

Title: A Conformal Window Into Quantum Gravity

Speakers: Eric Perlmutter

Series: Colloquium

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Abstract: Current progress in quantum field theory is largely driven by the conformal bootstrap program, which aims to classify the space and properties of conformal field theories using symmetries and other fundamental constraints. In the context of the AdS/CFT Correspondence, this increasingly sophisticated endeavor doubles as a probe of foundations of quantum gravity. I will describe the current era in the holographic application of conformal field theory methods, with particular focus on the use of the conformal bootstrap to describe the spectrum and dynamics of quantum gravity and string theory beyond the classical regime.

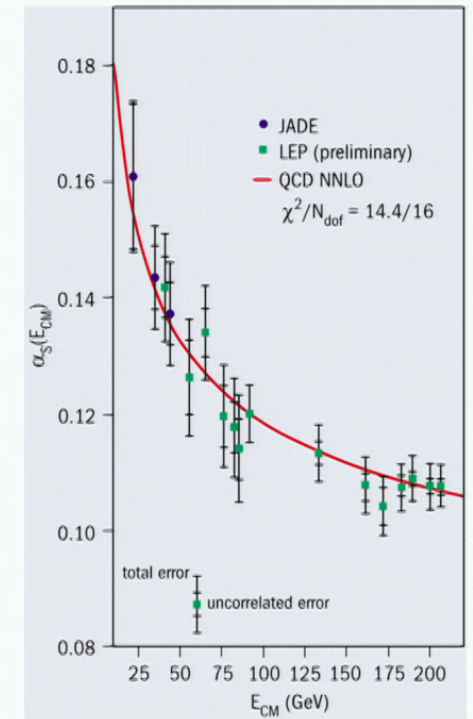
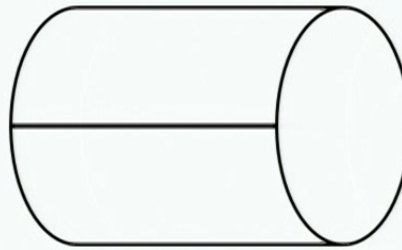
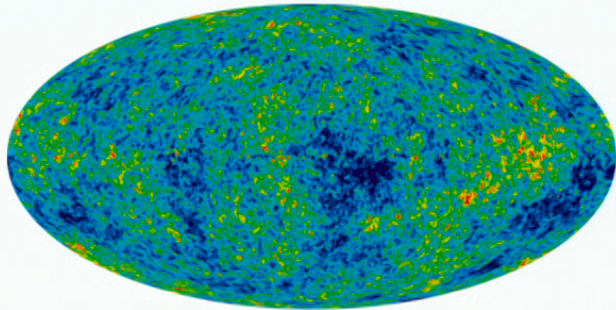
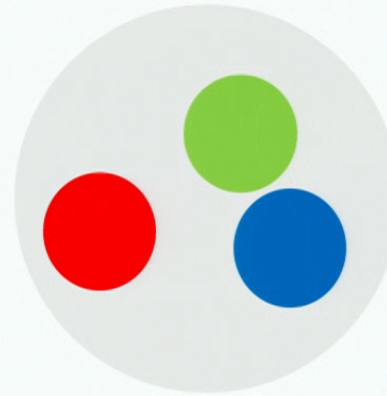
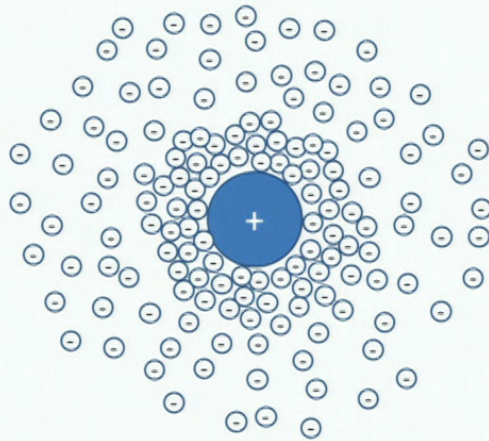
A Conformal Window Into Quantum Gravity

Eric Perlmutter

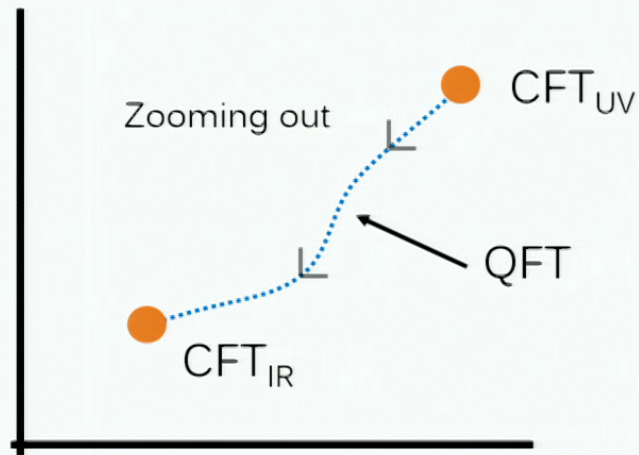
Caltech, Simons Collaboration on the
Nonperturbative Bootstrap

PI Colloquium, April 1, 2019

One of the world's most fascinating features is its dependence on scale.



In quantum field theory (QFT), scale-dependence is formally encoded in the equations of the renormalization group.



But not all theories flow. **Conformal field theories** (CFTs) are scale-invariant. More precisely, they are invariant under the full conformal group.

As fixed points of RG flows, CFTs are essential to the study of QFT in general.

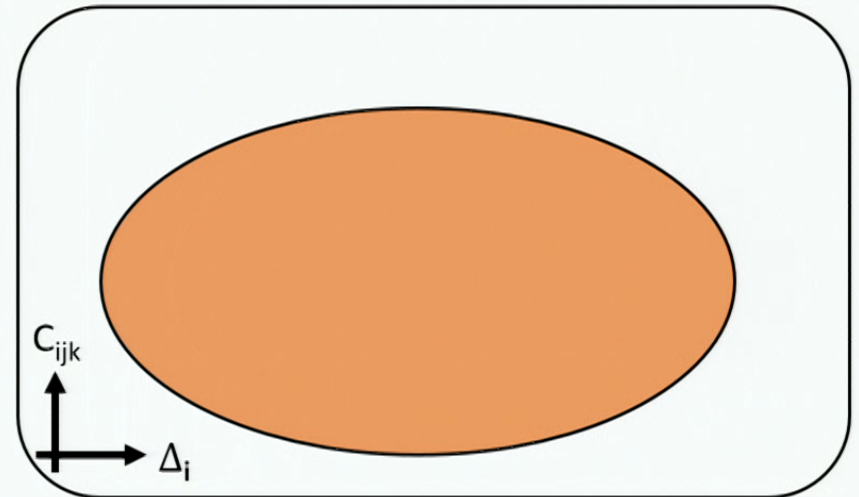
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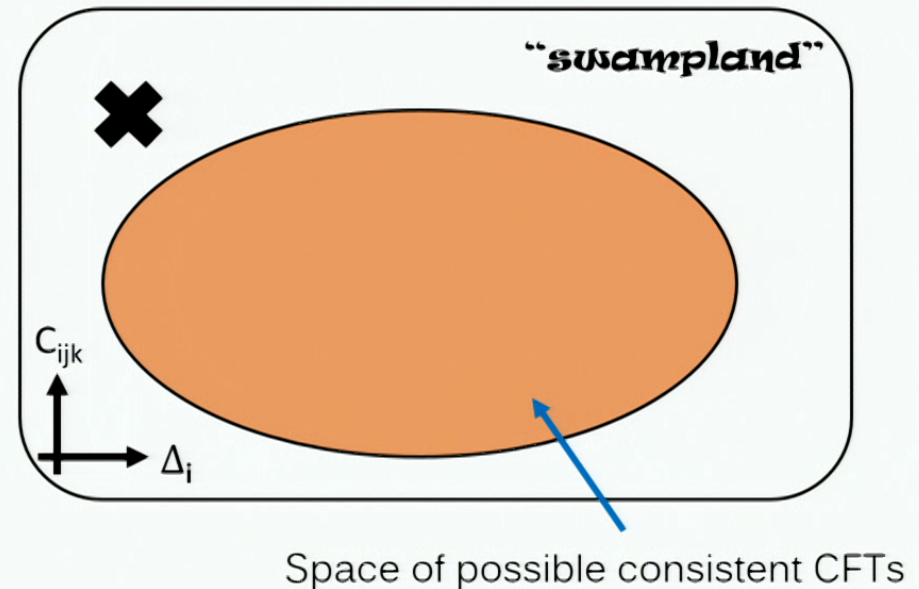
Conformal bootstrap: the program of classifying the space and properties of conformal field theories using symmetries and other abstract constraints.



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There has been a proliferation of new ideas about what, fundamentally, a CFT *is*, without reliance on weak coupling expansions, lattices, or, indeed, even quantum fields.

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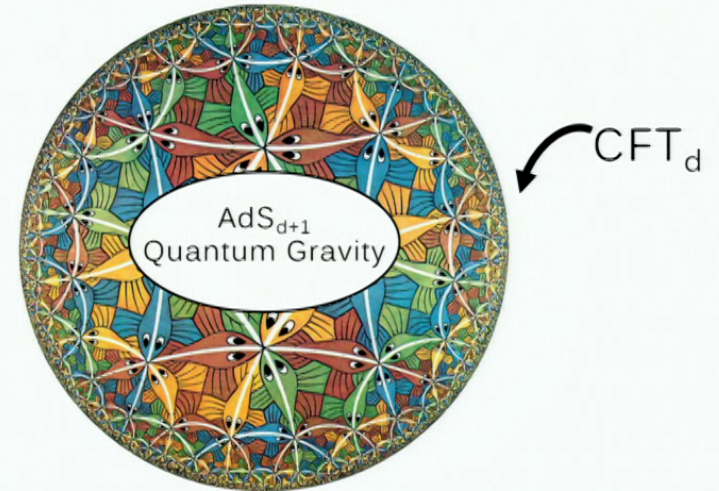
The bootstrap has three main prongs:

1. The **space** of CFTs
2. The **properties** of *all* CFTs
3. The **properties** within given *universality classes* of CFTs

Originally, these investigations were numerical. Now, **analytics** are exploding.

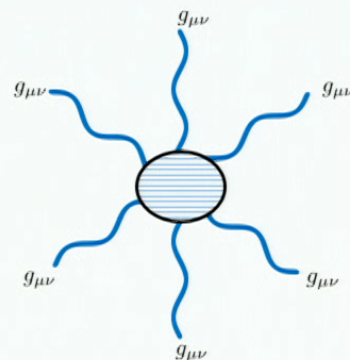
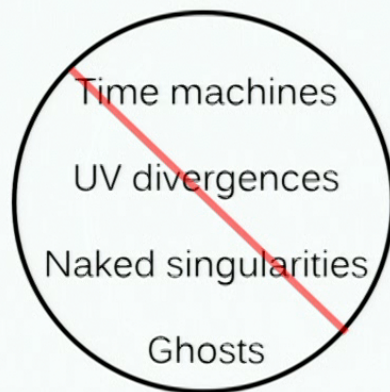
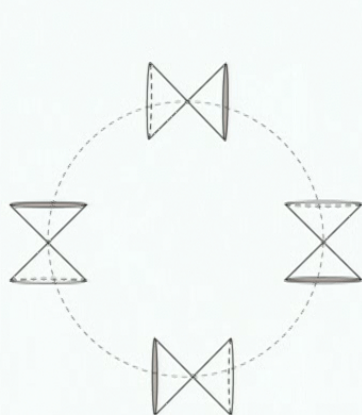
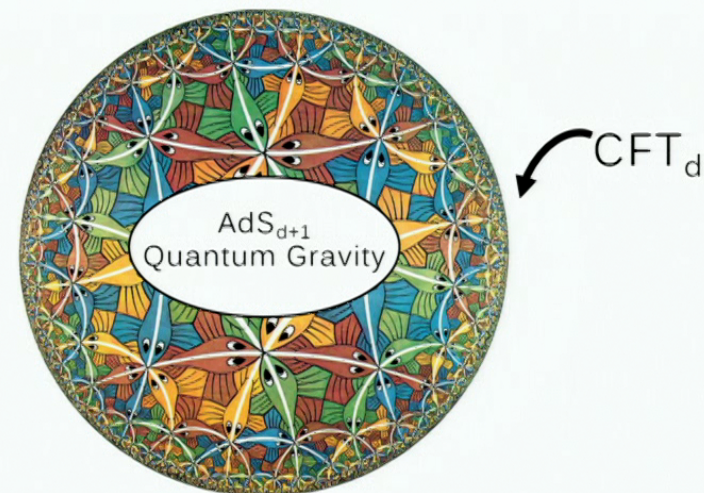
How the bootstrap works – i.e. what symmetries and abstract constraints are used – is time-dependent, as we discover new facts about field theory.

