Title: Covariant Quantum Error correction: an approximate Eastin-Knill theorem, reference frame encoding, and continuous symmetries in AdS/CFT

Speakers: Grant Salton

Series: Perimeter Institute Quantum Discussions

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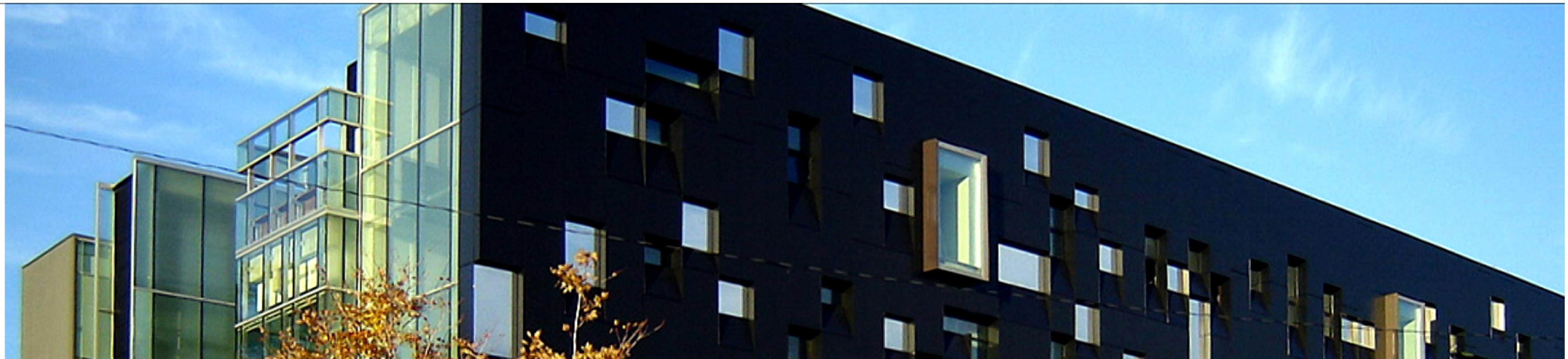
Abstract: In the usual paradigm of quantum error correction, the information to be protected can be encoded in a system of abstract qubits or modes. But how does this work for physical information, which cannot be described in this way? Just as direction information cannot be conveyed using a sequence of words if the parties involved do not share a reference frame, physical quantum information cannot be conveyed using a sequence of qubits or modes without a shared reference frame. Covariant quantum error correction is a procedure for protecting such physical information against noise in such a way that the encoding and decoding operations transform covariantly with respect to an external symmetry group. In this talk, we'll study covariant QEC, and we will see that there do not exist finite dimensional quantum codes that are covariant with respect to continuous symmetries. Conversely, we'll see that there do exist finite codes for finite groups, and continuous variable (CV) codes for continuous groups. This leads to a CV method of circumventing the Eastin-Knill theorem. By relaxing our requirements to allow for only approximate error correction and covariance, we'll find a fundamental tension between a code's ability to approximately correct errors and covariance with respect to a continuous symmetry. In this way, we'll arrive at an approximate version of the Eastin-Knill theorem, and we'll end by learning what covariant QEC tells us about continuous symmetries in AdS/CFT, among other applications.



Grant Salton

Caltech

Covariant Quantum Error Correction

Approximate Eastin-Knill Theorem, reference frame encoding, and continuous symmetries in AdS/CFT

PERIMETER INSTITUTE

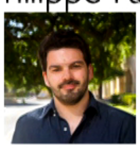
April 10, 2019

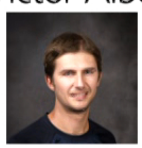
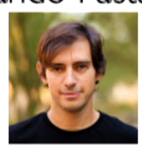
OUTLINE

COVARIANT QEC

- **arXiv:1709.04471** and **arXiv:1902.07714**
- Error correction of quantum reference frame information
- Covariant quantum error correction
- Approximate error correction
- Eastin-Knill Theorem
- Application to AdS/CFT and symmetries in quantum gravity
- With: Philippe Faist, Sepehr Nezami, Victor Albert, Fernando Pastawski, Patrick Hayden, Sandu Popescu, John Preskill

(Error correction of quantum reference frame information } Covariant quantum error correction: Equivalent)

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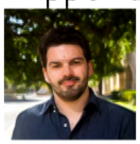





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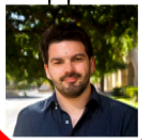
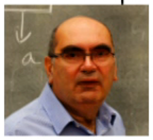





COVARIANT QEC

REFERENCE FRAME

QUANTUM ERROR CORRECTION

Abstract classical information can be described using a sequence of symbols (e.g., 0,1)

Abstract quantum information can be stored systems of qubits (or qudits, modes, etc.)

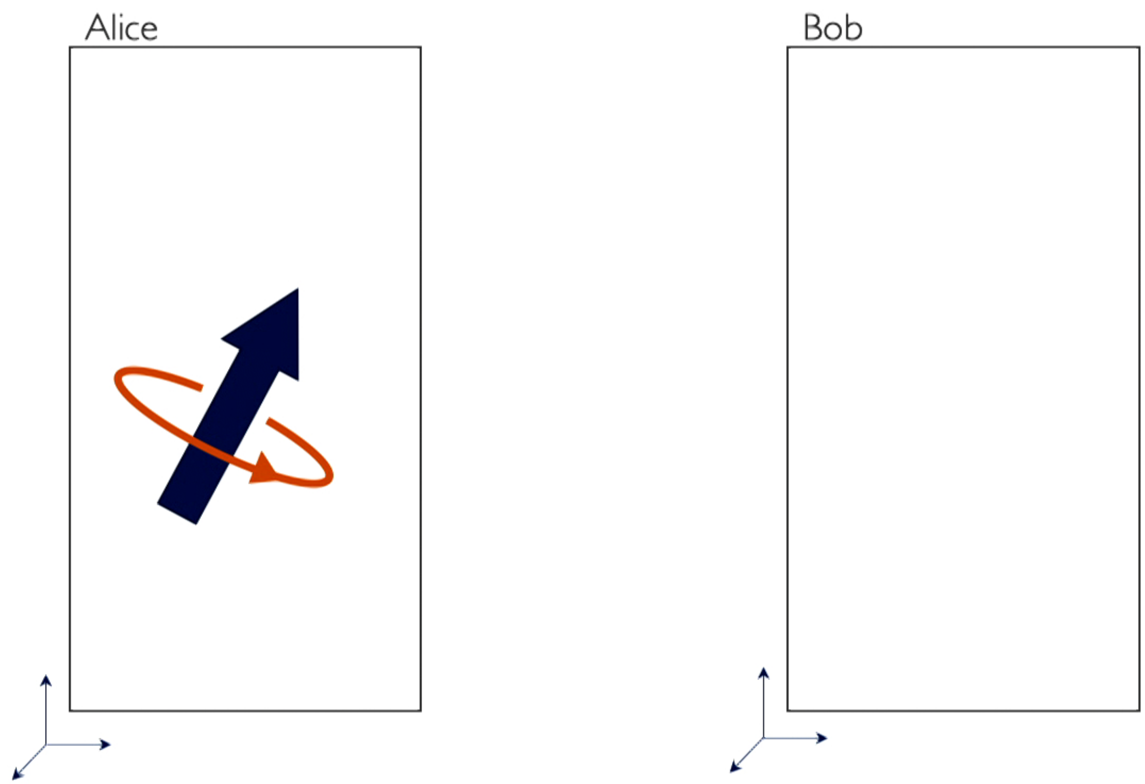
Physical information is any information that cannot be described in this abstract way (e.g., reference frames)

SHARED FRAMES

Alice wants to convey some "directional" information to Bob (e.g., the rotation axis of a gyroscope.)

Alice measures the the vector and describes the components to Bob in words

Bob can then create a gyroscope spinning along the same axis



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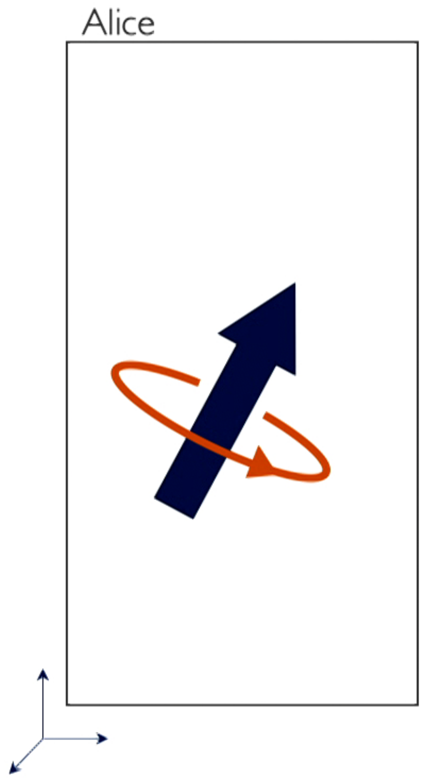
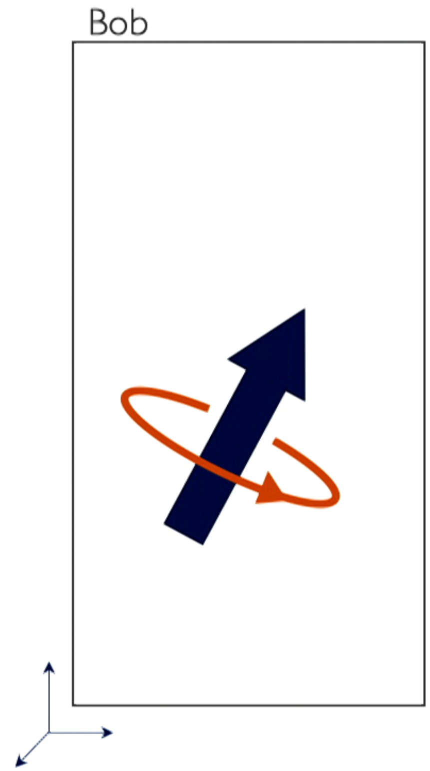




NO COMMON REFERENCE FRAME

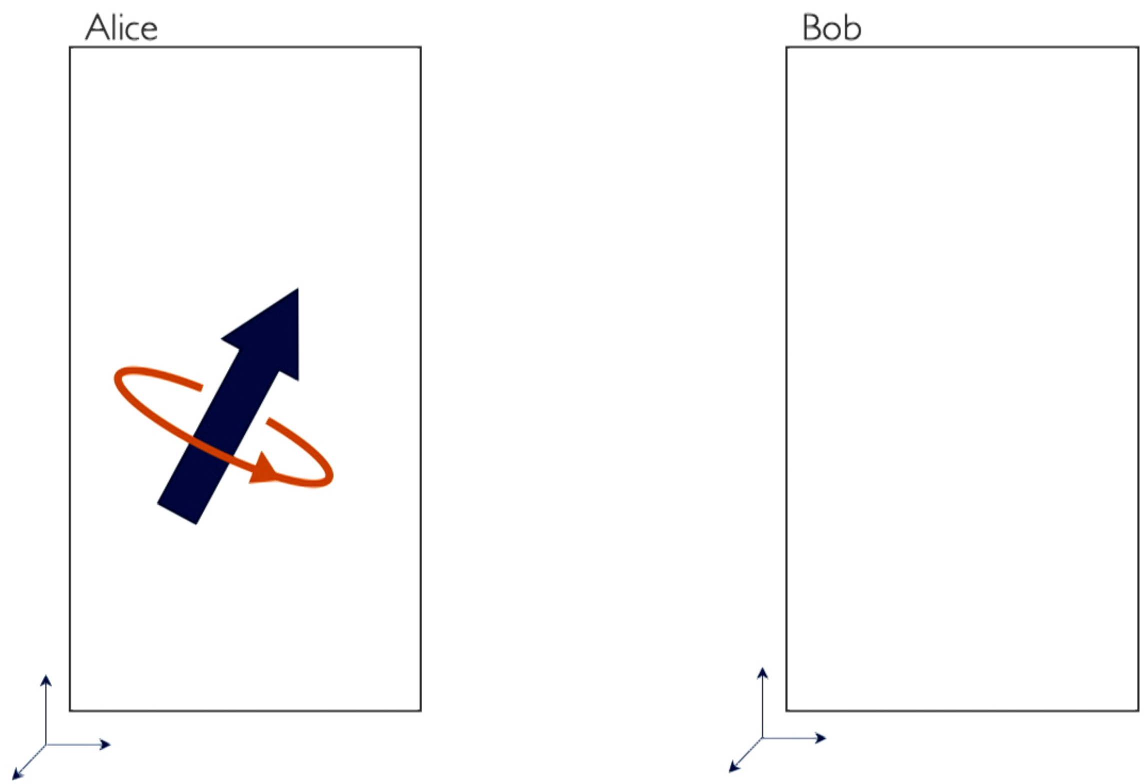
Now suppose Alice and Bob do not share a common reference frame

By sending a string of words to describe the vector, Bob will create the wrong vector

Alice and Bob fail this task.

What can we do in this situation?

What if the channel is noisy?



