Title: PSI 2018/2019 - Explorations in Cosmology - Lecture 6

Speakers: Kendrick Smith

Collection: PSI 2018/2019 - Explorations in Cosmology (Smith)

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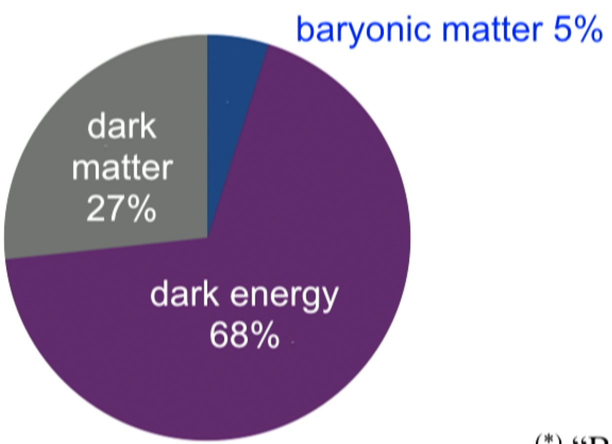
Standard Cosmological Model

Kendrick Smith

PSI 2019, Explorations in Cosmology

All current data can be fit by a 6-parameter cosmological model!

$h = 0.70 \pm 0.03$	Hubble parameter
$\Omega_b = 0.0486 \pm 0.0007$	Baryonic(*) matter abundance
$\Omega_c = 0.267 \pm 0.009$	Cold dark matter abundance
$\Delta_\zeta^2 = (2.11 \pm 0.05) \times 10^{-9}$	Initial power spectrum amplitude
$n_s = 0.967 \pm 0.004$	Spectral index
$\tau = 0.058 \pm 0.012$	CMB optical depth



(*) "Baryons" = protons + neutrons + electrons(!)

Ingredients in the standard cosmological model:

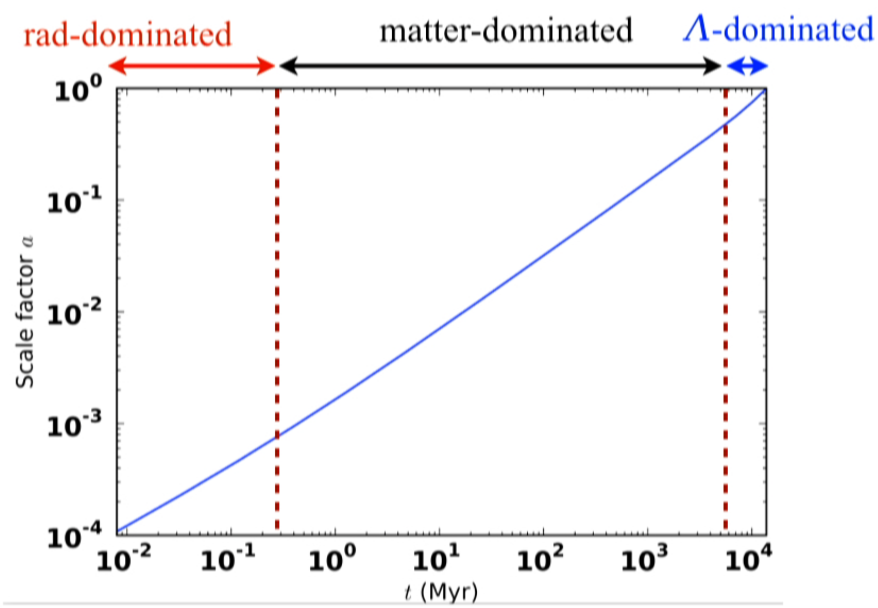
- Background metric is FRW
- Expansion history is ΛCDM
- Initial perturbations are Gaussian random
- Initial perturbations are scalar adiabatic
- Power spectrum of initial perturbations is a power law: $(k^3/2\pi^2)P(k) = \Delta_\zeta^2 (k/k_0)^{n_s - 1}$

In the next few slides, we'll describe these ingredients precisely. (Exception: "adiabatic" will involve a little hand-waving.)