The bootstrap paradigm is especially powerful in the context of the **AdS/CFT Correspondence**.

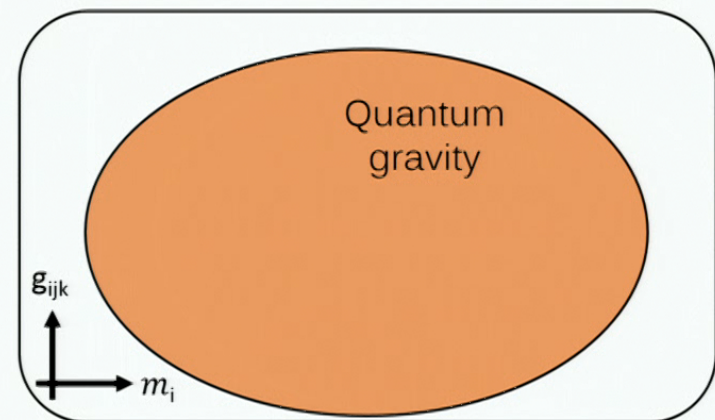
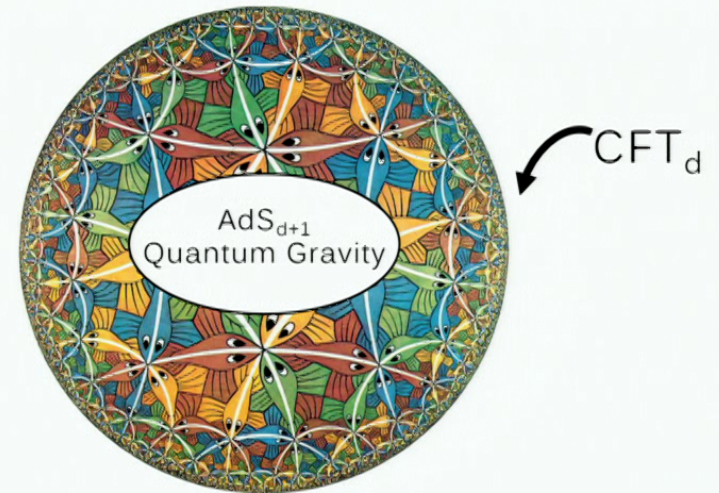


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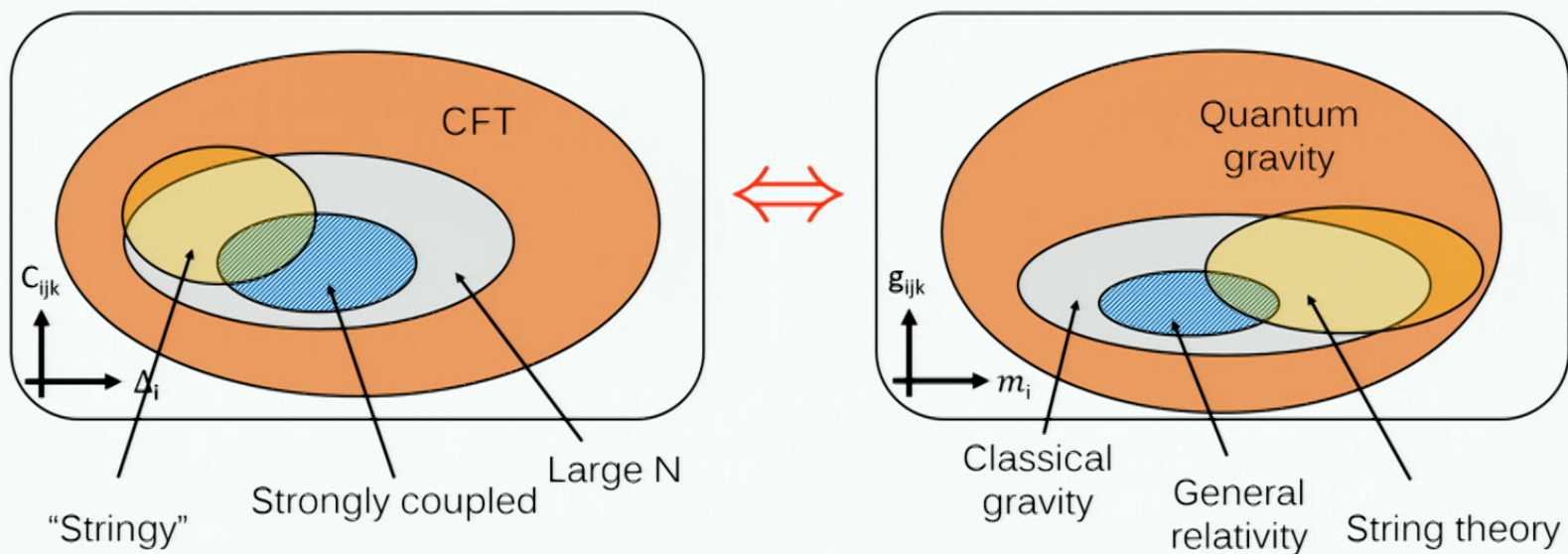
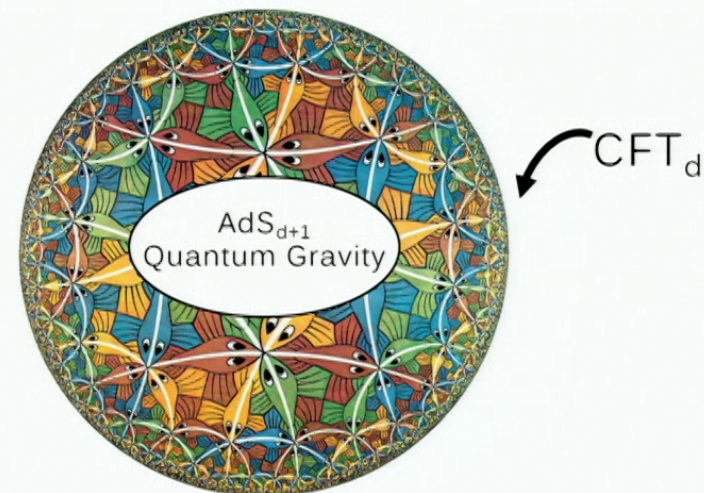
Theories of gravity have their own requirements:



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At the outset, AdS/CFT was mostly used as a tool for determining strongly coupled field theory dynamics from simple, semiclassical calculations in gravity.

AdS \rightarrow CFT

In the current era, CFT knowledge is sophisticated enough to reverse the arrow.

AdS \leftarrow CFT

We are learning about quantum gravity from insights and precision computations in CFT.

Three Core Questions:

What is the space of consistent quantum field theories,
and of theories of quantum gravity?

What are the underlying physical and functional structures of quantum
gravitational scattering amplitudes and field theory correlation functions?

Today's talk will be about some bootstrap-inspired CFT techniques, at both large and finite N , that shed light on the **spectrum** and **dynamics** of AdS quantum gravity.

This will include the development of a promising alternative to perturbative string theory.

I will try to emphasize the scope of the (ever-expanding!) conformal bootstrap ideal.

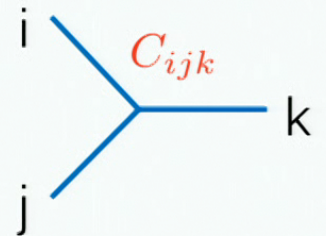
Outline

1. **Conformal Bootstrap: Then and Now**
 - 1.0: Crossing symmetry and numerics
 - 2.0: Analytics
2. **Conformal Bootstrap at Large N and AdS/CFT**
3. **The Bound State Spectrum of 3D Quantum Gravity**
4. **Modern Approaches to AdS Scattering Amplitudes**
 - The AdS Unitarity Method
 - A new alternative to string perturbation theory
5. **Visions for Future Research**

What are Conformal Field Theories (made of)?

I. Local operators: $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \dots$ 


These carry a conformal dimension (Δ), Lorentz spins, and maybe other symmetry charges.

II. Their interactions: $\mathcal{O}_i(x)\mathcal{O}_j(0) \sim \sum_k C_{ijk} \mathcal{O}_k(0) x^{\Delta_k - \Delta_i - \Delta_j}$ 

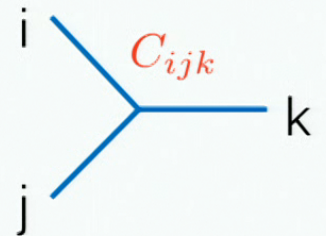
This is the operator product expansion (OPE).

“OPE data” $\{\Delta_i, C_{ijk}\}$ completely determine local operator dynamics of a CFT.

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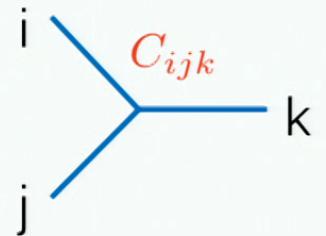
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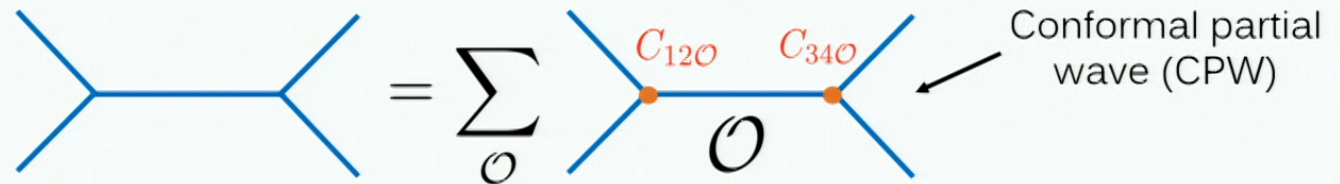
“OPE data” $\{\Delta_i, C_{ijk}\}$ completely determine local operator dynamics of a CFT.

Charting theory space = Constraining the sets $\{\Delta_i, C_{ijk}\}$

Note: No reference to Lagrangians!

What are Conformal Field Theories (made of)?

We can glue these vertices to make higher-point correlation functions.



The diagram shows an equation between two Feynman diagrams. On the left is a tree-level diagram with four external legs and a single internal propagator. On the right is a sum over operators \mathcal{O} of a diagram with two vertices connected by a horizontal line. Each vertex has two external legs. The left vertex is labeled with red text $C_{12\mathcal{O}}$ and the right vertex with $C_{34\mathcal{O}}$. The horizontal line is labeled with \mathcal{O} below it. An arrow points from the text "Conformal partial wave (CPW)" to the horizontal line.

$$\text{Tree diagram} = \sum_{\mathcal{O}} \text{CPW diagram}$$

These obey dynamical laws which constrain the underlying data $\{\Delta_i, C_{ijk}\}$.

