

Title: How to simulate problems from high energy physics on quantum computers

Speakers: Christine Muschik

Series: Colloquium

Date: April 26, 2019 - 2:30 PM

URL: <http://pirsa.org/19040080>

Abstract: Gauge theories are fundamental to our understanding of interactions between the elementary constituents of matter as mediated by gauge bosons. However, computing the real-time dynamics in gauge theories is a notorious challenge for classical computational methods. In the spirit of Feynman's vision of a quantum simulator, this has recently stimulated theoretical effort to devise schemes for simulating such theories on engineered quantum-mechanical devices, with the difficulty that gauge invariance and the associated local conservation laws (Gauss laws) need to be implemented. Here we report the first digital quantum simulation of a lattice gauge theory, by realising 1+1-dimensional quantum electrodynamics (Schwinger model) on a few-qubit trapped-ion quantum computer. We are interested in the real-time evolution of the Schwinger mechanism, describing the instability of the bare vacuum due to quantum fluctuations, which manifests itself in the spontaneous creation of electron-positron pairs. Our work represents a first step towards quantum simulating high-energy theories with atomic physics experiments, the long-term vision being the extension to real-time quantum simulations of non-Abelian lattice gauge theories.

Quantum simulations of models from high energy physics

Christine Muschik



UNIVERSITY OF
WATERLOO

IQC

Institute for
Quantum
Computing

Quantum Optics Theory



How can we use quantum systems to achieve a
quantum advantage?

How can this be done **in practice?**

Quantum Optics Theory



Quantum Networks

Quantum Simulations

Entanglement distribution

New design concepts for 2D quantum networks

Robust quantum repeater architectures

- ➡ Quest: faithfully transfer quantum states
- ➡ Vision: 'quantum internet'

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Self-stabilizing quantum systems

Autonomous quantum error correction

Nat. Commun. 8, 1822 (2017).

Entanglement distribution

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- ➡ Quest: faithfully transfer quantum states
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Self-stabilizing quantum systems

Autonomous quantum error correction

Nat. Commun. 8, 1822 (2017).

- ➡ Quest: keep a qubit alive
- ➡ Vision: self-correcting quantum technology

Quantum Optics Theory



Quantum Networks

Quantum Simulations

QUANTUM SIMULATIONS FOR HIGH ENERGY PHYSICS

We want to understand:

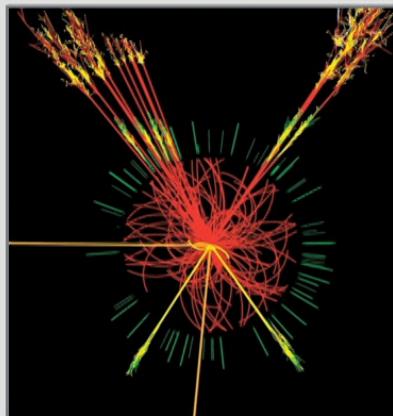
- Why is there more matter than antimatter in the universe?
- What happens inside neutron stars?
- What happened in the early universe?
- What happens in heavy ion collisions in particle accelerators?

Explore sign-problem free methods for lattice gauge theories

Dynamical problems:

What happens in heavy ion collisions

?



Topological terms:

How can we understand the large degree of CP violation in nature?

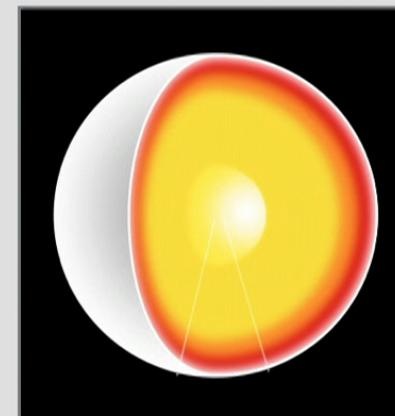
?



High baryon density:

What happens inside neutron stars

?



Gauge Theories:

Quest to find sign-problem free methods

- Quantum Simulations
- Numerical methods based on tensor network states



Karl Jansen (DESY)
Visitor at PI until end of May

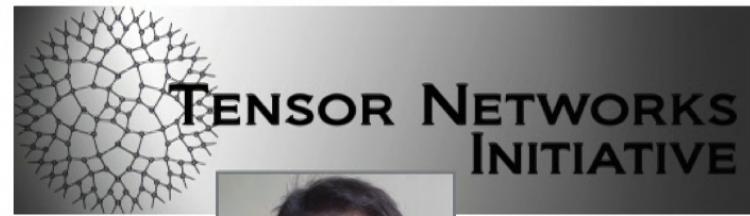
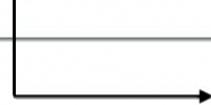
Work by:

- Ignacio Cirac
- Frank Verstraete
- Mari Carmen Bañuls
- Simone Montangero
- ...

Gauge Theories:

Quest to find sign-problem free methods

- Quantum Simulations
- Numerical methods based on tensor network states



Guifre Vidal



Gauge Theories:

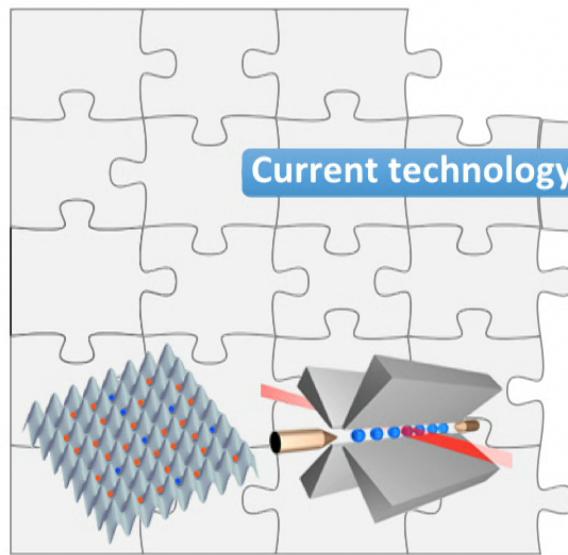
Quest to find sign-problem free methods

- Quantum Simulations
- Numerical methods based on tensor network states

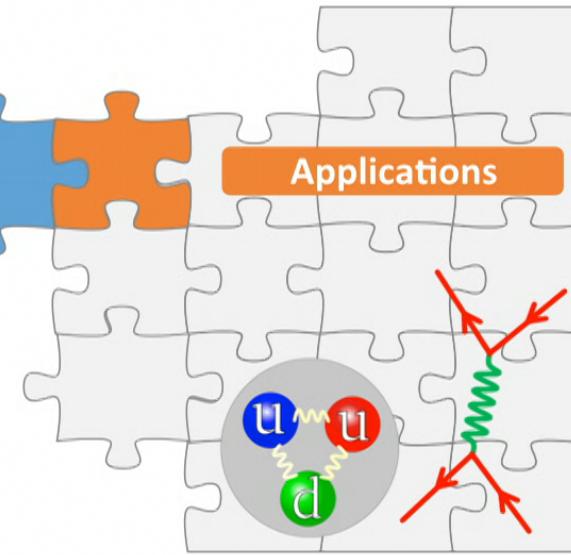
→ Two routes towards the same goal.
Both paths are actively explored.

→ This talk: Quantum simulations

Quantum information science



High energy physics



Review Articles: Ann. Phys. 525, 777 (2013); Rep. Prog. Phys. 79, 014401 (2016); Contemporary Physics 57 388 (2016).

