Title: Entropic uncertainty relations for quantum-information scrambling

Speakers: Nicole Yunger Halpern

Series: Perimeter Institute Quantum Discussions

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Abstract: How violently do two quantum operators disagree? Different subfields of physics feature different notions of incompatibility: i) In quantum information theory, uncertainty relations are cast in terms of entropies. These entropic uncertainty relations constrain measurement outcomes. ii) Condensed matter and high-energy physics feature interacting quantum many-body systems, such as spin chains. A local perturbation, such as a Pauli operator on one side of a chain, preads through many-body entanglement. The perturbation comes to overlap, and to disagree, with probes localized on the opposite side of the system. This disagreement signals that quantum information about the perturbation has scrambled, or become hidden in highly nonlocal correlations. I will unite these two notions of quantum operator disagreement, presenting an entropic uncertainty relation for quantum-information scrambling. The entropies are of distributions over weak and strong measurements' possible outcomes. The uncertainty bound strengthens when a spin chain scrambles in numerical simulations. Hence the subfields - quantum information, condensed matter, and high-energy physics - can agree about when quantum operations disagree. Our relation can be tested experimentally with superconducting qubits, trapped ions, and quantum dots.

NYH, Bartolotta, and Pollack, accepted by Comms. Phys. (in press) arXiv:1806.04147.

Uncertainty relations for quantum-information scrambling

Nicole Yunger Halpern

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QI Seminar, Perimeter Institute, 4/18/19

HOW MUCH DO TWO QUANTUM OPERATORS DISAGREE WITH EACH OTHER?

$\hat{A}$  $\hat{B}$

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2 quantum physicists

↓

2 ways of answering

HOW MUCH DO TWO QUANTUM OPERATORS DISAGREE WITH EACH OTHER?

(1) Pure quantum information theorist

- Stick *A* and *B* in an uncertainty relation.

$$\Delta A \, \Delta B \geq \frac{1}{2} \, | \langle [A, B] \rangle |$$

- The higher the uncertainty bound, the worse the disagreement.
  - (I'm dropping hats from operators.)

(2) Condensed-matter/high-energy physicist

(thinking about black holes, thermalization, holography, ...)

- Interacting quantum many-body system

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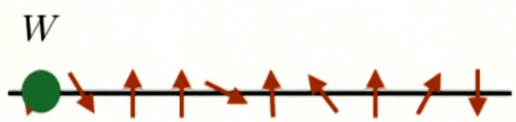
- Interacting quantum many-body system
  - Inject information locally



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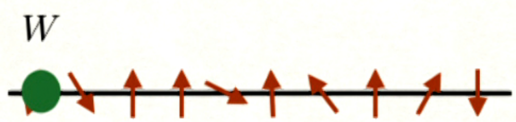
  - Inject information locally
- Interactions, especially nonlocal $\longrightarrow U$



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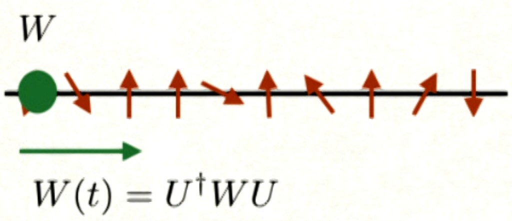

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- Information about W spreads through many-body entanglement.
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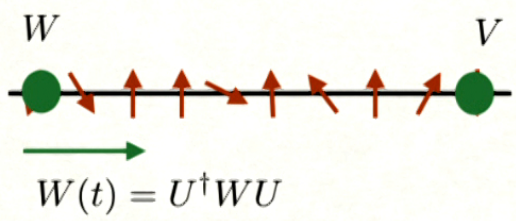

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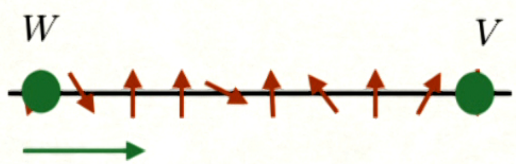
- Information scrambles when *W(t)* quits agreeing with *V*.



$W(t) = U^\dagger W U$

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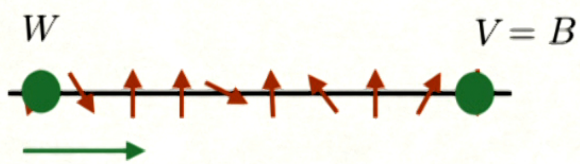

$$W(t) = U^\dagger W U$$
$$= A$$

$\hat{A}$

$\hat{B}$

could lead to

Pure quantum-information physicist

Condensed-matter/high-energy physicist

Desired

Reconciliation of the two notions of operator disagreement,

uncertainty relations and quantum-information scrambling

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- Uncertainty relation for scrambling
- Uncertainty bound tightens when system scrambles

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- Uncertainty relation for scrambling
- Uncertainty bound tightens when system scrambles
- Key: **quasiprobability distribution**
    - Like probability
    - But can behave nonclassically

Where we're headed

- Uncertainty relations 

↑ Entropic

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- Quantum-information scrambling 

- Entropic uncertainty relation for quantum-information scrambling 

Where we're headed

- **Uncertainty relations** 

  ↑ **Entropic**

- **Quantum-information scrambling** 

- **Entropic uncertainty relation for quantum-information scrambling** 
  - Construct via intuition
    - Behavior

Old-fashioned uncertainty relation

Old-fashioned uncertainty relation

Intuition: Heisenberg, Z. Phys. **43**, 172 (1927).

