Title: Abelian topological order on lattice with electromagnetic background

Speakers:

Series: Condensed Matter

Date: April 09, 2019 - 3:30 PM

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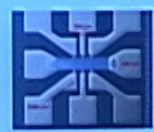
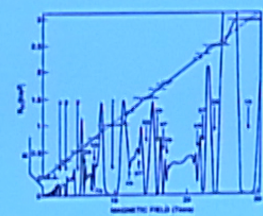
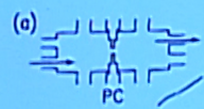
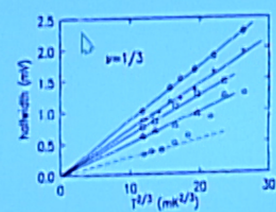
Abstract: The construction of soluble lattice toy models is an important theoretical approach in the study of strongly interacting topological phases of matter. On the other hand, the primary experimental probe to such systems is via electromagnetic response. Somewhat unsatisfactorily, the current systematic construction of the lattice toy models focuses on braiding statistics and does not admit coupling to an electromagnetic background. Thus there is a mismatch between our theoretical approach and experimental probe. In this talk I introduce how to systematically incorporate electromagnetic response into the soluble lattice toy models, for a large class of abelian topological phases. [Reference: 1902.06756]





Hall conductivity





fractional charge on edge

primary experimental probes via
electromagnetic response

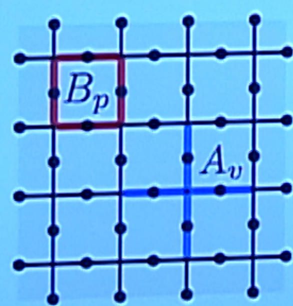
(** recent measurement of thermal Hall effect on edge is beyond EM response)

- EM probe is <u>missing</u>
  from a big theoretical framework
  of interacting topological phases

what do we mean?

exactly soluble lattice model

an important theoretical approach to interacting topological phases



e.g. Kitaev's toric code

exactly soluble lattice model

an important theoretical approach to interacting topological phases

- concrete computation (universality)
- explicit physical picture
- realizability in solid state systems (in principle)
- fun & beauty

...

exactly soluble lattice model
an important theoretical approach to interacting topological phases

A large class of them is systematically built
from the **renormalization fixed point** construction

in condensed matter

Levin-Wen
string-net condensation

in mathematics

Turaev-Viro
modular tensor category

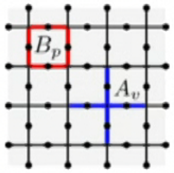
in quantum computation

Kitaev
quantum doubled model

exactly soluble lattice model
an important theoretical approach to interacting topological phases

A large class of them is systematically built
from the **renormalization fixed point** construction

this framework is based on **anyon braiding & fusion**
no natural appearance of **coupling to EM background**



EM background
in toric code ??

(** there are examples with EM, but no systematic build-in)

- EM response is the primary **experimental probe** to interacting topological phases

- EM is missing in our systematic **theoretical approach** of exactly soluble lattice model

In this talk:

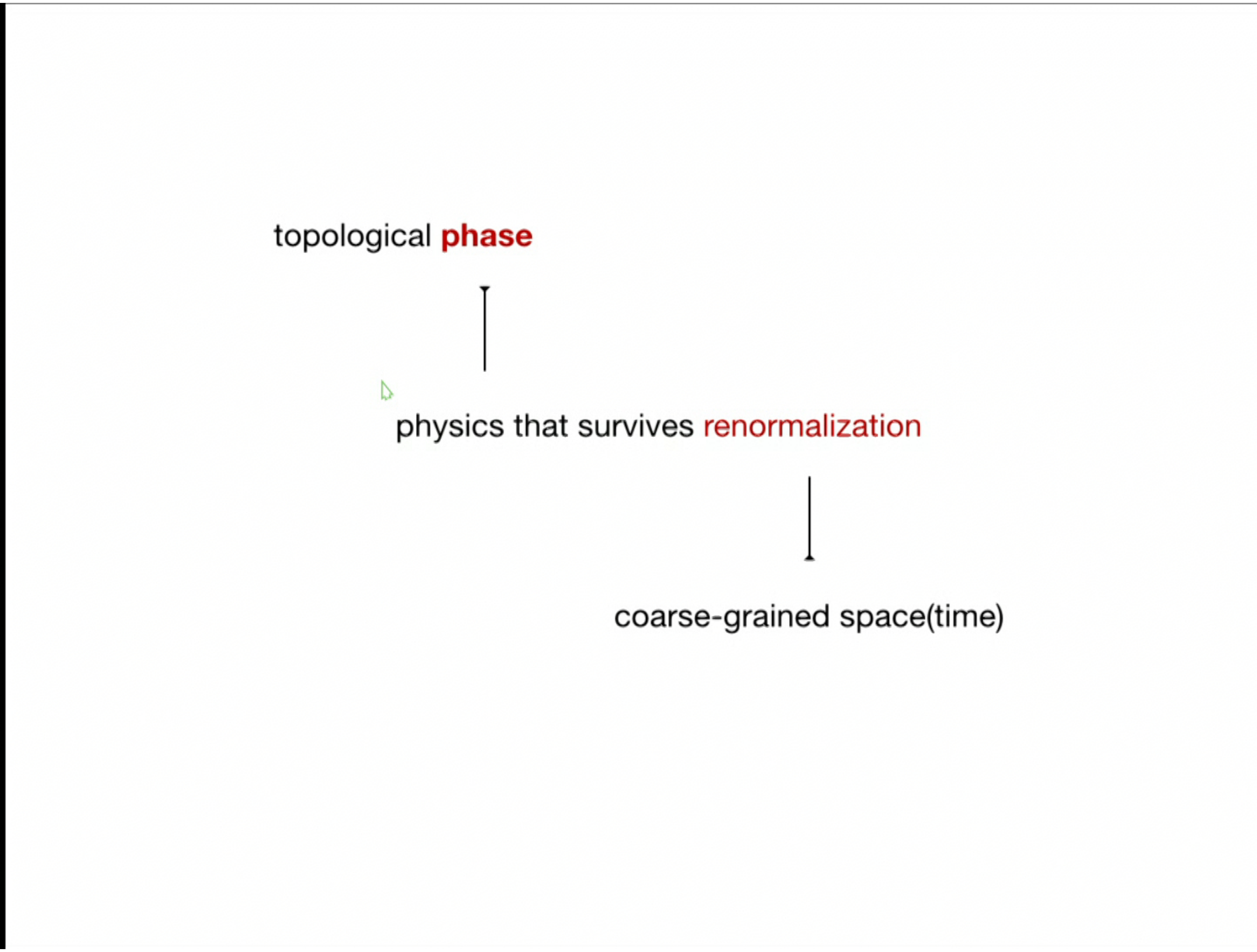
We systematically build-in EM background for *abelian* topological phases in (2+1)d

Invitation

Idea of Fix-Point Construction
& why EM is missing

Build-In EM

Outlook


topological **phase**
physics that survives renormalization
coarse-grained space(time)

in fixed-point construction

deep IR **effective theory** on
“lattice” from coarse-graining

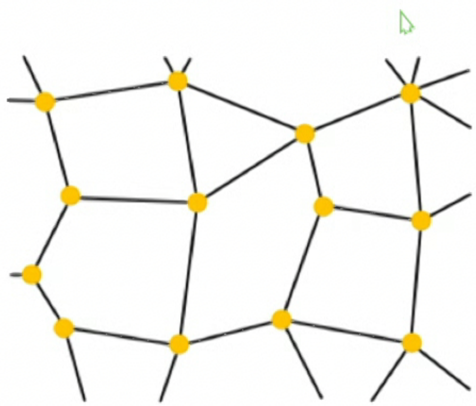
=

a **microscopic theory** on
a microscopic lattice

demonstrate the idea with the simplest example

toric code

microscopic theory



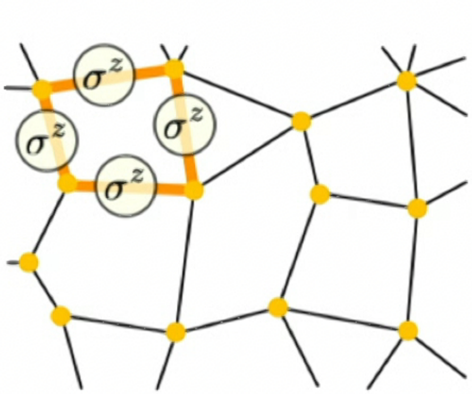
one spin on each link

$$H = -\sum_{\text{plaq. p}} \prod_{\text{links l around p}} \sigma^z_l - \sum_{\text{vert. v}} \prod_{\text{links l around v}} \sigma^x_l$$

