ultraviolet. At low energies these theories reduce to a special type of scalar â€” tensor gravity. I will show that this approach allows us to construct renormalizable gravitational theories in any number of spacetime dimensions. The study of a (2+1) dimensional model reveals its asymptotic freedom and suggests that this property may be generic for gravity with anisotropic scaling. Relevance of these results for gravity in the real world will be discussed.


Renormalizable quantum gravity with anisotropic scaling
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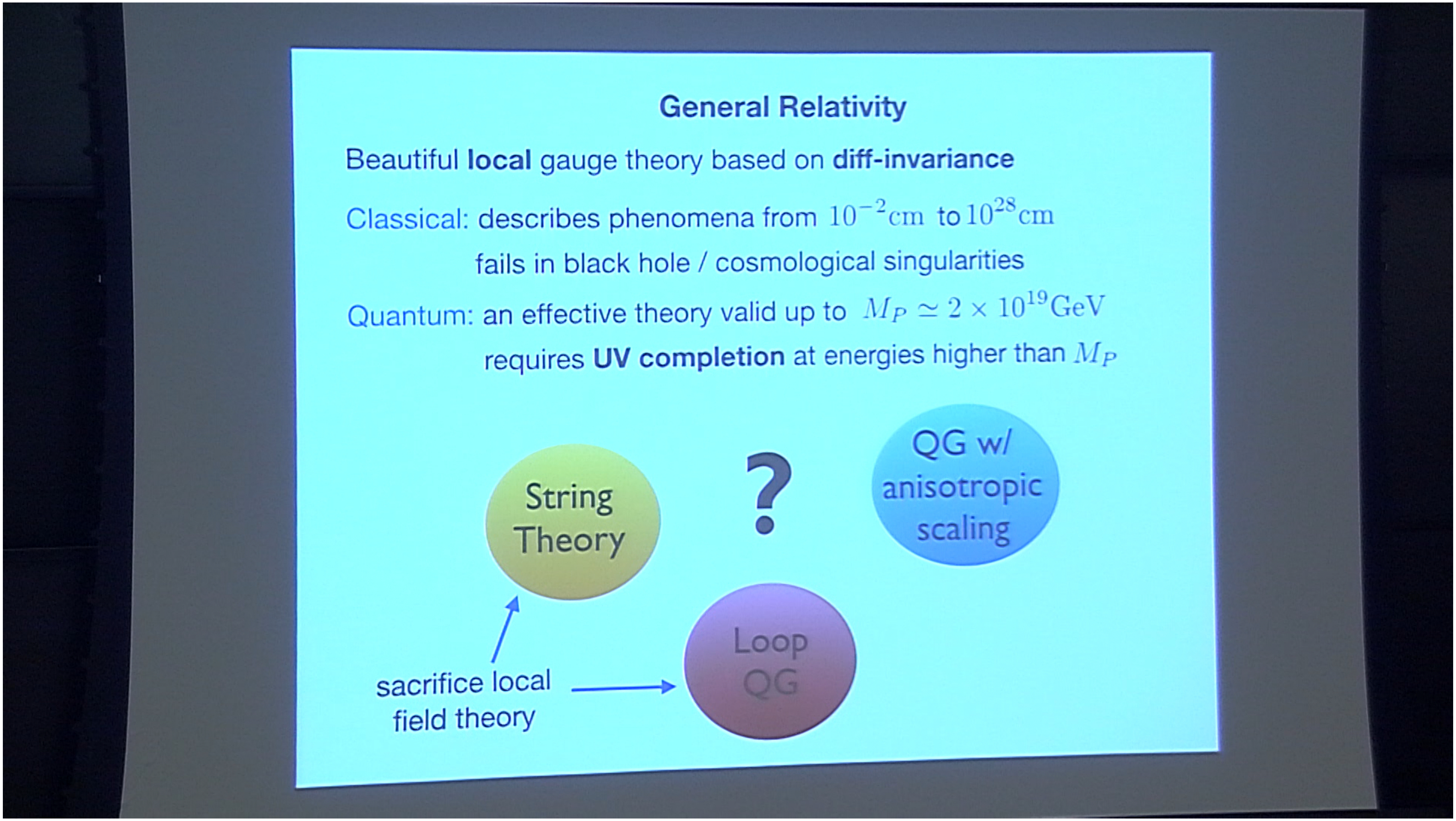
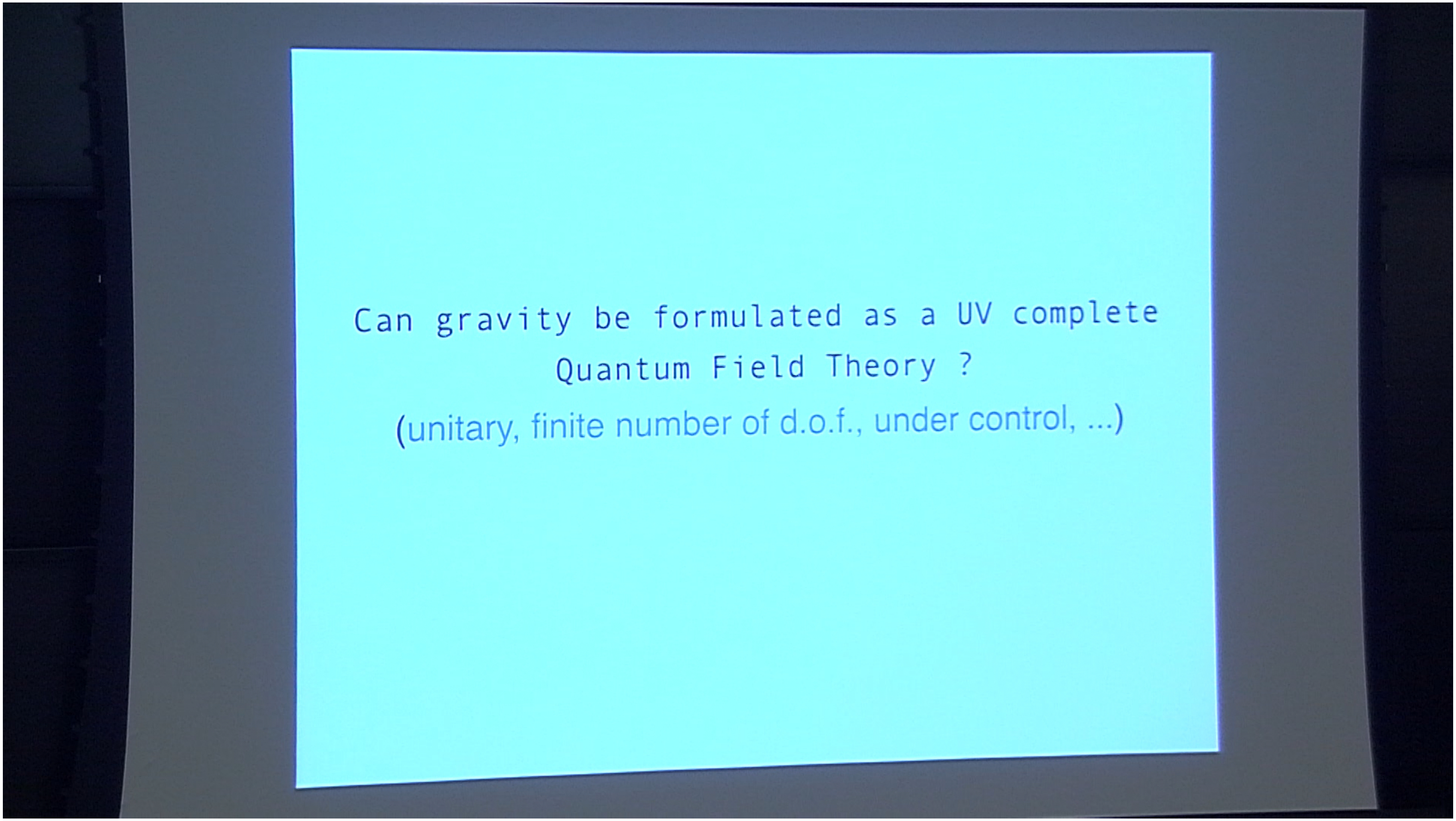