Robertson, Phys. Rev. **34**, 163 (1929).

$$\Delta A \, \Delta B \geq \frac{1}{2} \, | \langle [A, B] \rangle |$$

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Standard deviation in state $|\psi\rangle$

Old-fashioned uncertainty relation

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Infelicities

- Bound depends on a particular quantum state

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<u>Infelicities</u>

- Bound depends on a particular quantum state
- Variances depend on the eigenvalues of *A* and *B*
- Not maximally amenable to quantum information theory

Modern (entropic) uncertainty relations

Review: Coles, Berta, Tomamichel, and Wehner,
Rev. Mod. Phys. **89**, 015002 (2017).

Exemplar entropic uncertainty relation

- Maassen and Uffink, PRL **60**, 1103 (1988).

$$H(A) + H(B) \geq -\log c$$

Left-hand side

- $A = \sum_a a|a\rangle\langle a|$

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- **Shannon entropy:** $H(A) := -\sum_a p_a \log p_a$ ←
  - Analogous for $B = \sum_b b|b\rangle\langle b|$

- Uncertainty about measurement outcome
- Operational significances from information theory (e.g., data compression)

Exemplar entropic uncertainty relation

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<u>Right-hand side</u>

Exemplar entropic uncertainty relation

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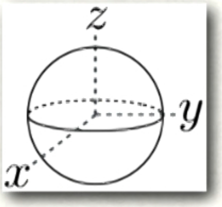
Right-hand side

- **Maximum overlap**
  - $c := \max_{a,b} |\langle a | b \rangle|^2$
- Best-case probability that, if you prepare a $|b\rangle$, then measure A, the outcome *a* matches your prediction
- Independent of any quantum state

Example application

$$H(A) + H(B) \geq -\log c$$

- Qubit (2-level quantum system)



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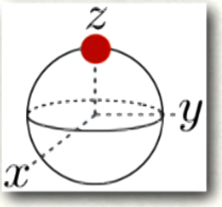

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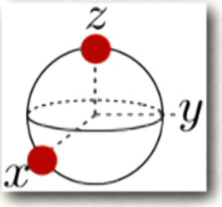

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- $B = \sigma^x = \sum_{\pm} \pm |x\pm\rangle\langle x\pm|$

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  - No matter which σ^z eigenstate you prepare, you never have any idea which state the σ^x measurement will output.
  - σ^z and σ^x "fail maximally to agree."

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⇒ Uncertainty bound large

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$$H(\sigma^x) = \log 2$$

Example application

$$H(A) + H(B) \geq -\log c$$

<u>Left-hand side</u>

- If you minimize one of the *H*'s, the other *H* maximizes.

$$\rho = |z+\rangle\langle z+| \quad \Rightarrow \quad H(\sigma^z) = 0$$

$$H(\sigma^x) = \log 2$$

- So the left-hand side is large, as the right-hand side is.

Quantum-information scrambling

Setup

Quantum many-body system

Setup

<u>Quantum many-body system</u>

- Boundary dual of black hole 

→ Black-hole-information paradox, quantum gravity, space-time from entanglement, …

Setup

Quantum many-body system

- Boundary dual of black hole 

→ Black-hole-information paradox, quantum gravity, space-time from entanglement, …

- Spin chain in condensed matter 

Evolution

Local operators

- W, V

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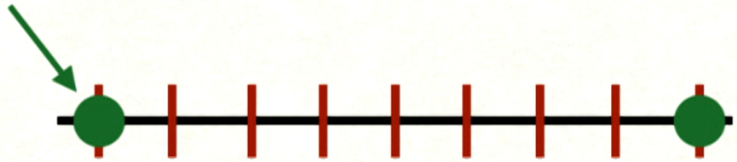
$$\mathcal{W} = \sigma_z \otimes \mathbf{1}^{\otimes(N-1)}$$



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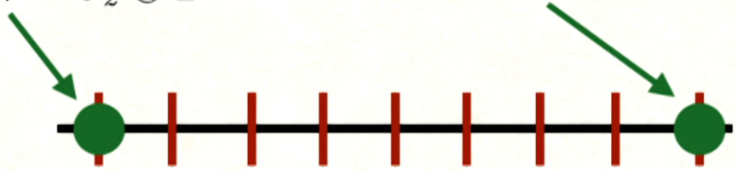
$$\mathcal{W} = \sigma_z \otimes \mathbf{1}^{\otimes(N-1)} \qquad V = \mathbf{1}^{\otimes(N-1)} \otimes \sigma_x$$



Local operators

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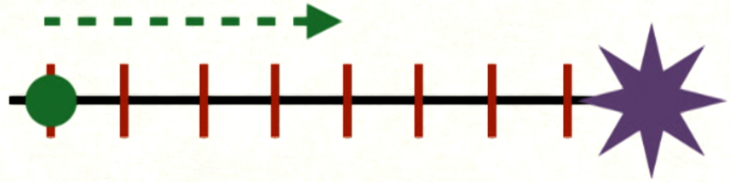
  - Commute
- Heisenberg picture: $W(t) := U^{\dagger} W U$



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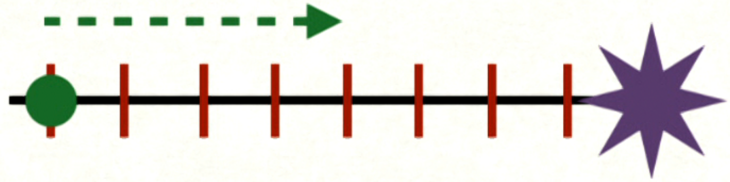

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$$\mathbf{Tr}\left(\left[W(t), V\right]^{\dagger}\left[W(t), V\right]\rho\right)$$

- Information scrambles as the commutator grows.

$$\mathrm{Tr}\left(\left[W(t),V\right]^{\dagger}\left[W(t),V\right]\rho\right) \propto \boxed{\mathrm{Tr}\left(W^{\dagger}(t)V^{\dagger}W(t)V\rho\right)}$$

- Information scrambles as the commutator grows.