What are Conformal Field Theories (made of)?

We can glue these vertices to make higher-point correlation functions.

The diagram shows a four-point function (two external lines on the left, two on the right) equal to a sum over an operator \mathcal{O} . The summand consists of two three-point vertices connected by a horizontal line representing the operator \mathcal{O} . The left vertex has two external lines and a red label $C_{12\mathcal{O}}$ on the internal line. The right vertex has two external lines and a red label $C_{34\mathcal{O}}$ on the internal line. An arrow points to the internal line with the text "Conformal partial wave (CPW)".

$$\text{Four-point function} = \sum_{\mathcal{O}} \text{CPW}(\mathcal{O})$$

These obey dynamical laws which constrain the underlying data $\{\Delta_i, C_{ijk}\}$.

- **Unitarity:** $\Delta_i \geq \Delta_* \geq 0$ and $C_{ijk}^2 \geq 0$

- **Associativity:** $\overbrace{\mathcal{O}_1 \mathcal{O}_2} \mathcal{O}_3 = \mathcal{O}_1 \overbrace{\mathcal{O}_2 \mathcal{O}_3}$

Soon we will discuss other, more recently discovered, constraints.

Bootstrap 1.0: Crossing symmetry + numerics

Inside four-point functions, associativity implies crossing symmetry:

$$\sum_{\mathcal{O}} \text{[s-channel diagram]} = \sum_{\mathcal{O}'} \text{[t-channel diagram]}$$

Non-trivial: partial waves are not symmetric!

Unitarity is key: expansion coefficients are **positive**

In $d=2$: Minimal models, WZW models, Liouville CFT

In $d>2$: Basically nothing until 2008

[Belavin, Polyakov, Zamolodchikov; Rattazzi, Rychkov, Tonni, Vichi; Ferrara, Gatto, Grillo, Parisi; Zamolodchikov; Dolan, Osborn; El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi]

Some classic bootstrap questions:

Is there an upper bound on the **dimension of the lightest operator** in any CFT? In a given OPE?

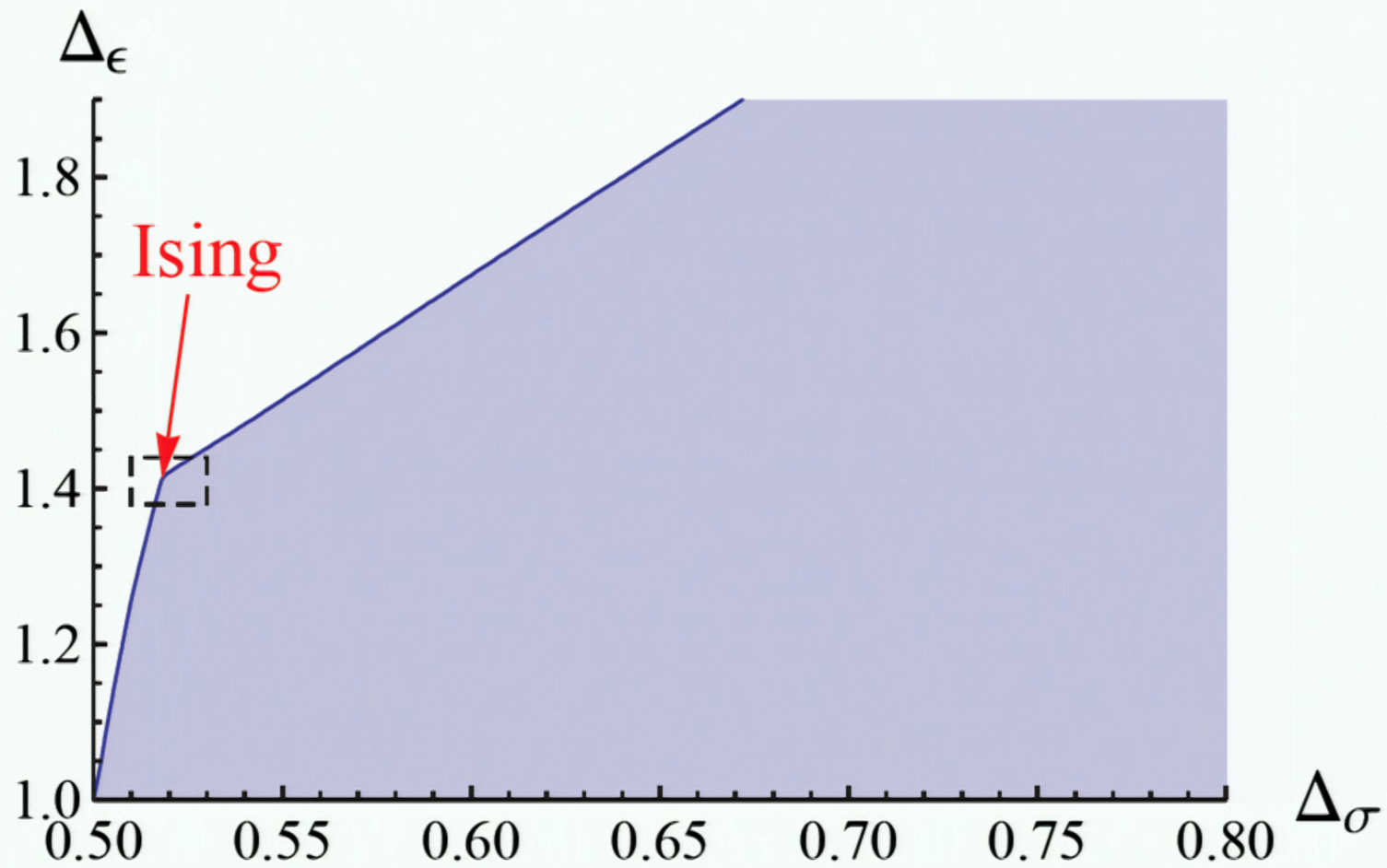
Are there bounds on **OPE coefficients** – for example, central charges or anomaly coefficients?

Assuming certain features, **is there a CFT at all**? If so, can we determine the precise value of its critical exponents, etc?

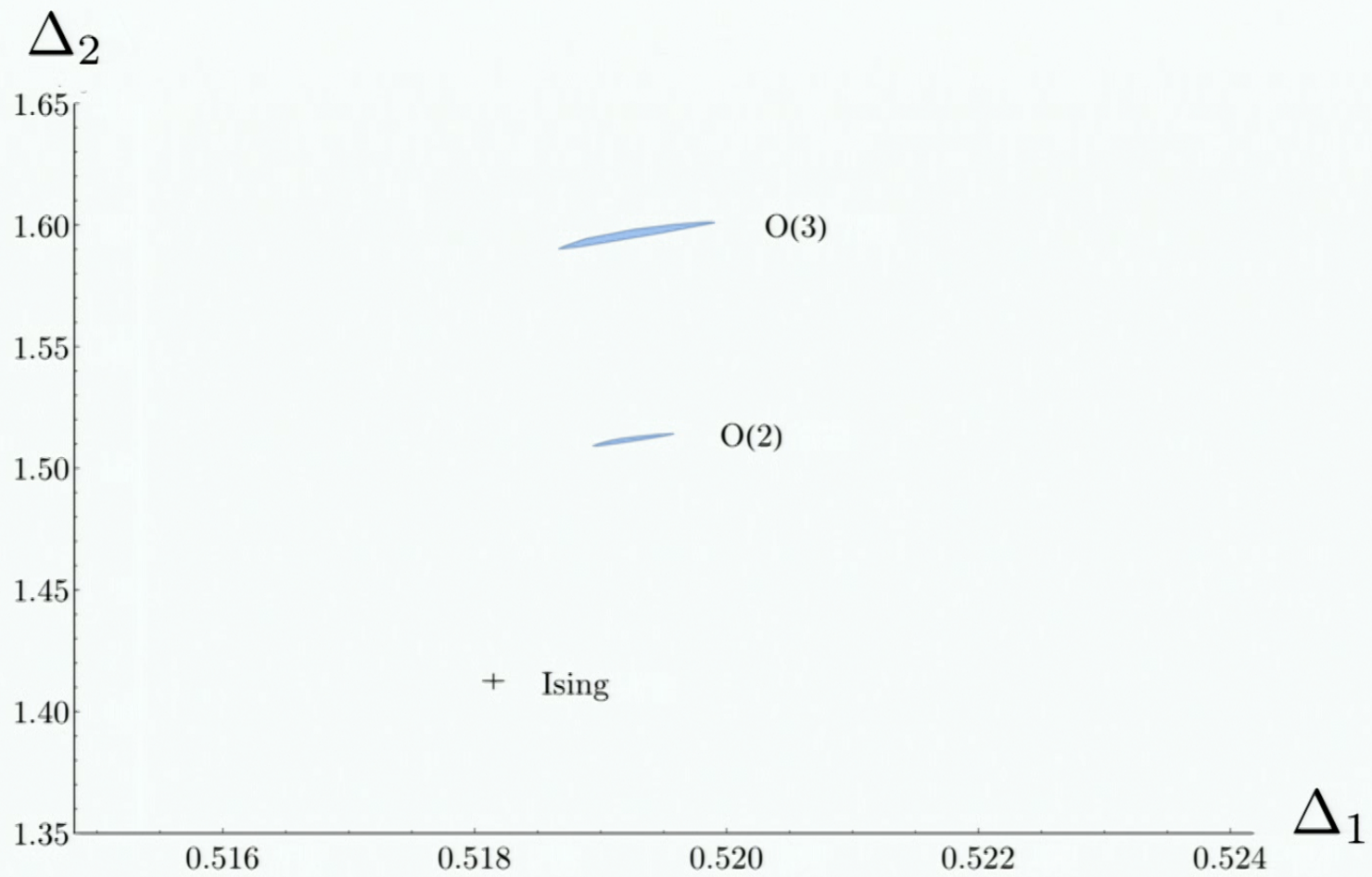
Why numerics?

The crossing equations are functional equations for an infinite set of OPE data $\{\Delta_i, C_{ijk}\}$.