demonstrate the idea with the simplest example

toric code

microscopic theory



one spin on each link

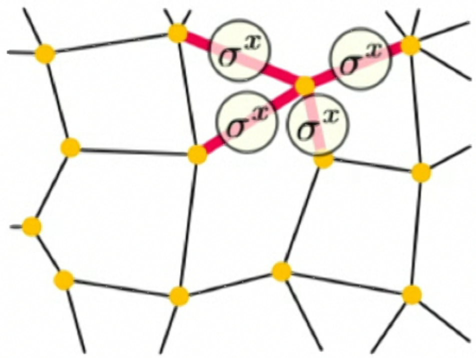
$$H = -\sum_{\text{plaq. p}} \prod_{\text{links l around p}} \sigma_l^z - \sum_{\text{vert. v}} \prod_{\text{links l around v}} \sigma_l^x$$

- suppresses **Z**2 flux (π flux)

demonstrate the idea with the simplest example

toric code

microscopic theory



one spin on each link

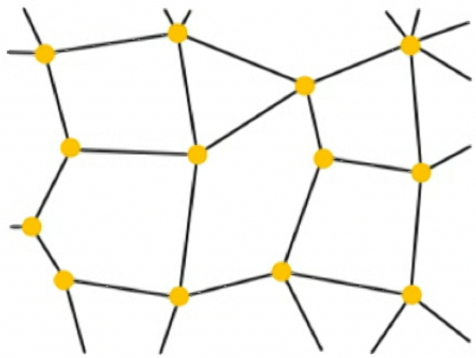
$$H = -\sum_{\text{plaq. p}} \prod_{\text{links l around p}} \sigma_l^z - \sum_{\text{vert. v}} \prod_{\text{links l around v}} \sigma_l^x$$

- suppresses **Z**2 flux (π flux)
- favors **Z**2 gauge equiv configs

demonstrate the idea with the simplest example

toric code

microscopic theory



ground state
equal superposition of gauge equiv configs with zero gauge flux

one spin on each link

$$H = -\sum_{\text{plaq. p}} \prod_{\text{links l around p}} \sigma^z_l - \sum_{\text{vert. v}} \prod_{\text{links l around v}} \sigma^x_l$$

- suppresses Z_2 flux (π flux)
- favors Z_2 gauge equiv configs

the two terms commute
(flux is gauge invariant)

demonstrate the idea with the simplest example
toric code

When does it arise as an **effective theory?**

type-II superconducting thin film
coupled to a <u>*fictitious*</u> **dynamical** *U(1)* gauge field

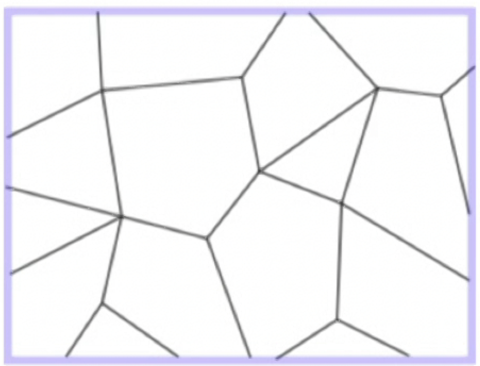
** Can equally well think of this dynamical *U(1)* as dynamical EM.
In this talk we assume EM is background since we can manipulate it.
To avoid notion conflict, we say the dynamical *U(1)* is fictitious for now.

(a familiar physical system
stressed by Hansson, Oganesyan, Sondhi; and Wen)

demonstrate the idea with the simplest example

toric code

effective theory of type-II SC thin film



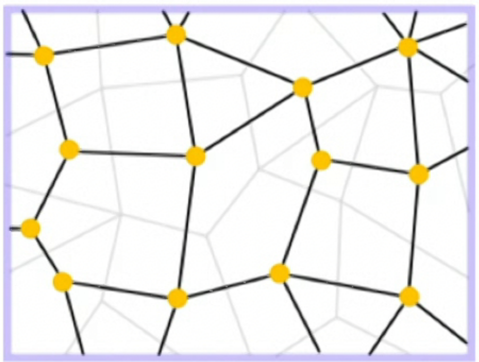
deep IR ~ coarse-grain into patches

type-II SC ~ between patches,
gauge transition by 0 or π

demonstrate the idea with the simplest example

toric code

effective theory of type-II SC thin film



take “dual lattice” from the patches

type-II SC ~ between patches,
gauge transition by 0 or π

hence $\mathcal{A}_{\text{link}} = 0 \text{ or } \pi$

~ $\mathbf{Z}_2$ “spin”