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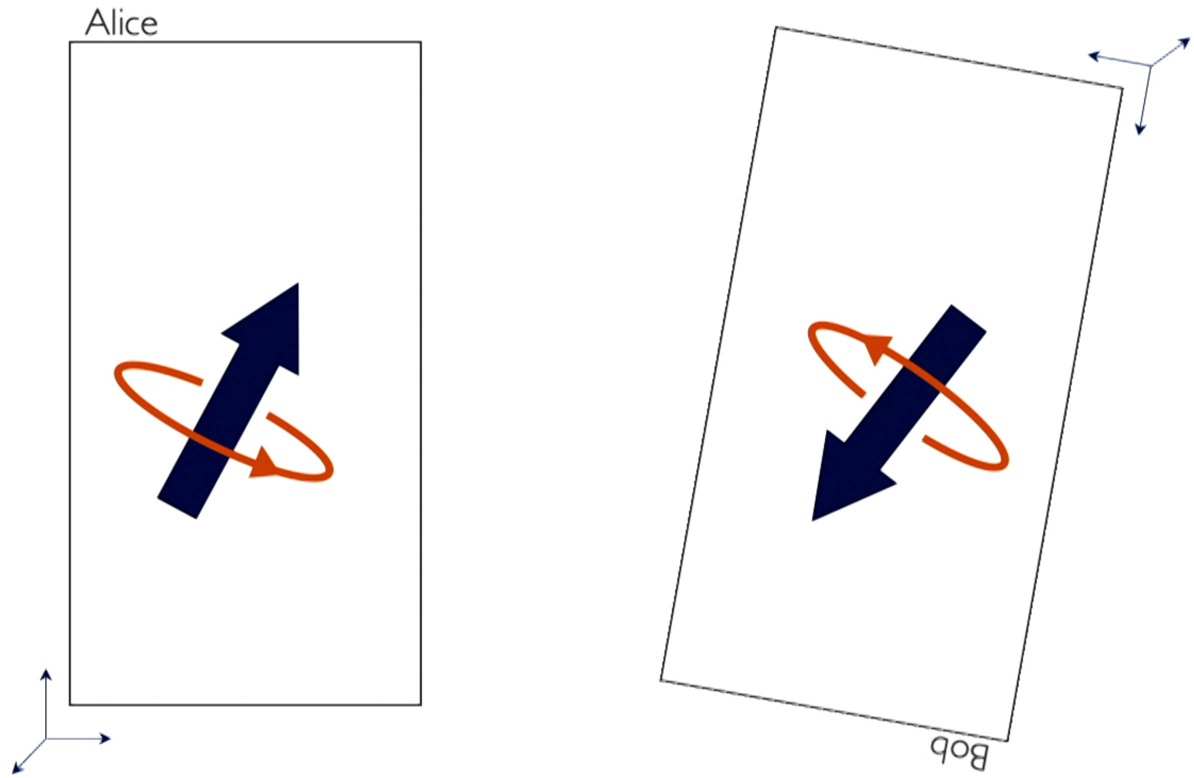

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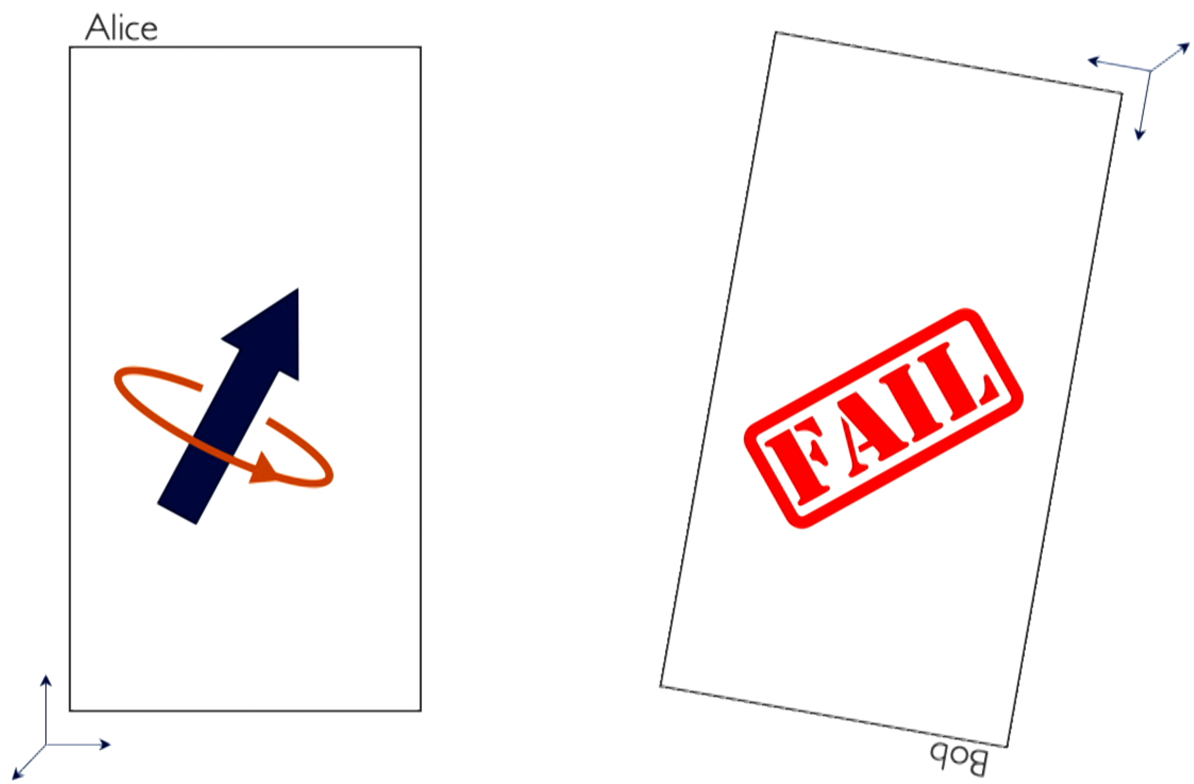

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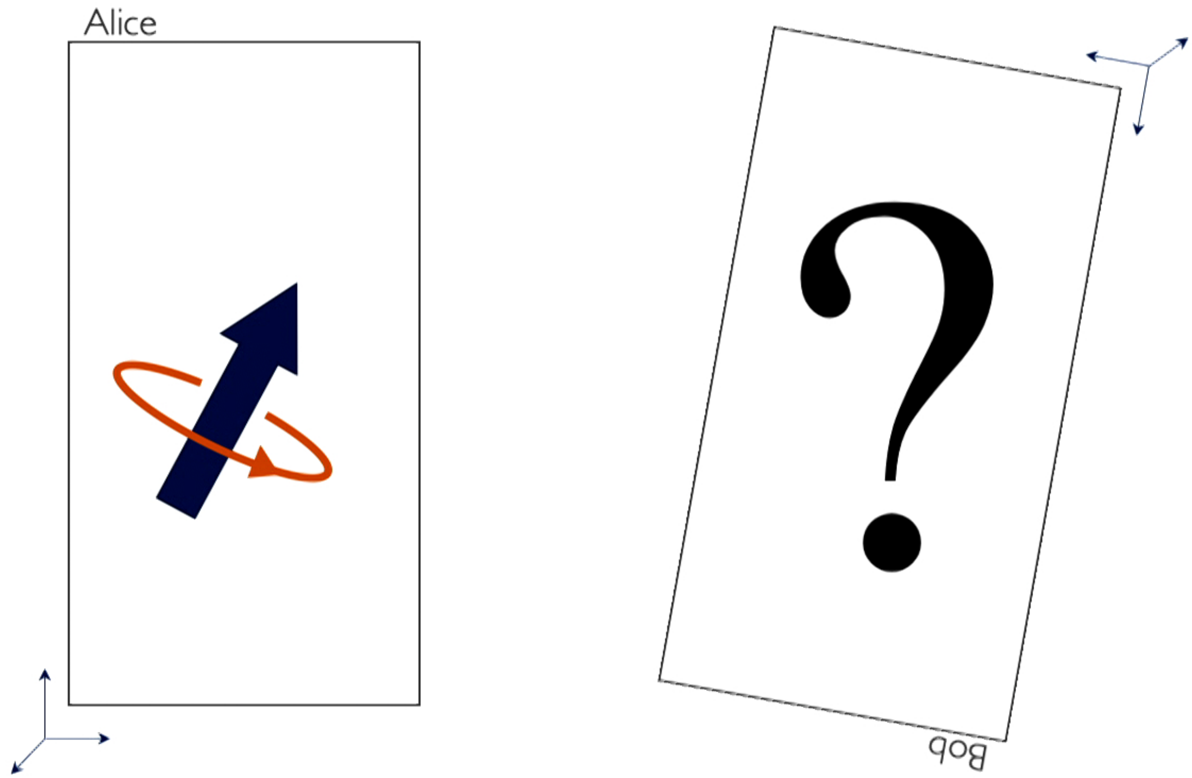

REFERENCE FRAME QEC

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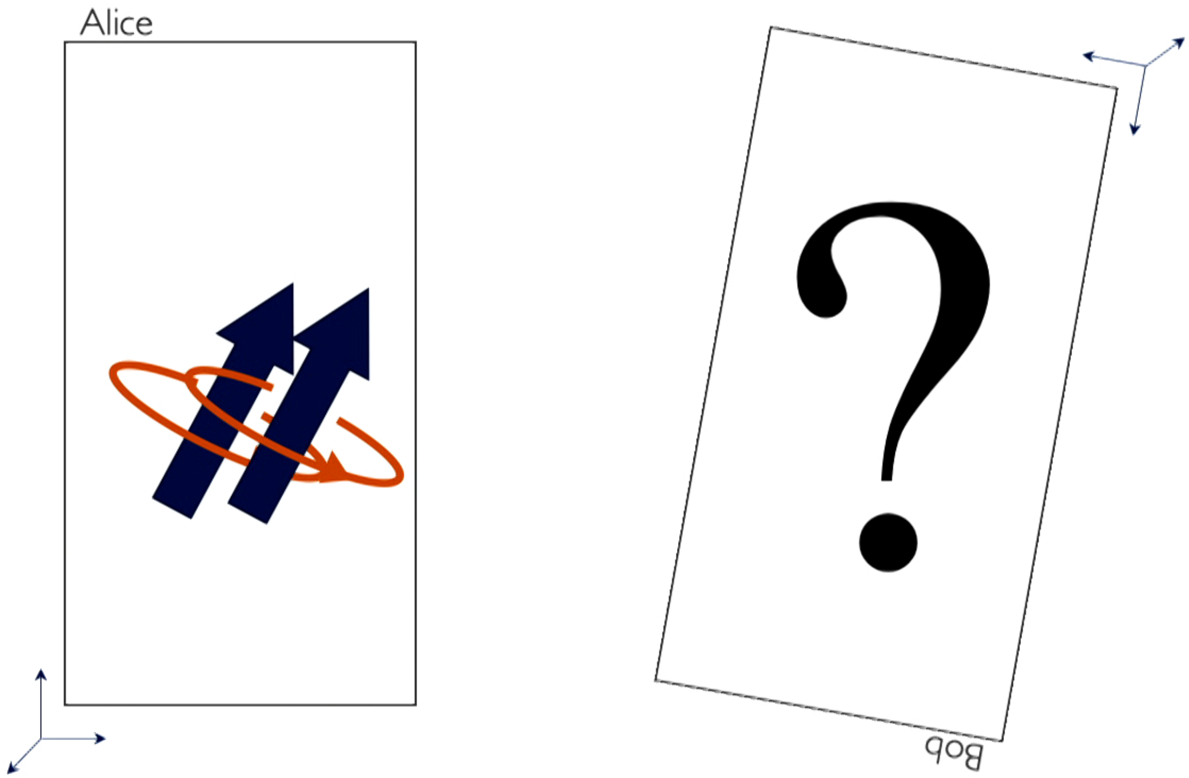

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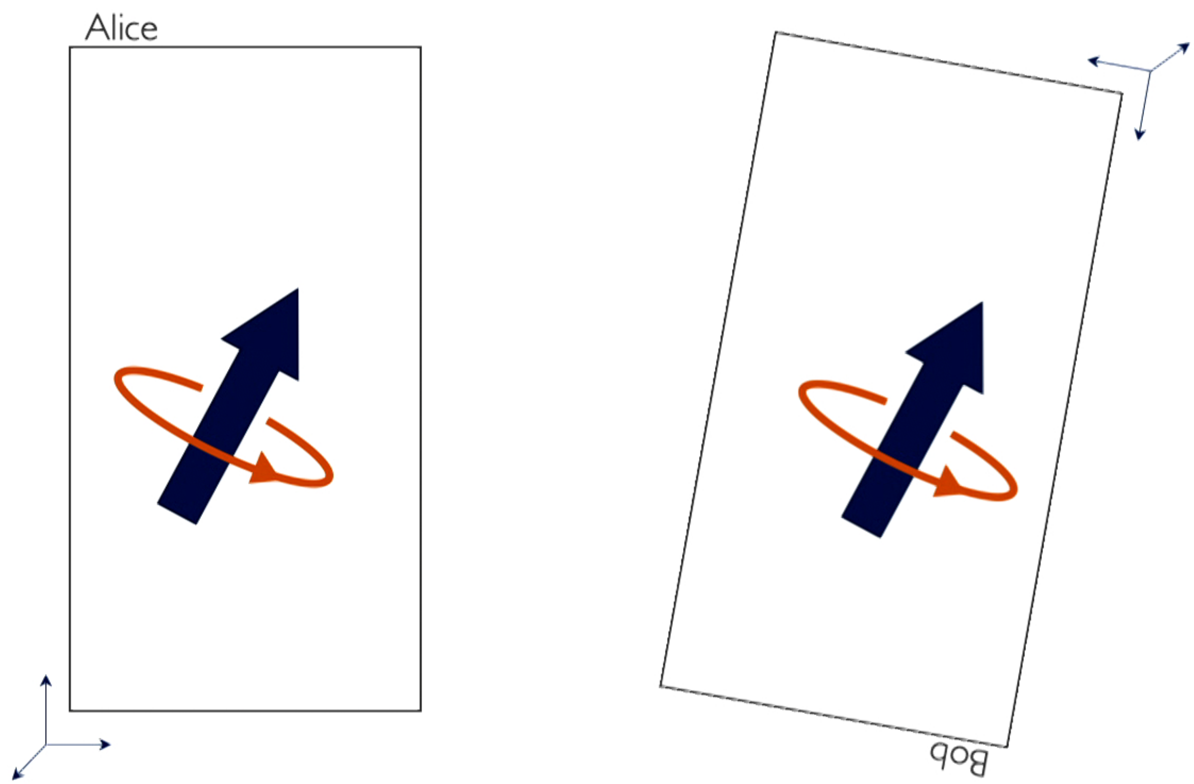