Short-term goal:

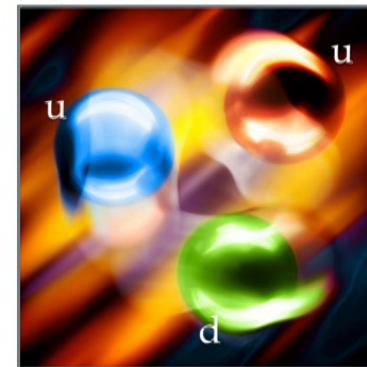
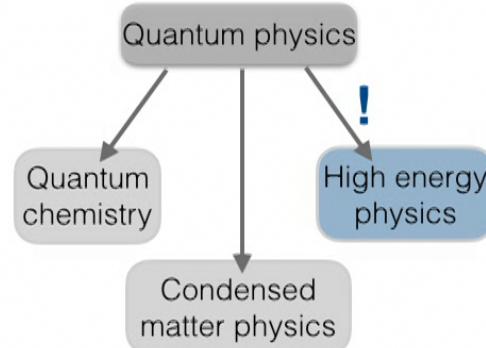
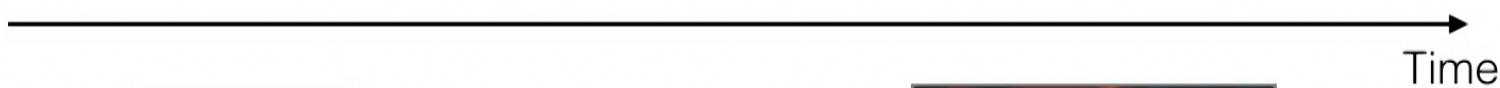
Develop a new type of
Quantum Simulator

Perform proof-of-concept
Experiments

Long-term vision:

Simulate
Quantum Chromo Dynamics

Answer questions that
can not be tackled
numerically



Develop a new type of quantum simulator

Simulated states and dynamics must be gauge-invariant

Review Articles: Ann. Phys. 525, 777 (2013); Rep. Prog. Phys. 79, 014401 (2016); Contemporary Physics 57 388 (2016).

Develop a new type of quantum simulator

Simulated states and dynamics must be gauge-invariant

Difficulty for realizing quantum simulations of lattice gauge theories:

Implement a quantum many-body Hamiltonian
and a large set of local constraints ('Gauss law', in the case of QED: $\nabla E(r) = \rho(r)$)

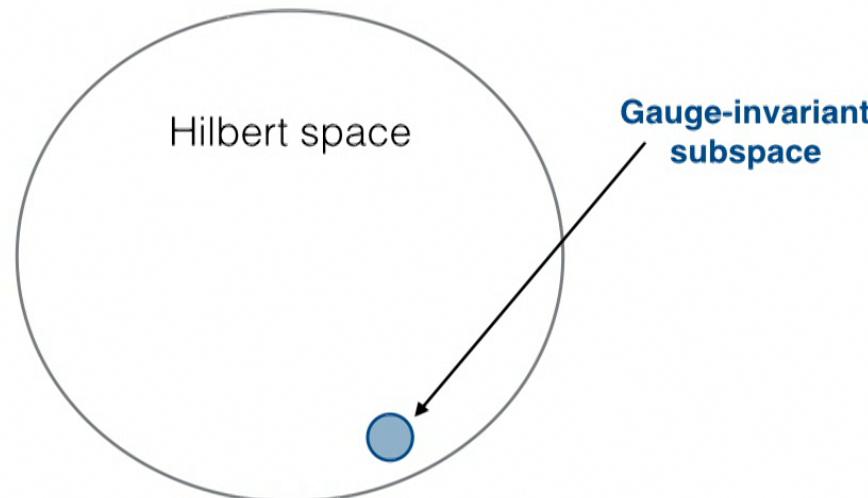
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Develop a new type of quantum simulator

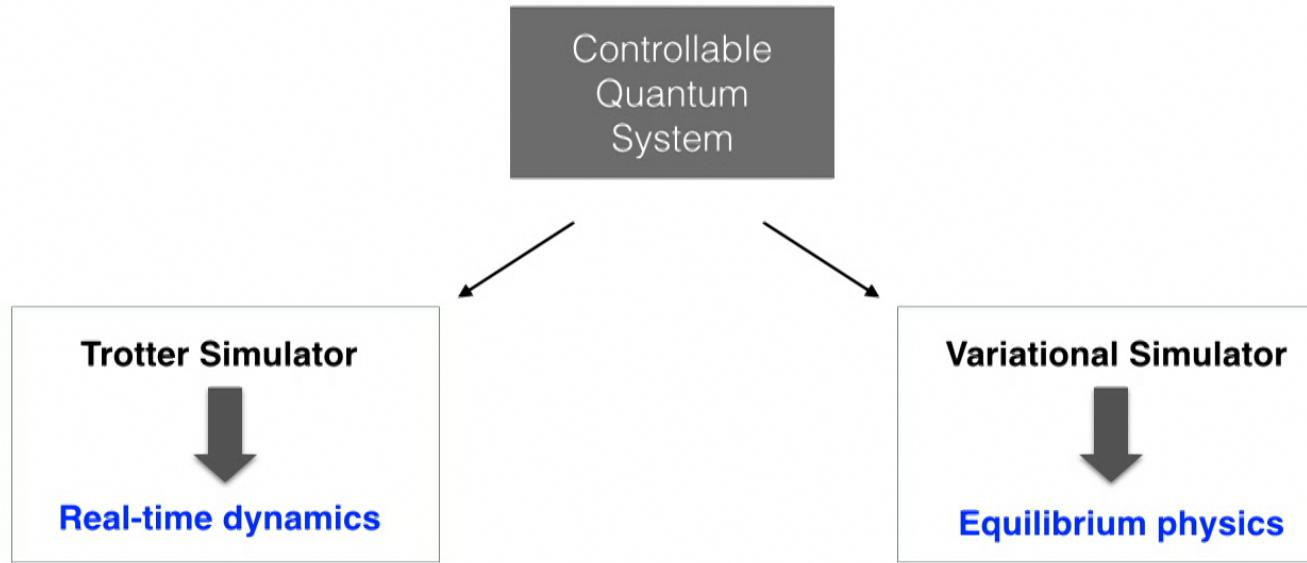
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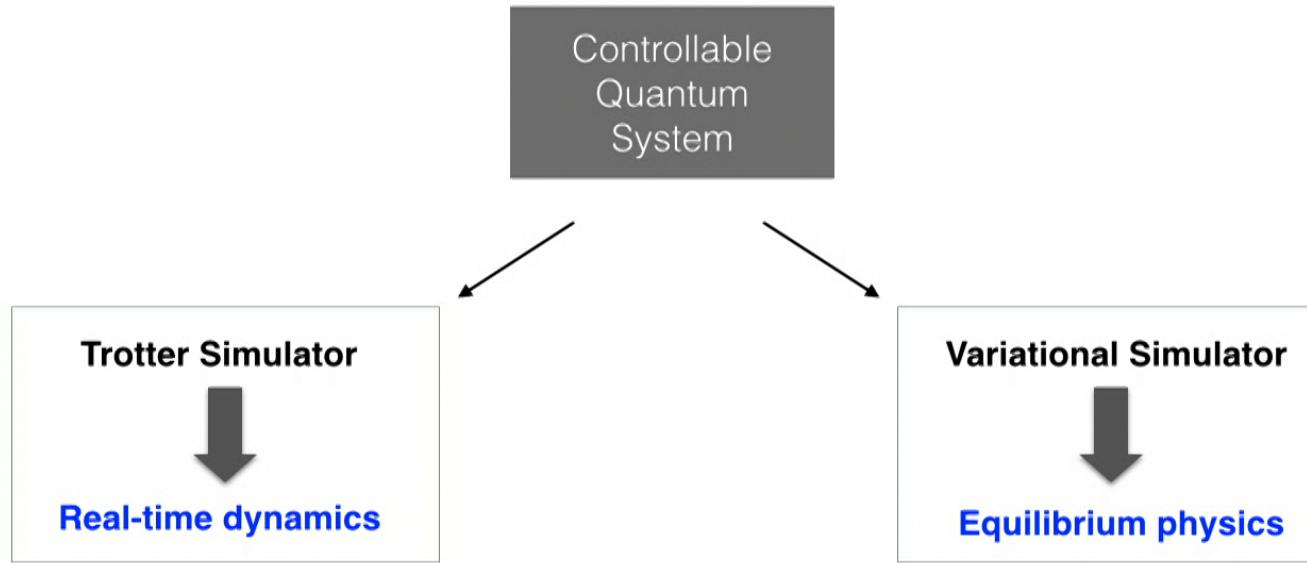


