Title: Soft Heisenberg hair

Speakers: Daniel Grumiller

Series: Quantum Gravity

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Abstract: Gravity theories naturally allow for edge states generated by non-trivial boundary-condition preserving diffeomorphisms. I present a specific set of boundary conditions inspired by near horizon physics, show that it leads to soft hair excitations of black hole solutions and discuss implications for black hole entropy.

Soft Heisenberg Hair

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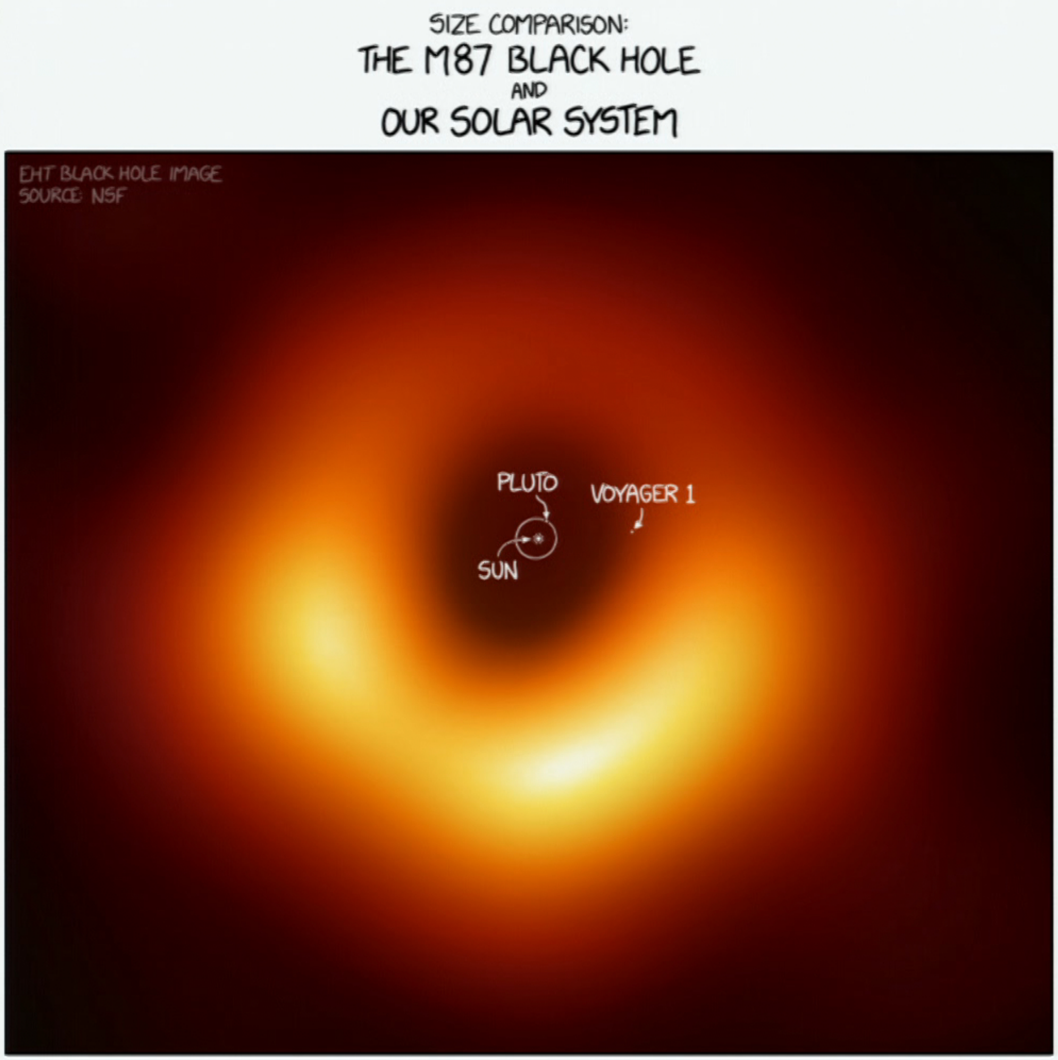
PI, Seminar Talk, April 2019