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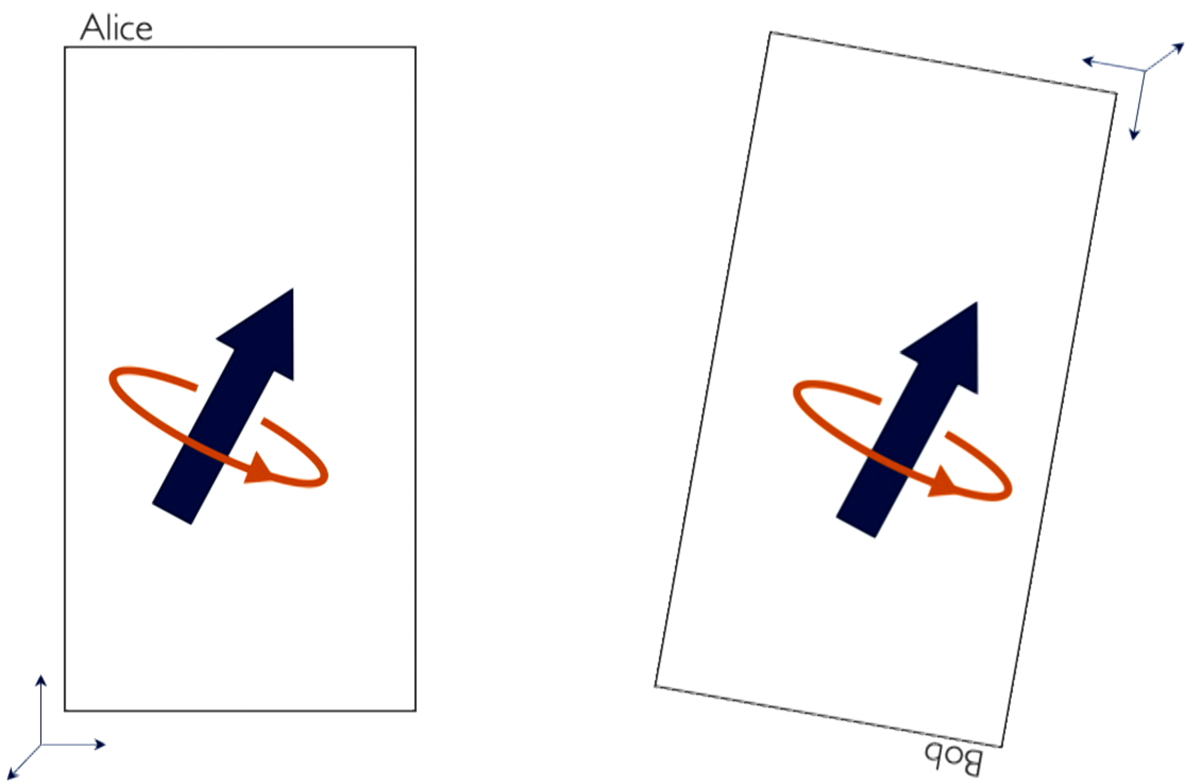

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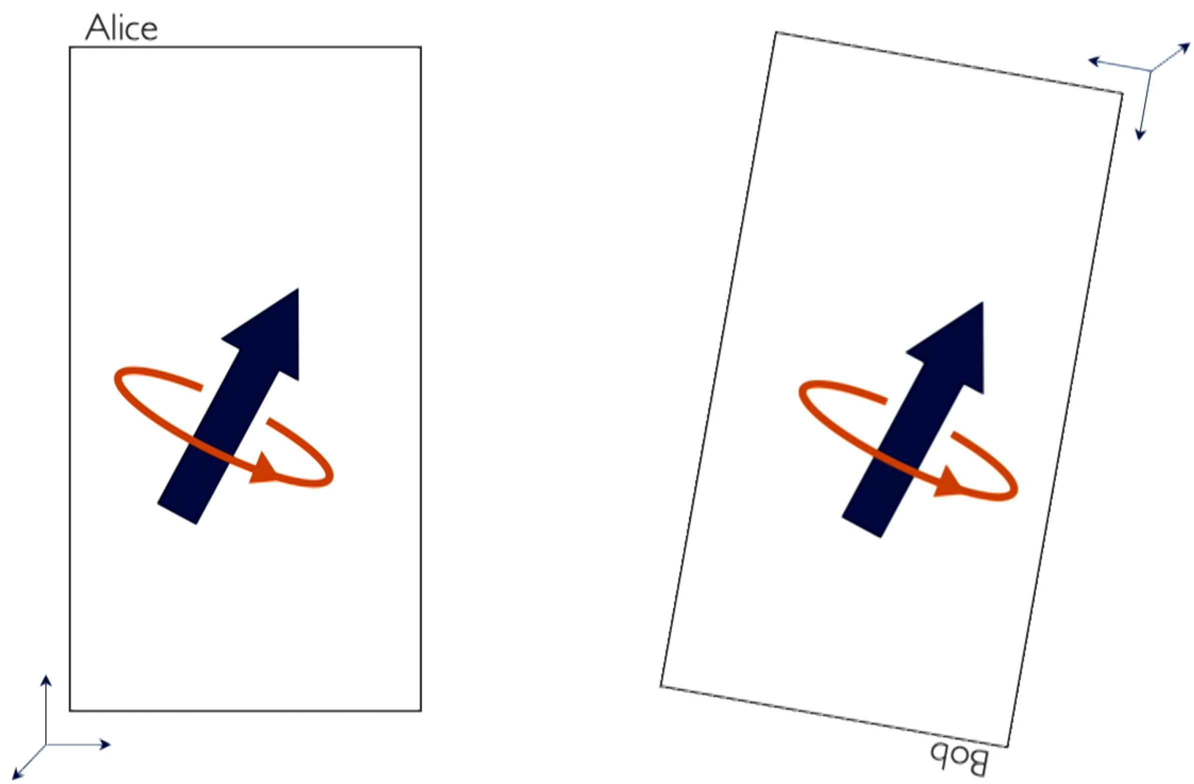

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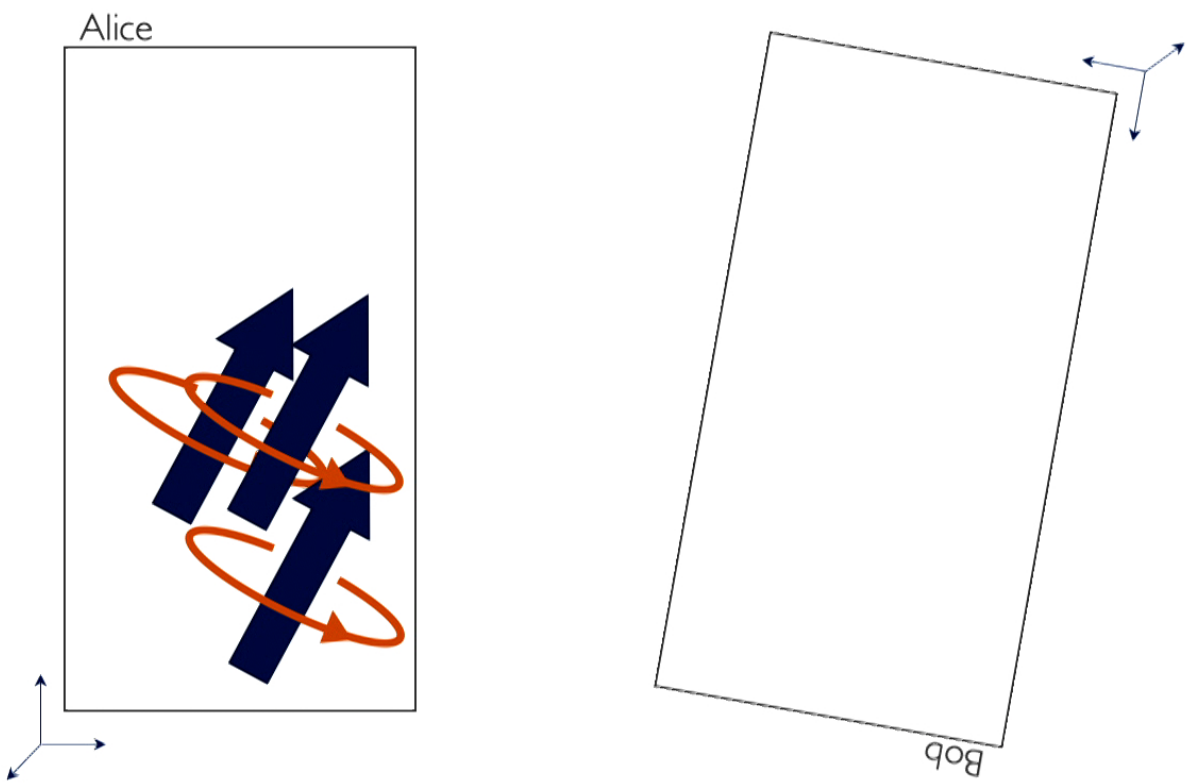

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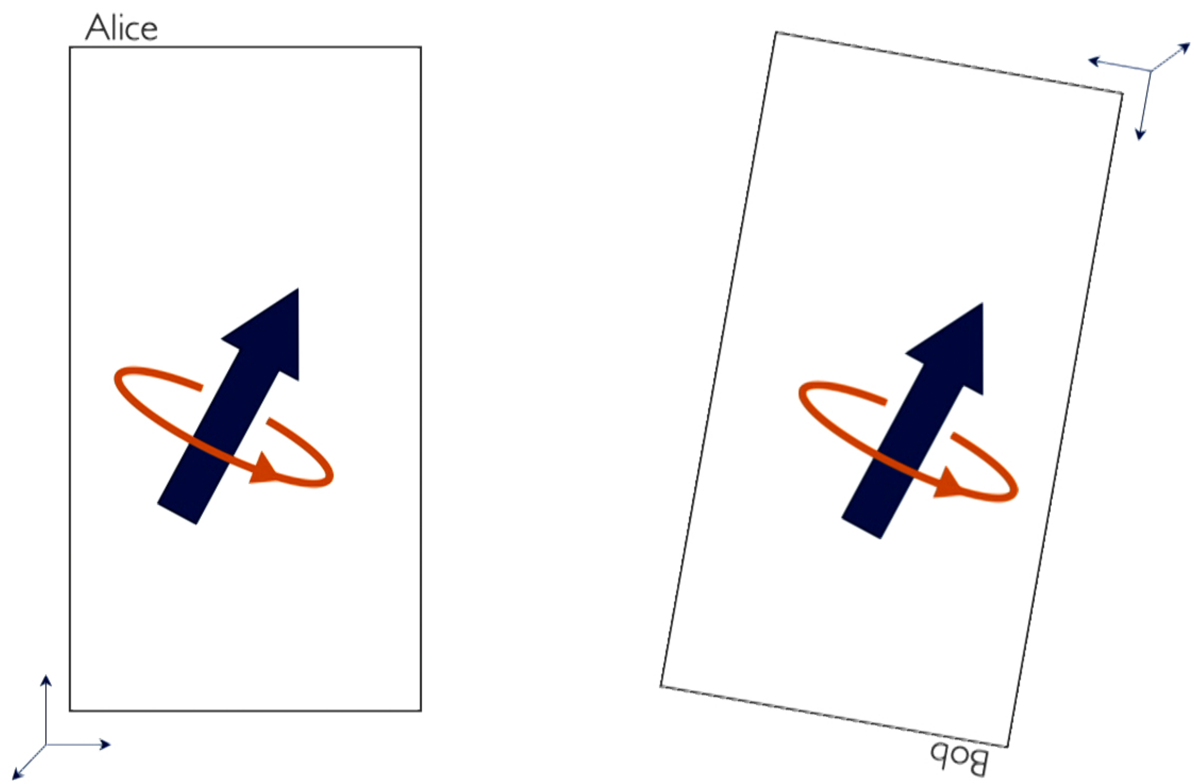