Out-of-time-ordered-correlator (OTOC)

$$F(t) := \langle W^\dagger(t) V^\dagger W(t) V \rangle \equiv \mathrm{Tr}\left(W^\dagger(t) V^\dagger W(t) V \rho \right)$$

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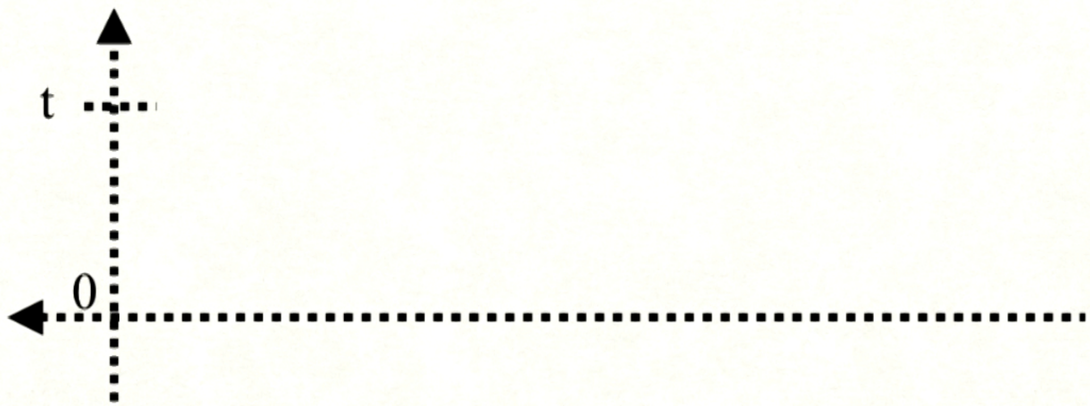

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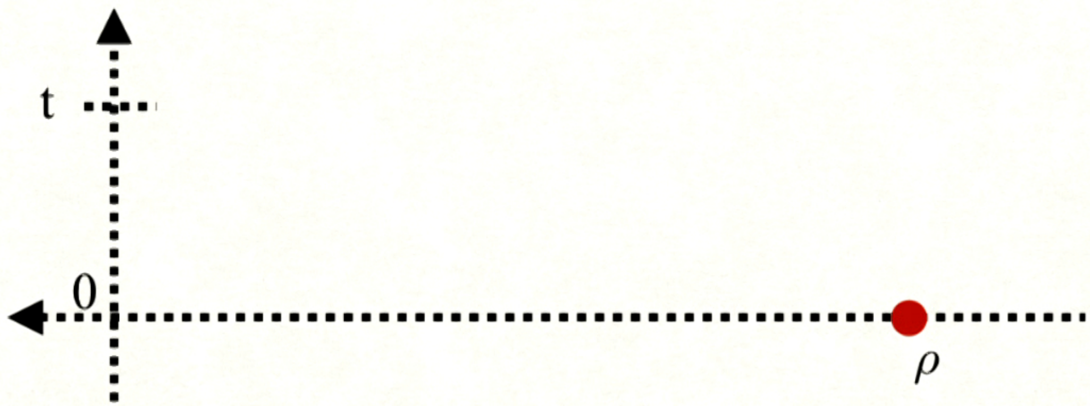

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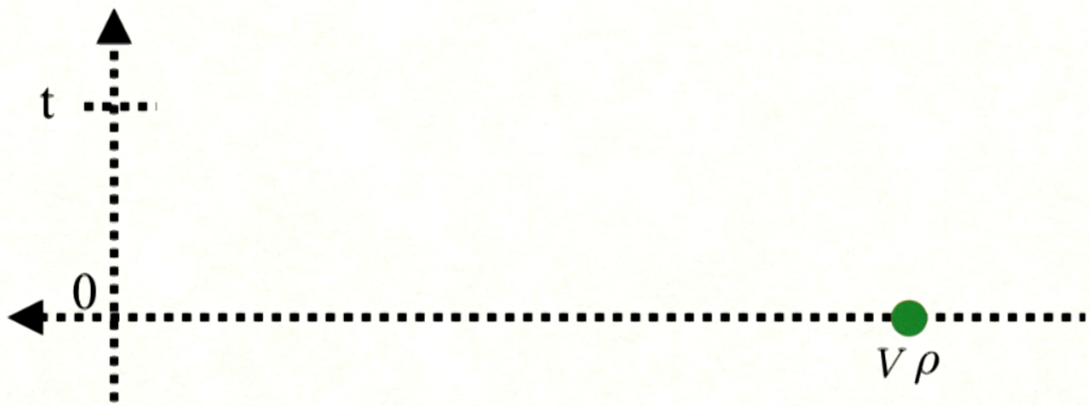