"Background metric is FRW"

$$ds^2 = -dt^2 + a(t)^2\, dx^2$$

$$\frac{d\log a}{dt} = \left(\frac{8\pi G}{3}\rho_{\rm tot}\right)^{1/2} = \left(\frac{8\pi G}{3}\left(\rho_{\rm de} + \rho_{\rm m}(a) + \rho_{\rm rad}(a)\right)\right)^{1/2}$$



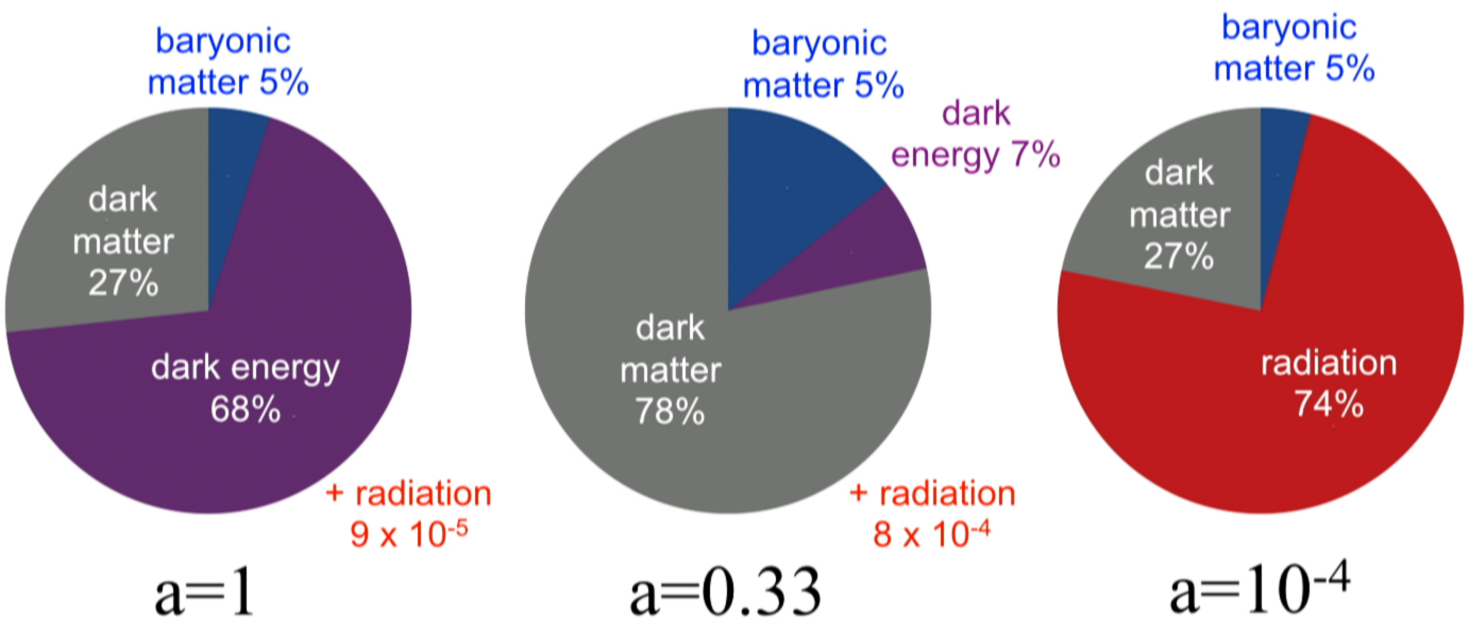
"Expansion history is ΛCDM"

Energy densities evolve with scale factor a(t):

$$\rho_{\rm de} = \text{constant}$$ dark energy (assuming it is a c.c.!)

$$\rho_{\rm m} \propto a(t)^{-3}$$ nonrelativistic matter (dark + baryonic)

$$\rho_{\rm rad} \propto a(t)^{-4}$$ relativistic particles (photons, neutrinos)



So far, the first three parameters (h, Ω_b, Ω_c) describe the expansion history.

$h = 0.70 \pm 0.03$	Hubble parameter
$\Omega_b = 0.0486 \pm 0.0007$	Baryonic matter abundance
$\Omega_c = 0.267 \pm 0.009$	Cold dark matter abundance
$\Delta_\zeta^2 = (2.11 \pm 0.05) \times 10^{-9}$	Initial power spectrum amplitude
$n_s = 0.967 \pm 0.004$	Spectral index
$\tau = 0.058 \pm 0.012$	CMB optical depth

The next two parameters (Δ_ζ^2, n_s) specify the statistics of the *initial* perturbations.