General Relativity
Beautiful local gauge theory based on diff-invariance
Classical: describes phenomena from 10^{-2}cm to 10^{28}cm
fails in black hole / cosmological singularities
Quantum: an effective theory valid up to $M_P \simeq 2 \times 10^{19}$GeV
requires UV completion at energies higher than M_P
String Theory
?
QG w/ anisotropic scaling
Loop QG
sacrifice local field theory


Can gravity be formulated as a UV complete
Quantum Field Theory ?
(unitary, finite number of d.o.f., under control, ...)

Why GR is not UV complete ?

$$S_{EH} = \frac{M_P^2}{2} \int d^{d+1}x \, \sqrt{g} R$$

quadratic action sets the amplitude of fluctuations

$$\Rightarrow \frac{M_P^2}{2} \int d^{d+1}x \, \left(\overbrace{h_{ij} \Box h_{ij}} + \underbrace{h^2 \Box h + \dots} \right)$$

interaction terms

Zoom in on shorter scales: $x^\mu \mapsto b^{-1} x^\mu$

To preserve the quadratic action scale the metric:

$$h_{\mu\nu} \mapsto b^{(d-1)/2} h_{\mu\nu}$$

scaling dimension

Look at the interactions: $S_{int} \mapsto b^{(d-1)/2} S_{int}$

For $d > 1$ the interaction strength grows unboundedly at short distances ($b \to \infty$)

Generation of higher-order operators $\Rightarrow$ loss of predictive power

A failed attempt

Stelle (1977)

Interactions contain arbitrarily high powers of the metric

Different from Yang-Mills theory, similar to sigma models

→ If we want to bound the interactions in UV we need to reduce the scaling dimension of h_{ij} to zero

$$\int d^4x \sqrt{g}\left(M_P^2 R + R_{\mu\nu}R^{\mu\nu} + R^2\right)$$

$$\rightarrow \int d^4x \left(M_P^2 h_{ij}\Box h_{ij} + \overbrace{h_{ij}\Box^2 h_{ij}}^{\text{dominates at high energies, determines the scaling dim of the metric in UV}} + \dots\right)$$

Fast decrease of the graviton propagator $\langle h\, h\rangle \propto 1/k^4$ improves convergence of the loop integrals. The theory is renormalizable and asymptotically free !

Fradkin, Tseytlin (1981)

Avramidi, Barvinsky (1985)

But higher time derivatives give **ghost poles**

→ no unitary interpretation

$$\langle h\, h\rangle \sim \frac{1}{k^2} - \frac{1}{k^2 + M_P^2}$$

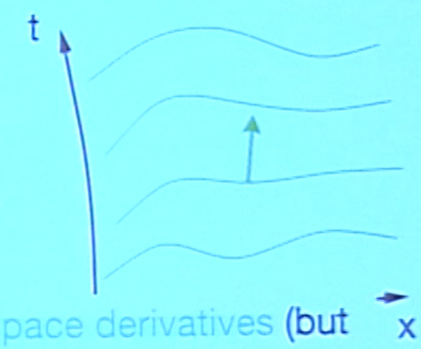
Never give up

Horava (2009)