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QUANTUM REFERENCE FRAMES

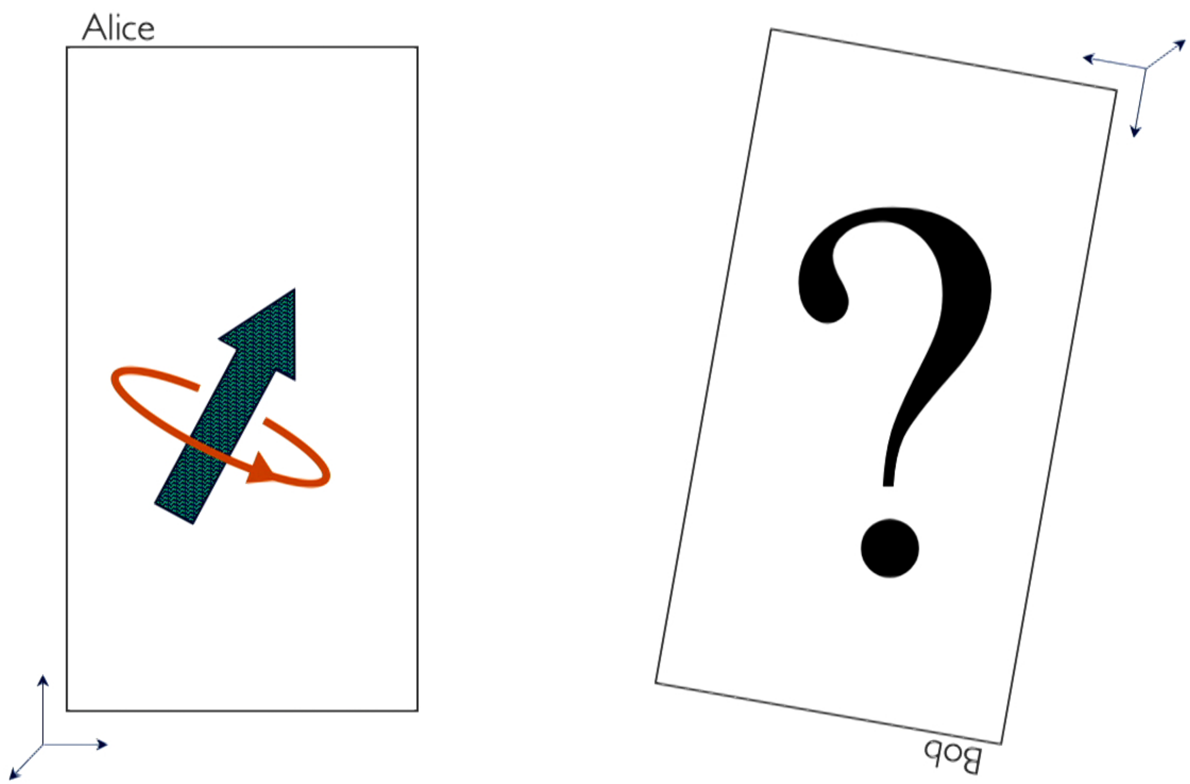
Suppose now that the information Alice wants to send is quantum (e.g., a spin)

Is a quantum error correction scheme for this type of information possible?

In which frame do we encode and decode?

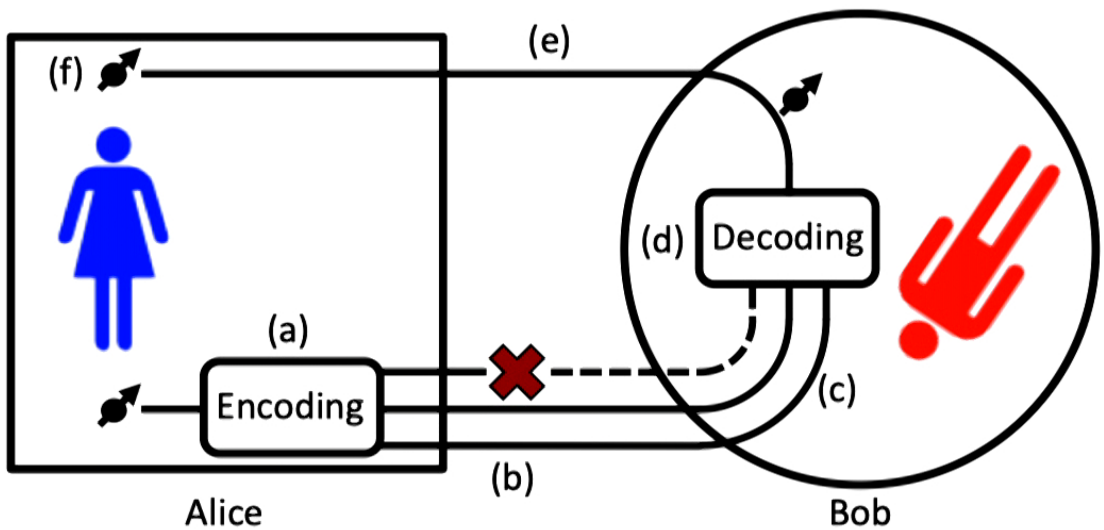
In full generality, Alice and Bob are related by an *unknown* element of a symmetry group G

e.g., the rotation group SO(3) for reference frames, and U(1) for a shared time standard



REFERENCE FRAME QEC

- In full generality, Alice and Bob are related by an *unknown* element of a symmetry group G
- The protocol we wish to accomplish is:
  a) Alice encodes an unknown quantum state in her lab
  b) The encoded state is sent to bob and subjected to noise
  c) Bob receives the state with an unknown unitary applied
  d) Bob applies a G-independent decoding in his lab
  e) The decoded state can be verified against Alice's input



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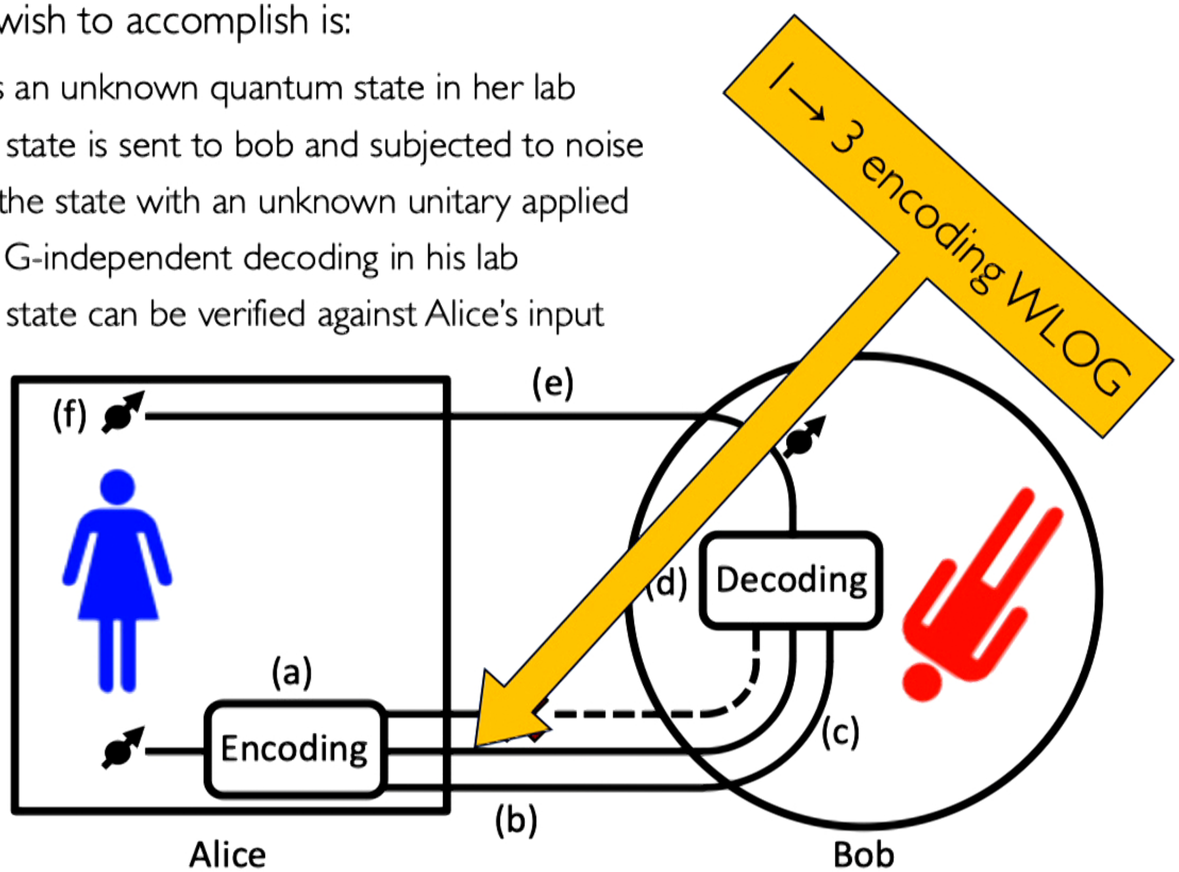


REFERENCE FRAME QEC

- In full generality, Alice and Bob are related by an *unknown* element of a symmetry group G
- The protocol we wish to accomplish is:

a) Alice encodes using $\mathcal{E}_A$ to get the encoded state $\mathcal{E}_A(\rho_{\rm in}) = \sigma_{123}$

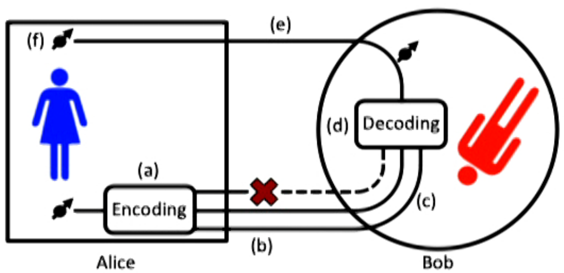
b) Spin j is erased, but Bob can infer which system was erased.

c) Bob receives the state $\mathrm{Tr}_j(U_1 \otimes U_2 \otimes U_3 \sigma_{123} U_1^\dagger \otimes U_2^\dagger \otimes U_3^\dagger)$ in his frame, where $U_i = U_i(R)$ is the unitary representation of the unknown rotation mapping Alice's frame to Bob's frame.

d) Bob decodes using $\mathcal{R}_j$ to get $\mathcal{R}_j[\mathrm{Tr}_j(U_1 \otimes U_2 \otimes U_3 \mathcal{E}(\rho_{\rm in}) U_1^\dagger \otimes U_2^\dagger \otimes U_3^\dagger)] = \tilde{\rho}_{\rm in}$

e) $\tilde{\rho}_{\rm in}$ is sent through a perfect channel to Alice for verification

f) Success is claimed if $\tilde{\rho}_{\rm in} = U_{\rm in} \rho_{\rm in} U_{\rm in}^\dagger$, so that the final state is the same as Alice's input state in her frame.



COVARIANT QEC

COVARIANT

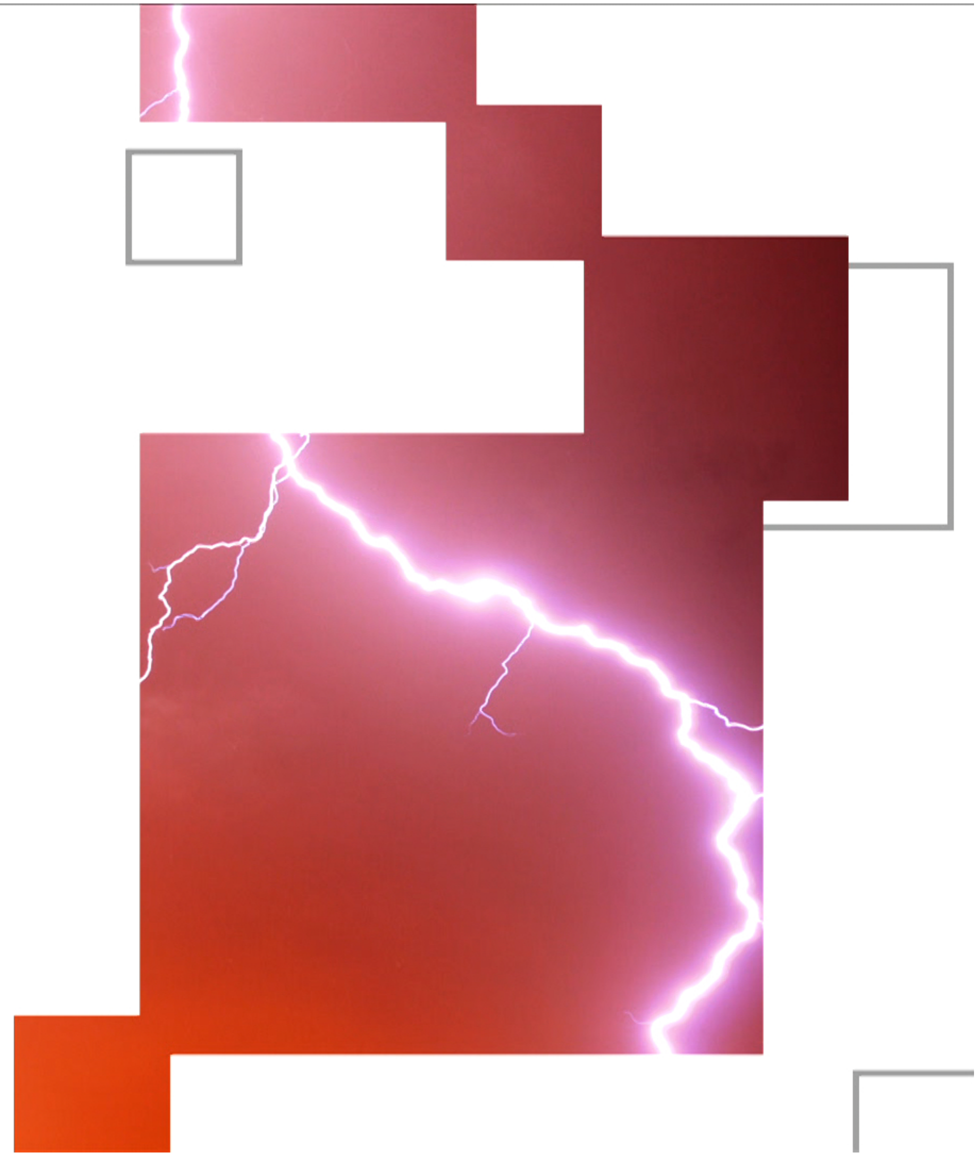
QUANTUM ERROR CORRECTION

We want to find error correcting encodings that are covariant with respect to a symmetry group G