Initial conditions: at early times, the FRW metric has small perturbations.

$$ds^2 = -dt^2 + a(t)^2 e^{2\zeta(x)} dx^2$$

The field $\zeta(x)$ is called the "adiabatic curvature" or the "initial curvature". This is a Gaussian random field with power spectrum:

$$\frac{k^3}{2\pi^2} P_\zeta(k) = \Delta_\zeta^2 \left(\frac{k}{0.05 \ h \ \mathrm{Mpc}^{-1}} \right)^{n_s - 1}$$

with free parameters

$\Delta_\zeta^2 = (2.11 \pm 0.05) \times 10^{-9}$	Initial power spectrum amplitude
$n_s = 0.967 \pm 0.004$	Spectral index

Initial conditions: at early times, the FRW metric has small perturbations.

$$ds^2 = -dt^2 + a(t)^2 e^{2\zeta(x)} dx^2$$

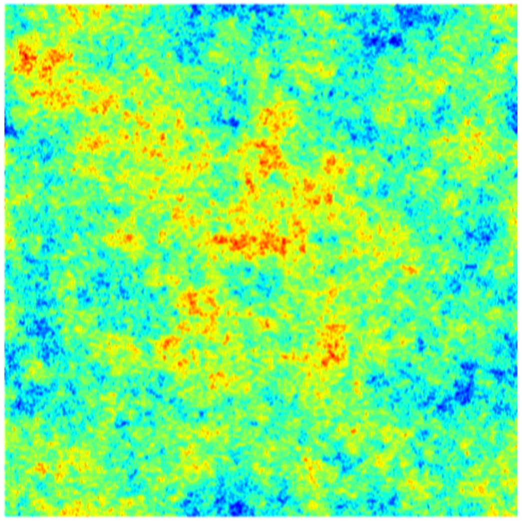
$$\frac{k^3}{2\pi^2} P_\zeta(k) = \Delta_\zeta^2 \left(\frac{k}{0.05\ h\ \mathrm{Mpc}^{-1}} \right)^{n_s - 1}$$

$\Delta_\zeta^2 = (2.11 \pm 0.05) \times 10^{-9}$

$n_s = 0.967 \pm 0.004$

Informally:

- Initial perturbations are self-similar (no preferred scale)
- Almost scale-invariant, small trend toward more power on large scales.
- Characteristic size of fluctuations is $\Delta_\zeta \sim (5 \times 10^{-5})$



"Initial perturbations are scalar adiabatic".

- "Scalar" means that there are no gravity wave perturbations in the initial metric. (Some models of inflation predict this, but so far it has not been observed.)

$$ds^2 = -dt^2 + a(t)^2 e^{2\zeta(x)}(\delta_{ij} + h_{ij}(x))$$

absent

- "Adiabatic" is more technical. It means that the ζ field also completely determines the perturbations in the stress-energy tensor, by a universal set of rules which will be explained later!

$$\rho(\mathbf{x}, t) = \bar{\rho}(t)\left(1 + \frac{4}{7}\zeta(\mathbf{x})\right)$$

$$\ldots$$

So far:

- Expansion history (h, Ω_b, Ω_c)
- Initial conditions (Δ_ζ^2, n_s)

$h = 0.70 \pm 0.03$	Hubble parameter
$\Omega_b = 0.0486 \pm 0.0007$	Baryonic matter abundance
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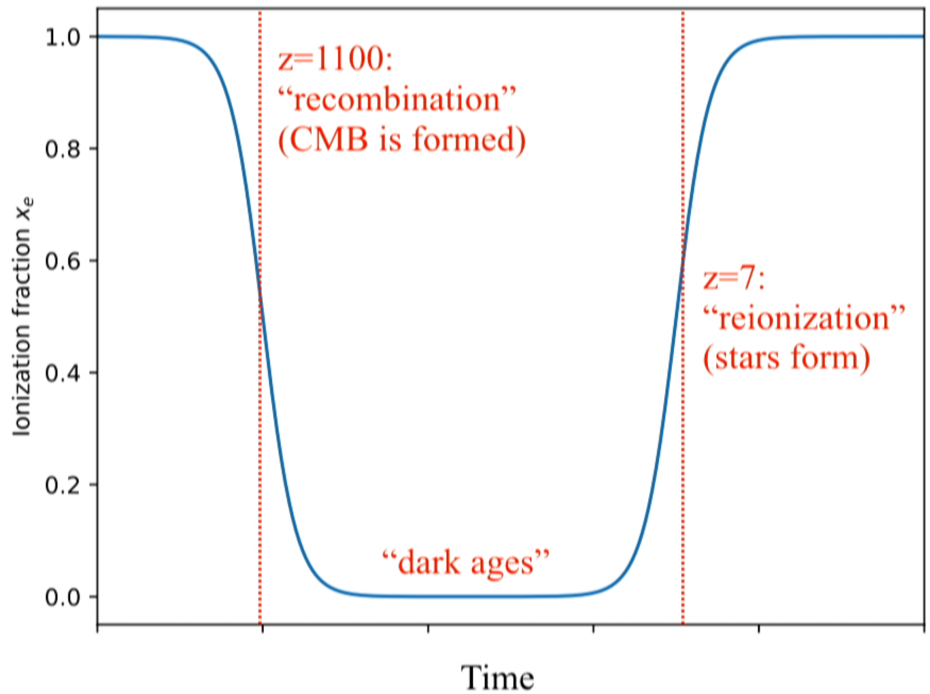
The final parameter τ is an astrophysical nuisance parameter which we define for completeness.