Imagine that spacetime is endowed with a preferred spacelike foliation

⇕

General covariance is reduced to foliation-preserving diffeomorphisms



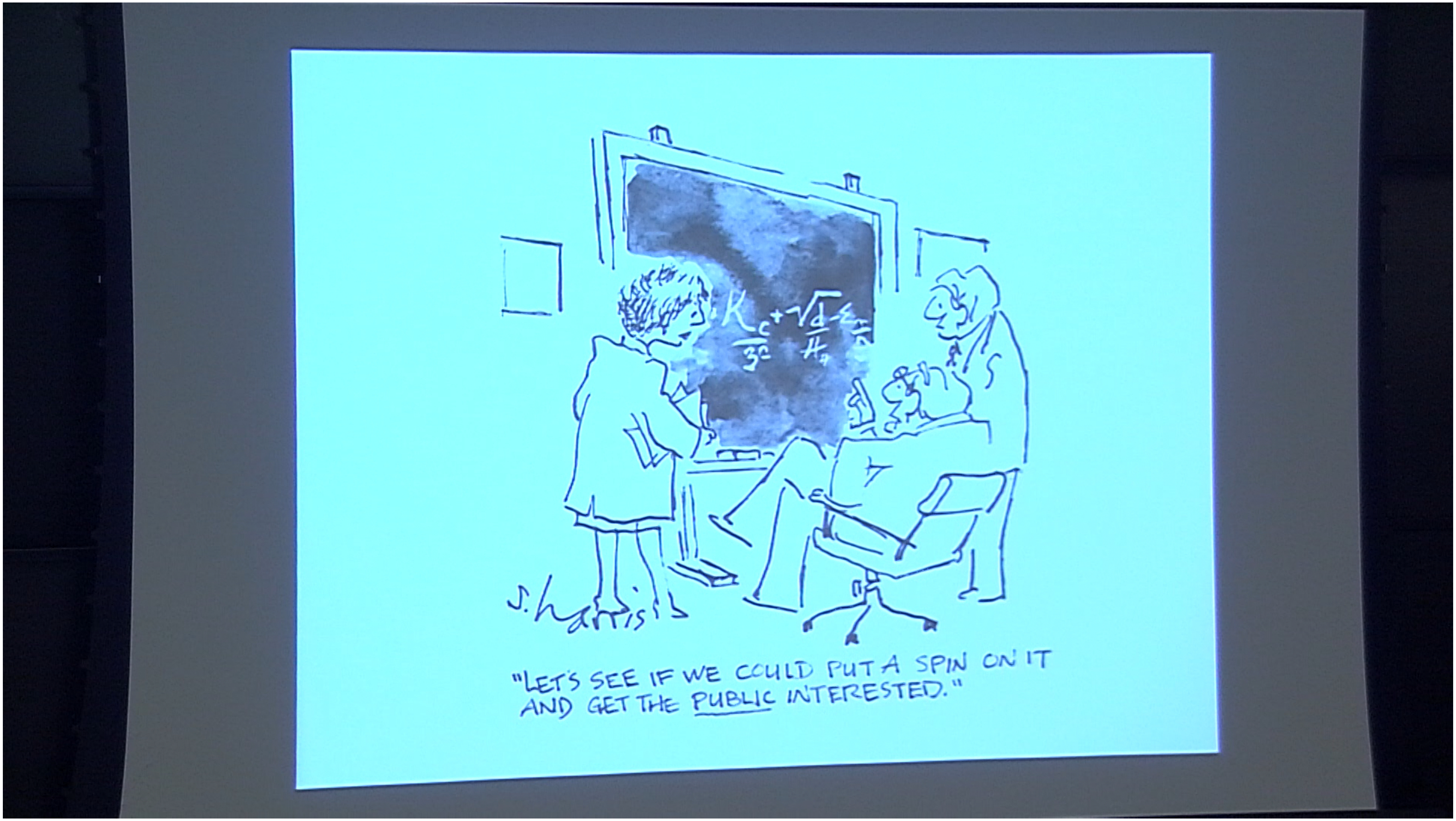
Write Lagrangians that have more than 2 space derivatives (but still 2 time derivatives). Use different scaling of time and space (*Lifshitz scaling*)

$$\int \underbrace{dt\, d^d x}_{\propto\, b^{-(z+d)}} \left(\dot{h}_{ij} \dot{h}_{ij} - h_{ij} (-\Delta)^z h_{ij} + \ldots \right)$$

$$\mathbf{x} \mapsto b^{-1} \mathbf{x}\,, \qquad t \mapsto b^{-z} t$$

➡ $h_{ij} \mapsto b^{(d-z)/2} h_{ij}$

critical theory in $z = d$


S. Harris
"LET'S SEE IF WE COULD PUT A SPIN ON IT AND GET THE PUBLIC INTERESTED."

Field content and low-E limit

We want to preserve as many symmetries, as possible

foliation-preserving diffeos
$$\begin{cases} x^i \mapsto \tilde{x}^i(\mathbf{x}, t) & \Rightarrow \quad \gamma_{ij}, \quad N^i \\ t \mapsto \tilde{t}(t) & \Rightarrow \quad N \end{cases}$$

$$\mathcal{L} = M_P^2 \sqrt{\gamma} N \left(K_{ij} K^{ij} - \lambda K^2 - \mathcal{V}[\gamma_{ij}, N] \right)$$

$\mathrm{dim}\gamma_{ij} = \mathrm{dim}N = 0$

$\mathrm{dim}N^i = d - 1$

$$K_{ij} = \frac{\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i}{2N}$$

contains terms with up to $2d$ spatial derivatives

Reduces to a scalar-tensor gravity at low energies

Blas, Pujolas, S.S. (2009a, 2009b, 2010)

$$\Rightarrow \quad \mathcal{L} = M_P^2 \sqrt{g} R + \mathcal{L}_\chi[g_{\mu\nu}, \chi]$$

What about renormalizability
or why power-counting is not enough ?

A naive "proof":

$$\mathcal{Z} = \int [dh] e^{-S}$$

$$= \sum \frac{(-1)^n}{n!} \int [dh] e^{-S_0} \int \underbrace{dx_1 \dots dx_n \mathcal{L}_{int}(x_1) \dots \mathcal{L}_{int}(x_n)}_{dim \leq 0}$$

Divergences are local and are removed by local counterterms of $dim \leq 2d$ that are already present in the action

this is not guaranteed because of gauge invariance

Toy model: d=2 "projectable"

"projectability condition" $N = N(t)$ → set $N = 1$ by gauge-fixing time and forget

$\dim \gamma_{ij} = 0 \quad \dim N^i = 2$

$$\mathcal{L} = \frac{1}{2G}\left(K_{ij}K^{ij} - \lambda K^2 - \mu R_{\rm sp}^2\right)$$

- is fully paramaterized by 3 couplings
- unlike GR in 3d, has propagating d.o.f., a single scalar
- is well-behaved for $G, \mu > 0$ and $\lambda < 1/2$ or $\lambda > 1$

Gauge fixing

We need to fix spatial diffeos

$$\mathcal{L}_{gf} = \frac{\sigma}{2G} F^i \mathcal{O}_{ij} F^j$$

linear combination of the fields

must have dim=4 to preserve power-counting

invertible operator

Local $\mathcal{O}_{ij}$ → $F^i = N^i$, $F^i = \partial_j h_{ij} + \sigma' \partial_i h$ → $\langle N^i N^j \rangle \ni \delta_{ij} \frac{G}{k^2}$

no dependence on energy

$$\langle N^i(t,x) N^j(0) \rangle \ni \frac{\delta_{ij}}{4\pi} \delta(t) \log |\mathbf{x}|$$

the singularity is **non-local** (in space)
Hard to keep track of divergences

Similar to the Coulomb gauge in YM. What is the analog of covariant gauges ?