SIZE COMPARISON:
THE M87 BLACK HOLE
AND
OUR SOLAR SYSTEM
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Outline

Boundary charges

Near horizon boundary conditions

Soft Heisenberg hair and black hole entropy

Generalizations and perspective

Physics with boundaries

Science is a differential equation. Religion is a boundary condition. — Alan Turing

- Many QFT applications employ "natural boundary conditions": fields and fluctuations tend to zero asymptotically
- Notable exceptions exist in gauge theories with boundaries: e.g. in Quantum Hall effect
- Natural boundary conditions not applicable in gravity: metric must not vanish asymptotically
- Gauge or gravity theories in presence of (asymptotic) boundaries: asymptotic symmetries

Definition of asymptotic symmetries

All boundary condition preserving gauge transformations (bcpgt's) modulo trivial gauge transformations

Asymptotic symmetries in gravity

- Impose some bc's at (asymptotic or actual) boundary:

$$\lim_{r\to r_b} g_{\mu\nu}(r,\, x^i) = \bar{g}_{\mu\nu}(r_b,\, x^i) + \delta g_{\mu\nu}(r_b,\, x^i)$$

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r: some convenient ("radial") coordinate
r_b: value of r at boundary (could be ∞)
x^i: remaining coordinates
$g_{\mu\nu}$: metric compatible with bc's

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r_b: value of r at boundary (could be ∞)
x^i: remaining coordinates
$g_{\mu\nu}$: metric compatible with bc's
$\bar{g}_{\mu\nu}$: (asymptotic) background metric
$\delta g_{\mu\nu}$: fluctuations permitted by bc's

- bcpgt's generated by asymptotic Killing vectors ξ:

$$\mathcal{L}_\xi g_{\mu\nu} \stackrel{!}{=} \mathcal{O}(\delta g_{\mu\nu})$$

- typically, Killing vectors can be expanded radially

$$\xi^\mu(r_b,\,x^i) = \xi^\mu_{(0)}(r_b,\,x^i) + \text{subleading terms}$$

$\xi^\mu_{(0)}(r_b,\,x^i)$: generates asymptotic symmetries
subleading terms: generate trivial diffeos

Simple example (based on unpublished notes with Salzer)

Asymptotic Rindler$_2$ spacetimes (in Eddington–Finkelstein gauge)

- Consider class of 2d metrics, partially gauge-fixed

$$g_{rr}(r,\,u) = 0$$
$$g_{ur}(r,\,u) = -1$$
$$g_{uu}(r,\,u) = \delta g(u) r + \mathcal{O}(1)$$

  expanded for large r

- bcpt's generated by asymptotic Killing vectors

$$\xi = \epsilon(u)\partial_u + \big(\eta(u) - \epsilon'(u) r\big)\,\partial_r$$

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$$\xi = \epsilon(u)\partial_u + \big(\eta(u) - \epsilon'(u)r\big)\,\partial_r$$

- asymptotic symmetry algebra ("BMS$_2$"):

$$\big[\xi(\epsilon_1,\,\eta_1),\,\xi(\epsilon_2,\,\eta_2)\big]_{\text{Lie}} = \xi\big(\epsilon_1\epsilon_2' - \epsilon_2\epsilon_1',\,(\epsilon_1\eta_2 - \epsilon_2\eta_1)'\big)$$

Lie bracket algebra of asymptotic Killing vectors is infinite dimensional here

Canonical boundary charges

God made the bulk; surfaces were invented by the devil — Wolfgang Pauli

- changing boundary conditions can change physical spectrum

  simple example: quantum mechanics of free particle on half-line $x \geq 0$

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  time-independent Schrödinger equation:

$$-\frac{\mathrm{d}^2}{\mathrm{d}x^2}\psi(x) = E\psi(x)$$

  look for (normalizable) bound state solutions, $E < 0$
  - Dirichlet bc's: no bound states
  - Neumann bc's: no bound states