Ionization history of the universe

$x_e(t)$ = electron ionization fraction

= probability that a random electron in the universe is ionized (rather than being part of an atom)



Ionization history of the universe

τ = CMB optical depth

= probability that a CMB photon emitted at $z \sim 1100$ scatters from an electron at low redshift, before being observed at $z=0$.

Astrophysical nuisance parameter: τ affects the CMB power spectrum.

When fitting cosmological parameters from the CMB, we need to include τ in the fit, and account for uncertainty in τ when assigning errors to other parameters.

Standard model of cosmology:

- Background metric is FRW
- Expansion history is ΛCDM
- Initial perturbations are Gaussian random
- Initial perturbations are scalar adiabatic
- Power spectrum of initial perturbations is a power law: $(k^3/2\pi^2)P(k) = \Delta_\zeta^2(k/k_0)^{n_s-1}$

Six parameters:

$h = 0.70 \pm 0.03$	Hubble parameter
$\Omega_b = 0.0486 \pm 0.0007$	Baryonic(*) matter abundance
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(*) "Baryons" = protons + neutrons + electrons(!)

Fundamental physics →

Hubble parameter h
Baryon abundance Ω_b
Dark matter abundance Ω_c
Initial amplitude Δ_ζ^2
Spectral index n_s
CMB optical depth τ

→ Data analysis

Challenge for observers: which model fits the data?

~1930: Expanding universe
1965: Big bang (discovery of CMB)
~1970: Dark matter
1992: Gaussian, nearly scale-invariant perturbations (COBE)
1998: Cosmological constant
2006: Deviation from scale invariance ($n_s < 1$)

Fundamental physics →

Hubble parameter h
Baryon abundance Ω_b
Dark matter abundance Ω_c
Initial amplitude Δ_ζ^2
Spectral index n_s
CMB optical depth τ

→ Data analysis

Some interesting extensions of the standard cosmological model:

- Non-Gaussian initial conditions ($f_{NL} = 2.5 \pm 5.7$)
- Non-minimal neutrino mass ($m_\nu < 0.15$ eV at 95% CL)
- Extra neutrino species or other light relics ($N_{eff} = 3.04 \pm 0.18$)
- Nonzero spatial curvature ($\Omega_K = 0.000 \pm 0.005$)
- Time-dependent dark energy density ($w = -1.02 \pm 0.08$)
- Cosmological gravity waves ($r < 0.12$ at 95% CL)
- + many more!

Fundamental physics →

Hubble parameter h
Baryon abundance Ω_b
Dark matter abundance Ω_c
Initial amplitude Δ_ζ^2
Spectral index n_s
CMB optical depth τ

→ Data analysis

Challenge for theorists: explain this model at a fundamental level

- What is dark matter?
- Why is the cosmological constant so fine-tuned?
  (if late-time accelerated expansion is indeed a c.c.!)
- What physics is responsible for generating the initial Gaussian, nearly scale invariant fluctuations?