Regular propagators

$[\Phi_1] = r_1\ ,\quad [\Phi_2] = r_2$ under Lifshitz scaling with $z = d$

$$\langle \Phi_1 \Phi_2 \rangle = \sum \frac{P(\omega, k)}{D(\omega, k)}$$

$$D = \prod_{m=1}^{M} \left(A_m \omega^2 + B_m k^{2d} + \ldots \right), \quad A_m, B_m > 0$$

P polynomial of degree $r_1 + r_2 + 2(M-1)d$ (to ensure the correct scaling at short distances)

Regular gauges

Barvinsky, Blas, Herrero-Valea, S.S., Steinwachs (2016)

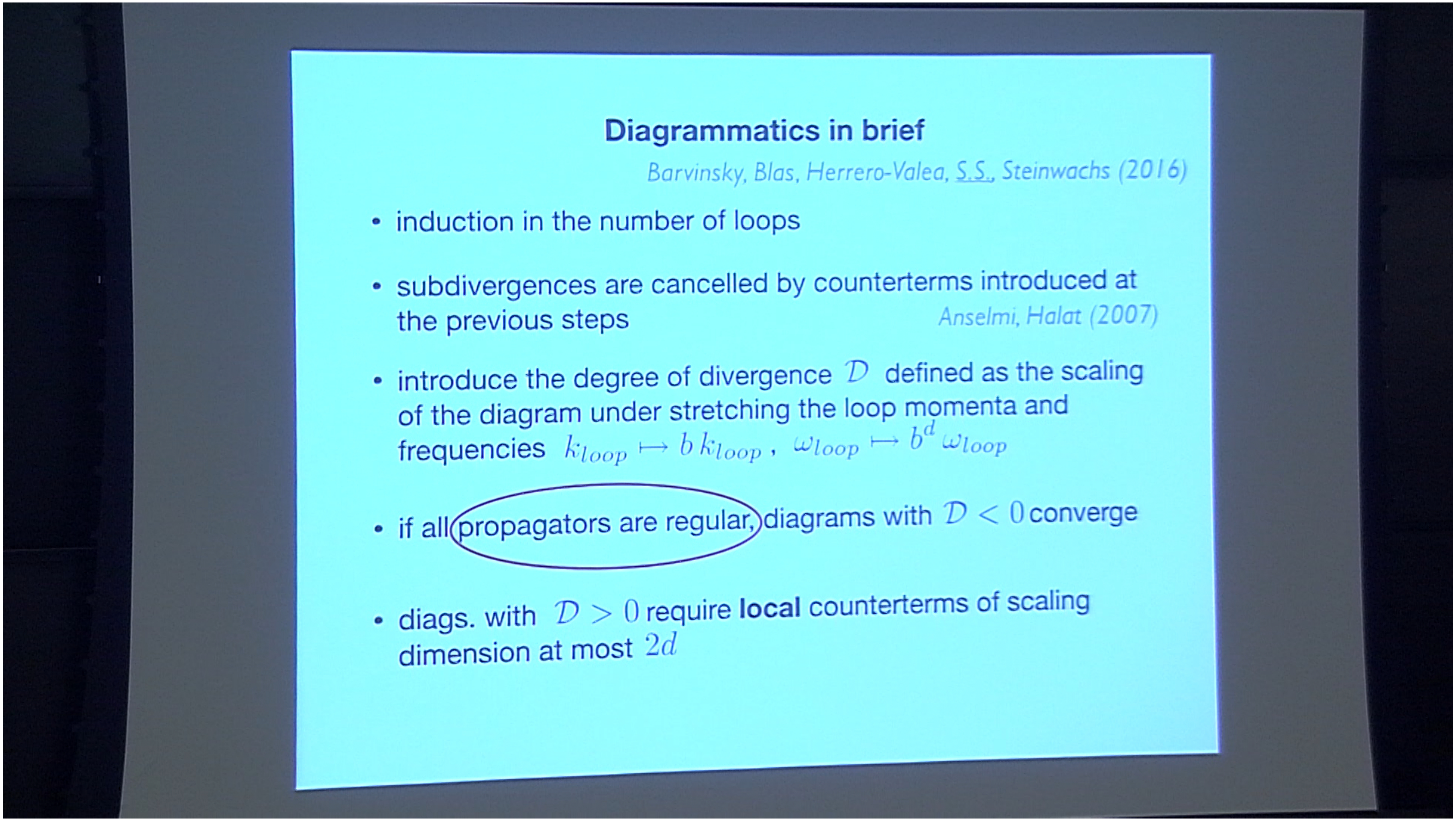
We have to allow for non-local gf. Lagrangian. Good choice:

$$\mathcal{O}_{ij} = -\left(\delta_{ij}\Delta + \xi\partial_i\partial_j\right)^{-1}$$

$$F^i = \dot{N}^i + \frac{1}{2\sigma}\mathcal{O}^{-1}_{ij}\partial_k h_{jk} - \frac{\lambda}{2\sigma}\mathcal{O}^{-1}_{ij}\partial_j h$$

- disentangle h_{ij} from N^i in the quadratic action
- regular propagators for all fields (including Faddeev-Popov ghosts)
- two free gf. parameters σ, ξ
- straightforward generalization to $d > 2$, e.g.

$$\mathcal{O}^{d=3}_{ij} = \Delta^{-1}\left(\delta_{ij}\Delta + \xi\partial_i\partial_j\right)^{-1}$$


Diagrammatics in brief
Barvinsky, Blas, Herrero-Valea, S.S., Steinwachs (2016)
• induction in the number of loops
• subdivergences are cancelled by counterterms introduced at the previous steps
Anselmi, Halat (2007)
• introduce the degree of divergence $\mathcal{D}$ defined as the scaling of the diagram under stretching the loop momenta and frequencies $k_{loop} \mapsto b\, k_{loop}$, $\omega_{loop} \mapsto b^d\, \omega_{loop}$
• if all propagators are regular, diagrams with $\mathcal{D} < 0$ converge
• diags. with $\mathcal{D} > 0$ require **local** counterterms of scaling dimension at most $2d$

Comments

- Straightforward generalization to projectable HL gravity in any dimensions
- Does not work for non-projectable: additional variable $N = 1 + \phi$

$$\langle \phi\phi \rangle = \text{regular} + \frac{1}{k^{2d}}$$

present even in $\sigma\xi$ - gauges
physical: shows up in the interaction of local sources

Blas, Pujolas, S.S. (2010)
Blas, S.S. (2011)

Cancellation of non-local divergence due to time-reparameterization ???

Gauge invariance ?

GI is explicitly broken by the gauge-fixing. Instead, we have to rely on the BRST symmetry (Slavnov-Taylor identities)

Non-linearity of BRST → deformation by quantum corrections. To restore original BRST, the gauge fields must be redefined at every loop order

To restore original BRST, the gauge fields must be redefined at every loop order.