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  - Dirichlet bc's: no bound states
  - Neumann bc's: no bound states
  - Robin bc's

    $$(\psi + \alpha\psi')\big|_{x=0^+} = 0 \qquad \alpha \in \mathbb{R}^+$$

    lead to one bound state

    $$\psi(x)\big|_{x\geq 0} = \sqrt{\frac{2}{\alpha}}\, e^{-x/\alpha}$$

    with energy $E = -1/\alpha^2$, localized exponentially near $x = 0$

Canonical boundary charges

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- changing boundary conditions can change physical spectrum
- to distinguish asymptotic symmetries from trivial gauge trafos: either use Noether's second theorem and covariant phase space analysis or perform Hamiltonian analysis in presence of boundaries

Some references:

- covariant phase space: Lee, Wald '90, Iyer, Wald '94 and Barnich, Brandt '02
- review: see Compère, Fiorucci '18 and refs. therein
- canonical analysis: Arnowitt, Deser, Misner '59, Regge, Teitelboim '74 and Brown, Henneaux '86
- review: see Bañados, Reyes '16 and refs. therein

Canonical boundary charges

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- changing boundary conditions can change physical spectrum
- to distinguish asymptotic symmetries from trivial gauge trafos: perform Hamiltonian analysis in presence of boundaries
- in Hamiltonian language: gauge generator $G[\epsilon]$ varies as

$$\delta G[\epsilon] = \int_\Sigma (\text{bulk term})\,\epsilon\,\delta\Phi - \int_{\partial\Sigma} (\text{boundary term})\,\epsilon\,\delta\Phi$$

  not functionally differentiable in general (Σ: constant time slice)
- add boundary term to restore functional differentiability

$$\delta\Gamma[\epsilon] = \delta G[\epsilon] + \delta Q[\epsilon] \stackrel{!}{=} \int_\Sigma (\text{bulk term})\,\epsilon\,\delta\Phi$$

Canonical realization of asymptotic symmetries

- canonical gauge generator generates gauge trafos on phase space

$$\delta_\epsilon f(\Phi) = \{\Gamma[\epsilon],\, f(\Phi)\}$$

- in particular:

$$\delta_{\epsilon_1}\Gamma[\epsilon_2] = \{\Gamma[\epsilon_1],\, \Gamma[\epsilon_2]\}$$

- on constraint surface $\Gamma[\epsilon] = Q[\epsilon]$, hence

$$\delta_{\epsilon_1}Q[\epsilon_2] = \{Q[\epsilon_1],\, Q[\epsilon_2]\} = Q[\epsilon_1 \circ \epsilon_2] + Z[\epsilon_1,\, \epsilon_2]$$

Z: possible central extension of asymptotic symmetry algebra

Canonical realization of asymptotic symmetries

Poisson (or Dirac) bracket algebra of canonical boundary charges

Simple example: abelian Chern–Simons

- abelian Chern–Simons action (on cylinder)

$$I[A] = \frac{k}{4\pi} \int_{\mathbb{R}\times\Sigma} A \wedge \mathrm{d}A$$

- gauge trafos $\delta_\epsilon A = \mathrm{d}\epsilon$
- canonical analysis yields boundary charges (background independent)

$$Q[\epsilon] = \frac{k}{2\pi} \oint_{\partial\Sigma} \epsilon\, A$$

- choice of bc's

$$\lim_{r\to\infty} A = \mathcal{J}(\varphi)\; \mathrm{d}\varphi + \mu\; \mathrm{d}t$$

preserved by $\epsilon = \eta(\varphi) +$ subleading

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- asymptotic symmetry algebra has non-trivial central term

$$\{Q[\eta_1],\, Q[\eta_2]\} = \delta_{\eta_1} Q[\eta_2] = \frac{k}{2\pi} \oint_{\partial\Sigma} \eta_2\, \eta_1'\, \mathrm{d}\varphi$$