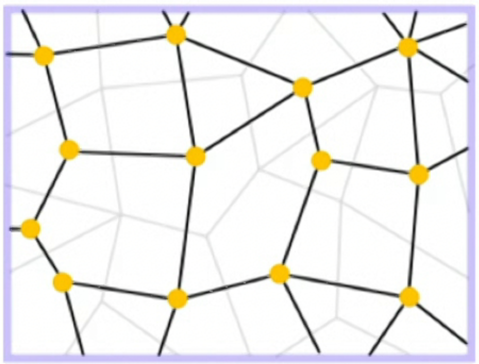
demonstrate the idea with the simplest example

toric code

effective theory of type-II SC thin film

$\mathcal{A}_{\text{link}} = 0$ or π ~ **Z2** "spin"

$\mathcal{A}_{\text{link}}$ and $\mathcal{E}_{\text{link}}$ are canonical



- π flux costs vortex core energy
- single charge costs BCS gap energy,

$\text{div}\, \mathcal{E}_{\text{link}} = \text{charge}$

generates gauge transformation,

so gauge inequiv costs energy

toric code after change of variables!

in fixed-point construction

deep IR **effective theory** on "lattice" from coarse-graining = a **microscopic theory** on a microscopic lattice

We have demonstrated, in what sense is the **toric code** the **effective theory (fixed-point)** of a physical system

in fixed-point construction

deep IR **effective theory** on "lattice" from coarse-graining = a **microscopic theory** on a microscopic lattice

simple rules under further coarse-graining → consistency conditions → solubility

survived in deep IR: braiding of anyons

further coarse-graining: fusion of anyons

(Levin-Wen's philosophy of string-net)

- “fixed-point” requires certain consistency conditions:
  Levin-Wen’s string-net Hamiltonian

- solutions given by the math structure “tensor category”:
  Turaev-Viro’s state sum is the path integral of string-net

- Kitaev’s quantum doubled;
  Dijkgraaf-Witten;
  fermionic versions …

confluence of different motivations

Why background EM is not built in?

need new degree of freedom

excitations with trivial braiding but carry electric charge

(e.g. electron excitation in FQH)

not really "fixed-point"

— EM background field smears macroscopically

— narrow 2π EM flux invisible on microscopic lattice

but visible in deep IR limit: "narrow" 2π EM flux creates anyon

Why background EM is not built in?

in math machinery?

symmetry (global or gauge) incorporated via "grading",

suited for discrete symmetry groups

not continuous ones (which include EM)

Invitation

Idea of Fix-Point Construction
& why EM is missing

Build-In EM

Outlook

In this talk:

We systematically build-in EM background for *abelian* topological phases in (2+1)d

but do these phases possess non-trivial EM properties?

they do!

continuum description of toric code phase:

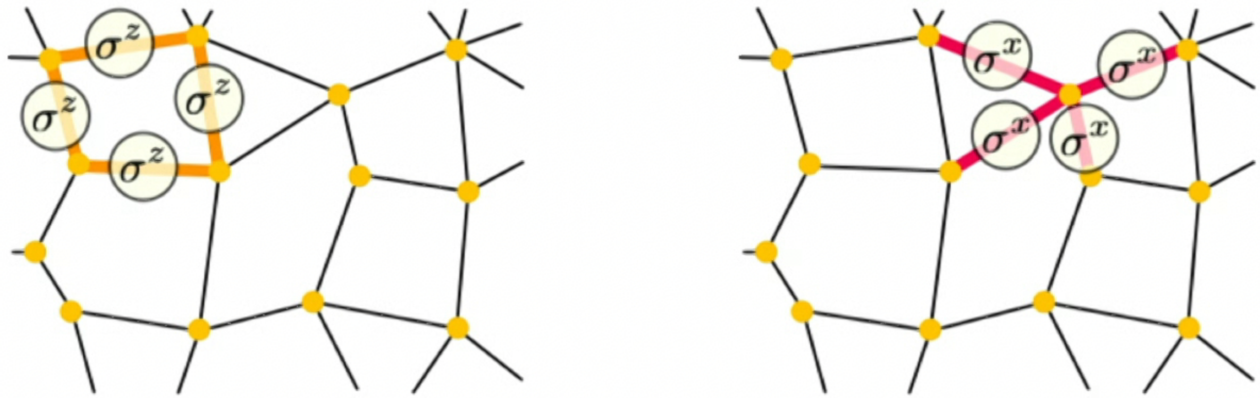
doubled Chern-Simons Lagrangian

$$\frac{1}{4\pi}\begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} d\begin{bmatrix} a \\ b \end{bmatrix}$$

admits EM coupling $\frac{1}{2\pi}\begin{bmatrix} pA & qA \end{bmatrix} d\begin{bmatrix} a \\ b \end{bmatrix}$