Fundamental physics →

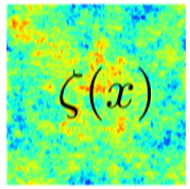
Hubble parameter h
Baryon abundance Ω_b
Dark matter abundance Ω_c
Initial amplitude Δ_ζ^2
Spectral index n_s
CMB optical depth τ

→ Data analysis

Challenge for theorists: find all physically motivated "surprises"

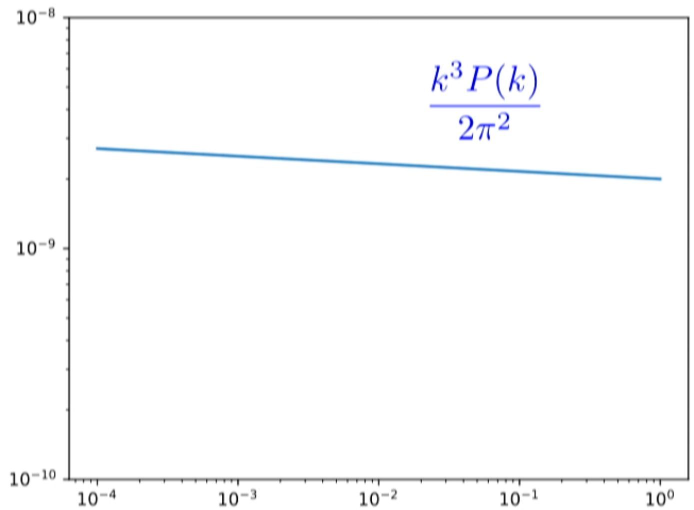
- Is our list of interesting standard model extensions complete?
- For a given standard model extension, how do we compute the effect on its observables? (E.g. the CMB power spectrum.)

inflation?
cyclic universe?
something else?



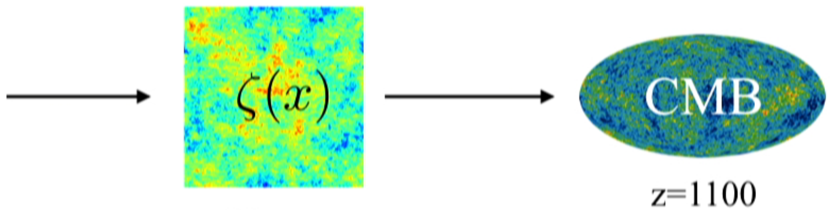
High-z

The standard cosmological model specifies the perturbations at very early times (high-z). They are fairly simple, and parameterized by a Gaussian random field ζ(x) with a featureless power spectrum.



wavenumber k (Mpc^{-1})

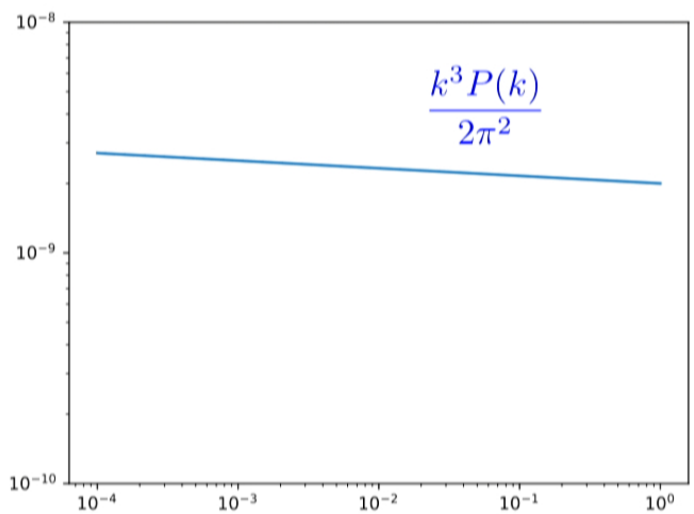
inflation?
cyclic universe?
something else?



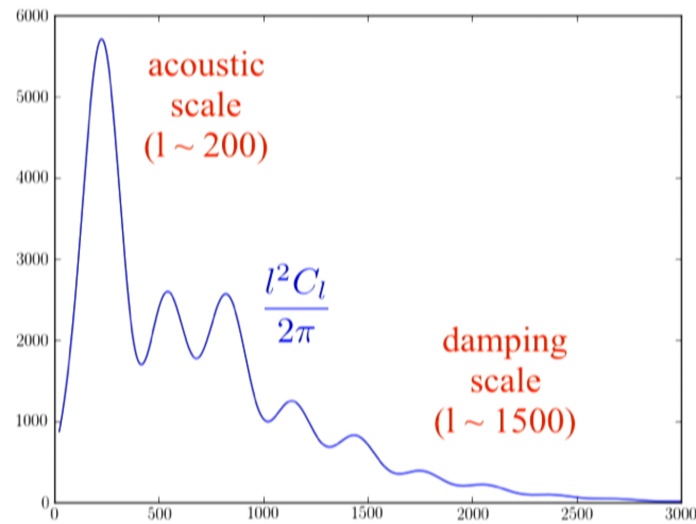
High-z

z=1100

As time evolves, the perturbations become more complex. By the time the CMB is formed (z=1100), a lot of physics has been "imprinted" on the power spectrum.