- Fourier modes $J_n \sim \oint \mathcal{J} e^{in\varphi}$ yield $u(1)_k$ current algebra, $i\{J_n,\, J_m\} = \frac{k}{2}\, n\, \delta_{n+m,\,0}$

Edge states

see e.g. Halperin '82, Witten '89, or Balachandran, Chandar, Momen '94

- changing boundary charges changes physical state
- boundary charges (if non-trivial) thus generate edge states
- back to abelian Chern–Simons example:
  - asymptotic symmetry algebra

$$[J_n,\, J_m] = \frac{k}{2}\, n\, \delta_{n+m,\,0}$$

  - define vacuum

$$J_n|0\rangle = 0 \qquad \forall n \geq 0$$

  - descendants of vacuum are examples of edge states

$$|\text{edge}(\{n_i\})\rangle = \prod_{\{n_i>0\}} J_{-n_i}|0\rangle$$

e.g.

$$|\text{edge}(\{1,1,42\})\rangle = J_{-1}^2 J_{-42}|0\rangle$$

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- theories with no local physical degrees of freedom can have edge states! $\Rightarrow$ perhaps cleanest example of holography

Motivation for near horizon boundary conditions

Old idea by Strominger '97 and Carlip '98

Main idea

Impose existence of non-extremal horizon as boundary condition on state space

Motivations:

- Want to ask conditional questions "given a black hole, what are the probabilities for some scattering process"

Explicit form of near horizon boundary conditions

See Donnay, Giribet, Gonzalez, Pino '15 and Afshar et al '16

Postulates of near horizon boundary conditions:

1. Rindler approximation

$$ds^2 = -\kappa^2 r^2 \, dt^2 + dr^2 + \Omega_{ab}(t, x^c) \, dx^a \, dx^b + \dots$$

$r \to 0$: Rindler horizon
κ: surface gravity
Ω_{ab}: metric transversal to horizon
$\dots$: terms of higher order in r or rotation terms

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   $r \to 0$: Rindler horizon
   κ: surface gravity
   Ω_{ab}: metric transversal to horizon
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2. Surface gravity is state-independent