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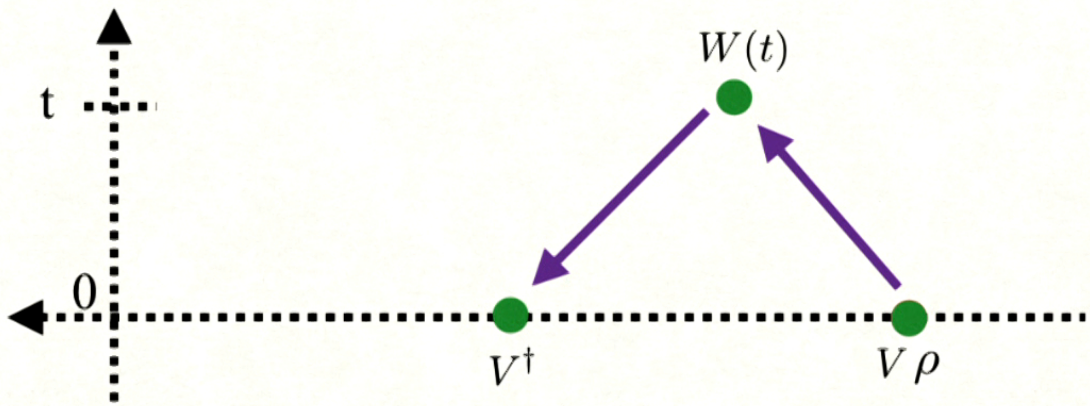

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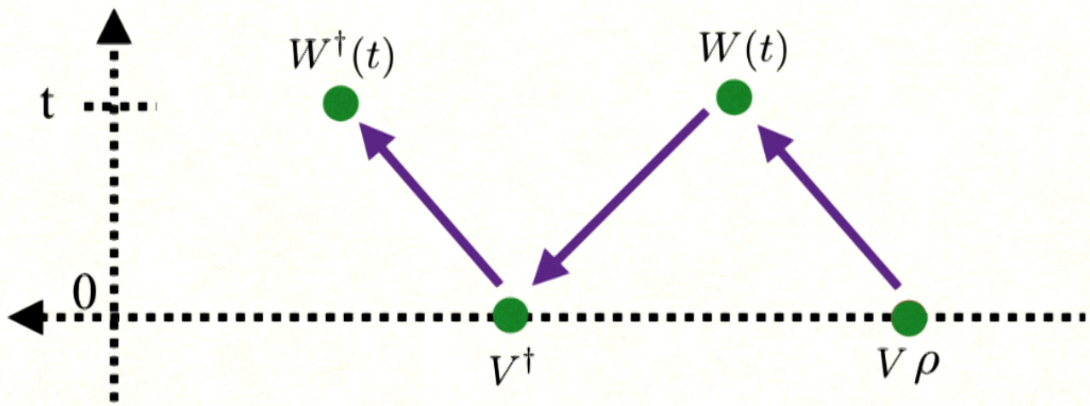

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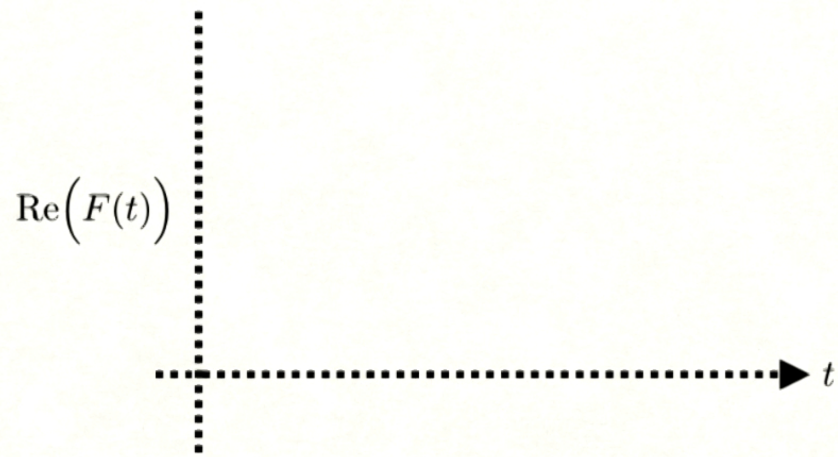

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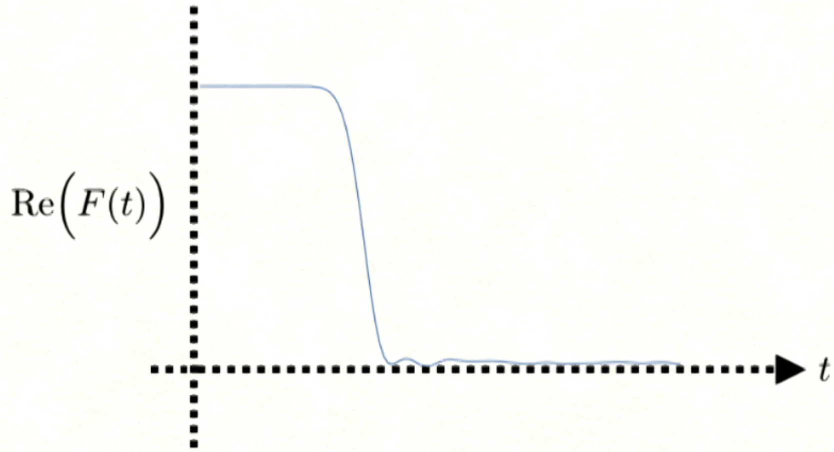