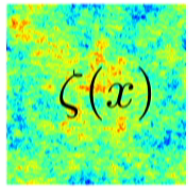


wavenumber k (Mpc^{-1})

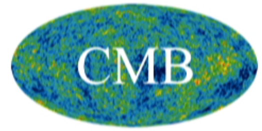


angular multipole l

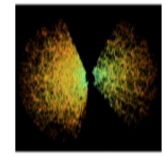
inflation?
cyclic universe?
something else?



High-z

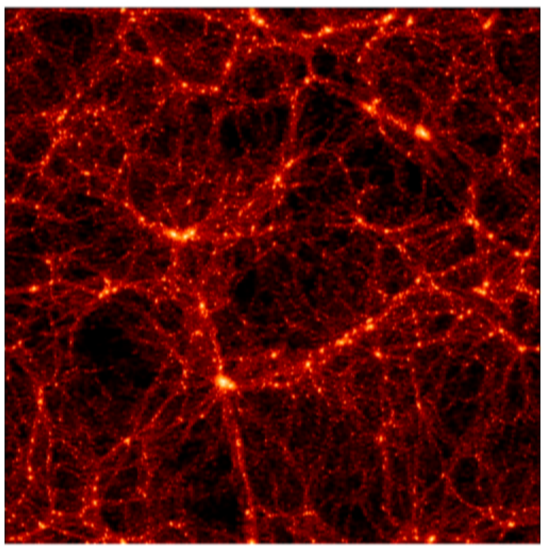


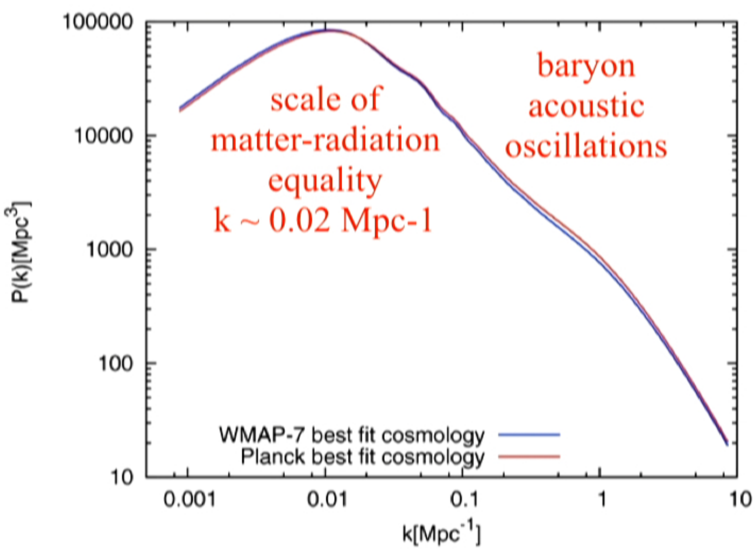
z=1100



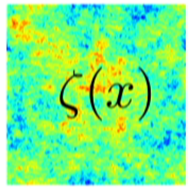
z~1

At late times (z~1), nonlinear effects are important and the perturbations are non-Gaussian. Data analysis becomes more complicated! Non-power-spectrum statistics are useful.

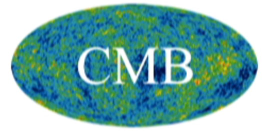




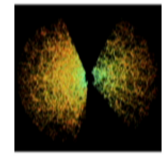
inflation?
cyclic universe?
something else?