$$\delta\kappa = 0$$

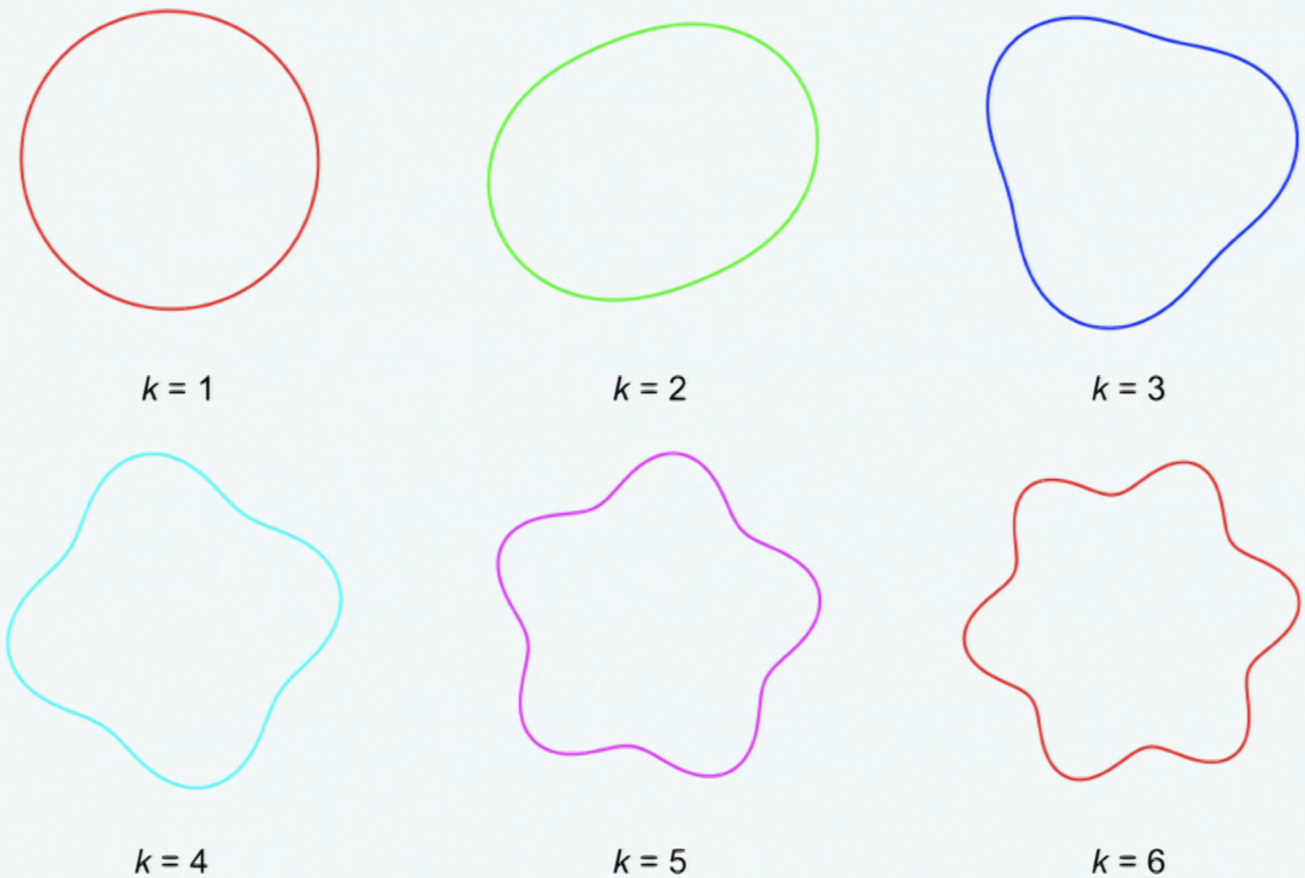
3. Metric transversal to horizon is state-dependent