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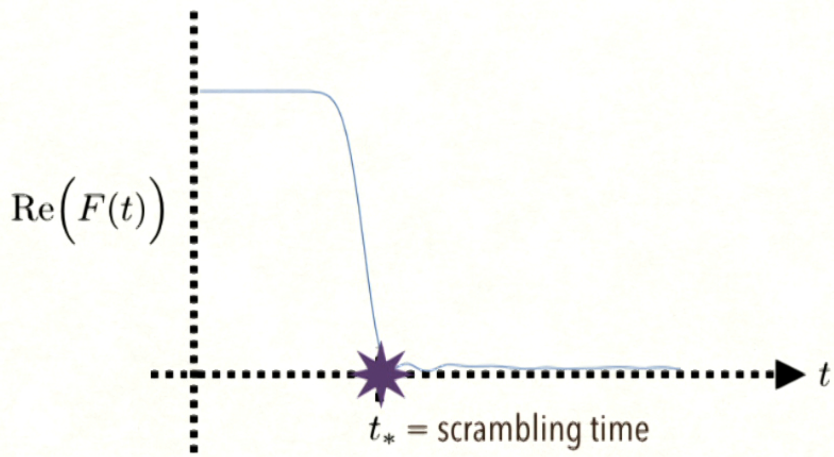

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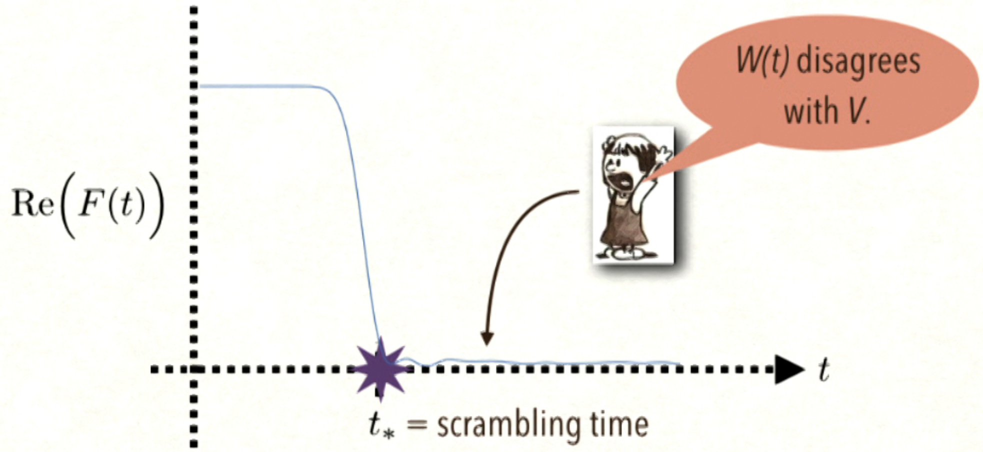

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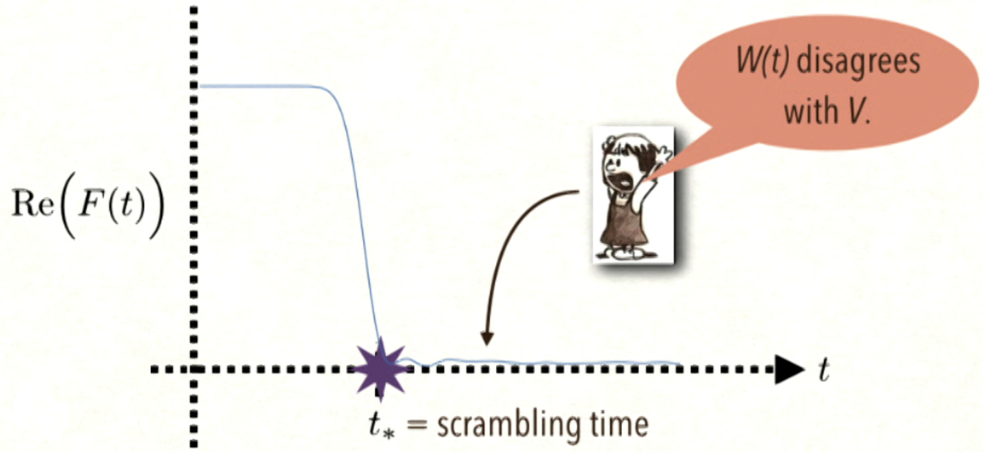

$\langle W(t) V \rangle$?

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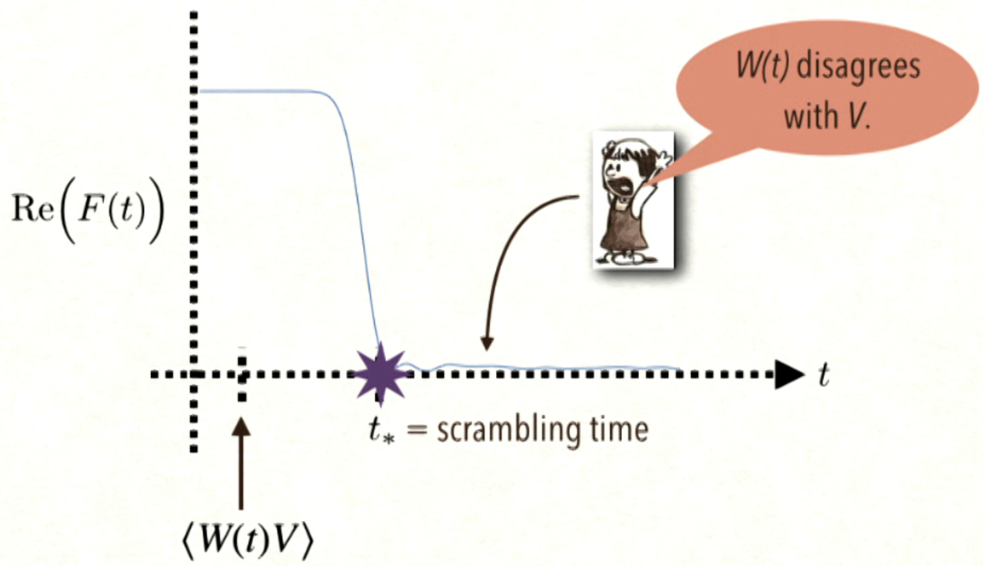

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  operator disagreement, ...