Nature 534, 516 (2016).

1D-QED:
Pair creation

arXiv:1810.03421 (Nature 2019)

1D-QED:
Parity breaking phase transition



Nature 534, 516 (2016).

1D-QED:
Pair creation



arXiv:1810.03421.

1D-QED:
Parity breaking phase transition



QED in (1+1) dimensions

Electromagnetic fields:

Vector potential: $A_0(x), A_1(x)$

Electric field: $E(x) = \partial_0 A_1(x)$

$$[E(x), A_1(x')] = -i\delta(x - x')$$

QED in (1+1) dimensions

Electromagnetic fields:

Vector potential: $A_0(x), A_1(x)$

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$$[E(x), A_1(x')] = -i\delta(x - x')$$

Matter fields:

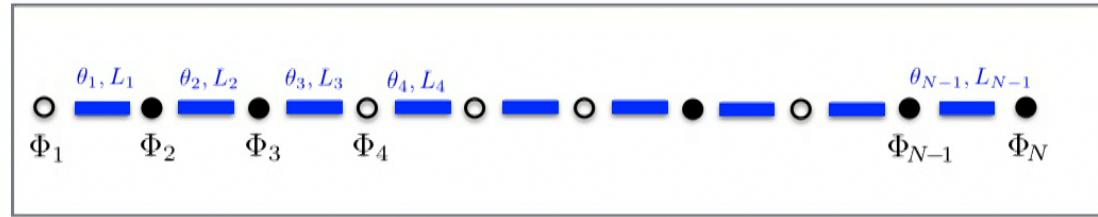
$$\Psi(x) = \begin{pmatrix} \Psi_1(x) \\ \Psi_2(x) \end{pmatrix}$$

Hamiltonian:

$$H_{\text{cont}} = \int dx \left[-i\Psi^\dagger(x)\gamma^1 (\delta_1 - igA_1) \Psi(x) + m\Psi^\dagger(x)\Psi(x) + \frac{1}{2}E^2(x) \right]$$

$\gamma_1 = -i\sigma_y$ coupling strength (charge) Fermion mass

The lattice Schwinger Model



Continuum

Vector potential $A_1(x)$

Electric field $E(x)$

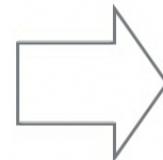
$$[E(x), A_1(x')] = -i\delta(x - x')$$

Lattice

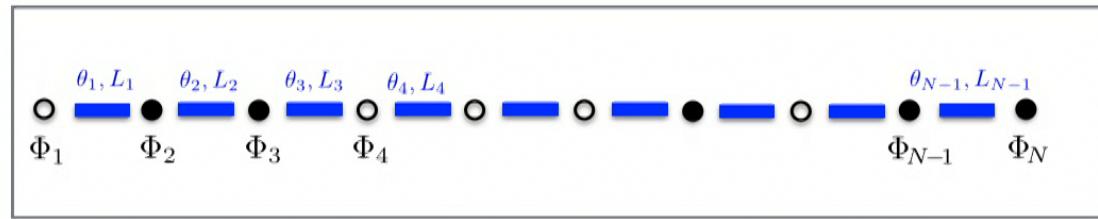
$$\theta_n = agA_1(x_n)$$

$$L_n = \frac{1}{g}E(x_n)$$

$$[\theta_n, L_m] = i\delta_{n,m}$$



The lattice Schwinger Model



Continuum

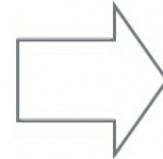
Vector potential $A_1(x)$

Electric field $E(x)$

$$[E(x), A_1(x')] = -i\delta(x - x')$$

Dirac spinor

$$\Psi(x) = \begin{pmatrix} \Psi_1(x) \\ \Psi_2(x) \end{pmatrix}$$



Lattice

$$\theta_n = agA_1(x_n)$$

$$L_n = \frac{1}{g}E(x_n)$$

$$[\theta_n, L_m] = i\delta_{n,m}$$

odd lattice sites:

$$\Phi_n = \sqrt{a}\Psi_1(x_n)$$

even lattice sites:

$$\Phi_n = \sqrt{a}\Psi_2(x_n)$$

Hamiltonian formulation of the Schwinger model:

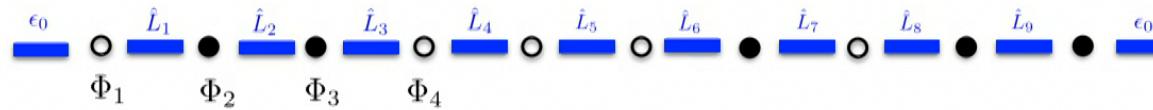
J. Kogut and L. Susskind, Phys. Rev. D 11, 395 (1975).

$$\hat{H} = -iw \sum_{n=1}^{N-1} \left[\hat{\Phi}_n^\dagger e^{i\hat{\theta}_n} \hat{\Phi}_{n+1} - \text{H.c.} \right] + J \sum_{n=1}^{N-1} \hat{L}_n^2 + m \sum_{n=1}^N (-1)^n \hat{\Phi}_n^\dagger \hat{\Phi}_n$$

The dynamics is constraint by the Gauss law:

In the continuum in 3D: $\nabla E = \rho$

Here: $\hat{L}_n - \hat{L}_{n-1} = \hat{\Phi}_n^\dagger \hat{\Phi}_n - \frac{1}{2} [1 - (-1)^n]$



One-dimensional QED on a trapped ion quantum computer

We explore:

- Coherent real-time dynamics of particle-antiparticle creation
- Entanglement generation during pair creation

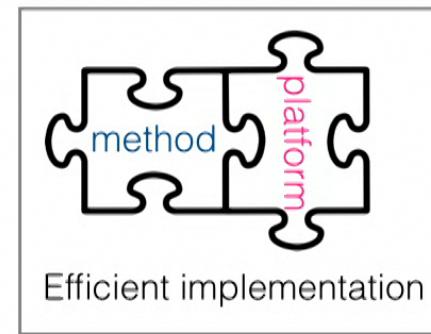


First experiment:

Real-time dynamics of lattice gauge theories on a few-qubit quantum computer
E. Martinez*, C. Muschik* et al, Nature 534, 516 (2016).