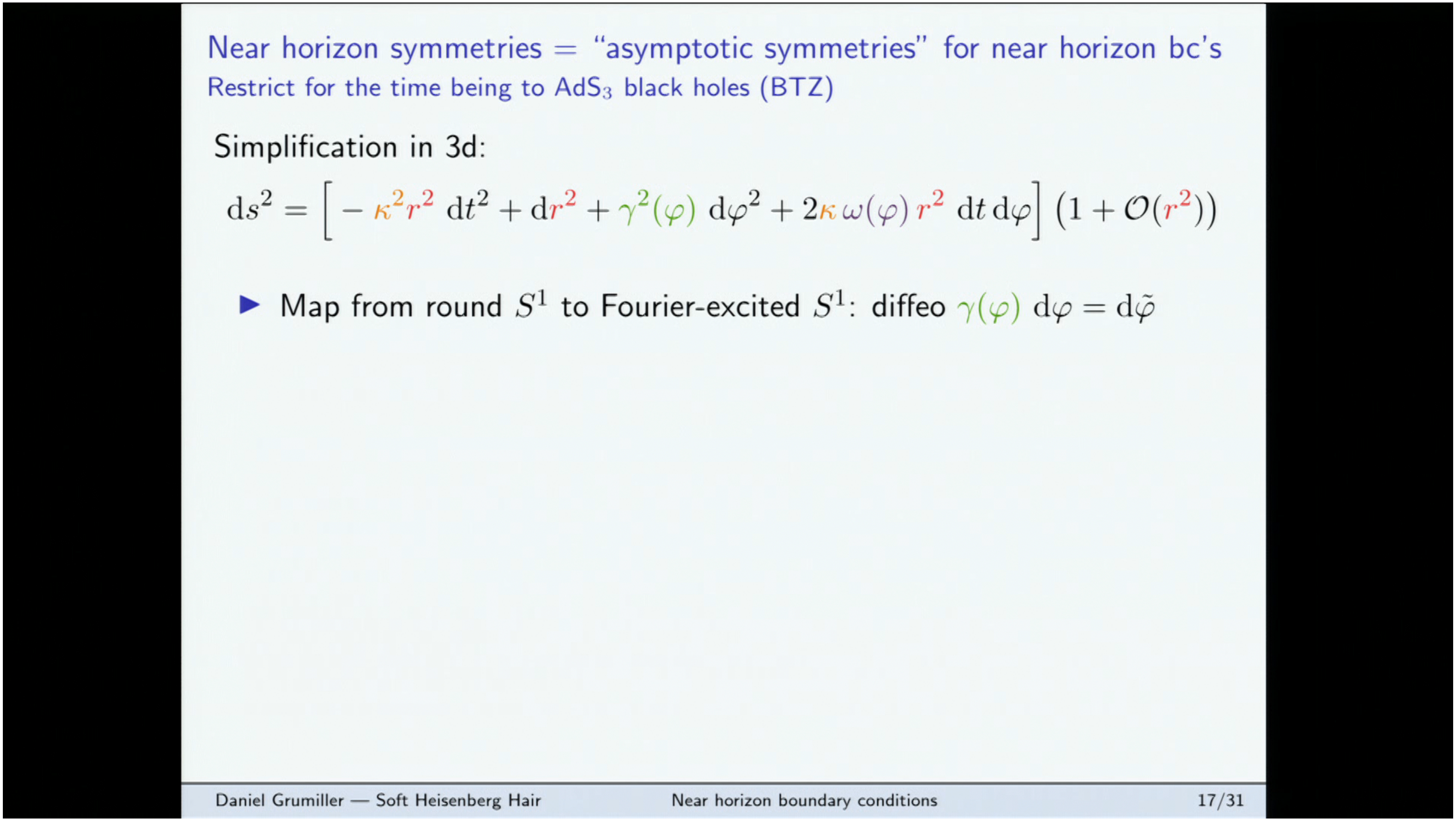
$$\delta\Omega_{ab} = \mathcal{O}(1)$$

4. Remaining terms fixed by consistency of canonical boundary charges

Black holes can be deformed into black flowers Afshar et al. 16

Horizon can get excited by area preserving shear-deformations




Near horizon symmetries = "asymptotic symmetries" for near horizon bc's
Restrict for the time being to AdS_3 black holes (BTZ)
Simplification in 3d:
$$\mathrm{d}s^2 = \Big[-\kappa^2 r^2\,\mathrm{d}t^2 + \mathrm{d}r^2 + \gamma^2(\varphi)\,\mathrm{d}\varphi^2 + 2\kappa\,\omega(\varphi)\,r^2\,\mathrm{d}t\,\mathrm{d}\varphi\Big]\big(1+\mathcal{O}(r^2)\big)$$
▶ Map from round S^1 to Fourier-excited S^1: diffeo $\gamma(\varphi)\,\mathrm{d}\varphi = \mathrm{d}\tilde\varphi$
Daniel Grumiller — Soft Heisenberg Hair
Near horizon boundary conditions
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- Map from round S^1 to Fourier-excited S^1: diffeo $\gamma(\varphi) \, \mathrm{d}\varphi = \mathrm{d}\tilde{\varphi}$
- Non-trivial diffeo!
- Canonical analysis yields

$$Q^\pm[\epsilon^\pm] \sim \oint \mathrm{d}\varphi \, \epsilon^\pm(\varphi) \, \big(\gamma(\varphi) \pm \omega(\varphi)\big)$$

where $\epsilon^\pm$ are functions appearing in asymptotic Killing vectors

charge conservation follows from on-shell relations $\partial_t \gamma = 0 = \partial_t \omega$

explains last word in title: γ and ω are hair of black hole

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- Near horizon symmetry algebra Fourier modes $\mathcal{J}_n^\pm = Q^\pm[\epsilon^\pm = e^{in\varphi}]$