$\mathrm{dim} h_{ij} = 0$ → the redefinition can be non-linear (unlike YM)

Textbook YM treatment: relies on explicit power counting

Beyond PC: relies on relativistic invariance and the explicit structure of the gauge group *Barnich, Brandt, Henneaux (1994)*

Renormalization in the background-field method

Barvinsky, Blas, Herrero-Valea, S.S., Steinwachs (2017a)

Decompose the fields in the "background" and "quantum fluctuations"

$$\gamma_{ij} = \bar{\gamma}_{ij} + h_{ij} \ , \quad N^i = \bar{N}^i + n^i$$

Doubles the number of GI:

- BRST transformations acting on fluctuations
- **Backgroung GI** acting on both. Acts linearly !

⇒ one-loop counterterms are manifestly gauge-invariant

at higher loops BGI helps to explicitly separate redefinition of quantum fields from renormalization of couplings

General result: BRST structure is preserved in any non-anomalous gauge theory admitting sensible BF formulation

- YM, GR and their higher-derivative extensions
- non-relativistic gauge theories
- theories with U(1) subgroups

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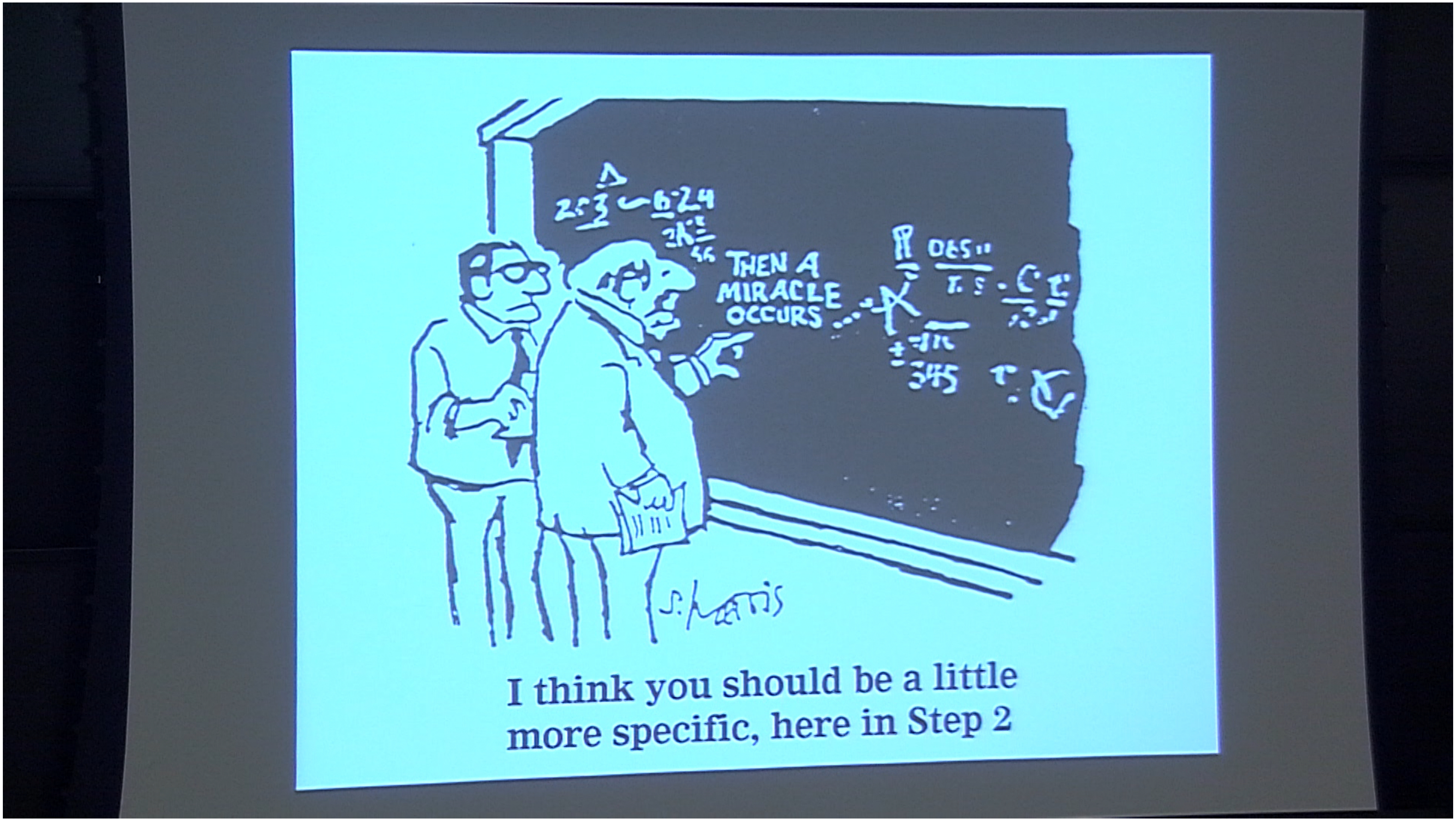
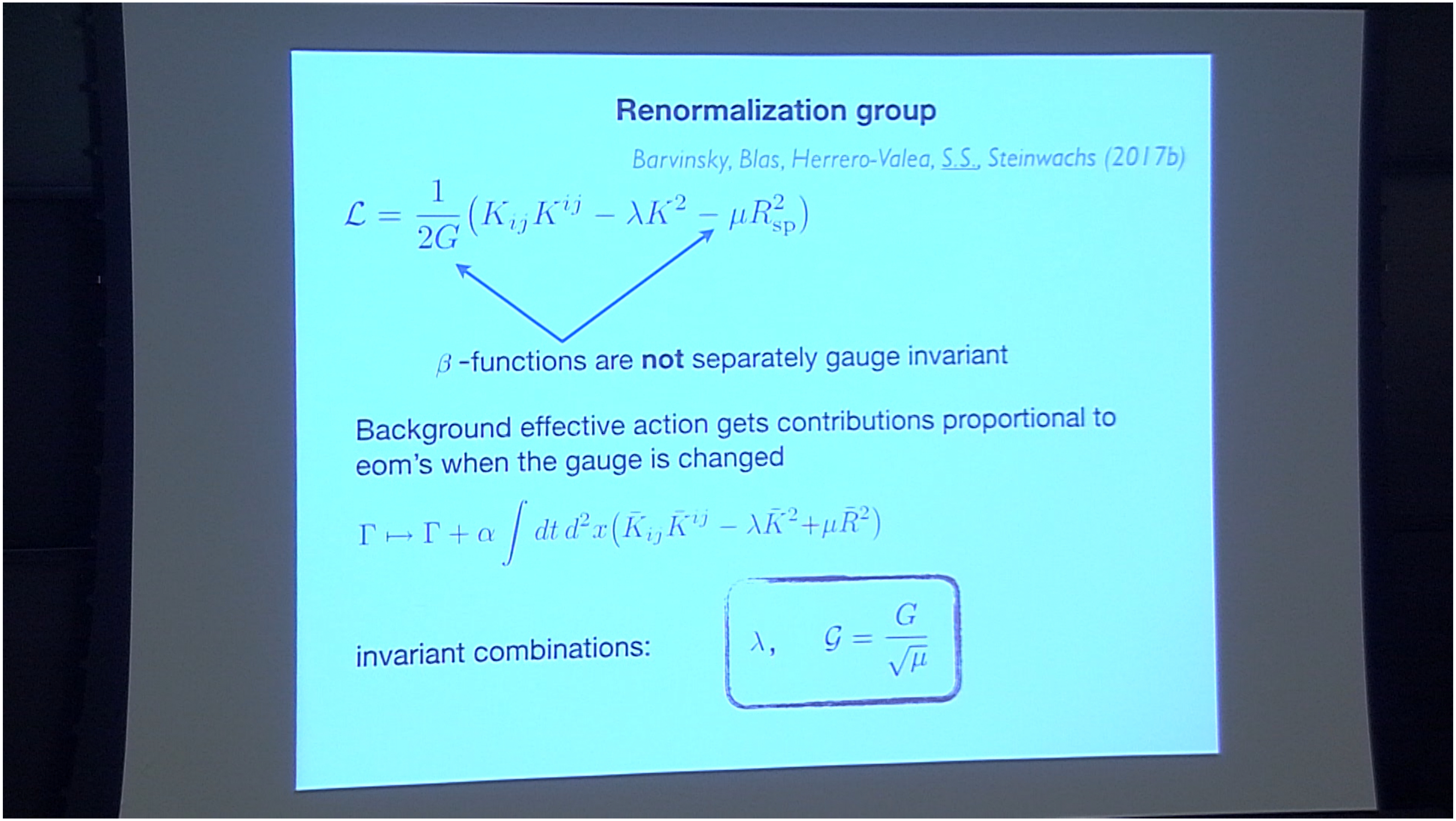