U(1) Wilson lattice gauge theories in digital quantum simulators
C. Muschik et al. New J. Phys. 19 103020 (2017).

Physics world: one of the top ten Breakthroughs in physics 2016



Our approach

Our scheme:

- (1) Mapping of the Schwinger Hamiltonian to a pure spin model with long range interactions
- (2) Realization of the required interactions with an efficient digital simulation scheme using "shaking methods".

Important features of the scheme

- Exact gauge invariance at all energy scales (by construction)
- Very efficient use of resources

Elimination of the gauge fields  **Pure spin model with long-range interactions**

The gauge fields don't appear explicitly in the encoded description. Instead, they act in the form of a non-local interaction that corresponds to the Coulomb-interaction between the simulated charged particles.

The Schwinger model as exotic spin model

$$\hat{H}_S = w \sum_i (\hat{\sigma}_i^+ \hat{\sigma}_{i+1}^- + \hat{\sigma}_{i+1}^+ \hat{\sigma}_i^-)$$

particle - antiparticle creation/annihilation

$$+ J \sum_{i < j} c_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z$$

long - range interaction

$$+ m \sum_i c_i \hat{\sigma}_i^z + J \sum_i \tilde{c}_i \hat{\sigma}_i^z$$

effective particle masses

The Schwinger model as exotic spin model

$$\hat{H}_S = w \sum_i (\hat{\sigma}_i^+ \hat{\sigma}_{i+1}^- + \hat{\sigma}_{i+1}^+ \hat{\sigma}_i^-)$$

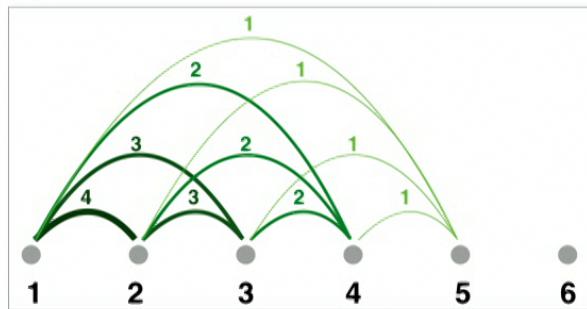
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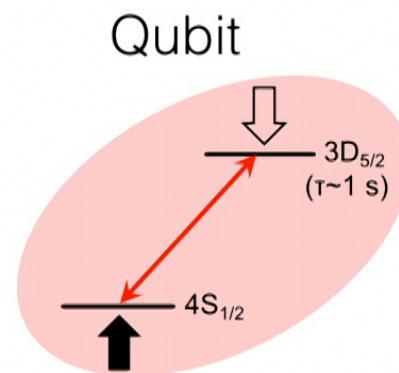
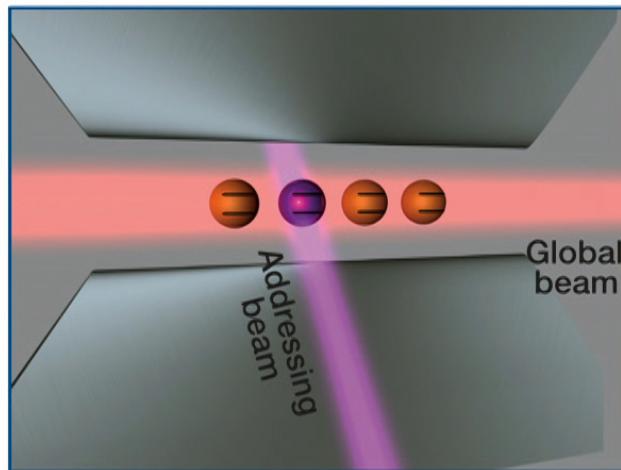
$$+ m \sum_i c_i \hat{\sigma}_i^z + J \sum_i \tilde{c}_i \hat{\sigma}_i^z$$

effective particle masses



Experiment

E. Martinez, P. Schindler, D. Nigg, A. Erhard, T. Monz, and R. Blatt



Tools for universal digital quantum simulation are available:

B. Lanyon, et al. Science 334, 57 (2011).

- High fidelity local rotations ✓
- Entangling gates ✓

Mølmer-Sørensen interaction

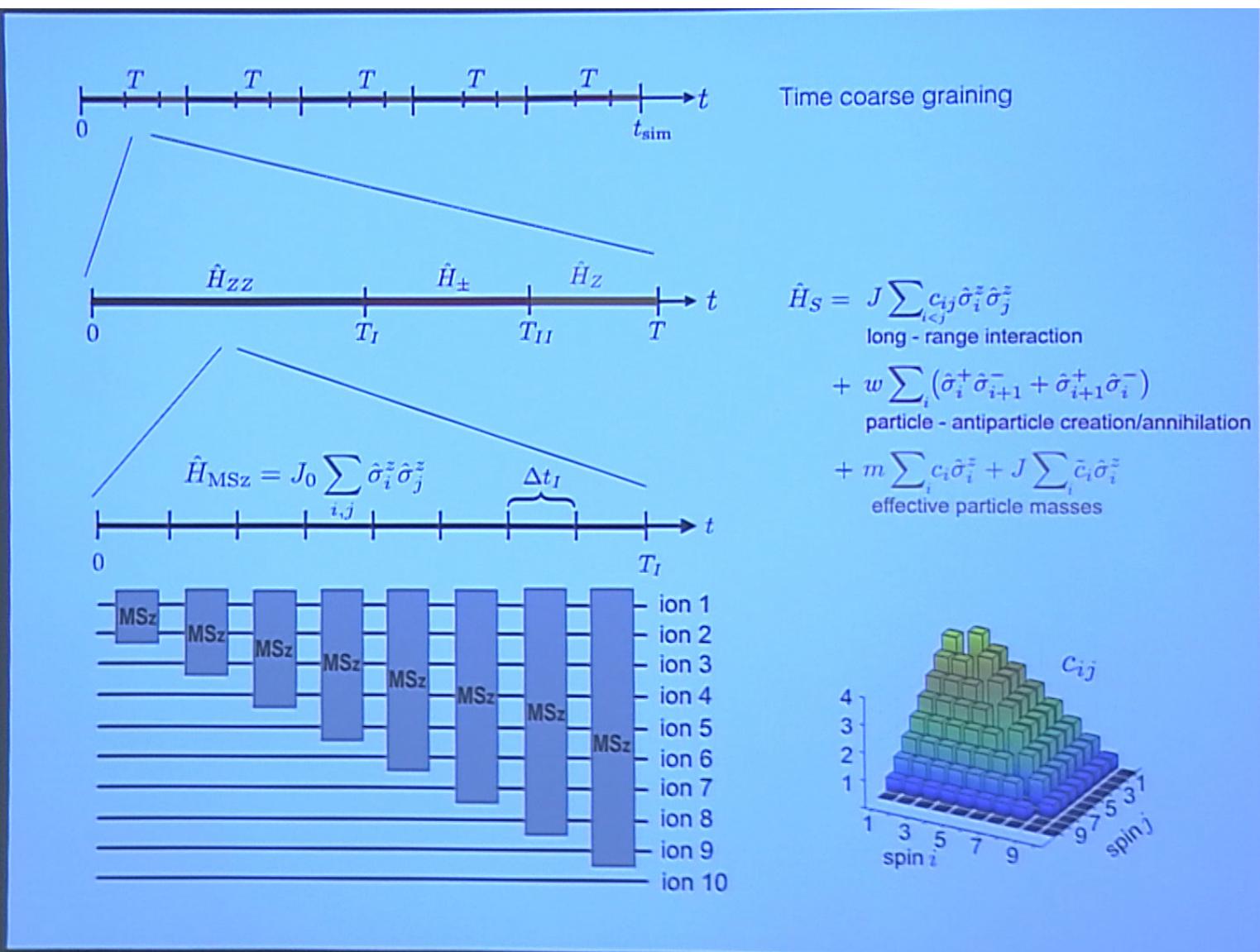
$$H_0 = J_0 \sum_{i,j} \sigma_i^x \sigma_j^x$$