$$[\mathcal{J}_n^\pm,\, \mathcal{J}_m^\pm] = \tfrac{1}{2}\,n\,\delta_{n+m,\,0}$$

- Isomorphic to Heisenberg algebras plus center

$$[X_n,\, P_m] = i\,\delta_{n,m} \qquad\qquad [P_0,\, X_n] = 0 = [X_0,\, P_n]$$

$P_0 = \mathcal{J}_0^+ + \mathcal{J}_0^-$, $X_n = \mathcal{J}_n^+ - \mathcal{J}_{-n}^-$, $P_n = 2i/n(\mathcal{J}_{-n}^+ + \mathcal{J}_n^-)$ for $n \neq 0$

Unique features of near horizon boundary conditions

1. All states allowed by bc's have same temperature
2. All states allowed by bc's are regular

   (in particular, they have no conical singularities at the horizon in the Euclidean formulation)

   By contrast: for given temperature not all states in theories with asymptotically AdS or flat space bc's are free from conical singularities; usually a unique black hole state is picked

Unique features of near horizon boundary conditions

1. All states allowed by bc's have same temperature
2. All states allowed by bc's are regular
   (in particular, they have no conical singularities at the horizon in the Euclidean formulation)
3. There is a non-trivial reducibility parameter (= Killing vector)
4. Technical feature: in Chern–Simons formulation of 3d gravity simple expressions in diagonal gauge

$$A^{\pm} = b^{\mp 1}\big(\mathrm{d} + a^{\pm}\big)\, b^{\pm 1}$$

$$a^{\pm} = L_0 \Big(\big(\gamma(\varphi) \pm \omega(\varphi)\big)\, \mathrm{d}\varphi + \kappa\, \mathrm{d}t\Big)$$

$$b = \exp\big[\big(L_+ - L_-\big)\, r/2\big]$$

$L_{\pm}$ are $sl(2,\,\mathbb{R})$ raising/lowering generators
L_0 is $sl(2,\,\mathbb{R})$ Cartan subalgebra generator

Soft Heisenberg hair for BTZ

- Black flower excitations = hair of black holes
  Algebraically, excitations from descendants

$$|\text{black flower}\rangle \sim \prod_{n_i^\pm > 0} \mathcal{J}^+_{-n_i^+} \mathcal{J}^-_{-n_i^-} |\text{black hole}\rangle$$

- What is energy of such excitations?
- Near horizon Hamiltonian = boundary charge associated with unit time-translations

$$H = Q[\partial_t] = \kappa\, P_0$$

  commutes with all generators $\mathcal{J}^\pm_n$
- H-eigenvalue of black flower = H-eigenvalue of black hole
- Black flower excitations do not change energy of black hole!

> Black flower excitations = soft hair in sense of Hawking, Perry and Strominger '16

New entropy formula

Express entropy in terms of near horizon charges:

$$S = 2\pi \, P_0$$

- Entropy = parity inv. combination of near horizon charge zero modes
- Obeys simple near horizon first law

$$\delta S = \frac{2\pi}{\kappa} \, \delta\big(\kappa P_0\big) \qquad \Rightarrow \qquad T \, \delta S = \delta H$$

with Hawking–Unruh-temperature

$$T = \frac{\kappa}{2\pi}$$

δ refers to any variation of phase space variables allowed by the boundary conditions

Semi-classical microstates?

Given our soft Heisenberg hair, attack now entropy questions

1. Why only semi-classical input for entropy?
2. What are microstates?
3. Semi-classical construction of microstates?
4. Does counting of microstates reproduce S_{BH}?

Regarding 1. and 3.: may expect decoupling of scales so that description of microstates does not need info about UV completion, but rather only some semi-classical "Bohr-like" input

Evidence for this: universality of BH entropy for large black holes

$$S_{\text{BH}} = \frac{A}{4G} + \dots$$

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Possible obstacles:

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- TMI: no upper bound on soft hair excitations

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4. Does counting of microstates reproduce $S_{\rm BH}$?