THEN A MIRACLE OCCURS
I think you should be a little more specific, here in Step 2

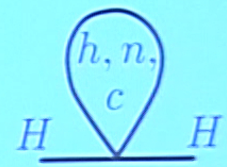

Renormalization group
Barvinsky, Blas, Herrero-Valea, S.S., Steinwachs (2017b)
$\mathcal{L} = \frac{1}{2G}\left(K_{ij}K^{ij} - \lambda K^2 - \mu R_{\rm sp}^2\right)$
β -functions are **not** separately gauge invariant
Background effective action gets contributions proportional to eom's when the gauge is changed
$\Gamma \mapsto \Gamma + \alpha \int dt\, d^2x \left(\bar{K}_{ij}\bar{K}^{ij} - \lambda \bar{K}^2 + \mu \bar{R}^2\right)$
invariant combinations:
$\lambda, \quad \mathcal{G} = \frac{G}{\sqrt{\mu}}$

Sit down and calculate

- fix the background gauge

  3 gauge choices: 2 regular + conformal $h_{ij} = \bar{\gamma}_{ij} e^{2\zeta}$

- expand background: $\bar{\gamma}_{ij} = \delta_{ij} + H_{ij}$
- integrate out fluctuations



- extract divergent parts of coefficients in front of

$$\dot{H}_{ij}\dot{H}_{ij} \quad , \quad (\dot{H}_{ii})^2 \quad , \quad (\partial_i \partial_j H_{ij})^2$$

$$\updownarrow \qquad \updownarrow \qquad \updownarrow$$

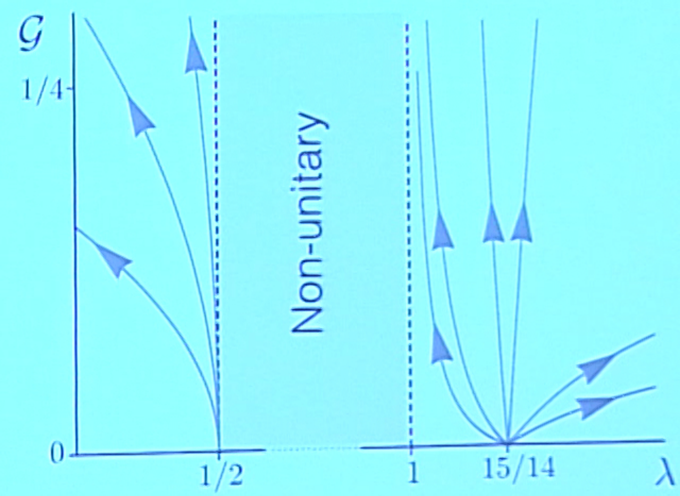
$$G \qquad \lambda \qquad \mu$$

$$\frac{d\lambda}{d\log\Lambda} = \frac{15 - 14\lambda}{64\pi}\sqrt{\frac{1-2\lambda}{1-\lambda}}\,\mathcal{G} \qquad \frac{d\mathcal{G}}{d\log\Lambda} = -\frac{(16 - 33\lambda + 18\lambda^2)}{64\pi(1-\lambda)^2}\sqrt{\frac{1-\lambda}{1-2\lambda}}\,\mathcal{G}^2$$

RG portrait of Horava-Lifshitz gravity in (2+1)d

Barvinsky, Blas, Herrero-Valea, S.S., Steinwachs (2017b)

RG flow of essential couplings:



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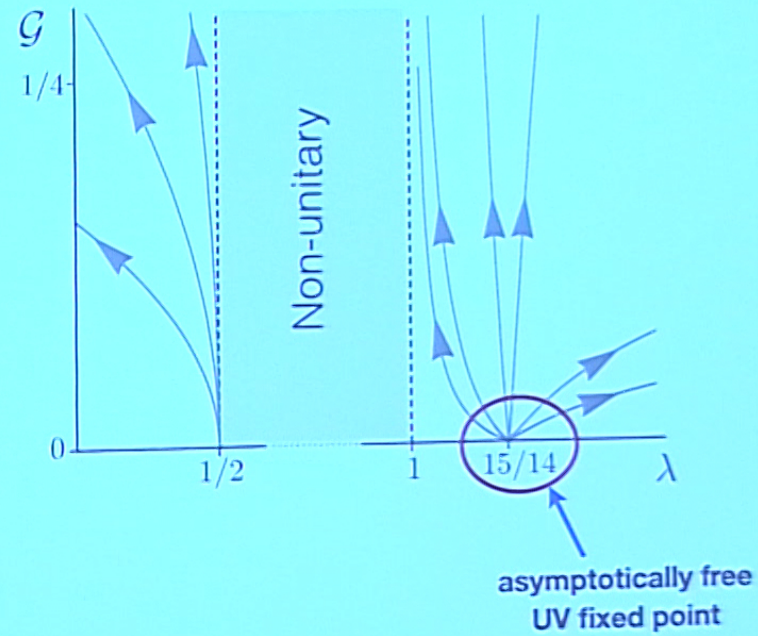