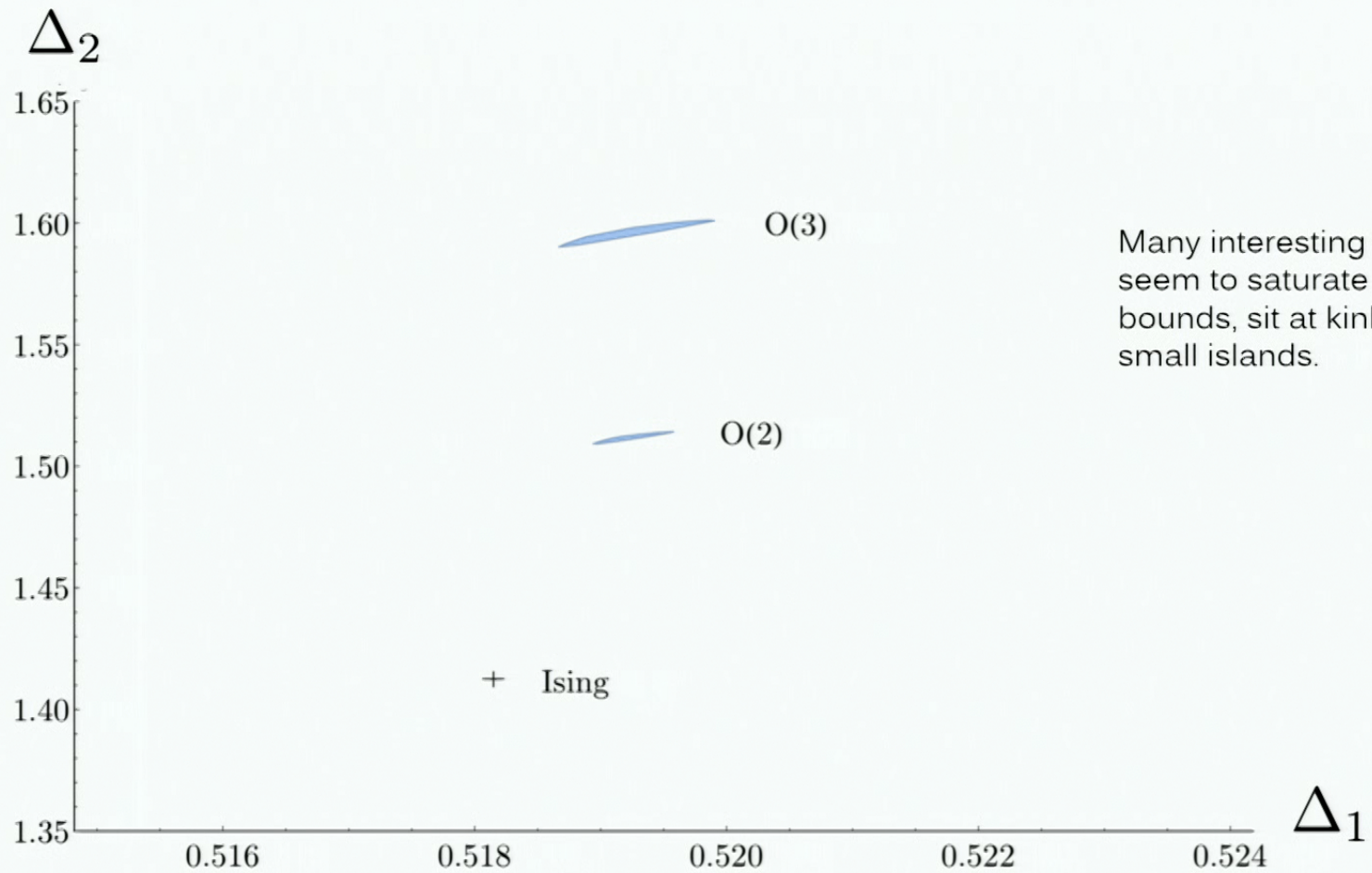
Their solution requires truncation, approximation, or (usually) both.



[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi]



[Kos, Poland, Simmons-Duffin, Vichi]



Many interesting known CFTs seem to saturate bootstrap bounds, sit at kinks, or lie inside small islands.

[Kos, Poland, Simmons-Duffin, Vichi]

Bootstrap 2.0: Analytics

Some landmark results:

- Every CFT has an infinite number of primaries.
- Every 2d CFT has a lightest primary below a universal upper bound.
- CFTs with higher spin currents are free.
- Central charges – measures of anomalies and/or degrees of freedom – are lower-bounded.
- Many classes of superconformal theories have soluble subsectors that are completely determined by 2d chiral algebras.

[Komargodski, Zhiboedov; Fitzpatrick, Kaplan, Poland, Simmons-Duffin; Hellerman; Maldacena, Zhiboedov; Hofman, Maldacena; Beem, Rastelli, van Rees; Afkhami-Jeddi, Hartman, Kundu, Jain; Caron-Huot]

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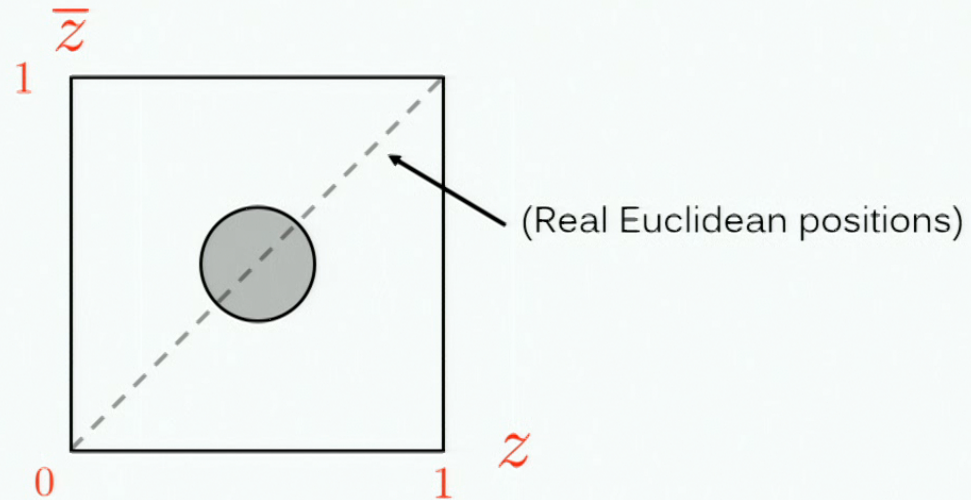
Some of these proven using new approaches, not traditional crossing symmetry!

- Causality and analyticity
- Regge physics/quantum chaos
- Energy conditions (e.g. ANEC)
- In 2d, modular invariance

[Komargodski, Zhiboedov; Fitzpatrick, Kaplan, Poland, Simmons-Duffin; Hellerman; Maldacena, Zhiboedov; Hofman, Maldacena; Beem, Rastelli, van Rees; Afkhami-Jeddi, Hartman, Kundu, Jain; Caron-Huot]

Bootstrap 2.0: Analytics

Numerical bootstrap typically expands in a Lorentzian neighborhood of the crossing-symmetric point.



Much analytic progress has come from deriving constraints in real-time kinematics.

I. Lightcone Bootstrap

Consider a *lightcone limit*.

In all $d > 2$, crossing symmetry + existence of ground state implies:

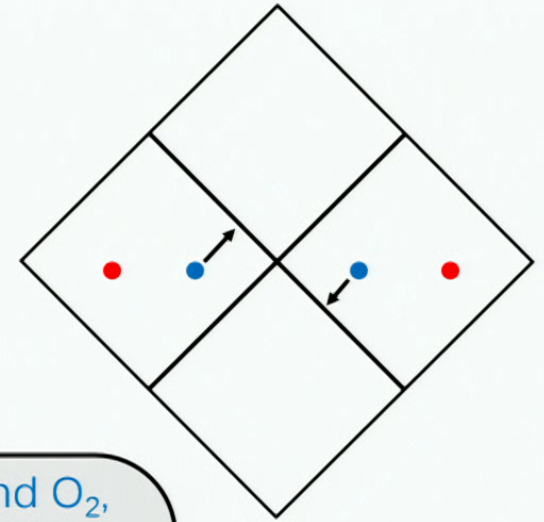
Given two local conformal primary operators \mathcal{O}_1 and \mathcal{O}_2 , there exists an infinite set of *large-spin* composite primaries,

$$[\mathcal{O}_1 \mathcal{O}_2]_{n,\ell} \equiv \mathcal{O}_1 \square^n \partial_{\mu_1} \dots \partial_{\mu_\ell} \mathcal{O}_2$$

with vanishing anomalous dimension at infinite spin.

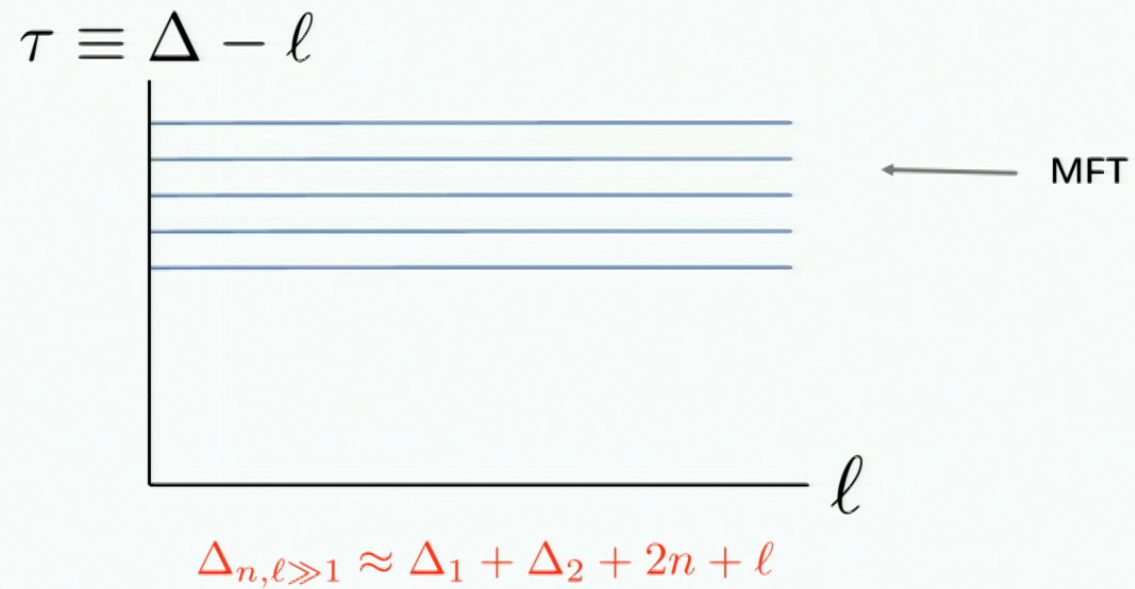
$$\Delta_{n,\ell \gg 1} \approx \Delta_1 + \Delta_2 + 2n + \ell$$

→ CFT at large spin \supset Mean field theory/“Generalized free field theory”

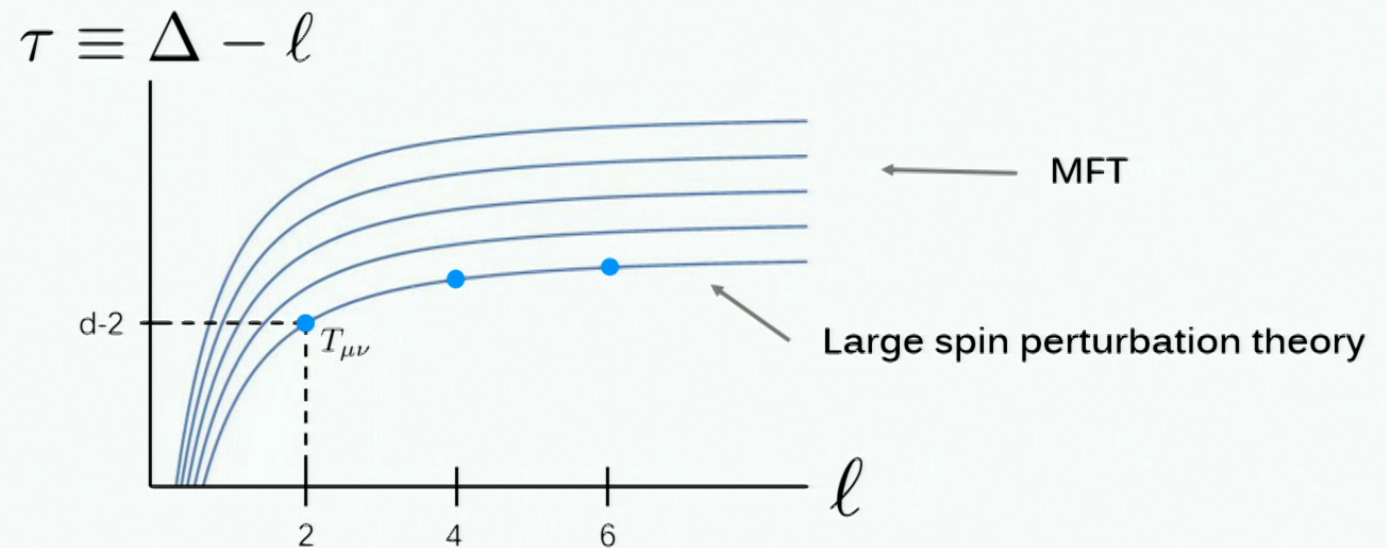


[Komargodski,
Zhiboedov; Fitzpatrick,
Kaplan, Poland,
Simmons-Duffin]

I. Lightcone Bootstrap



I. Lightcone Bootstrap



Regge trajectories in $\text{CFT}_{d>2}$ = **Analytic** families of operator data

[Caron-Huot;
Alday]

Their precise shape depends on the operator content of the theory.