frac charges: $p/2,\ q/2$ Hall cond: pq

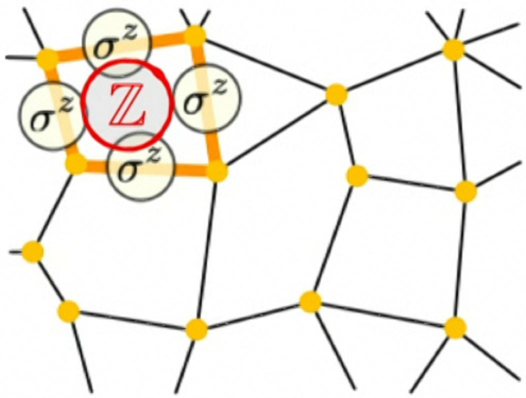
coupling to EM



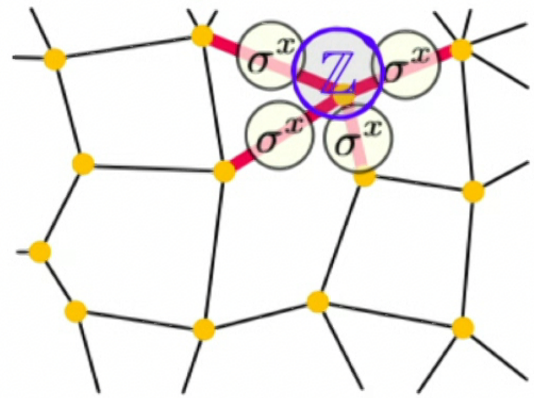
need new degree of freedom

excitations with trivial braiding but carry electric charge

coupling to EM — new degree of freedom



electric charge $q\mathbf{Z}$
on plaquette



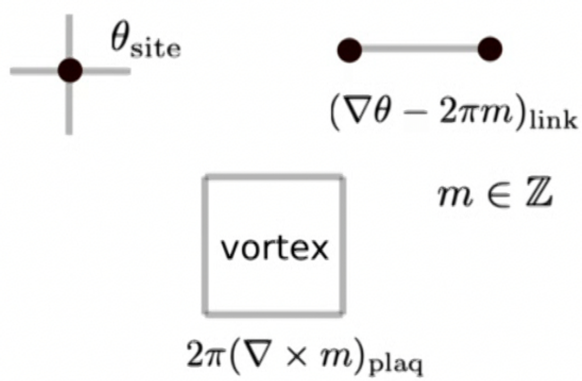
electric charge $p\mathbf{Z}$
on vertex

coupling to EM — new degree of freedom

underlying math

gauging 1-form **Z** symmetry

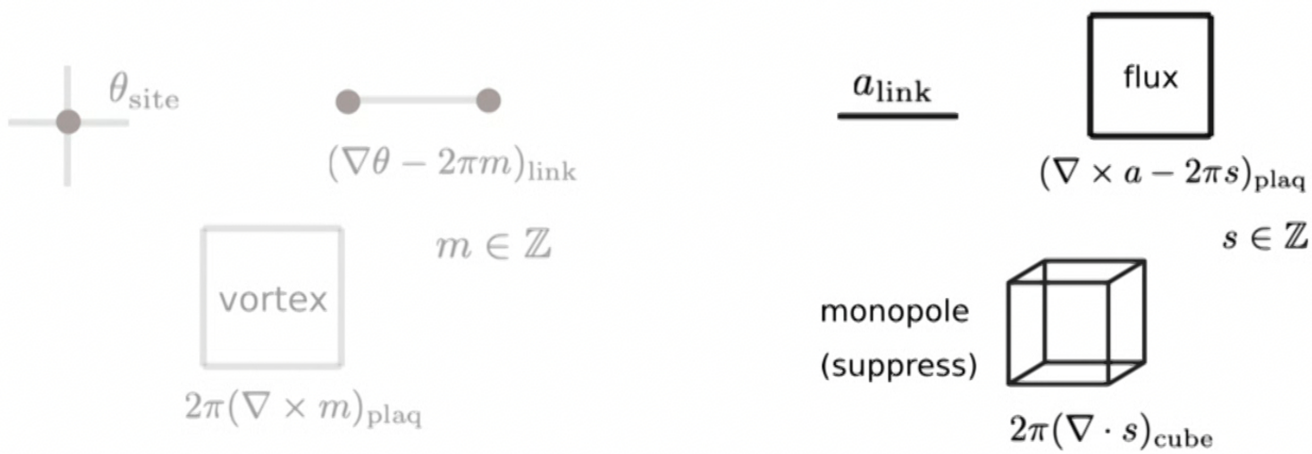
recall: gauging (0-form) **Z** symmetry in Villain model



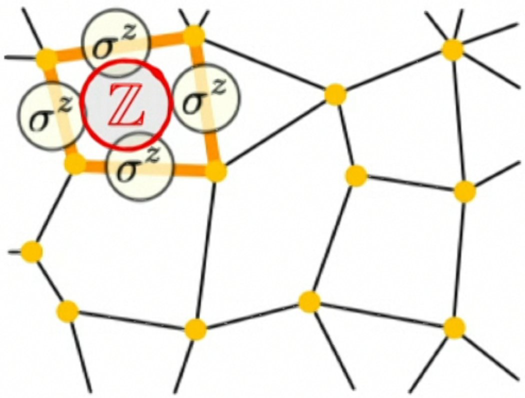
coupling to EM — new degree of freedom

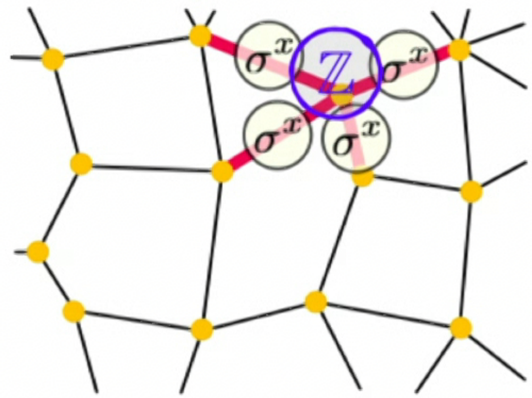
underlying math

gauging 1-form **Z** symmetry



coupling to EM — new degree of freedom





electric charge $q\mathbf{Z}$
on plaquette

electric charge $p\mathbf{Z}$
on vertex

also, deal with the subtlety…

not really "fixed-point"

— narrow 2π EM flux invisible on microscopic lattice

but visible in deep IR limit: "narrow" 2π EM flux creates anyon

after taking care of it…

no Hall conductivity
Hamiltonian exactly soluble (terms commute)

with Hall conductivity
Hamiltonian not exactly soluble (terms don't commute)