Our toolbox

Ion trap quantum computers:

- Fast and accurate single qubit operations
- Entangling gates: Mølmer-Sørensen interaction

$$\text{All-to-all 2-body interaction: } H_0 = J_0 \sum_{i,j} \sigma_i^x \sigma_j^x$$



Quantum Simulation of pair creation

Particle number density: $\nu(t) = \frac{1}{N} \sum_{n=1}^N \langle (-1)^n \sigma_n^z(t) + 1 \rangle$

Creation of a particle antiparticle pair:



$$\nu = 0$$

$$\nu = 0.5$$

Odd lattice sites:

$$\begin{array}{c} \uparrow \\ n \end{array} \cong \text{vac}$$
$$\begin{array}{c} \downarrow \\ n \end{array} \cong e^-$$

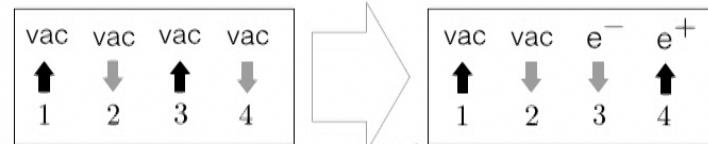
Even lattice sites:

$$\begin{array}{c} \uparrow \\ n \end{array} \cong e^+$$
$$\begin{array}{c} \downarrow \\ n \end{array} \cong \text{vac}$$

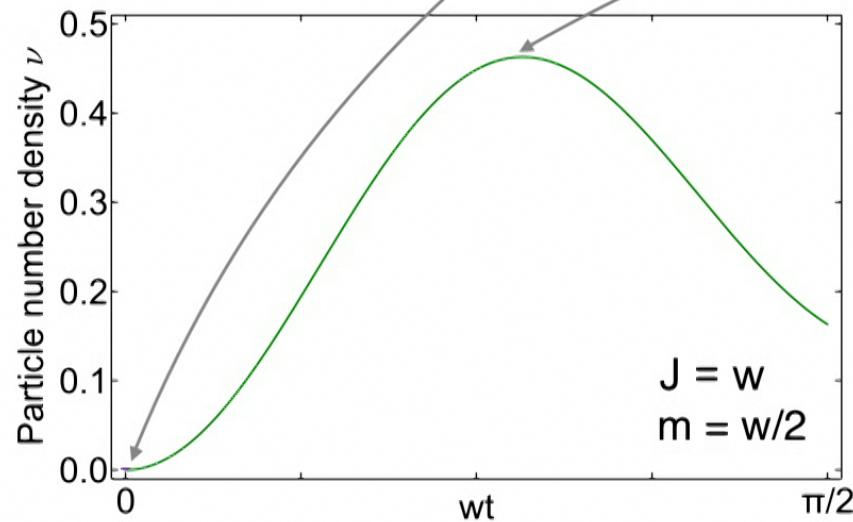
Schwinger Mechanism

Particle number density: $\nu(t) = \frac{1}{N} \sum_{n=1}^N \langle (-1)^n \sigma_n^z(t) + 1 \rangle$

Creation of a particle antiparticle pair:



In the ideal case ($N=4$):

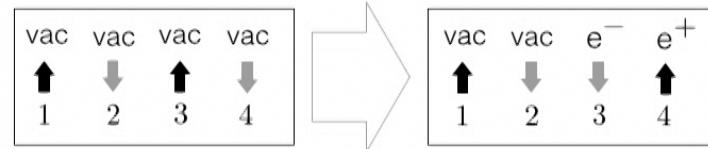


$$\begin{aligned}\hat{H}_S = & w \sum_i (\hat{\sigma}_i^+ \hat{\sigma}_{i+1}^- + \hat{\sigma}_{i+1}^+ \hat{\sigma}_i^-) \\ & \text{particle - antiparticle creation/annihilation} \\ & + J \sum_{i < j} c_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z \\ & \text{long - range interaction} \\ & + m \sum_i c_i \hat{\sigma}_i^z + J \sum_i \tilde{c}_i \hat{\sigma}_i^z \\ & \text{effective particle masses}\end{aligned}$$

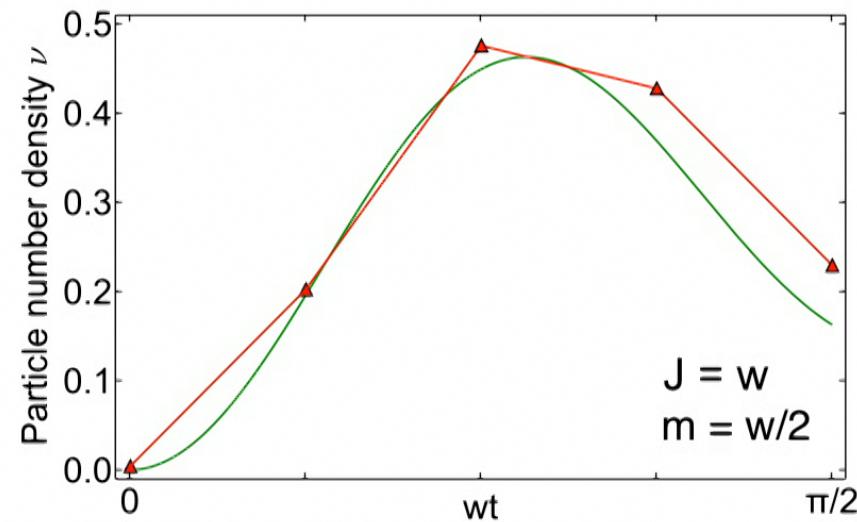
Schwinger Mechanism

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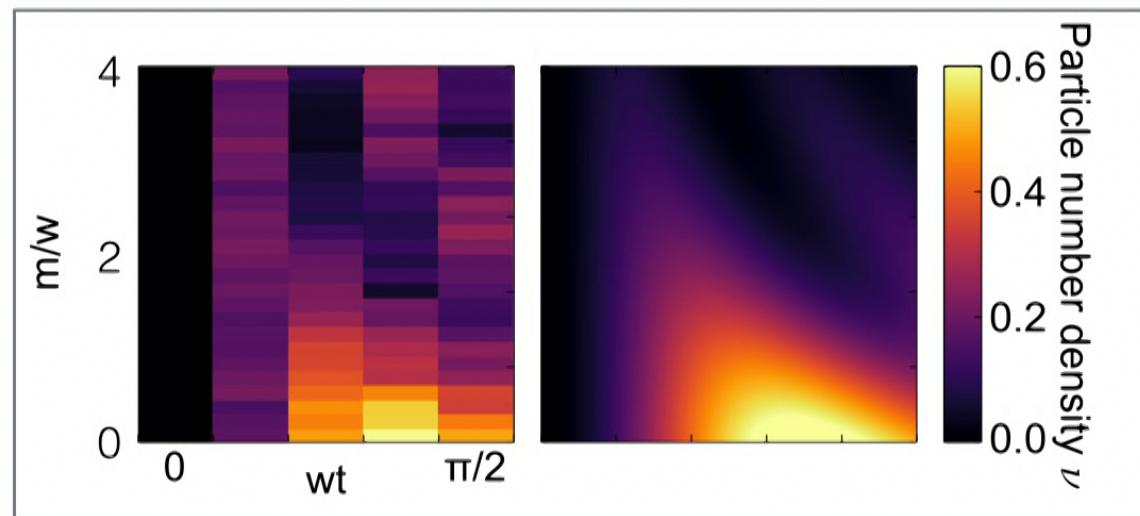