It shouldn't matter whether we transform and then encode, or encode and then transform

$$\mathcal{E}(U_{\text{in}}\rho_{\text{in}}U_{\text{in}}^{\dagger}) = U_1 \otimes U_2 \otimes U_3\, \mathcal{E}(\rho_{\text{in}})\, U_1^{\dagger} \otimes U_2^{\dagger} \otimes U_3^{\dagger}$$

In principle, this places harsh constraints on $\mathcal{E}$



COVARIANT QEC

- An encoding $\mathcal{E}$ from a logical input to $\boldsymbol{n}$ physical systems is *covariant* if

$$\mathcal{E}(U_{\rm in}\rho_{\rm in}U_{\rm in}^{\dagger}) = U^{\otimes n}\mathcal{E}(\rho_{\rm in})U^{\dagger\otimes n}$$

- The encoding operation *commutes* with the action of the symmetry group G.

- This requirement places severe constraints on $\mathcal{E}$, and in principle such a map need not exist

- If a covariant encoding *does* exist, it means we can correct errors:

$$\mathcal{R}_j[\mathrm{Tr}_j(\mathcal{E}(\rho_{\rm in}))] = \rho_{\rm in}$$

- It turns out that the existence of a covariant code for a group G implies that one can error correct physical information that transforms under G.

REFERENCE FRAME QEC

- Success condition:

$$\mathcal{R}_j(\mathrm{Tr}_j(U^{\otimes n}\mathcal{E}_A(U_{\mathrm{in}}^\dagger \tilde{\rho}_{\mathrm{in}} U_{\mathrm{in}})U^{\dagger\otimes n})) = \tilde{\rho}_{\mathrm{in}}$$
$$\tilde{\rho}_{\mathrm{in}} = U_{\mathrm{in}}\rho_{\mathrm{in}}U_{\mathrm{in}}^\dagger$$

- If reference frame QEC is possible, we can construct a covariant code by *averaging* over the group G (e.g., average over all rotations)

COVARIANT QEC

- If a covariant code exists:

$$\mathcal{E}(U_{\mathrm{in}}\rho_{\mathrm{in}}U_{\mathrm{in}}^\dagger) = U^{\otimes n}\mathcal{E}(\rho_{\mathrm{in}})U^{\dagger\otimes n}$$
$$\mathcal{R}_j[\mathrm{Tr}_j(\mathcal{E}(\rho_{\mathrm{in}}))] = \rho_{\mathrm{in}}$$

- Then using this code for reference frame QEC gives the success condition

When are these possible?

REFERENCE FRAME QEC

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These are equivalent

RESULTS

EXISTENCE OF COVARIANT QUANTUM CODES

Symmetry group: / Code dimension:	Lie group (continuous symmetry)	Finite group
Finite dimensional code	NO-GO THEOREM	Explicit construction
Infinite dimensional code	Explicit construction	Explicit construction

SKETCH OF NO-GO THEOREM

- Continuous symmetry (e.g., Lie Group), with at least one infinitesimal generator and conserved charge
- Logical generator T_L and physical generator T_A, assumed to be nontrivial
- On the physical space, the generator is a *sum of local terms* $T_A = \sum_i T_i$

- Physical subsystem j is erased and given to the environment
- The environment can measure T_j and *learn* some information about the charge
- But for an erasure correcting code, each reduced state ρ_j should be *independent* of the input

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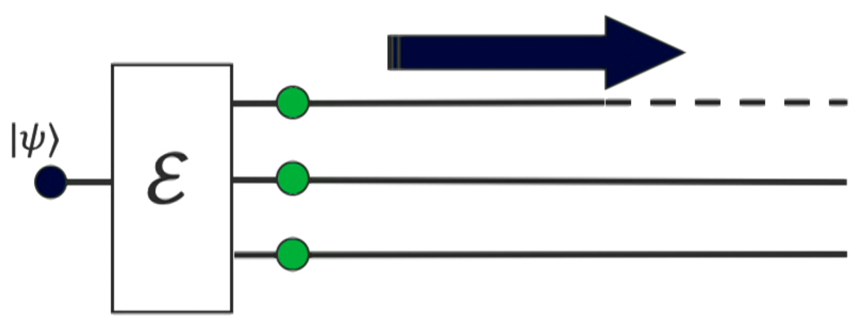

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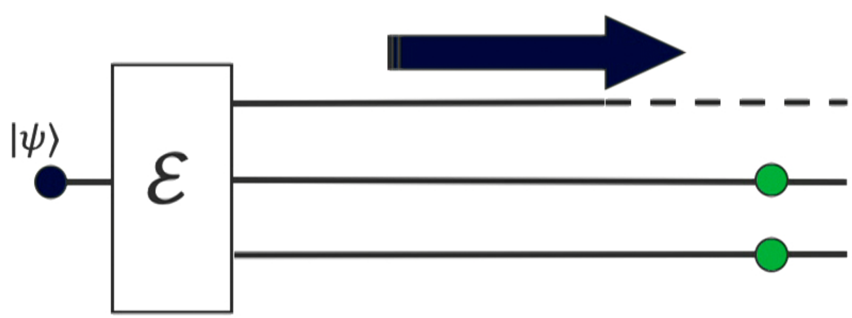

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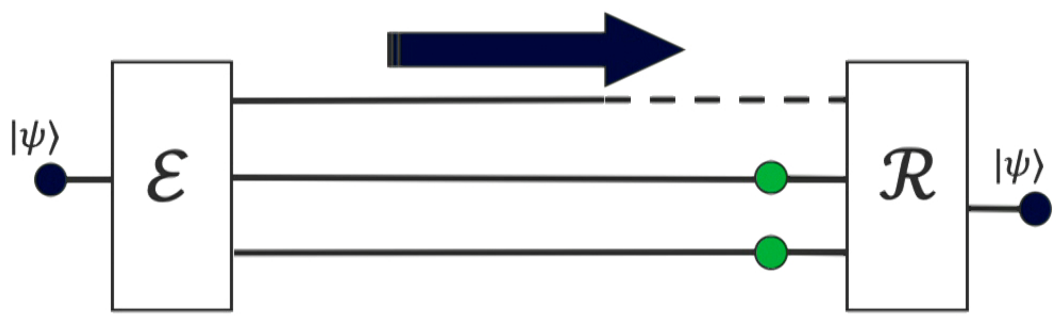

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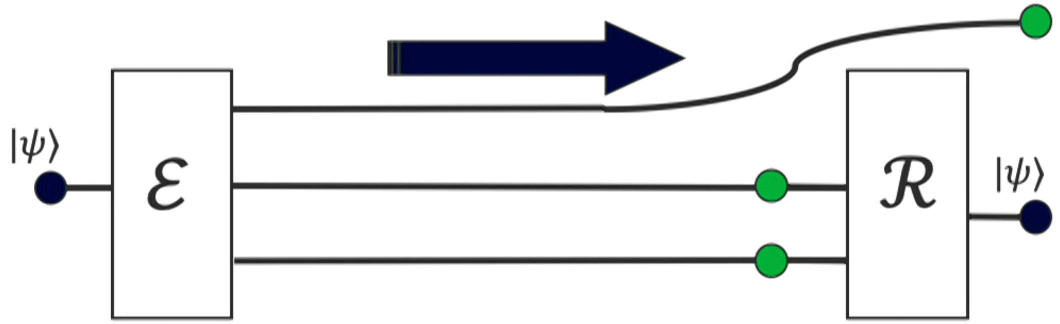

COVARIANT QEC

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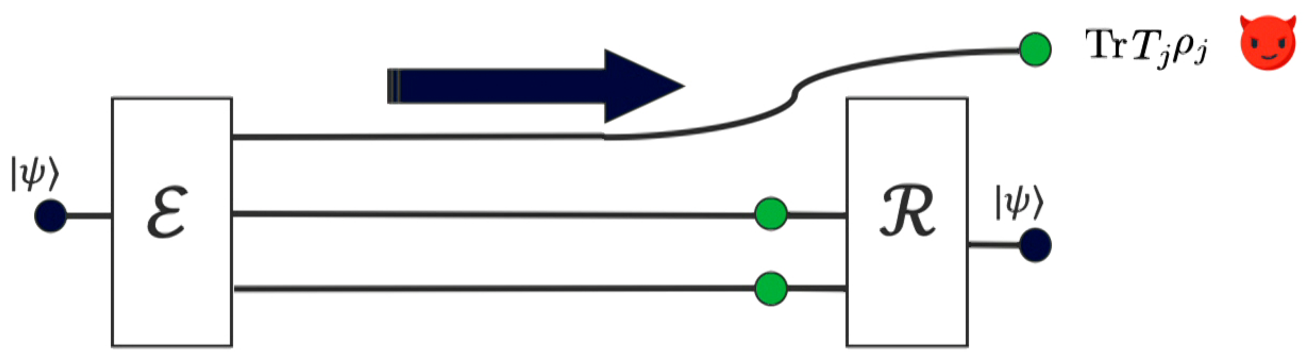

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$$\text{const}_i = \text{Tr}(T_i \rho_i) = \text{Tr}(T_i \mathcal{E}(\rho_{\text{in}})) = \text{Tr}(\mathcal{E}^\dagger(T_i)\rho_{\text{in}}) \; \forall \rho_{\text{in}}$$

$$\implies \mathcal{E}^\dagger(T_i) \propto I \implies \mathcal{E}^\dagger(T_A) \propto I$$

To avoid a contradiction, the generators must be trivial

$$T_L \propto I$$

INFINITE DIMENSIONAL CODES

- To see that it a covariant code possible, we return to the reference frame error correction picture:

- Alice encodes using her favourite (*non-covariant*) quantum code
- Alice appends a classical gyroscope encoding her reference frame
  - Since noise can happen to any subsystem, including the gyroscope, Alice actually just sends **two** gyroscopes

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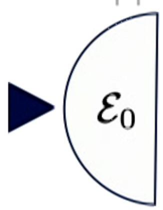

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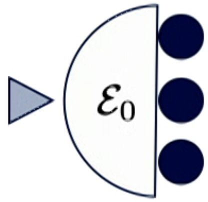

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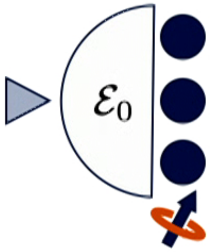

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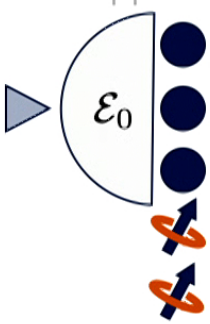

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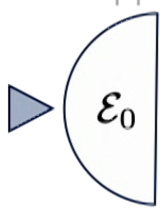

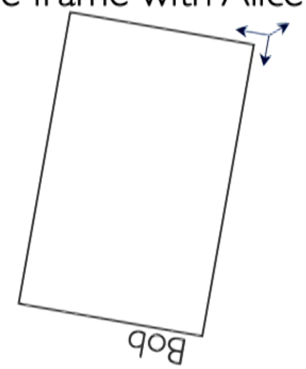

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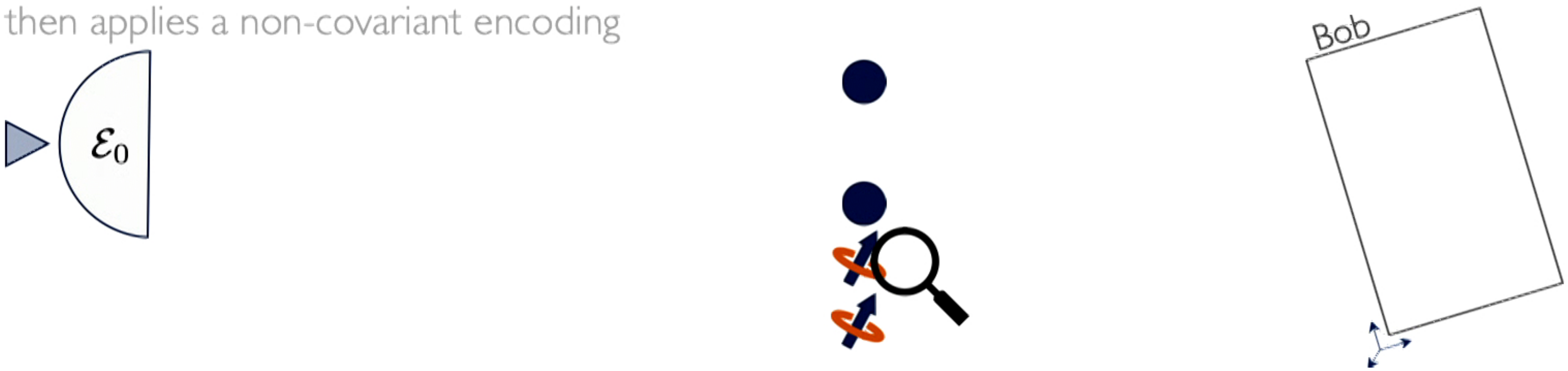