manifestation of recent theorem by Kapustin & Fidkowski

but our systematic method gives
controllably soluble Hamiltonians

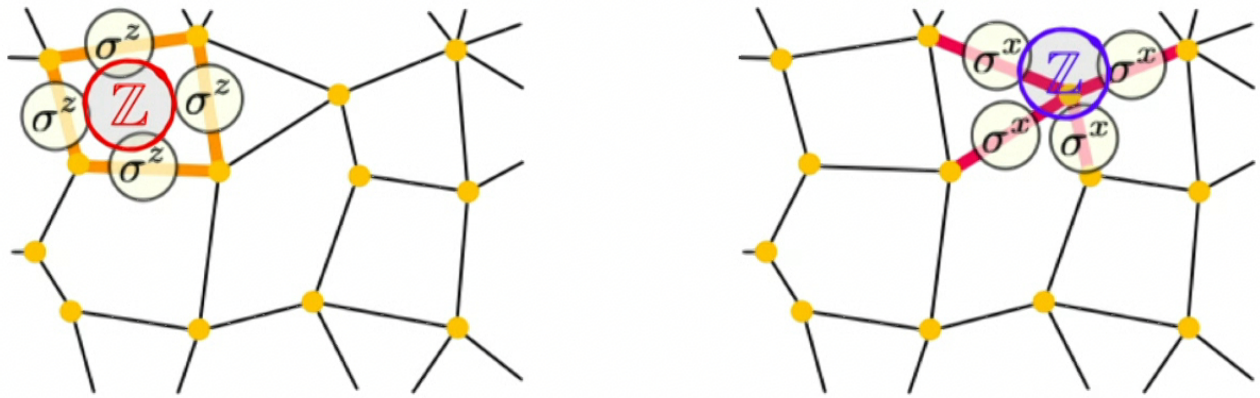
(non-commutativity *arbitrarily* small,
i.e. arbitrarily close to "exactly" soluble)

some other interesting math…

- fermionic topological phases
  EM field as spin-c connection

- equivalence to continuum doubled Chern-Simons
  via Deligne-Beilinson cohomology

We achieve
exactly or controllably soluble Hamiltonian with EM



some other interesting math…

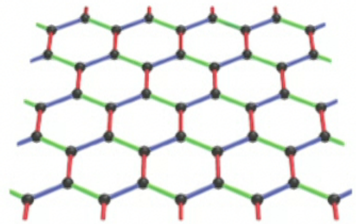
- fermionic topological phases
  EM field as spin-c connection
- equivalence to continuum doubled Chern-Simons
  via Deligne-Beilinson cohomology

We introduced EM response
into a major theoretical framework
that constructs (abelian) topological phases

General math framework of
continuous symmetry group
in tensor category?

Physically, more interesting topological phases?

EM response in Kitaev honeycomb model



SU(2) non-abelian anyons

chiral edge states

bosonic cousin of Moore-Read Pfaffian state

New EM response?

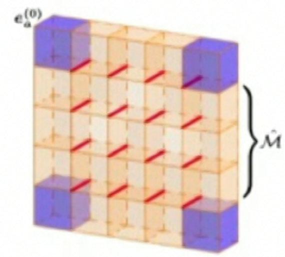
In phases that detect Borromean rings

(as opposed to previous abelian phases that detect linked rings)

non-abelian anyons from abelian gauge fields

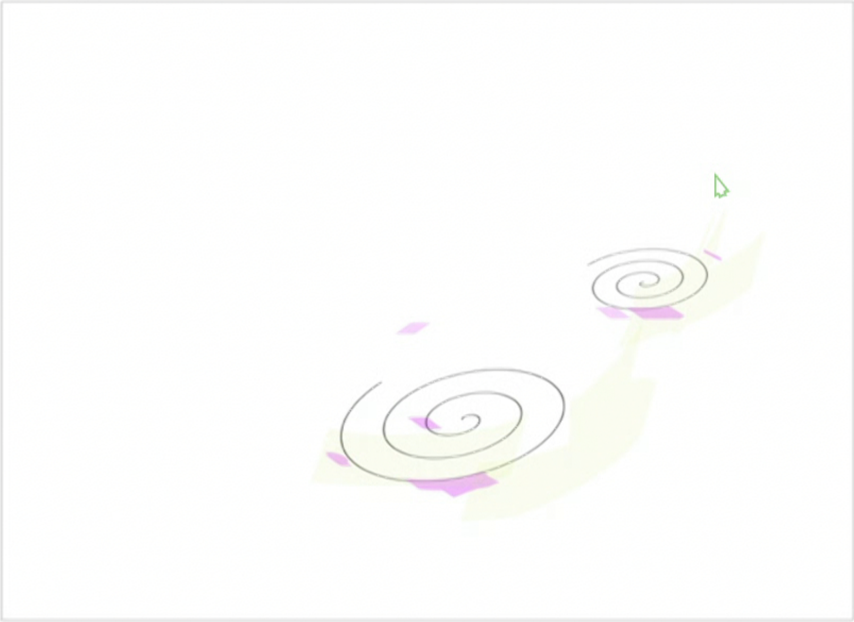
New EM response?

In some fracton models



some fracton models (e.g. X-cube) involve stacked toric code

admits abelian gauge field description in continuum



thank you