I. Lightcone Bootstrap

We can build up trajectories operator-by-operator.

Q: If an operator O couples to O_1 and O_2 , what is its contribution to trajectories $(O_1 O_2)$?

In a large spin expansion,

$$\gamma_{n,\ell}|_O \sim \ell^{-\tau_O}$$

At finite spin?

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At finite spin?

$$\begin{array}{c} 1 \quad 2 \\ \diagdown \quad \diagup \\ \mathcal{O} \\ \diagup \quad \diagdown \\ 1 \quad 2 \end{array} = \sum_{\ell} \int d\Delta K_{\mathcal{O}|\mathcal{O}_{\Delta,\ell}} \begin{array}{c} 1 \quad 2 \\ \diagdown \quad \diagup \\ \mathcal{O}_{\Delta,\ell} \\ \diagup \quad \diagdown \\ 1 \quad 2 \end{array}$$

A: $(\mathcal{O}_1\mathcal{O}_2)$ data = Residues of **crossing kernel** for CPW at double-twist poles.

$$K_{\mathcal{O}|\mathcal{O}_{\Delta,\ell}} = \left(\begin{array}{c} 1 \quad 2 \\ \diagdown \quad \diagup \\ \mathcal{O} \\ \diagup \quad \diagdown \\ 1 \quad 2 \end{array}, \begin{array}{c} 1 \quad 2 \\ \diagdown \quad \diagup \\ \mathcal{O}_{\Delta,\ell} \\ \diagup \quad \diagdown \\ 1 \quad 2 \end{array} \right)$$

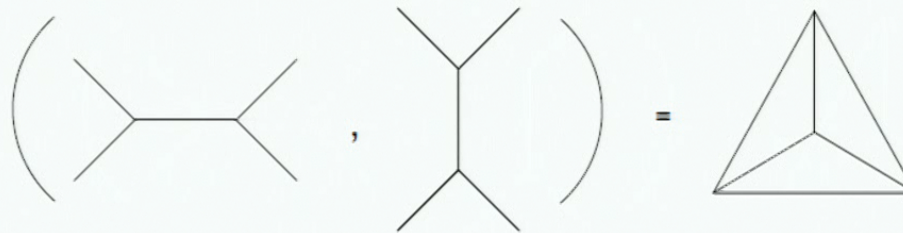
I. Lightcone Bootstrap

The crossing kernel is equivalent to the **6j symbol** for the conformal group.

Its definition generalizes the addition of angular momenta in quantum mechanics:

$$3j \text{ symbol} = J_1 + J_2$$

$$6j \text{ symbol} = J_1 + J_2 + J_3$$



Computed in $d=1,2,4 \rightarrow {}_7F_6$ hypergeometric (!?)

[Hogervorst, van Rees; Gadde]

[Liu, EP, Rosenhaus, Simmons-Duffin]

Its poles lie at MFT twists \rightarrow Efficient encoding of $(O_1 O_2)$ Regge trajectories.

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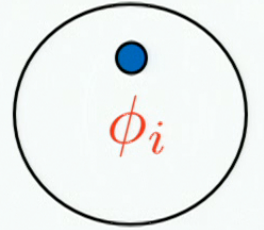
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Large N Conformal Field Theory



CFT

"Single-trace" operators

\mathcal{O}_i

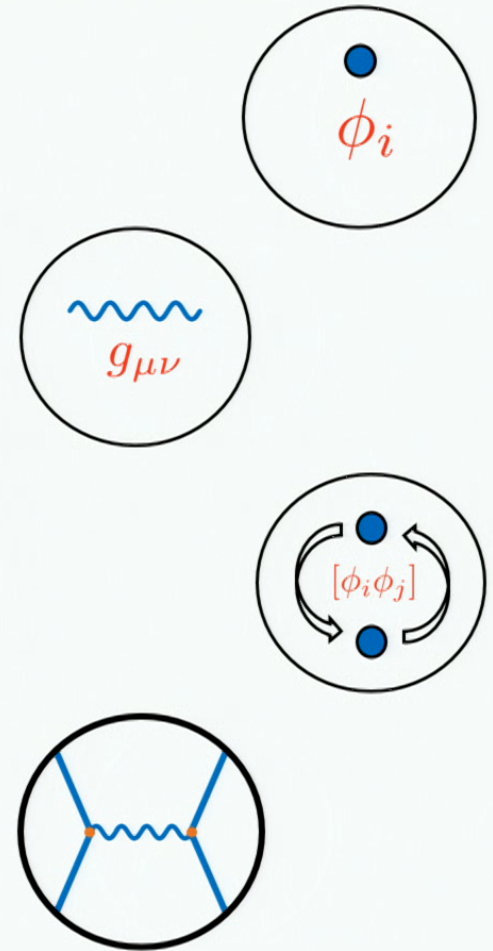
AdS

Elementary fields

ϕ_i

Large N Conformal Field Theory

CFT		AdS	
"Single-trace" operators	\mathcal{O}_i	Elementary fields	ϕ_i
Stress tensor	$T_{\mu\nu}$	Graviton	$g_{\mu\nu}$
"Multi-trace" composites	$[\mathcal{O}_i\mathcal{O}_j],$ $[\mathcal{O}_i\mathcal{O}_j\mathcal{O}_k], \dots$	Multi-particle states	$[\phi_i\phi_j],$ $[\phi_i\phi_j\phi_k], \dots$
Conformal dimensions	Δ_i	Masses	$m_i^2 = \Delta_i(\Delta_i - d)$
Central charge	$c \sim N^\#$	Planck scale (loop expansion)	
Correlation function		Amplitude	



Strongly-coupled
quark-gluon
plasma

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Area law
entanglement

$$S_{EE} = \frac{A_{RT}}{4G_N}$$

AdS \rightarrow CFT

Huge landscape of
non-Lagrangian CFTs

Strongly coupled
anomalous
dimensions

$$\Delta \sim M_{\text{string}} \sim \lambda^{\#>0}$$

PROBLEMS (STRINGS 99?)

• STRINGS ON R-R BACKGROUNDS

• quantization

MEYER
TSTYLIH
KALLOSH
RAJARAMAN

• HOW FAR CAN WE GET WITH SUPERGRAVITY?

• NEW SOLUTIONS

PURE: $U(N)$ $N=2, 1, 0$?

DO THEY INCLUDE A
GRAVITY REGION?

• BLACK HOLES

• ESTABLISH PRECISELY HOW THE GRAVITY

APPROXIMATION FAILS TO TAKE INTO

ACCOUNT THE QUANTUM CORRELATIONS

(IF INFORMATION WERE LOST THESE CONJECTURES
WOULD BE WRONG)

• OTHER BACKGROUNDS?

• COSMOLOGICAL SOLUTIONS

de Sitter $(N \sim 10^{160})$

HOROWITZ
MABOUF
DOUGLAS
HULL

[Maldacena, Strings '98, slide 22]

Lightcone limit in gravity

Double-trace composites

$$[\mathcal{O}_1 \mathcal{O}_2]_{n,\ell}$$



Two-particle bound states

$$[\phi_1 \phi_2]_{n,\ell}$$

Anomalous dimensions

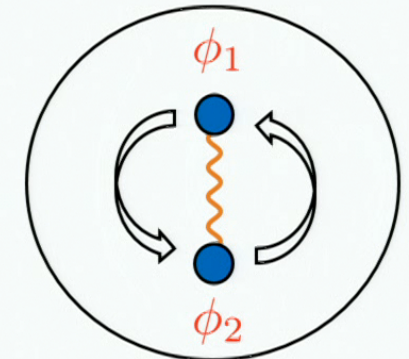
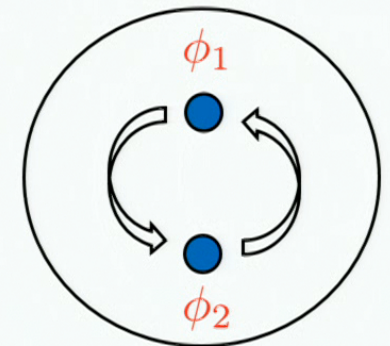


Binding energies

Large spin falloff



Gravitational force falloff



n = Radial quantum number

ℓ = Angular momentum

Lightcone limit in gravity

Double-trace composites

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Binding energies

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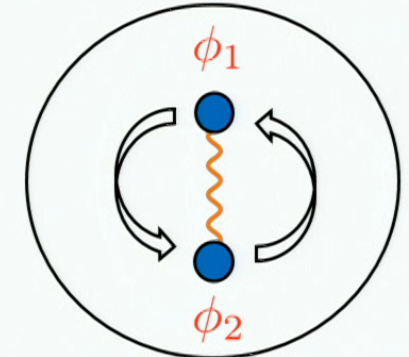
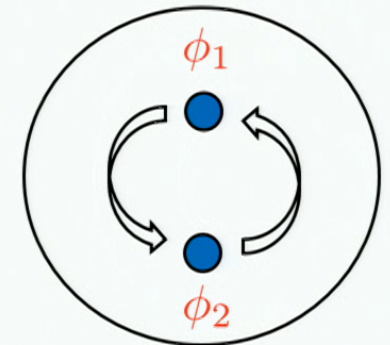
Gravitational force falloff

In CFT_2 , T and its composites $[T\dots T]$ have zero twist.

What do CFT_2 double-twist Regge trajectories look like?



What is the bound state spectrum of 3D quantum gravity?



n = Radial quantum number

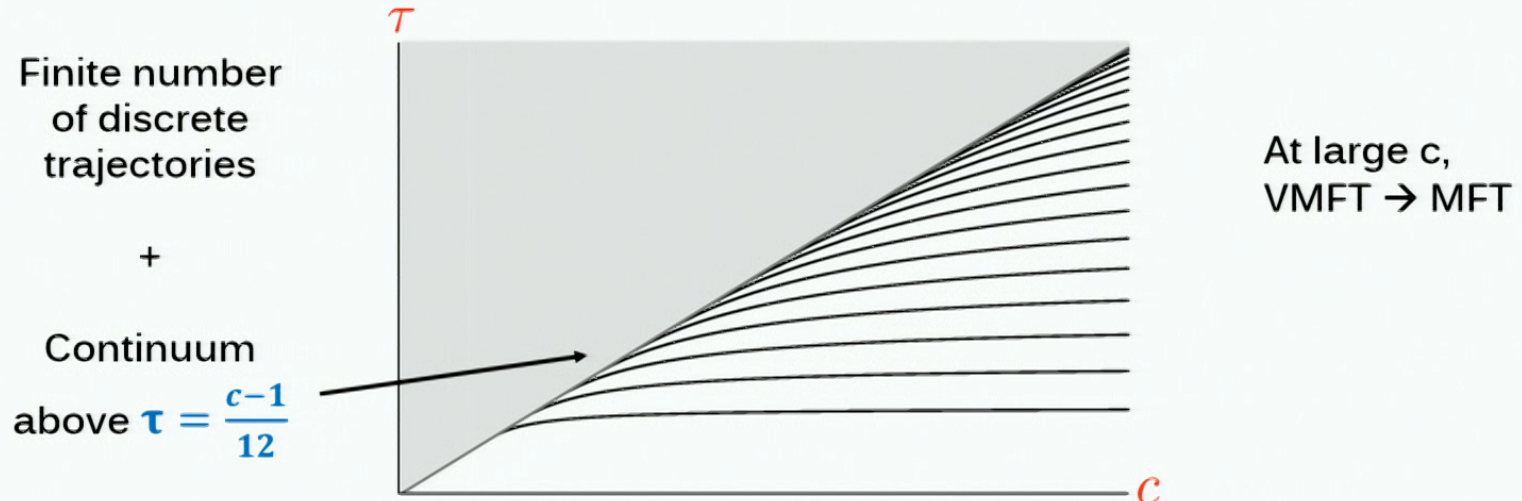
ℓ = Angular momentum

Bound State Spectrum of 3D Gravity

“Virasoro Mean Field Theory”: the complete contribution to composite operator data ($O_1 O_2$) from the Virasoro vacuum module = 1, T , $[TT]$,

Bound State Spectrum of 3D Gravity

“Virasoro Mean Field Theory”: the complete contribution to composite operator data ($O_1 O_2$) from the Virasoro vacuum module = $1, T, [TT], \dots$



Bound State Spectrum of 3D Gravity

The computation uses the [Virasoro crossing kernel](#), computed by Ponsot + Teschner.