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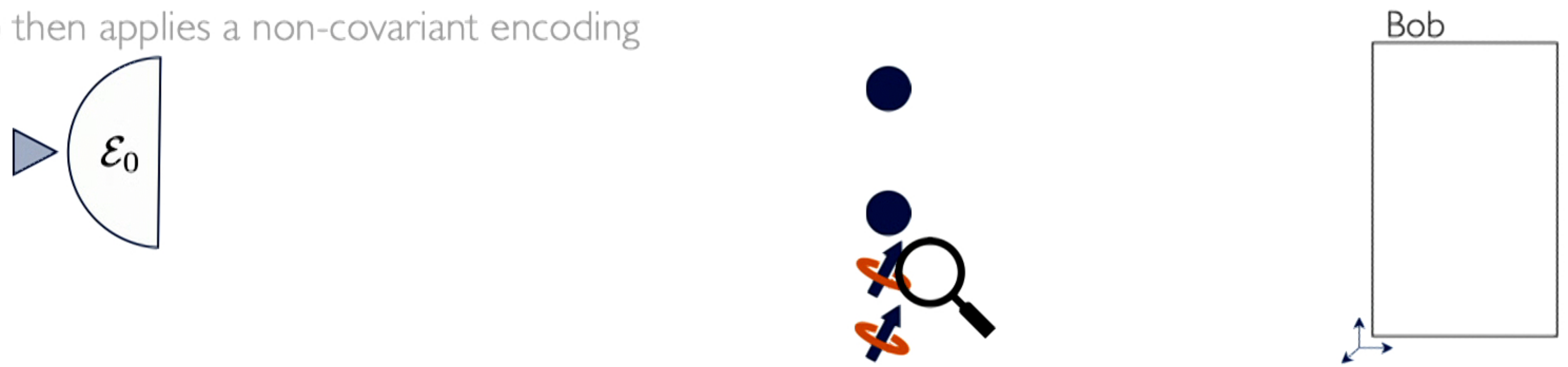

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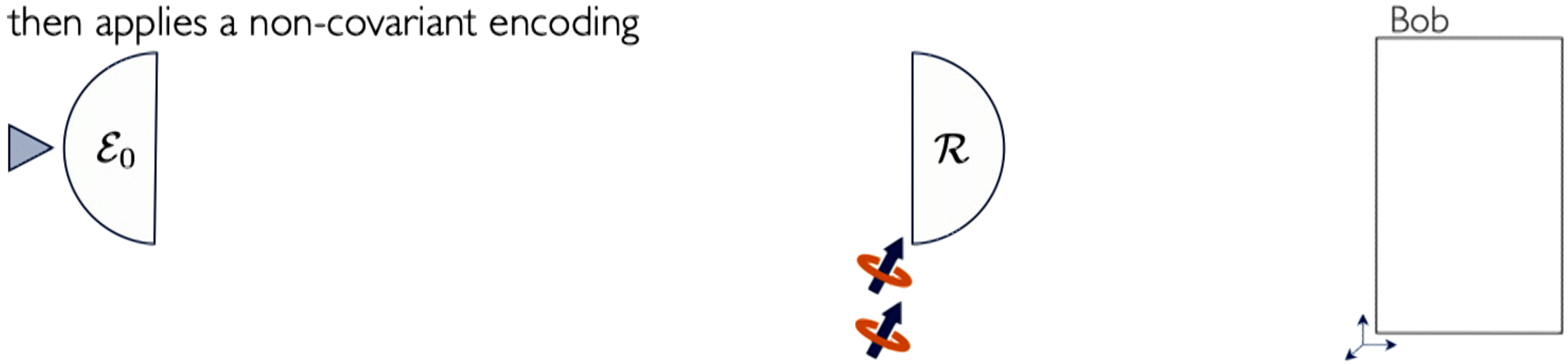

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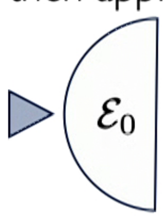
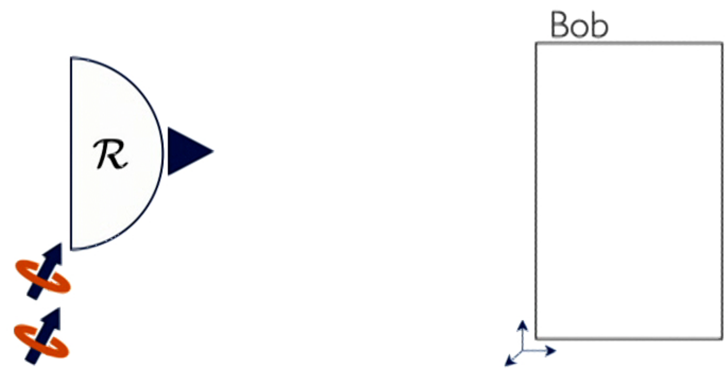




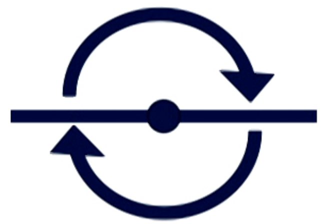
INFINITE DIMENSIONAL CODES

- Infinite dimensional systems are needed to perfectly describe Alice's reference frame in the gyroscope
- To find the covariant quantum code:
  - Let $\mathcal{H}_G = \mathrm{span}\{|g\rangle\}$ for all $g \in G$ such that $U(g)\,|h\rangle = |gh\rangle$
  - Non-covariant encoding $\mathcal{E}_0$ and reference frame gyroscope Ancilla $|e\rangle\langle e| \otimes |e\rangle\langle e|$
  - Encoded state $\mathcal{E}_0(\rho_{\mathrm{in}}) \otimes |e\rangle\langle e|^{\otimes 2}$
  - In Bob's frame: $U(g)^{\otimes 3}\mathcal{E}_0[U^\dagger(g)\rho_{\mathrm{in}}U(g)]U(g)^{\dagger\otimes 3} \otimes |g\rangle\langle g|^{\otimes 2}$
- The covariant code is made by averaging over the group G:

$$\mathcal{E}(\rho_{\mathrm{in}}) = \int_{g\in G} dg\, U(g)^{\otimes 3}\mathcal{E}_0[U^\dagger(g)\rho_{\mathrm{in}}U(g)]U(g)^{\dagger\otimes 3} \otimes |g\rangle\langle g|^{\otimes 2}$$

EXPLICIT EXAMPLE: ROTOR CODE

- Rotor: compact 'position' observable, and discrete momentum observables



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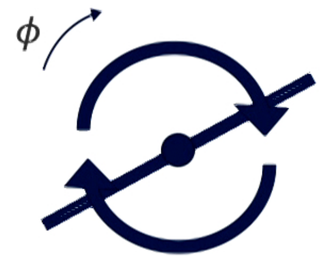

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COVARIANT QEC

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- Isometric encoding $\mathcal{E}_{U(1)}(\rho) = E_{U(1)} \rho E_{U(1)}^{\dagger}$ given by $E_{U(1)} = \sum_{x,y \in \mathbb{Z}} |-3y, -x+y, 2(x+y)\rangle_{123} \langle x|_{\text{in}}$

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- Covariant w.r.t $U(1)$ which acts as $e^{i\theta\hat{k}}$

COVARIANT QEC

Covariance:

OTOR CODE

n observables

$$E_{U(1)}\, U_{\text{in}}^{\dagger} = \sum_{x,y\in\mathbb{Z}} |{-3y}, -x+y, 2(x+y)\rangle_{123}\, \langle x|_{\text{in}}\; e^{-i\theta \hat{k}_{\text{in}}}$$

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$$= \sum_{x,y\in\mathbb{Z}} e^{i\theta(-3y\, -x+y\, +2(x+y))}\; |{-3y}, -x+y, 2(x+y)\rangle_{123}\, \langle x|_{\text{in}}$$

$$= \sum_{x,y\in\mathbb{Z}} e^{i\theta(\hat{k}_1+\hat{k}_2+\hat{k}_3)}\; |{-3y}, -x+y, 2(x+y)\rangle_{123}\, \langle x|_{\text{in}}$$

$$= U_1 \otimes U_2 \otimes U_3\, E_{U(1)}$$

$$_{(1)} = \sum_{x,y\in\mathbb{Z}} |{-3y}, -x+y, 2(x+y)\rangle_{123}\, \langle x|_{\text{in}}$$

EASTIN-KNILL THEOREM

For *any* finite quantum error correcting code:

The **logical** group generated by **transversal** physical gates is **finite**.

COVARIANT QEC

- Can you implement a **universal** set of **encoded** gates **transversally**?
- No. Eastin-Knill: "the ability of a quantum code to detect an arbitrary error on any single physical subsystem is incompatible with the existence of a universal, transversal encoded gate set for the code."
- In our language, set $G = U(d_L)$ and ask if $\mathcal{E}(U_L \rho U_L^\dagger) \stackrel{?}{=} U_A^{\otimes n}\, \mathcal{E}(\rho) U_A^{\dagger \otimes n}$. Are there $U(d_L)$ covariant codes?

  - Our no-go theorem reproduces the main thrust of Eastin-Knill – no, there are no such codes
- We can circumvent the theorem using infinite dimensional systems!

Eastin, Bryan, and Emanuel Knill. "Restrictions on transversal encoded quantum gate sets." *Physical review letters* 102.11 (2009)

APPROXIMATE

QUANTUM ERROR CORRECTION

What if we only need to recover *approximately*?

What if we only need approximate covariance?

Can we find an approximate Eastin-Knill Theorem?

$\approx$

MEASURES

- We need to quantify how poorly an approximate code can recover

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- Recall the covariant 3-rotor code: $$E_{U(1)} = \sum_{x,y\in\mathbb{Z}} |-3y, -x+y, 2(x+y)\rangle_{123} \langle x|_{\text{in}}$$

- Truncate the sum with a hard cutoff:

- Weight terms with a Gaussian envelope?

- In both cases, the encoding remains covariant, but error correction suffers:

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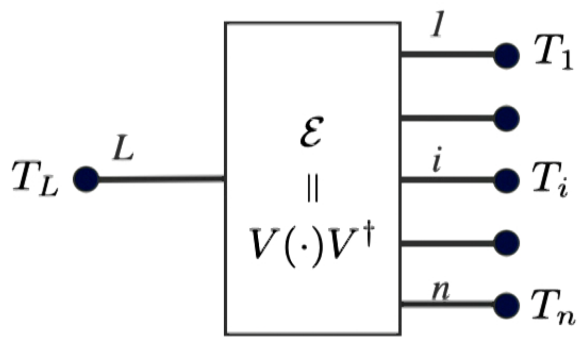
APPROXIMATE QEC

MAIN THEOREM (one of several)



$T_A = \sum_i T_i$

noise channel $\mathcal{N}$

The error of the isometric encoding $\mathcal{E}(\cdot) = V(\cdot)V^\dagger$ is bounded below by

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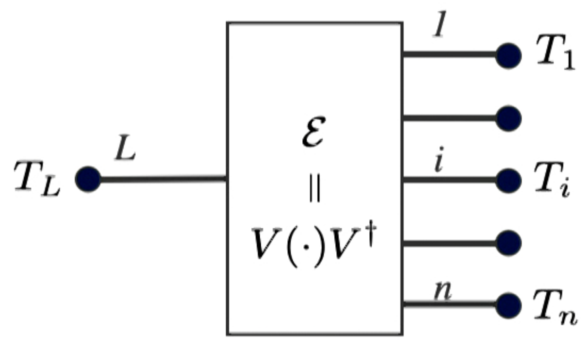

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Good codes: $\epsilon \approx 0$, $f \approx 1$

$\approx$

$$\epsilon_{\text{e}}(\mathcal{N} \circ \mathcal{E}) \geq \frac{1}{n} \frac{\|T_L - \text{Tr}(T_L)\mathbb{1}_L/d_L\|_1/(2d_L)}{\max_i \Delta T_i}$$

$$\Delta T_i = \max(\text{eig}(T_i)) - \min(\text{eig}(T_i))$$

APPROXIMATE QEC

≈

Complementary Channels:

Given a channel:

$$\mathcal{F}_{A\to B}(\cdot) = \mathrm{Tr}_C(W_{A\to BC}(\cdot)W^\dagger_{A\to BC})$$

where $W_{A\to BC}$ is a Stinespring dilation of $\mathcal{F}_{A\to B}$

Complementary channel:

$$\widehat{\mathcal{F}}_{A\to C}(\cdot) = \mathrm{Tr}_B(W_{A\to BC}(\cdot)W^\dagger_{A\to BC})$$

(It is the channel from input to the environment)

nentary channel to the **constant channel**

$\widehat{\ \circ \mathcal{E}}, \mathcal{T}_\zeta)$

$\mathcal{T}_\zeta(\cdot) = \mathrm{Tr}(\cdot)\zeta$

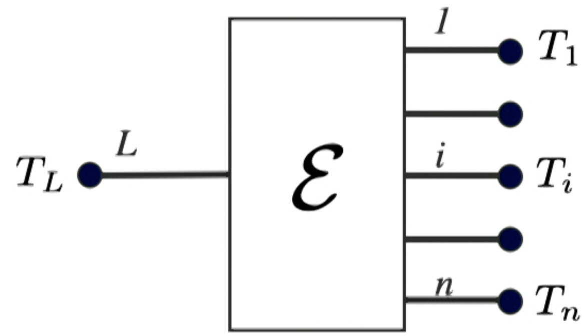
$(\widehat{\mathcal{N} \circ \mathcal{E}}, \mathcal{T}_\zeta)$

$(|x'\rangle\langle x'|_L)\Big\|_1 \geq \frac{1}{2n}\frac{\Delta T_L}{\max_i \Delta T_i}$

imal charge eigenvalues.