> Assume it is possible to construct microstates for large black holes semi-classically using soft-hair excitations

Possible obstacles:

- TMI: no upper bound on soft hair excitations
- possible resolution: cut-off on soft hair spectrum!
- TLI Mirbabayi, Porrati '16; Bousso, Porrati '17; Donnelly, Giddings '17: for asymptotic observer no information from soft hair states

Fluff proposal (with Afshar, Sheikh-Jabbari '16 and also with Yavartanoo '17)

Semi-classical BTZ black hole microstates as near horizon descendants of vacuum

Highest weight vacuum $|0\rangle$

$$\mathcal{J}_n^{\pm}|0\rangle = 0 \quad \forall n \geq 0$$

Black hole microstates:

$$|\mathcal{B}(\{n_i^{\pm}\})\rangle = \prod_{\{n_i^{\pm}>0\}} \left(\mathcal{J}_{-n_i^+}^{+} \cdot \mathcal{J}_{-n_i^-}^{-}\right)|0\rangle$$

subject to spectral constraint depending on black hole mass M and angular momentum J (measured by asymptotic observer)

$$\sum_i n_i^{\pm} = \frac{c}{2}\,(M \pm J)$$

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$$\sum_i n_i^{\pm} = \frac{c}{2}\,(M \pm J)$$

derived from Bohr-type quantization conditions

- quantization of central charge $c = 3/(2G)$ in integers
- quantization of conical deficit angles in integers over c
- black hole/particle correspondence
  (black hole $=$ gas of coherent states of particles on AdS_3)

Generalizations

- Near horizon boundary conditions

  works in any dimension, for any local geometry, for any reasonable theory* (with metric) and for any type of non-extremal horizon

* theories checked so far:
Einstein gravity with negative cosmological constant ($d \geq 3$)
Einstein gravity with vanishing cosmological constant ($d \geq 3$)
higher spin gravity ($d = 3$, principal embedding of $sl(2)$)
various massive gravity theories ($d = 3$)

Generalizations

- Near horizon boundary conditions

  works in any dimension, for any local geometry, for any reasonable theory (with metric) and for any type of non-extremal horizon

- Soft Heisenberg hair

  works for Einstein gravity, higher derivative gravity and higher spin gravity in three dimensions and Einstein gravity in higher dimensions

Generalizations

- Near horizon boundary conditions

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- Soft Heisenberg hair

  works for Einstein gravity, higher derivative gravity and higher spin gravity in three dimensions and Einstein gravity in higher dimensions
- Entropy formula

  works for Einstein gravity, higher derivative gravity and higher spin gravity in three dimensions and Einstein gravity in higher dimensions*

* for instance, for Schwarzschild

$$\{Q_{lm},\, P_{l'm'}\} = \frac{1}{8\pi G}\,\delta_{ll'}\,\delta_{mm'} \qquad l>0 \qquad\qquad \{P_{00},\, \bullet\} = 0$$

Q_{lm}: spherical harmonics of area preserving shear deformations
P_{lm}: spherical harmonics of near horizon supertranslations
Entropy given by $S = 2\pi\, P_{00}$

Kerr has additional generators: area preserving twist deformations

Outlook

Take-away messages:

- Near horizon boundary conditions useful for black hole description
- Soft Heisenberg hair generic consequence
- Universal entropy formula depends only on (semi-)classical input

$$S = 2\pi\, P_0$$

- Semi-classical microstate construction may work (at least for BTZ)

$$|\mathcal{B}(\{n_i^\pm\})\rangle = \prod_{\{n_i^\pm > 0\}} \left(\mathcal{J}^+_{-n_i^+} \cdot \mathcal{J}^-_{-n_i^-}\right)|0\rangle \qquad \sum_i n_i^\pm = \text{fixed by } M,\, J$$

Thanks for your attention!