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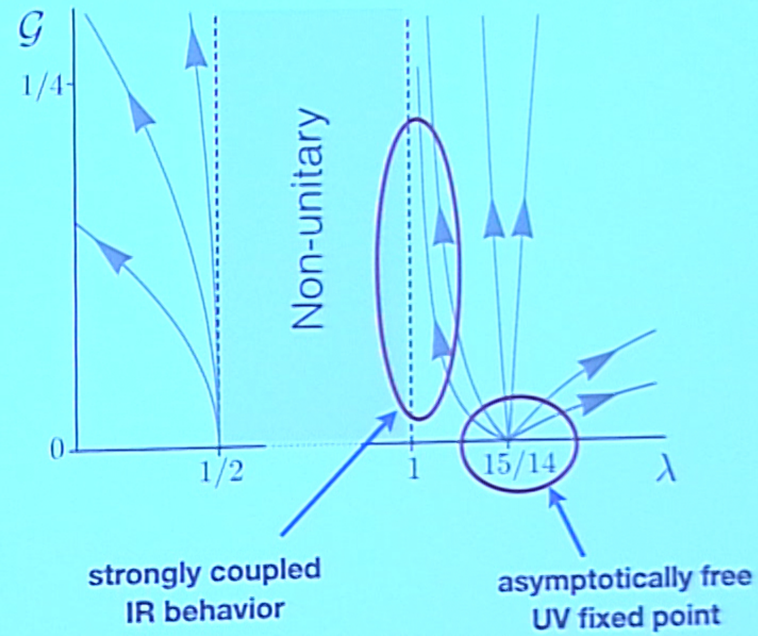


Smth interesting is going on here

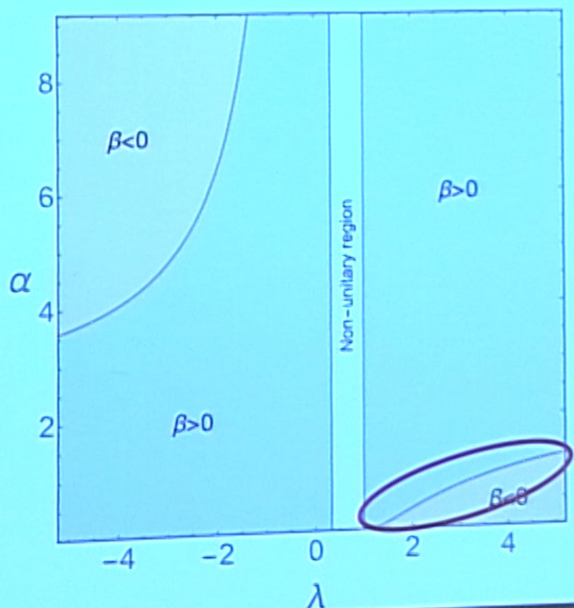
Towards RG flows in (3+1)d

Barvinsky, Herrero-Valea, S.S. (in preparation)

Contains a dynamical graviton (transverse-traceless tensor mode)

7 couplings, 6 essential

$$\beta_\lambda = \frac{27(1-\lambda)^2 + 3\sqrt{\alpha}(11-3\lambda)(1-\lambda) - 2\alpha(1-3\lambda)^2}{120\pi^2(1+\sqrt{\alpha})\sqrt{\alpha}(1-\lambda)}\,\mathcal{G}$$



$\alpha = \nu_s/\nu_{tt}$

$$\omega_s^2 = \nu_s k^6$$

$$\omega_{tt}^2 = \nu_{tt} k^6$$

candidate UV fixed points

Horava-Lifshitz gravity: Theory summary

- Projectable models represent a class of renormalizable gravity theories ← no local gauge-invariant observables, spin-2 d.o.f.

- In (2+1)d asymptotically free (UV complete); possibly also in (3+1)

- In IR goes into strong coupling — What is it ? (gravitational confinement ?? non-trivial fixed point ???)

- No definitive answer abour renormalizability of the non-projectable version

Horava-Lifshitz gravity: pheno status

- In **projectable** HL gravity in (3+1)d the scalar mode is unstable at low- (at weak coupling)
- Phenomenology of **non-projectable** HL gravity can be close to GR

  Blas, Pujolas, S.S., (2009, 2010, 2011)

- Lorentz invariance is fundamentally broken. Can it emerge as low-energy property ?