Including discretisation errors ($N=4$):

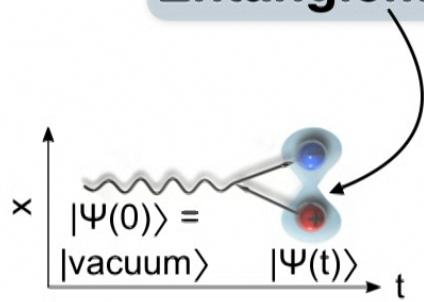


Schwinger Mechanism

Time evolution for different values of the particle mass m

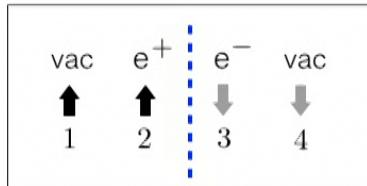


Entanglement in the Schwinger mechanism

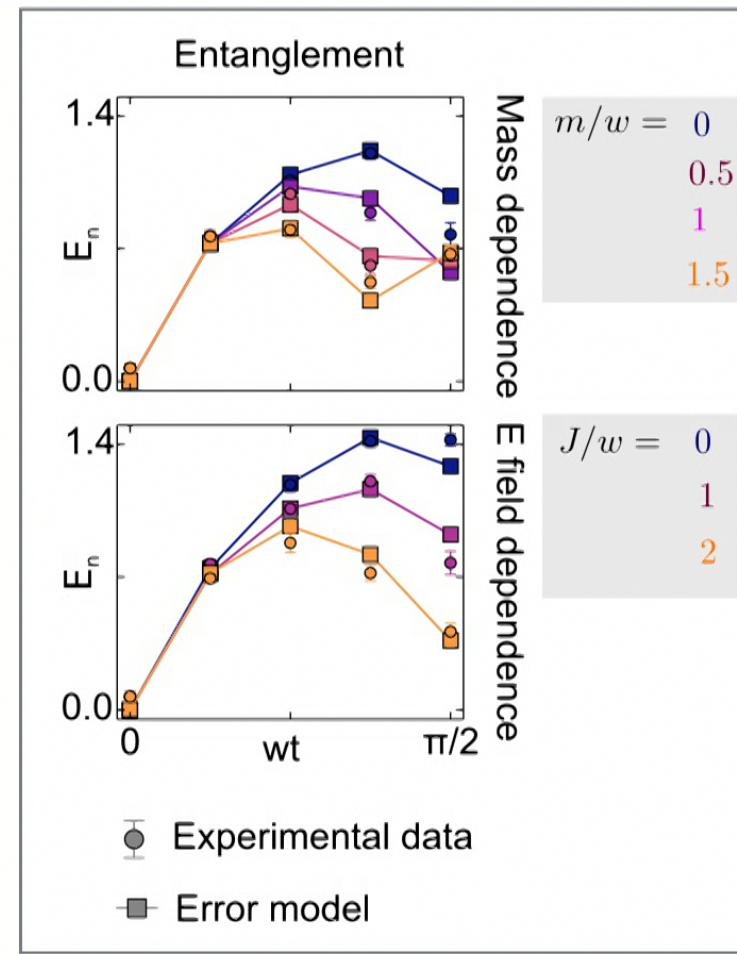


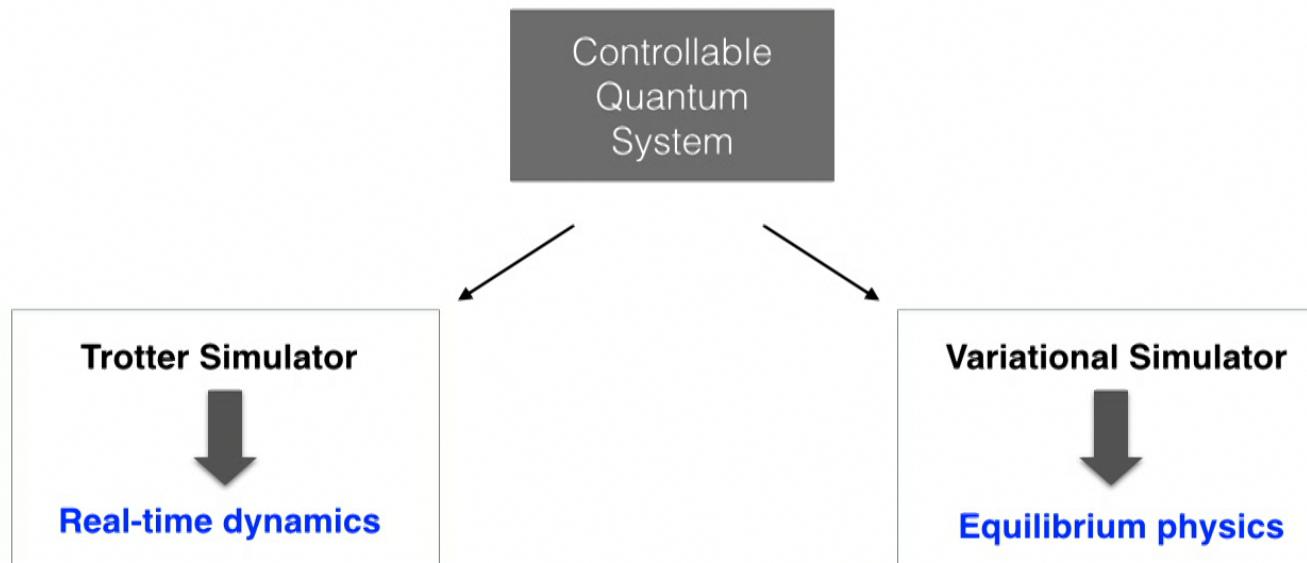
State tomography:
access to the full density matrix

E_n : logarithmic negativity
evaluated with respect to this bipartition:



Entanglement between the two
halves of the system.





Nature 534, 516 (2016).

1D-QED:
Pair creation



arXiv:1810.03421 (Nature 2019).