Bény, Cédric, and Ognyan Oreshkov. "General conditions for approximate quantum error correction and near-optimal recovery channels." *Physical review letters* 104.12 (2010):120501

SECOND MAIN THEOREM



Noise model: erasure of subsystems labelled by α with probability q_α

$$\|(T_L - \nu \mathbb{1}_L) - \mathcal{E}^\dagger(T_A)\|_\infty \leq \delta$$

The code is approximately covariant

$$\left|\mathrm{Tr}\left(\sum (T_\alpha - t_\alpha)\Pi_\alpha^\perp \mathcal{E}(\sigma_L)\right)\right| \leq \eta$$

Most of the charge falls within the range $[t_\alpha^-, t_\alpha^+]$

$$\epsilon_{\mathrm{worst}}(\mathcal{N} \circ \mathcal{E}) \geq \frac{\Delta T_L/2 - \delta - \eta}{\max_\alpha(\Delta T_\alpha / q_\alpha)}$$

$$\left.\begin{array}{c} \epsilon_e(\mathcal{N} \circ \mathcal{E}) \\ \langle \epsilon_e(\mathcal{N} \circ \mathcal{E}) \rangle_\alpha \end{array}\right\} \geq \frac{\|\Delta T_L - \mu(T_L)\mathbb{1}_L\|_1 / d_L - \delta - \eta}{\max_\alpha(\Delta T_\alpha / q_\alpha)}$$

$\Pi_\alpha^\perp$ projects onto eigenspaces of T_α with eigenvalues outside $[t_\alpha^-, t_\alpha^+]$

$\mu(T_L)$ is the median eigenvalue of T_L

$$\Delta T_\alpha = t_\alpha^+ - t_\alpha^-$$

$$t_\alpha = (t_\alpha^- + t_\alpha^+)/2$$

APPROXIMATE QEC

≈

SYMMETRIES IN ADS/CFT

BULK GLOBAL SYMMETRIES AND TIME

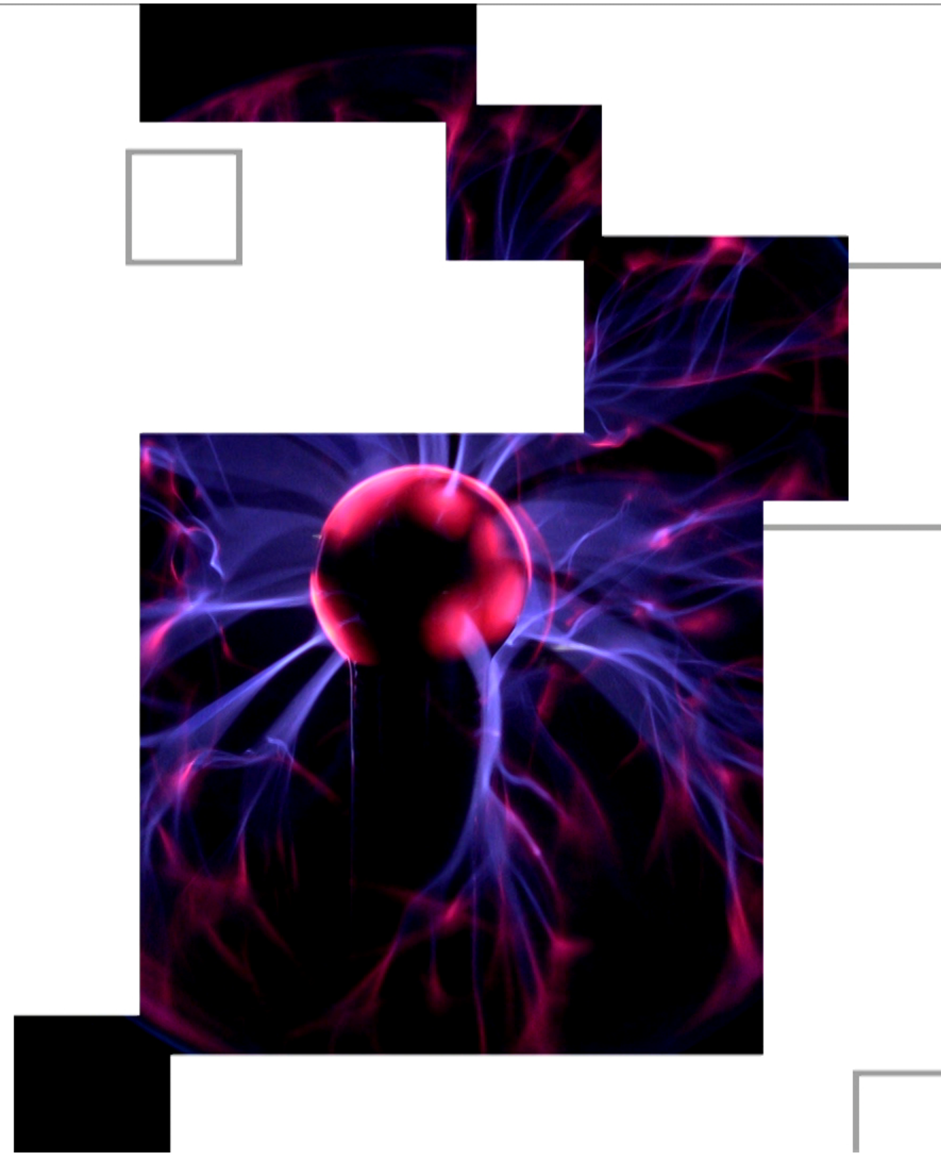
Standard lore in AdS/CFT:

"No bulk global symmetries"

Bulk-to-boundary map defines a QECC

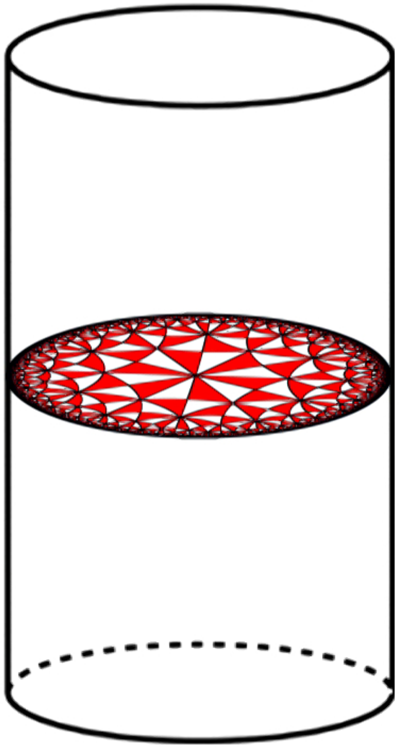
Bulk time evolution = boundary time evolution

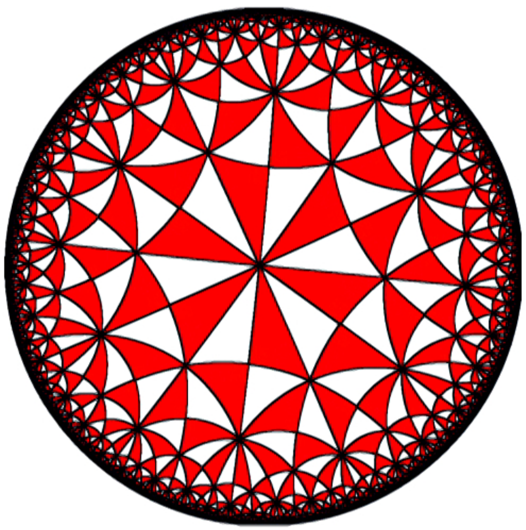
Apparent violation of Eastin-Knill?



ADS/CFT IN ONE SLIDE

Gravity in AdS in d+1 dimensions — dual to — Conformal field theory in d dimensions





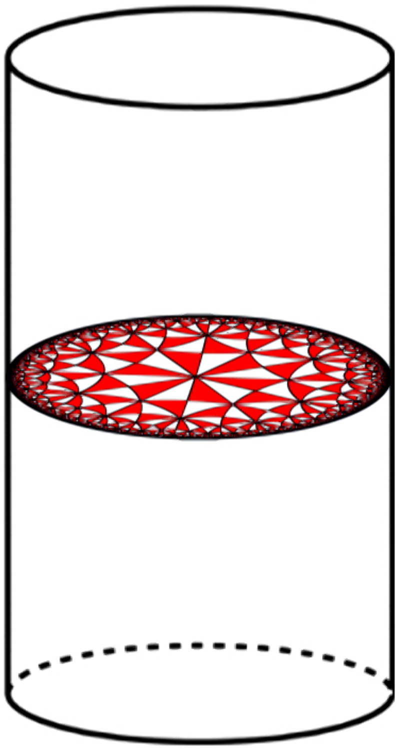
APPROXIMATE QEC

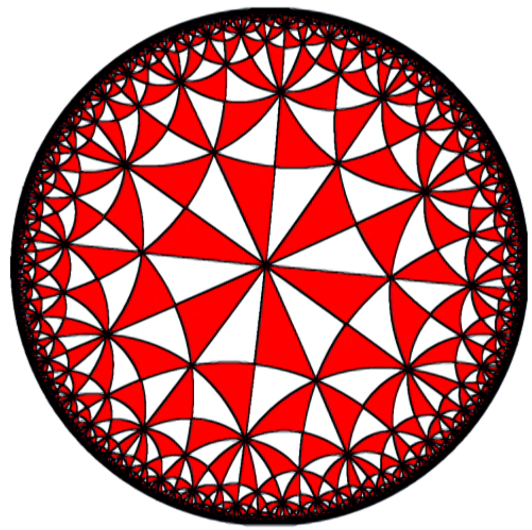
ADS/CFT IN ONE SLIDE

Gravity in AdS in d+1 dimensions

dual to

Conformal field theory in d dimensions





Consider a boundary subregion A, constructed by tracing over $\bar{A}$

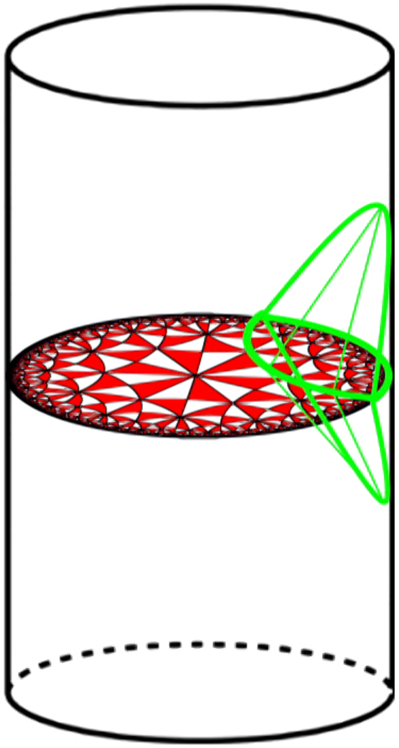
APPROXIMATE QEC

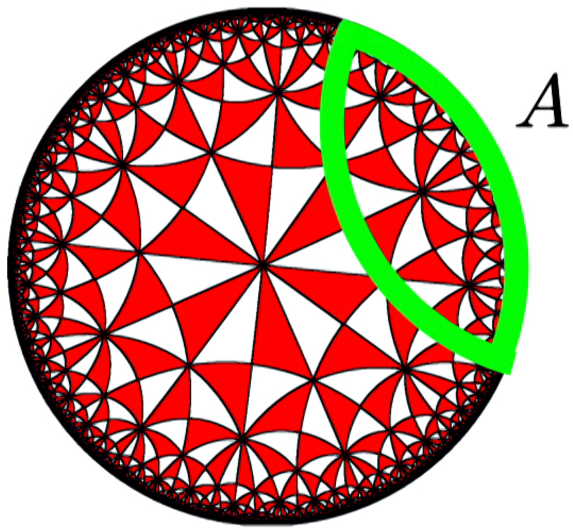
ADS/CFT IN ONE SLIDE

Gravity in AdS in d+1 dimensions

dual to

Conformal field theory in d dimensions





Consider a boundary subregion A, constructed by tracing over $\bar{A}$

We can try to find all bulk operators expressible with support only on A

There is a procedure for mapping bulk operators to boundary operators

This mapping from bulk to boundary defines a QECC!