How can we reconcile these two notions of quantum operator disagreement,

entropic uncertainty relations

and

quantum-information scrambling

?

Guess

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- The resulting uncertainty relation bears no signature of scrambling.

Clue

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- The entropic uncertainty relation $H(A) + H(B) \geq -\log c$

Clue

- The entropic uncertainty relation $H(A) + H(B) \geq -\log c$ replaced an uncertainty relation $\Delta A \, \Delta B \geq \frac{1}{2} |\langle [A, B] \rangle|$ that contains a commutator.

Clue

- The entropic uncertainty relation $H(A) + H(B) \geq -\log c$ replaced an uncertainty relation $\Delta A \, \Delta B \geq \frac{1}{2} \, | \langle [A, B] \rangle |$ that contains a commutator.
- The OTOC comes from a commutator's squared magnitude, $\left\langle \, | W(t), V \, |^2 \right\rangle$.

→ We should "square" the *A*, the *B*, and the c.

Clue

How can we "square" the left-hand side?

- $H(A) + H(B) \geq -\log c$

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- $A \sim W(t)V$
- A must $\neq B$, so try $B \sim VW(t)$
    - Consistent with a semiclassical interpretation of the OTOC in terms of chaos
        - Product isn't Hermitian, so not sure how to measure it
        - But run with the idea for now. A resolution might present itself.

Clue

How can we "square" the right-hand side?

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Clue

How can we "square" the right-hand side?

- $H(A) + H(B) \geq -\log c$
- $c = |\langle a | b \rangle|^2$ contains 2 inner products.
- So c should come to contain 4 inner products.

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- So *c* should come to contain 4 inner products.

$\downarrow$

- Such an object is known to characterize scrambling!

Decomposing the OTOC

$$F(t) = \text{Tr}\left(W^\dagger(t)\, V^\dagger\, W(t)\, V \rho\right)$$

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$$\sum_{v_1} v_1\, \Pi_{v_1}^V$$

$v_1 \leftarrow \pm 1$

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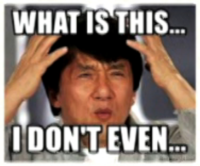
~product of 4 inner products

$$\mathrm{Tr}\left(\Pi_{w_2}^{W(t)}\,\Pi_{v_2}^{V}\,\Pi_{w_1}^{W(t)}\,\Pi_{v_1}^{V}\,\rho\right) \longrightarrow$$



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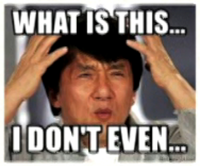

$\longrightarrow$ quasiprobability

Background: quasiprobabilities

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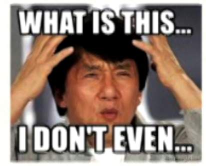

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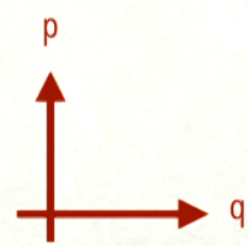

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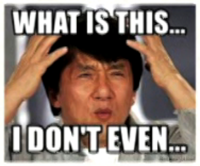

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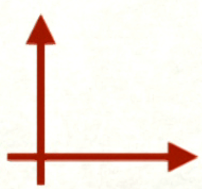
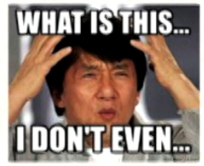
- Relax different axioms $\longrightarrow$ different quasiprobability representations of quantum state



$\mathrm{Tr}\left(\Pi^{W(t)}_{w_2}\,\Pi^{V}_{v_2}\,\Pi^{W(t)}_{w_1}\,\Pi^{V}_{v_1}\,\rho\right) \longrightarrow$ 

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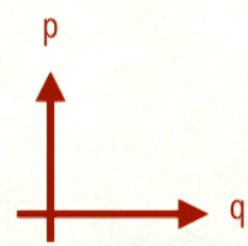
- <u>Example</u>: Wigner function



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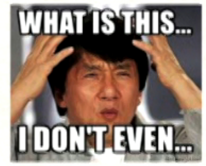

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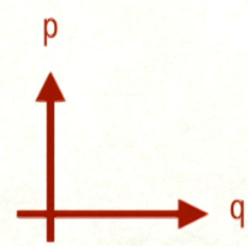
- <u>Example</u>: Wigner function
  - Negativity $\longrightarrow$
  - Can signal nonclassical physics



$\mathrm{Tr}\left(\Pi_{w_2}^{W(t)}\,\Pi_{v_2}^{V}\,\Pi_{w_1}^{W(t)}\,\Pi_{v_1}^{V}\,\rho\right)$ as the quasiprobability behind the OTOC

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- Recall, $F(t) \;=\; \sum_{v_1,w_1,v_2,w_2} w_2^* v_2^* w_1 v_1 \,\mathrm{Tr}\left(\Pi^{W(t)}_{w_2}\,\Pi^{V}_{v_2}\,\Pi^{W(t)}_{w_1}\,\Pi^{V}_{v_1}\,\rho\right).$

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OTOC = average over quasiprobability distribution

Contains more information about scrambling than the OTOC does

Theoretical and experimental applications of the OTOC quasiprobability

(1) NYH, Phys. Rev. A **95**, 012120 (2017).

(2) NYH, Swingle, and Dressel, Phys. Rev. A **97**, 042105 (2018).