If we include operators besides the vacuum and its descendants, the trajectories move.

However, in analogy with $d > 2$, all trajectories asymptote to [VMFT](#) at large spin!

Comparison to $d > 2$

$d > 2$	$d = 2$
Mean Field Theory	Virasoro Mean Field Theory
Infinite tower of discrete composites	Finite tower of discrete composites + continuum

Comparison to $d > 2$

$d > 2$	$d = 2$
Mean Field Theory	Virasoro Mean Field Theory
Infinite tower of discrete composites	Finite tower of discrete composites + continuum
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Large spin universality: MFT	Large spin universality: VMFT
$1/c$ effects not resumable, non-universal (i.e. T dynamics are theory-dependent)	$1/c$ effects resummed, universal

Bound State Spectrum of 3D Gravity

$$\mathcal{L} = R + 2\Lambda + (\partial\phi_1)^2 + m_1^2\phi_1^2 + (\partial\phi_2)^2 + m_2^2\phi_2^2$$

The result immediately translates into 3D gravity using the AdS/CFT dictionary.

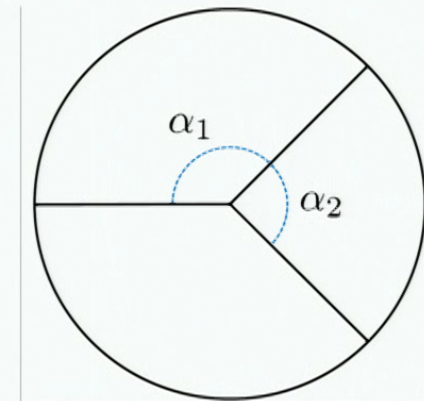
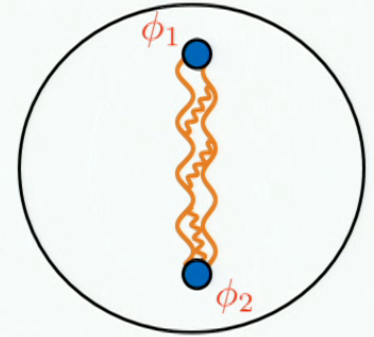
$$m_i^2 = \Delta_i(\Delta_i - 2) , \quad 1 + 6Q^2 = \frac{3}{2G_N}$$

Incorporates **quantum gravitational backreaction** in AdS_3 .

Finite cutoff on discrete tower \longleftrightarrow Black hole threshold

Addition of momenta \longleftrightarrow Addition of deficit angles

$$\Theta_{12} = \Theta_1 + \Theta_2 \propto \alpha_1 + \alpha_2$$



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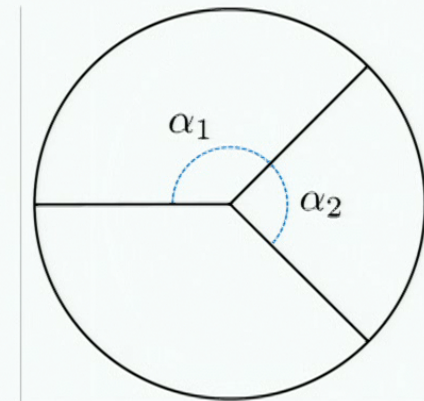
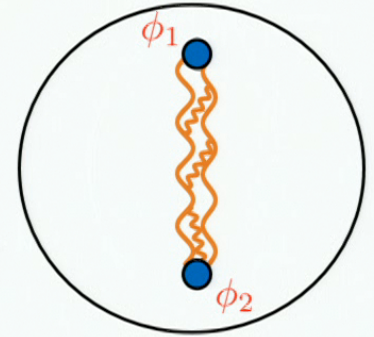
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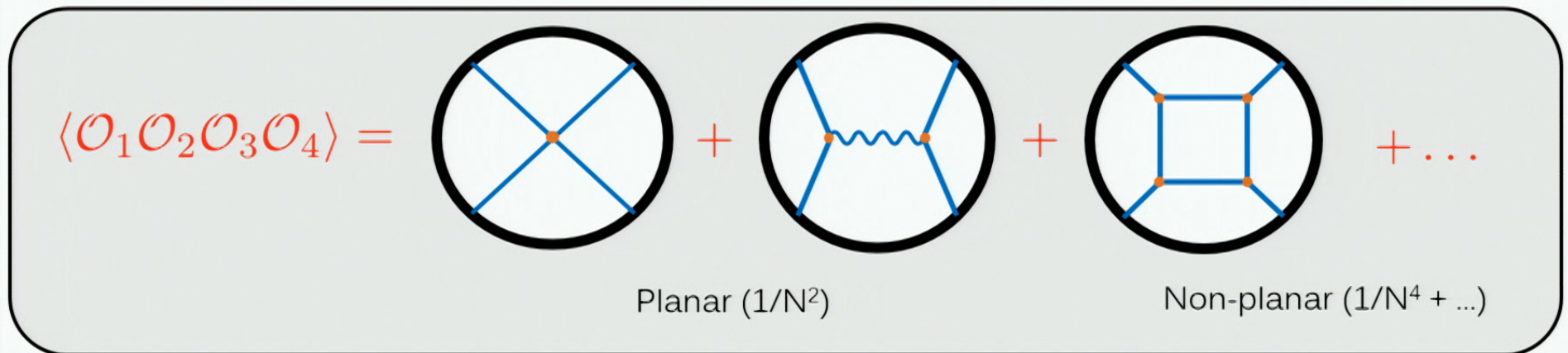
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The conformal bootstrap typically constrains CFT correlation functions.

AdS scattering amplitudes \longleftrightarrow CFT correlation functions

Loop expansion in AdS \longleftrightarrow $1/N$ expansion in CFT

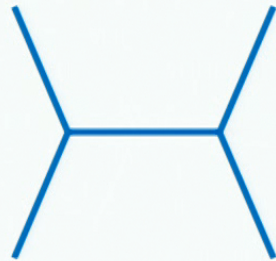


S-matrices in flat space are full of rich mathematics and physics.

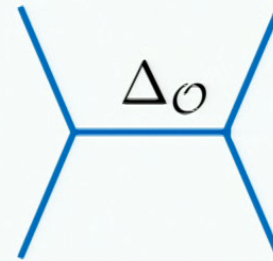
What happens in curved spacetimes? AdS? dS? Cosmological spacetimes?

Quite a bit is known about tree-level.

CFT:

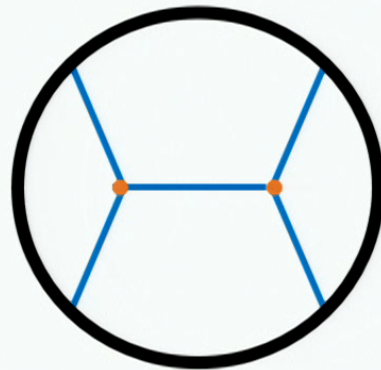


$$= \sum_{\mathcal{O}}$$



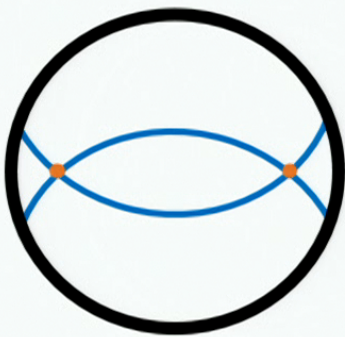
Conformal partial
wave

AdS:

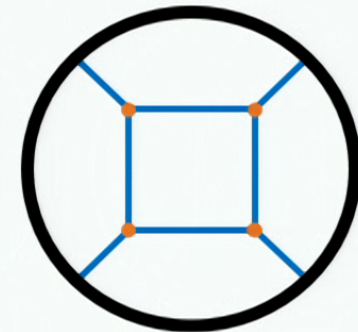
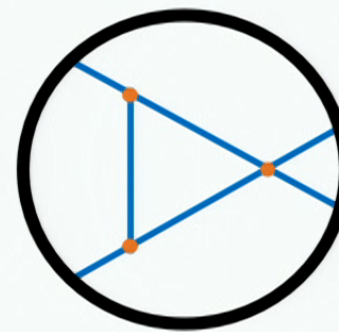
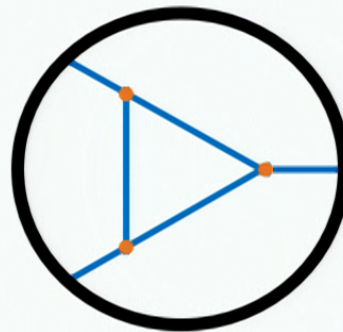


Much less is known about loop level.

Yes:



No:



...

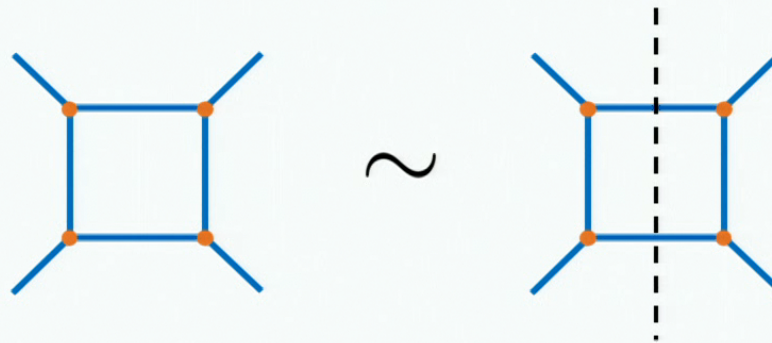
This is “just” perturbation theory!