1D-QED:
Parity breaking phase transition



Variational Quantum Simulation

arXiv:1810.03421

Self-Verifying Variational Quantum Simulation of the Lattice Schwinger Model

Authors: Christian Kokail, Christine Maier, Rick van Bijnen, Tiff Brydges, Manoj K. Joshi, Petar Jurcevic, Christine A. Muschik, Pietro Silvi, Rainer Blatt, Christian F. Roos, Peter Zoller



C. Kokail



C. Maier



C. Roos

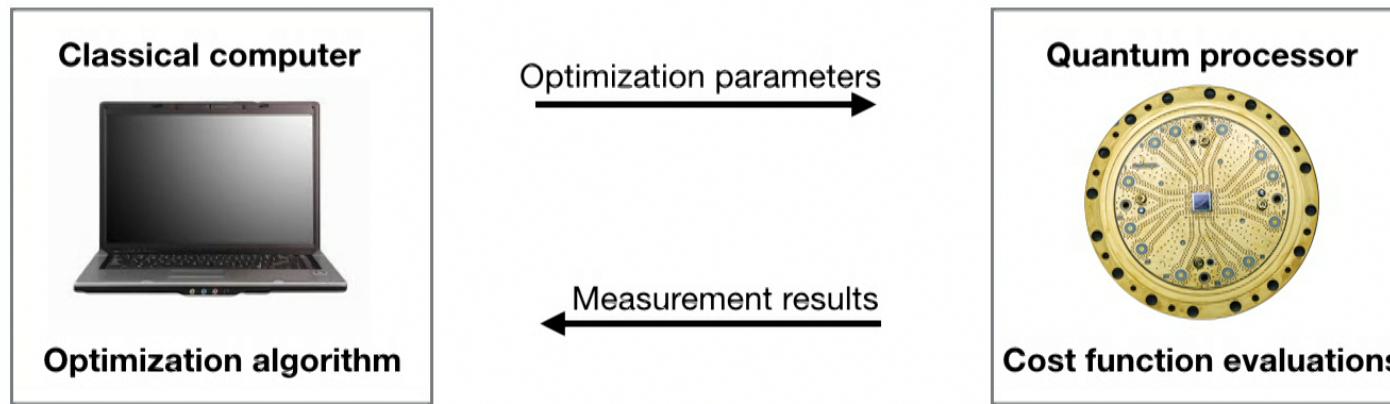


R. Blatt



P. Zoller

Variational Quantum Simulation

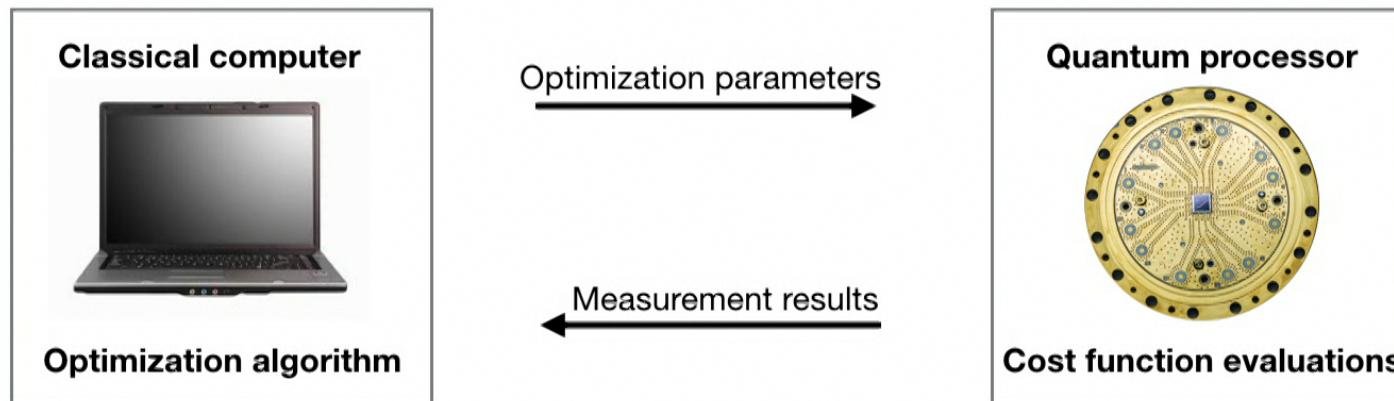


Hybrid quantum-classical optimization algorithms:

Inspiration: E. Farhi, J. Goldstone, S. Gutmann, H. Neven; MIT-CTP/4893 (2017).

Review: J. R. McClean, J. Romero, R. Babbush, and A. Aspuru-Guzik, New Journal of Physics 18, 023023 (2016).

Variational Quantum Simulation



Hybrid quantum-classical simulations for high energy and nuclear physics:

- Simulations of Subatomic Many-Body Physics on a Quantum Frequency Processor: arXiv:1810.03959
- Quantum-Classical Computation of Schwinger Model Dynamics using Quantum Computers: arXiv:1803.03326
- Cloud Quantum Computing of an Atomic Nucleus: Phys. Rev. Lett. 120, 210501 (2018).
- Toward convergence of effective field theory simulations on digital quantum computers: arXiv:1904.04338



Martin Savage



Karl Jansen



Phiala Shanahan



Guests at PI last week

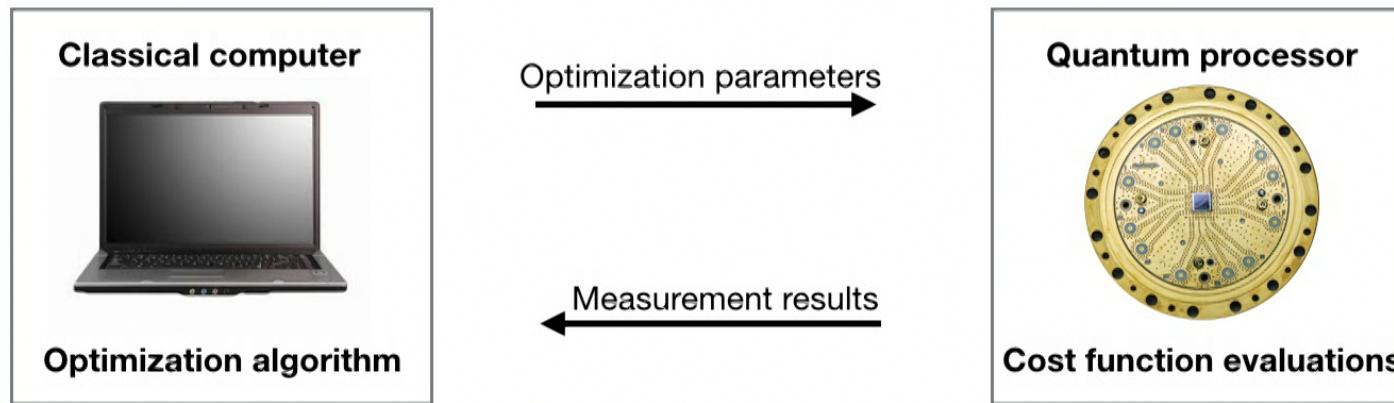
Variational Quantum Simulation

- Target Hamiltonian: H_T (contains e.g. 3-body terms or long-range interactions)
- Experimentally available resource Hamiltonians: $\{\dots, H_{\text{res}}^{(j)}, H_{\text{res}}^{(j+1)}, \dots\}$
- Create variational state: $|\psi(\Theta)\rangle = \dots e^{i\Theta_j H_{\text{res}}^{(j)}} e^{i\Theta_{j+1} H_{\text{res}}^{(j+1)}} \dots |\psi_{\text{init}}\rangle$
- The parameters Θ are varied such that $|\Psi(\Theta)\rangle$ becomes the ground state of a target Hamiltonian H_T :

$$\min_{\Theta} \frac{\langle \psi(\Theta) | H_T | \psi(\Theta) \rangle}{\langle \psi(\Theta) | \psi(\Theta) \rangle}$$