  Quite common in non-gravitational theories

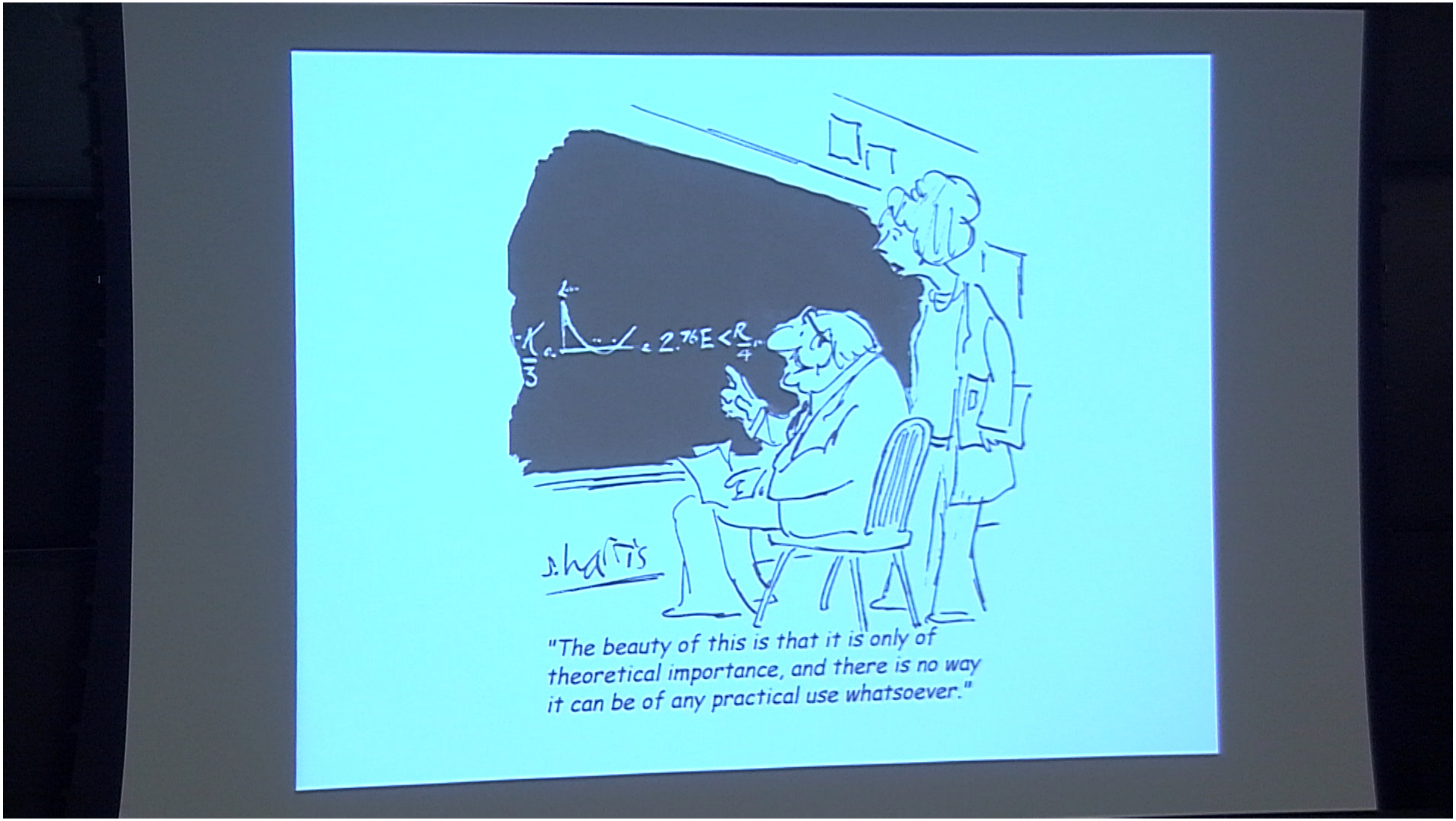
  Nielsen, Ninomiya (1978)

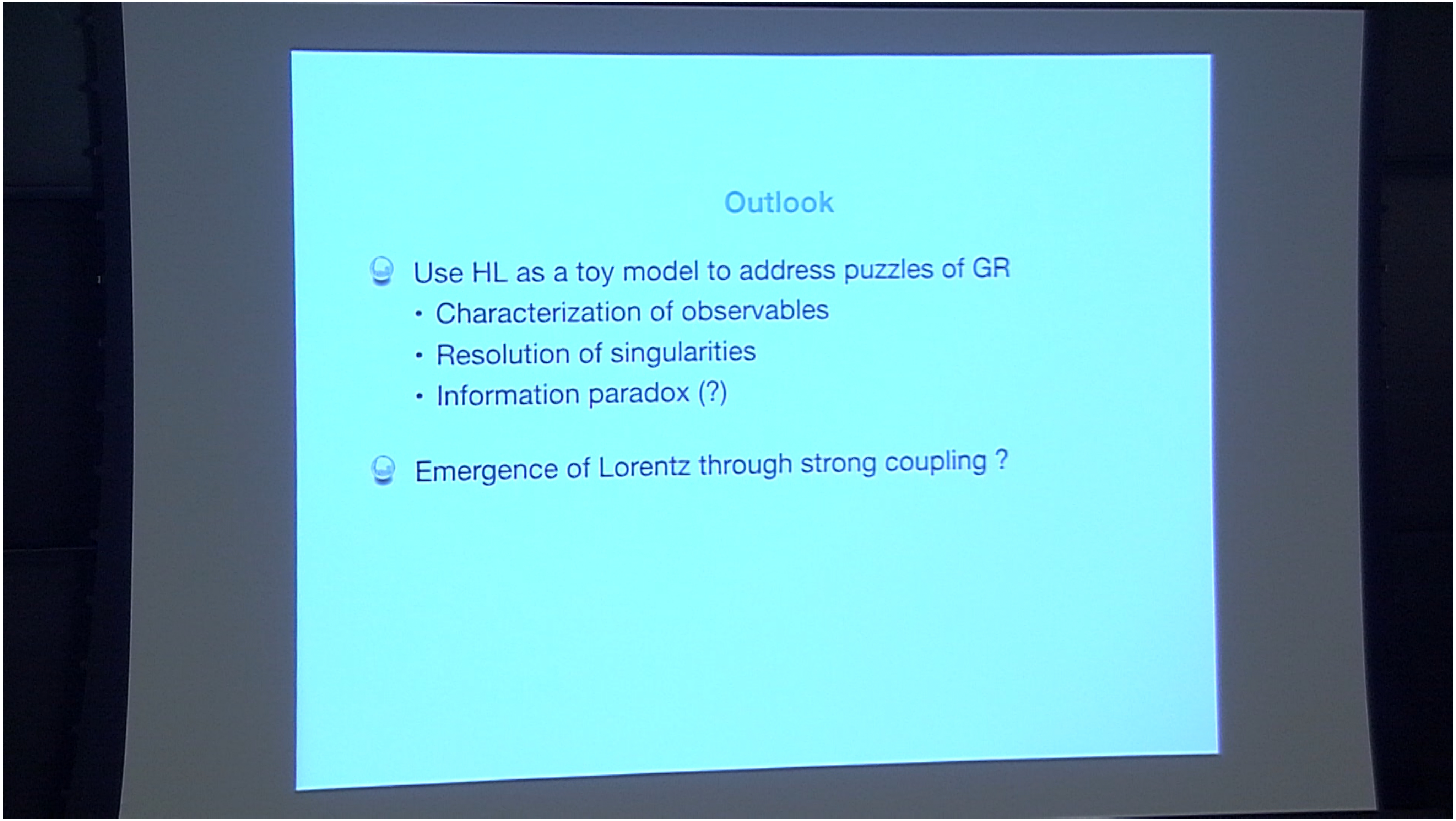
  ....

  Sundrum (2012)
  Bednik, Pujolas, S.S. (2013)
  S.S. (2014)

But satisfying $|c_g - c_\gamma| < 10^{-15}$ requires extreme fine-tuning

from GW170817 / GRB170817A


2.76E
S. harris
"The beauty of this is that it is only of
theoretical importance, and there is no way
it can be of any practical use whatsoever."


Outlook
Use HL as a toy model to address puzzles of GR
• Characterization of observables
• Resolution of singularities
• Information paradox (?)
Emergence of Lorentz through strong coupling ?