In the world of amplitudes, the dominant paradigm is that of “**unitarity methods**”.
Recall the optical theorem for an S-matrix:

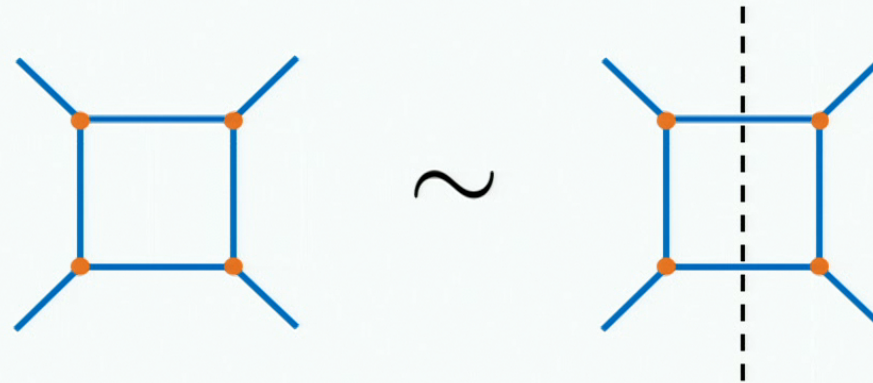
$$S = \mathbf{1} + iT \quad \Longrightarrow \quad \text{Disc}(T) = T^\dagger T$$

Unitarity of S

Important: this buys you one order in perturbation theory. e.g. at 1-loop,

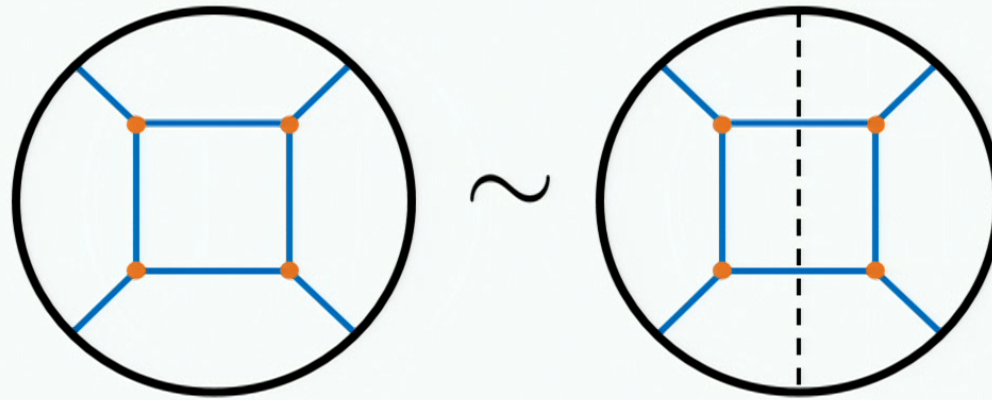


Unitarity method = constructing loop-level amplitudes from low-order ones by cutting.



Basic idea: A Lagrangian defines the set of tree-level amplitudes, so from these, one must be able to construct the S-matrix to all orders in perturbation theory.

“**AdS unitarity method**”: a prescription for constructing loop-level AdS amplitudes from OPE data of lower-order ones.

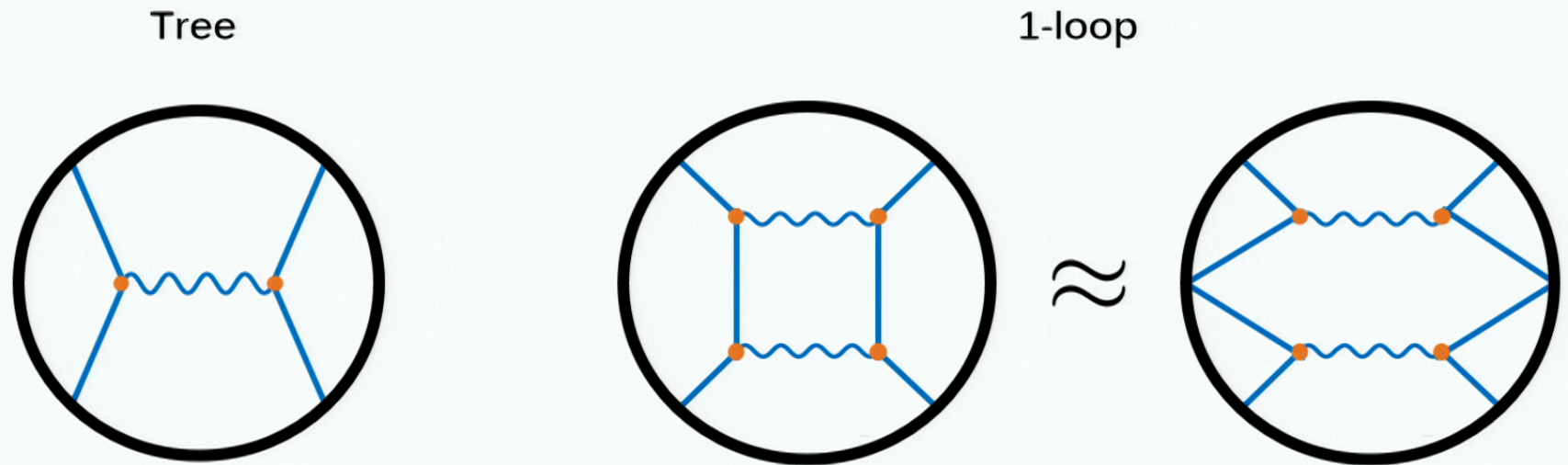


This implements the basic maxim of unitarity methods, but with a twist: AdS amplitudes are hard to deal with directly, we reconstruct them from the CFT instead.

[Aharony, Alday, Bissi, EP]

What we do, in words: “Glue together” CFT data at leading order in $1/N$ (dual to AdS tree amplitudes) to compute the data at lower orders in $1/N$ (dual to AdS loop amplitudes).

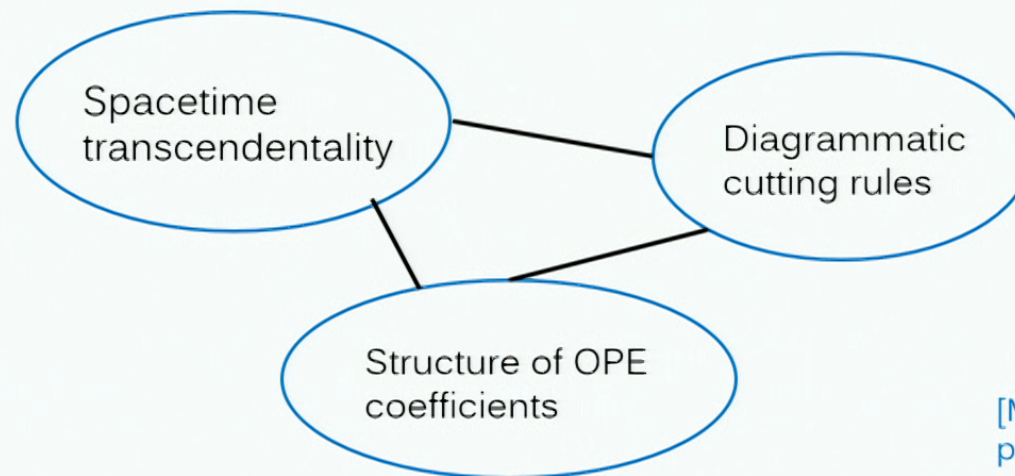
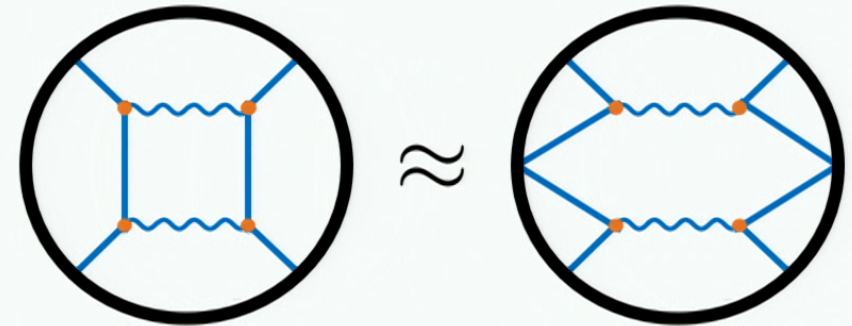
The gluing takes place in the coefficients of the conformal partial wave decomposition.



Back to the bulk

Emboldened, we have returned to the bulk, to discover clear parallels with flat space methods:

- **Cutting** = put internal propagators **on-shell**
- **Gluing** = reconstructing loops from its cuts
- **Transcendentality** properties in spacetime can be read off from diagrammatic cutting rules



[Meltzer, EP, Sivaramakrishnan (in progress)]

One of the most beautiful – and elemental – aspects of string/M-theory is that they predict specific corrections to general relativity.

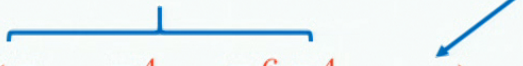
An experimentalist might rightfully ask: What are they?

In M-theory,

$$S_{\text{M-theory}} \sim \int d^{11}x \sqrt{g} (R + R^4 + D^6 R^4 + \dots)$$

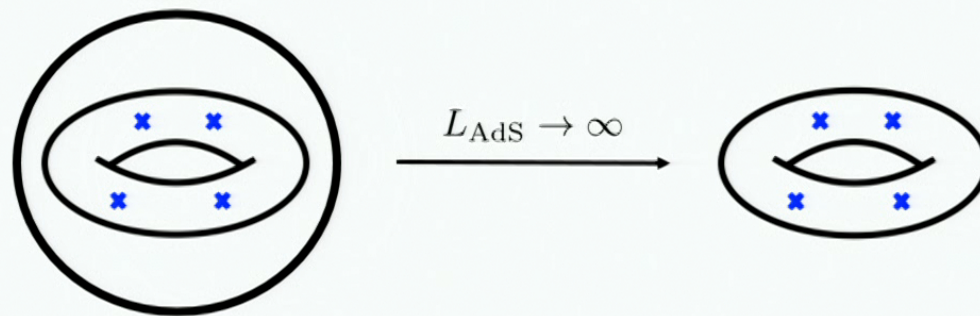
Known (fixed by SUSY)

Unknown



A new approach to string perturbation theory

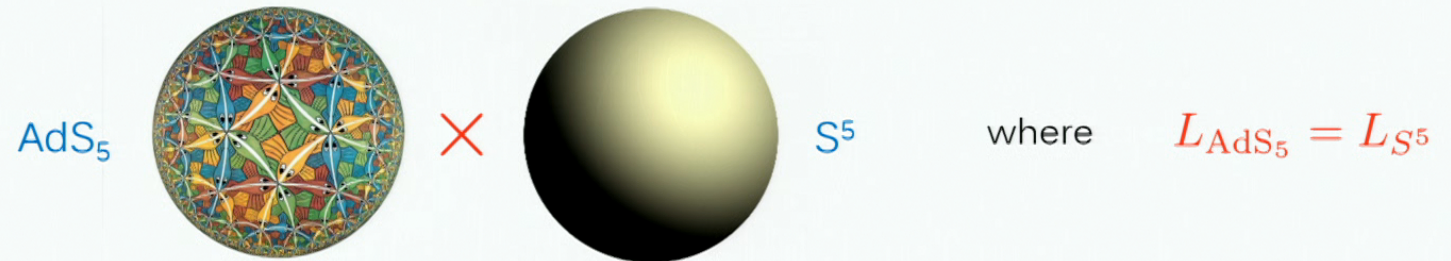
- 1) Holographically compute the one-loop amplitude for strings in AdS, as a nonplanar correlator in a dual CFT.
- 2) Take a “flat space limit”



[Alday, Bissi, EP]