Can be highly entangled,
yet parametrised with few parameters

Variational Quantum Simulation



Target Hamiltonian:

$$H_T$$

Resource Hamiltonians:

$$\{\dots, H_{\text{res}}^{(j)}, H_{\text{res}}^{(j+1)}, \dots\}$$

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Can be highly entangled,
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Ground state preparation for 1D-QED with trapped ions

Resource Gates:

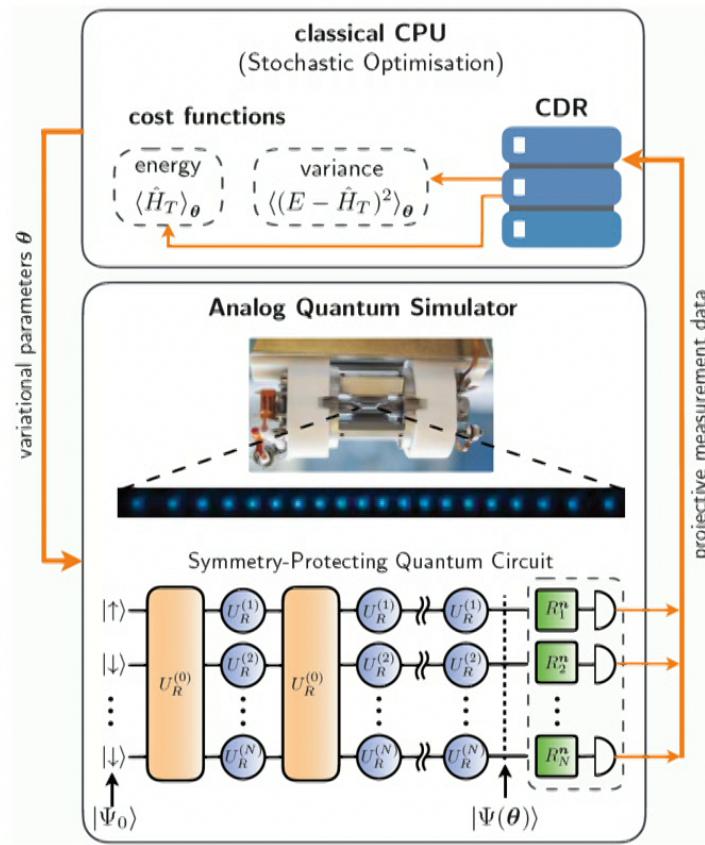
Spin-Spin Interaction Hamiltonian

$$H_{\text{spin-spin}} = \sum_{i=1}^{N-1} \sum_{j=i+1}^N J_{ij} \sigma_i^x \sigma_j^x + \sum_i B_i(t) \sigma_i^z$$

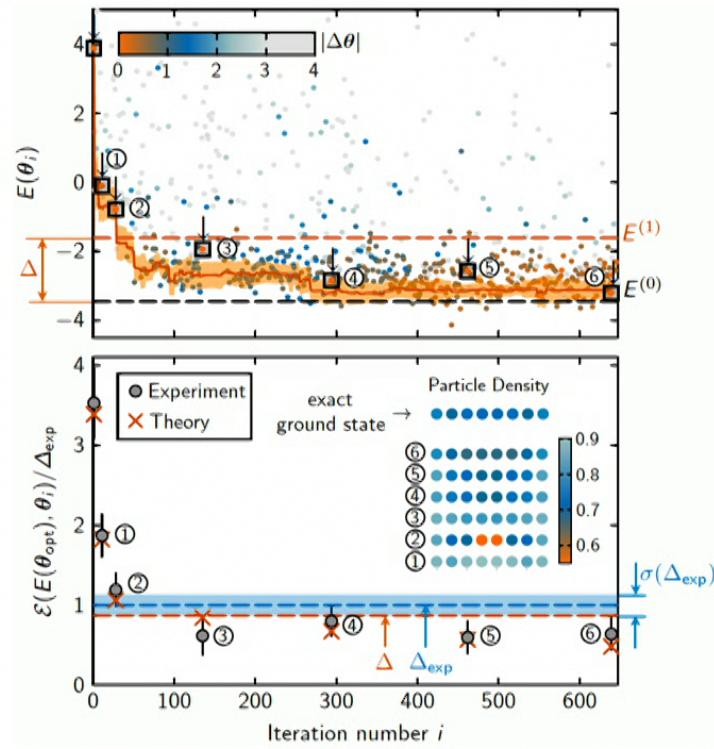
Single Qubit rotations on the Bloch sphere

$$R(\Theta) = \exp \left(-i \frac{\Theta}{2} \cdot \boldsymbol{\sigma} \right)$$

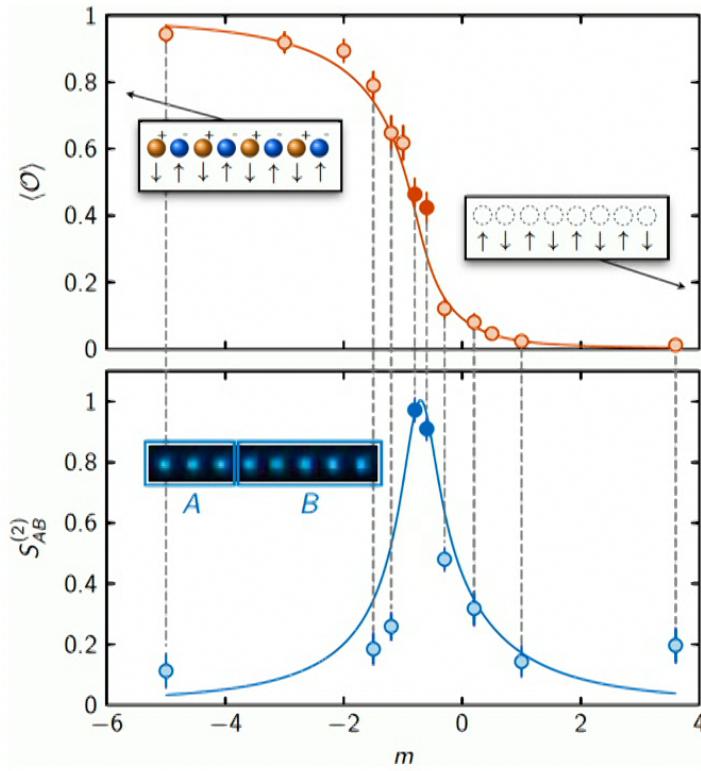
Ground state preparation for 1D-QED with trapped ions



Ground state preparation for 1D-QED with trapped ions



Ground state preparation for 1D-QED with trapped ions



Variational Quantum Simulation with trapped ions

Problem-adapted variational approach → resource-efficient

Resource Hamiltonian  Target Hamiltonian

New variational features:

- self-validation
- access excited states

Some related demonstrations:

Hybrid quantum-classical simulations:

- Rigetti, IBM: Deuteron (2 and 3 qubits)
- IBM: 1D-QED (2 and 3 qubits)
- IonQ: Deuteron (7 qubits)

Analog quantum simulations (experiments in progress)

- C. Wilson (IQC, Waterloo), 1D-QED with superconducting circuits
- M. Oberthaler and F. Jendrzejewski (Heidelberg) ,1D-QED with cold atoms

Next challenges:

- ➡ Realisation of 2D models
- ➡ Simulate increasingly complex dynamics
- ➡ Realisation of non-Abelian theories
- ➡ ...





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Quantum
Computing

PI PERIMETER
INSTITUTE



ARL

Thank you very much
for your attention!