(3) NYH, Bartolotta, and Pollack, accepted by Comms. Phys. (in press) arXiv:1806.04147.

(4) González Alonso, NYH, and Dressel, Phys. Rev. Lett. **122**, 040404 (2019).

(5) Swingle and NYH, Phys. Rev. A **97**, 062113 (2018).

(6) Dressel, Gonzàlez Alonso, Waegell, and NYH, Phys. Rev. A **98**, 012132 (2018).

(7) D.R.M. Arvidsson-Shukur, NYH, Lepage, Lasek, Barnes, and Lloyd, arXiv:1903.02563 (2019).

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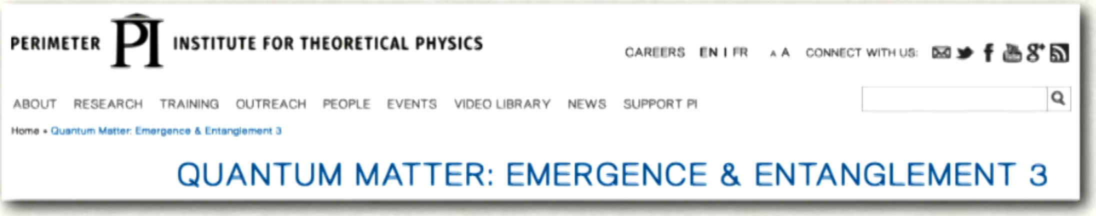
For more information



Tues., 4:00-4:45, Bob Room

Quasiprobability behind the OTOC

- Can be measured experimentally, with **weak measurements**

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Where has our clue brought us?

Conjecture

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Tightens the uncertainty bound around the scrambling time

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Can we prove it?

Proof sketch

(1) Find a generalized entropic uncertainty relation.

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- For generalized measurements (positive operator-valued measures)

Proof sketch

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Reverse process

Proof sketch

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- Taylor-approximate in the weak-coupling constant g, use the monotonicity of the Schatten p-norm

Entropic uncertainty relation for quantum-information scrambling

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$$H\big(VW(t)\big) + H\big(W(t)V\big)$$

$$\geq g\,(\text{classical factor})$$

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- Changes at the scrambling time, t_* ⟶
- Strengthens the uncertainty bound at the scrambling time, as you'd hope

The OTOC quasiprobability strengthens the uncertainty bound at the scrambling time.

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- Power-law quantum Ising model, nonintegrable

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- $V = \sigma_z^1, \quad W = \sigma_z^N$

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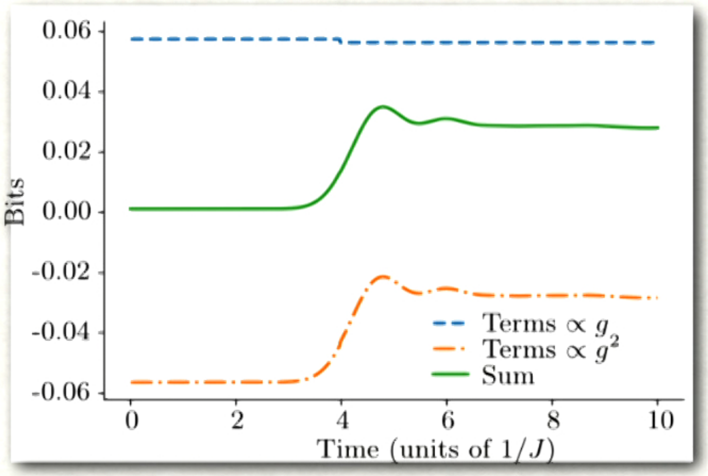

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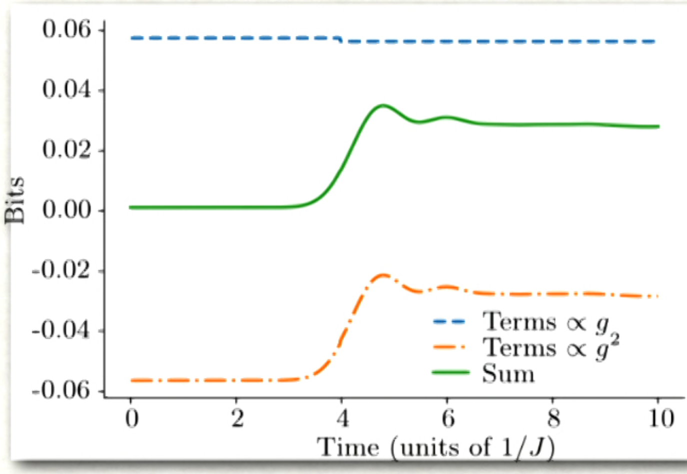

- The uncertainty bound strengthens at the scrambling time. ⟶
  - We've shown that entropic uncertainty relations  can reflect the quantum operator disagreement that underlies information scrambling. 

We've reconciled two notions of operator disagreement.


OPPORTUNITY
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Opportunities

(1) Explore the uncertainty relation for scrambling experimentally.

- Tool: weak-measurement scheme in NYH, PRA (2018) and NYH, Swingle, and Dressel, PRA (2018)
- Possible platforms: superconducting qubits, trapped ions, quantum dots, ultracold atoms, …

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- Simplification: entropic uncertainty relation for weak values

Opportunities

(2) Effects of the quasiprobability's nonclassicality on the uncertainty relation's strength.