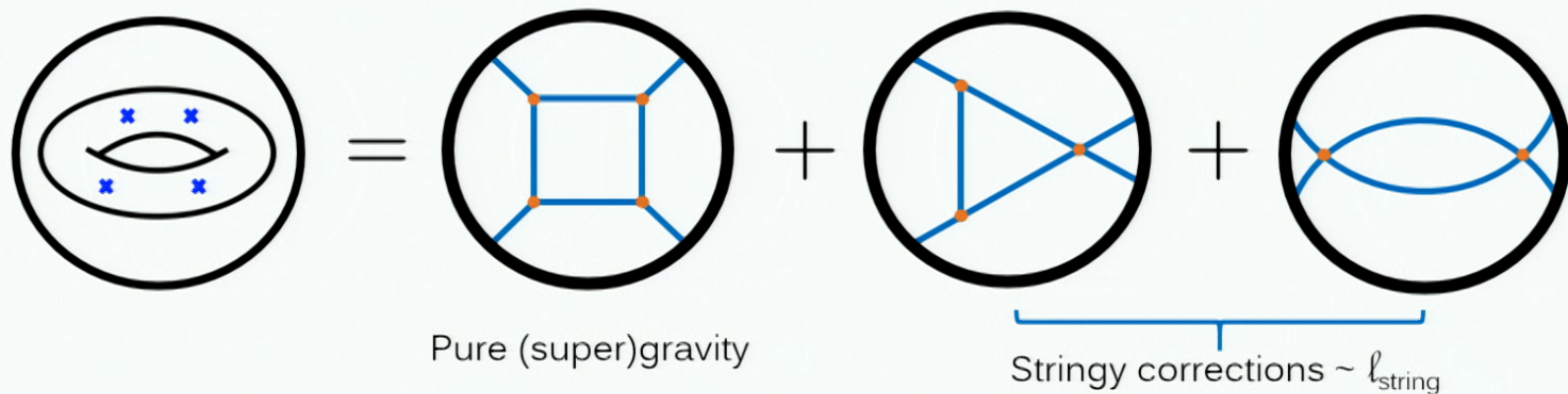
Application to string theory

The prototypical CFT with a string dual is 4d maximally supersymmetric Yang-Mills:

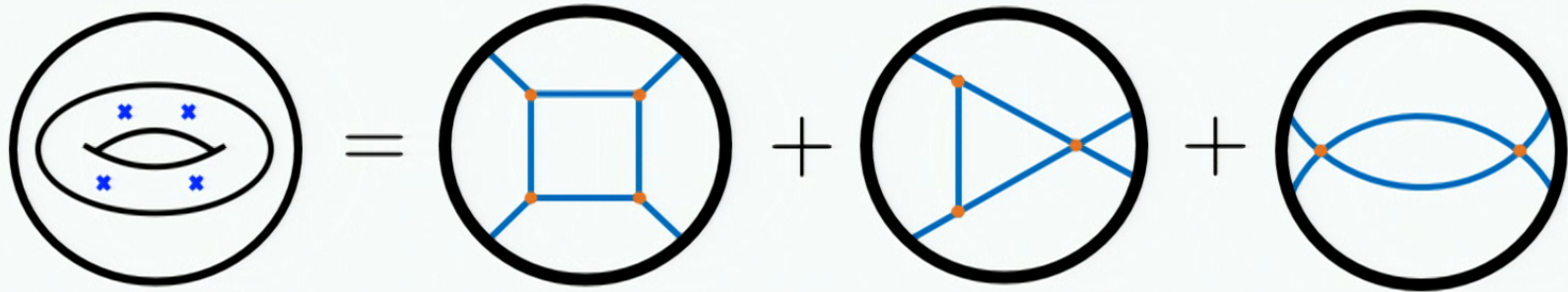


We compute the leading non-planar correction to a four-point function.

In practice, we take a low-energy limit = CFT strong coupling expansion.

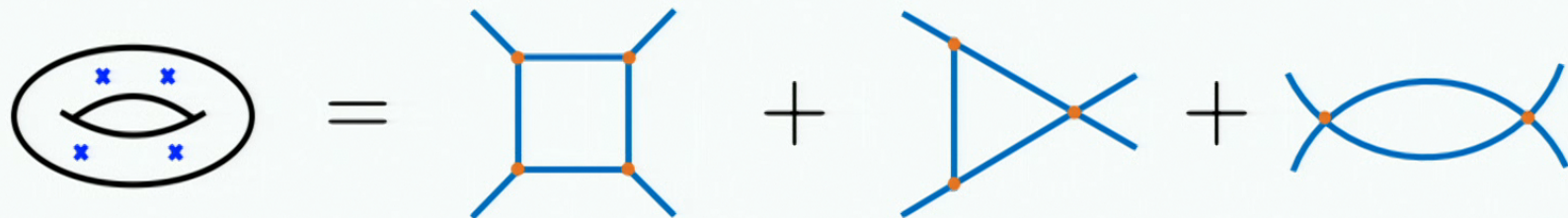


Application to string theory



Compute these diagrams using the strong-coupling expansion of the CFT.

Flat space limit \rightarrow Low-energy expansion of the genus-one string amplitude in **10d** flat space.



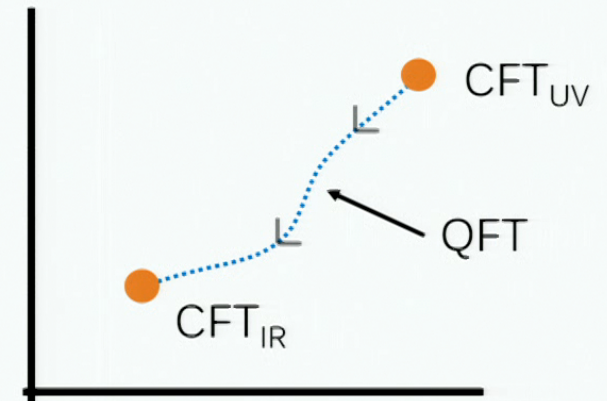
This matches the first several terms in genus-one string perturbation theory!

Analytic methods in conformal field theory...

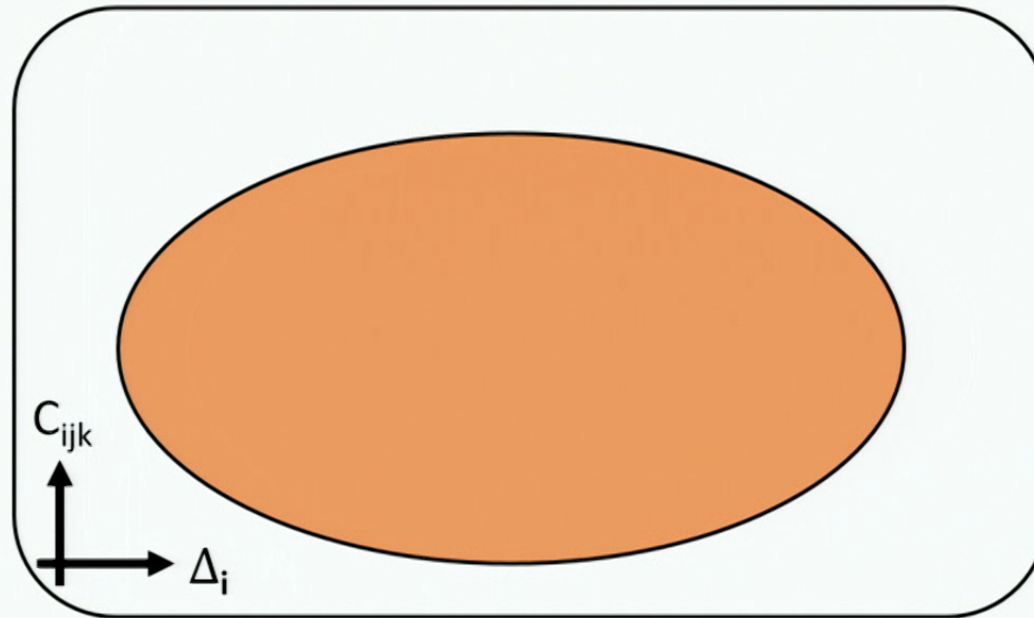
Goal: develop new analytic tools for exploring the **space** and **properties** of conformal field theories, by **extending the conformal bootstrap**.

- Seek **new fundamental constraints**: on vacuum correlators, and on other observables.

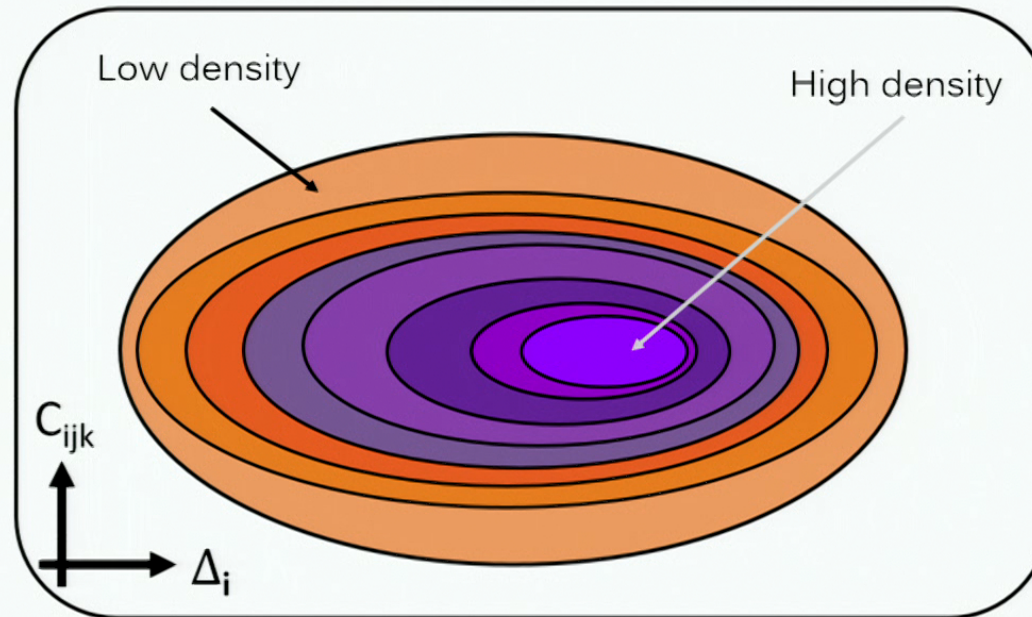
$$\sum_{\mathcal{O}} \text{[diagram of two vertices connected by a line labeled } \mathcal{O} \text{]} = \sum_{\mathcal{O}'} \text{[diagram of a single vertex with four external lines labeled } \mathcal{O}' \text{]}$$



The conformal bootstrap generates plots like this...



...but not like this:



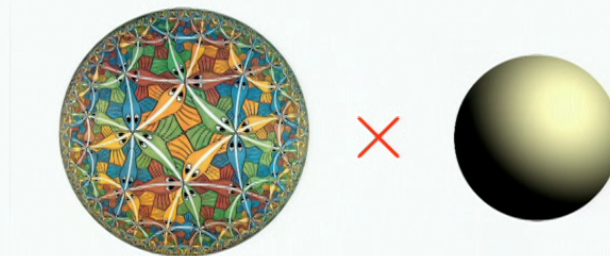
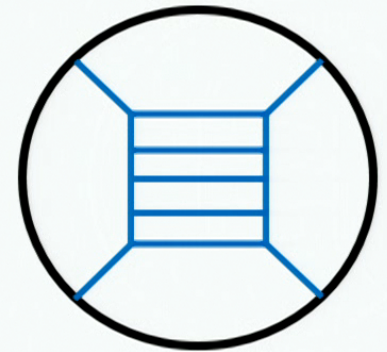
Can we make a **contour map** of theory space?

Can we give existence proofs/"proofs" for CFTs using crossing and nothing/little else? e.g. statistically?

... in the context of the AdS/CFT Correspondence

Goal: use analytic **CFT computations** to probe quantum gravity and string theory observables, coupled with direct bulk computation where possible.

- **All-order amplitudes**
 - Planck-scale scattering, scattering of/near black holes
- Construct **string/M-theory S-matrices**
- Bootstrap the **landscape of AdS string vacua**
 - Are there bounds on the shapes and sizes of extra dimensions?



Final thoughts

Tools for correlators in QFT/CFT have extremely **broad use**, including in cosmological correlations, quantum criticality in condensed matter, ...

AdS/**curved space amplitudes** are useful warmup/toy model for **cosmological** case, **astrophysical** scattering near black holes

It also seems vitally important to compute **basic observables of string/M-theory**.

The conformal bootstrap and holography are just getting started.