≈

NO GLOBAL BULK SYMMETRIES

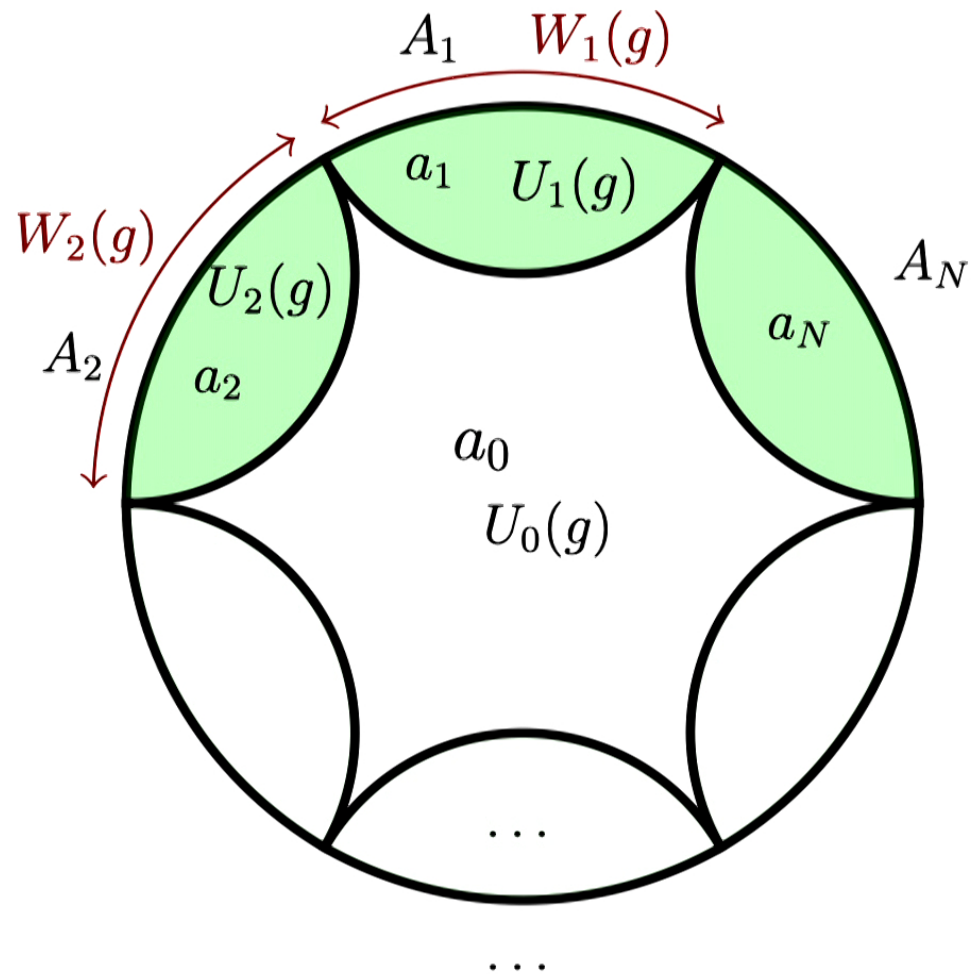
IN ADS/CFT

- Boundary CFT decomposed into N regions $\{A_i\}_{i=1}^{N}$
- Bulk has corresponding decomposition into entanglement wedges $\{a_i\}_{i=0}^{N}$
- Bulk global symmetry: product of local unitaries

$$U_L(g) = \bigotimes_{i=0}^{N} U_i(g)$$

- Mapped to boundary. Splittable CFT $\Rightarrow$ local unitaries

$$U_{\mathrm{CFT}}(g) = \bigotimes_{i=1}^{N} W_i(g)$$



Harlow, Daniel, and Hirosi Ooguri. "Symmetries in quantum field theory and quantum gravity." *arXiv preprint arXiv:1810.05338* (2018).

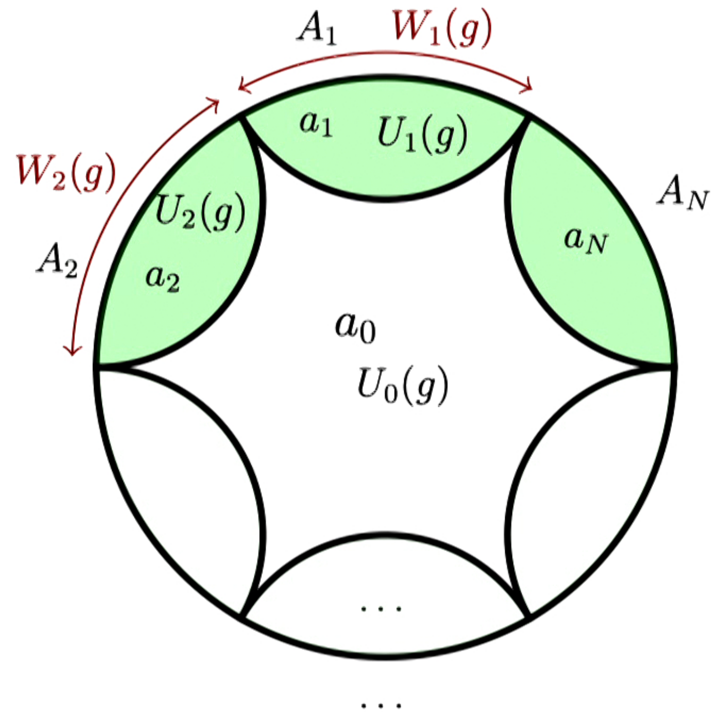
Harlow, Daniel, and Hirosi Ooguri. "Constraints on symmetry from holography." *arXiv preprint arXiv:1810.05337* (2018).

MORE DETAIL

- The boundary CFT is **splittable**: $U_{\mathrm{CFT}}(g) = \bigotimes_{i=1}^{N} W_i(g)$
  - Boundary operators can be decomposed
- **Erasure** of A_i is a **correctable error** for a_0.
- The code space for AdS/CFT is **low energy** excitations above the vacuum:

$$\mathcal{H}_{\mathrm{code}} = \mathrm{span}\{|\Omega\rangle, \hat{\phi}(x)|\Omega\rangle, \hat{\phi}(x)\hat{\phi}(y)|\Omega\rangle, \ldots\}$$

- The W_i are argued to be **low energy**. Therefore they preserve the code space and are **logical operators**
- **Logical operators** that are also **correctable errors** must be **trivial**



SYMMETRIES IN ADS/CFT

Harlow, Daniel, and Hirosi Ooguri. "Symmetries in quantum field theory and quantum gravity." *arXiv preprint arXiv:1810.05338* (2018).

Harlow, Daniel, and Hirosi Ooguri. "Constraints on symmetry from holography." *arXiv preprint arXiv:1810.05337* (2018).

What about gravitational dressings?

Each "local" bulk operator needs a dressing to be invariant under diffs.

The string (dressing) must connect to at least one A_k.

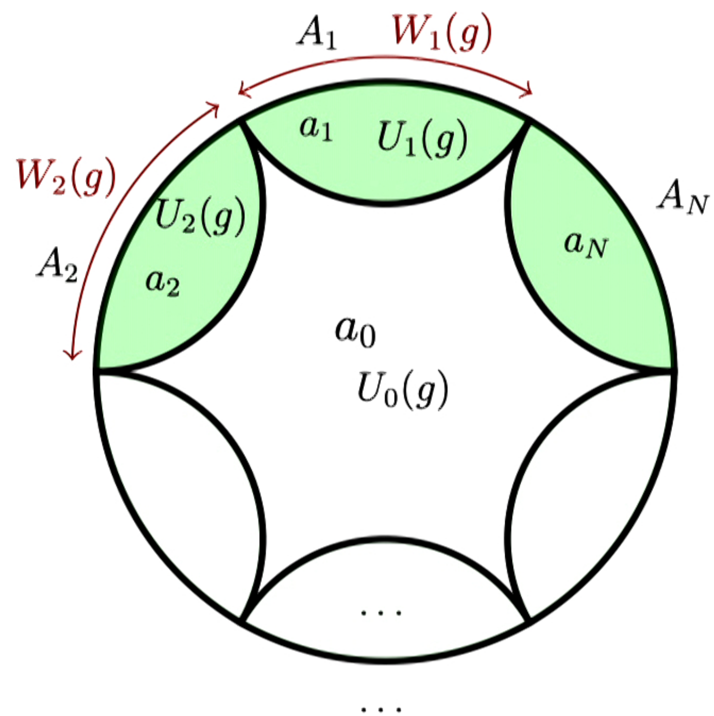
However, the dressing is only gravitational, and is blind to the charge in question

The dressing transforms trivially under the symmetry

$W_i(g)$

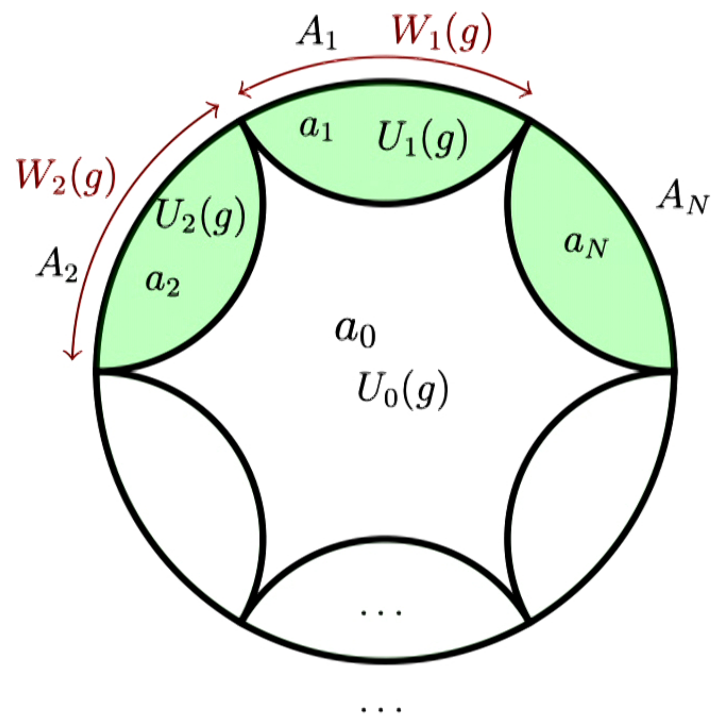
ns

rs



TIME EVOLUTION COVARIANCE

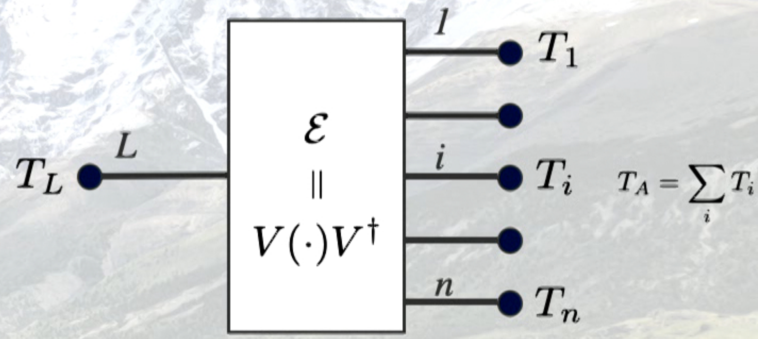
- The bulk and boundary theories are time translation invariant
- Bulk time evolution = boundary time evolution
- The mapping is covariant w.r.t. this symmetry
- Why does this not violate the previous slide?

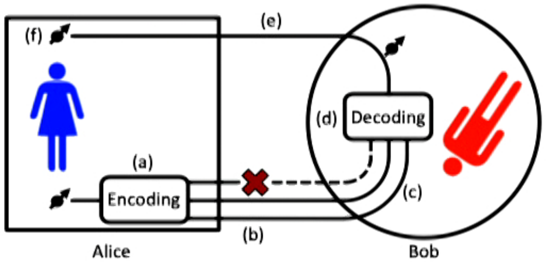


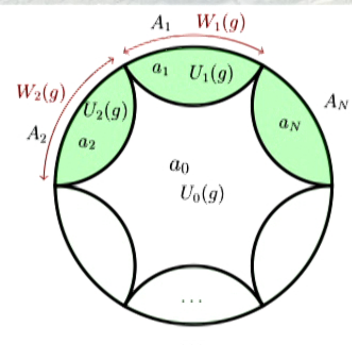
- The **gravitational dressing**, previously ignored, is crucial
- This dressing transforms non-trivially under the group action
  - This transformation implements bulk time evolution
- The dressing can be detected locally. Hence, the error correction must be approximate

COVARIANT QEC

Symmetry group: / Code dimension:	Lie group (continuous symmetry)	Finite group
Finite dimensional code	NO-GO THEOREM	Explicit construction
Infinite dimensional code	Explicit construction	Explicit construction







$$\epsilon(\mathcal{N}\circ\mathcal{E}) = \sqrt{1 - f^2(\mathcal{N}\circ\mathcal{E})}$$

$$\epsilon_{\text{worst}}(\mathcal{N}\circ\mathcal{E}) \geq \frac{1}{2n}\frac{\Delta T_L}{\max_i \Delta T_i}$$

$$E_{U(1)} = \sum_{x,y\in\mathbb{Z}} |-3y, -x+y, 2(x+y)\rangle_{123} \langle x|_{\text{in}}$$

THANK YOU!

QUESTIONS?