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(3) Extend to continuous systems ⟶ black holes

Recap

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  (1) Entropic uncertainty relations 

Recap

- **2 notions of quantum operator disagreement**

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  (2) Quantum-information scrambling 

- **Entropic uncertainty relation for quantum-information scrambling** 

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- **Entropic uncertainty relation for quantum-information scrambling** 
  - Constructed via intuition
    - Behavior

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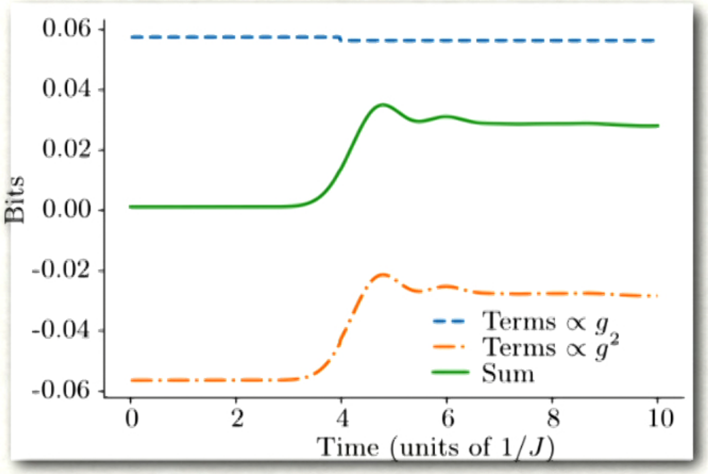
- **Entropic uncertainty relation for quantum-information scrambling** 
  - Constructed via intuition
    - Behavior
    - Experiments

Thanks for your time!



NYH, Bartolotta, and Pollack, accepted by Comms. Phys. (in press) arXiv:1806.04147.

The OTOC quasiprobability strengthens the uncertainty bound at the scrambling time.



- The uncertainty bound strengthens at the scrambling time. →
  - We've shown that entropic uncertainty relations  can reflect the quantum operator disagreement that underlies information scrambling. 

The OTOC as a signature of chaos

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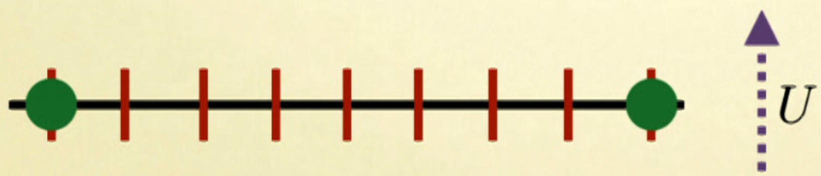

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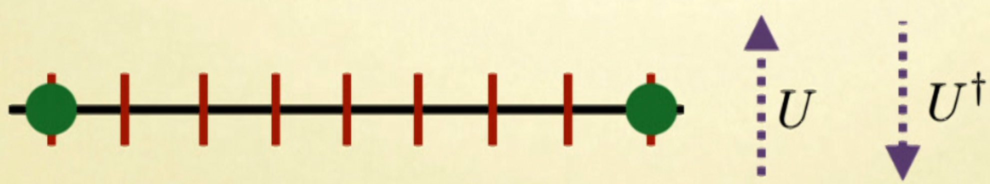

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(1) $|\psi\rangle \;\mapsto\; U^\dagger \mathcal{W} \, U \, V \, |\psi\rangle =: |\psi'\rangle$

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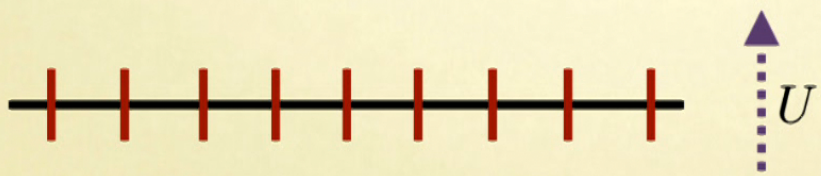

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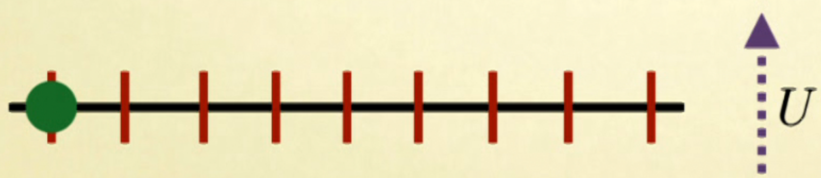

THE OTOC AS A SIGNATURE OF CHAOS

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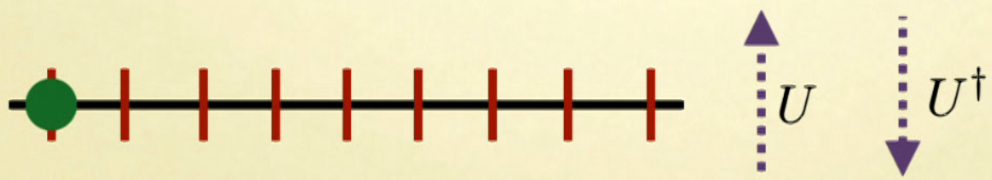

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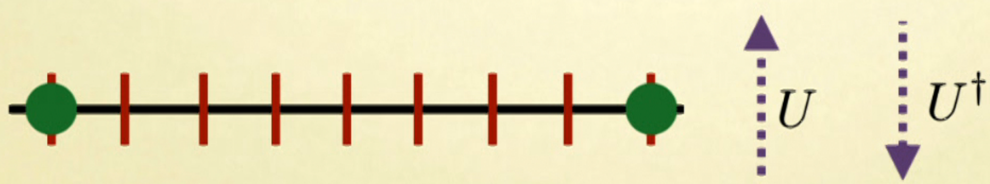

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Lyapunov-type exponent