High-z



z=1100

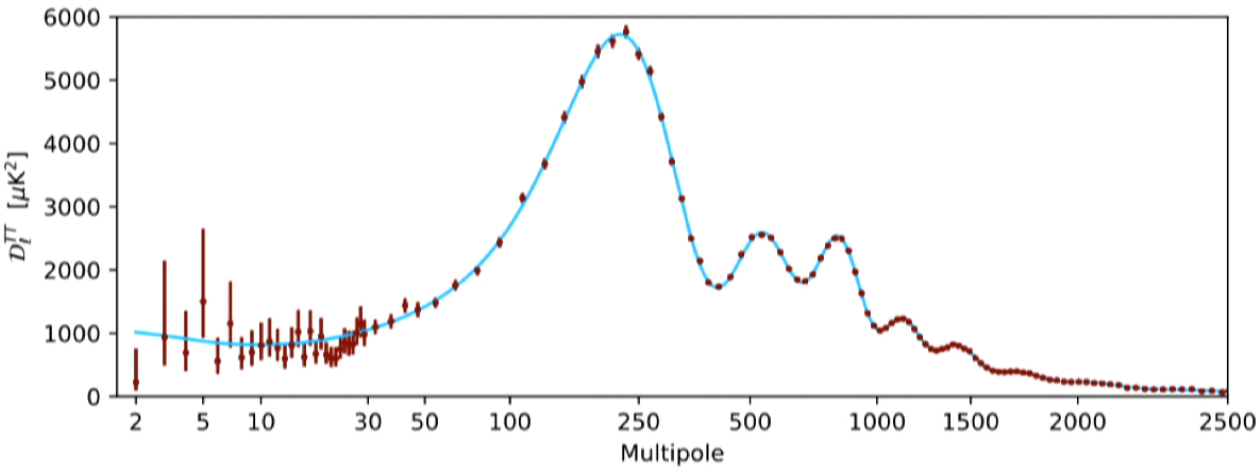


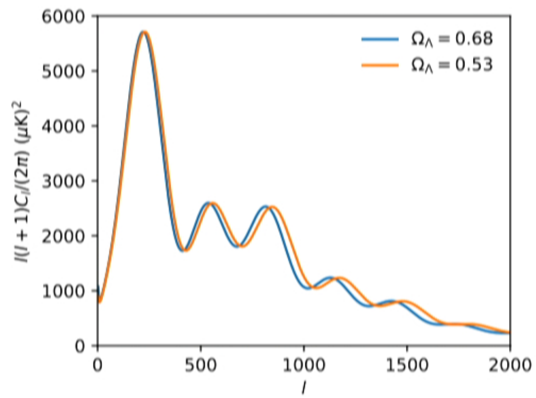
z~1

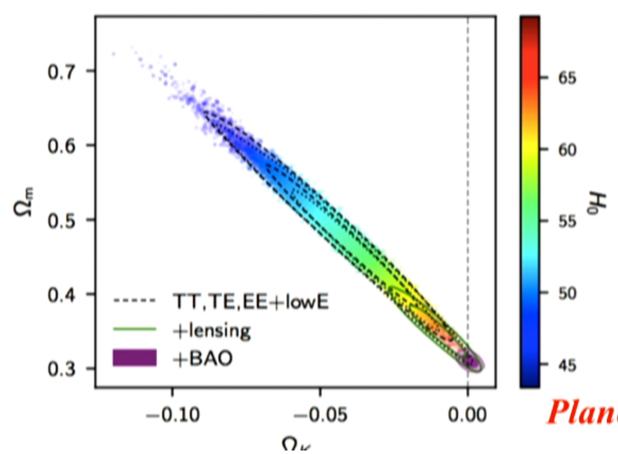
In each of these three stages, different physics is important:

- Early universe: Quantum mechanics in expanding spacetime generates Gaussian perturbations from vacuum
- Formation of the CMB: Linear perturbation theory in a plasma with multiple components (dark matter, baryons, photons, neutrinos) + metric degrees of freedom
- Late times: Gravitational N-body physics. Messy astrophysics! (galaxy formation, star formation, …)

“Most” cosmology data analysis consists of fitting cosmological parameters to measured power spectra.

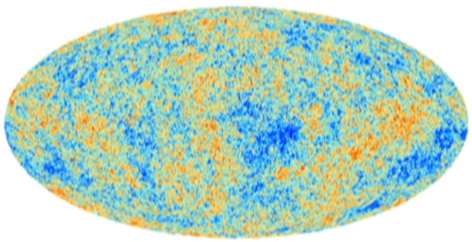




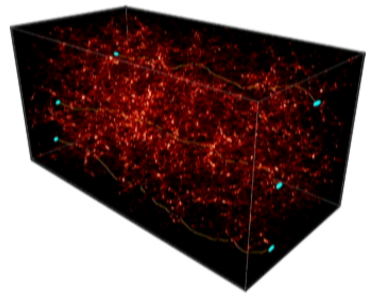


Planck 2015

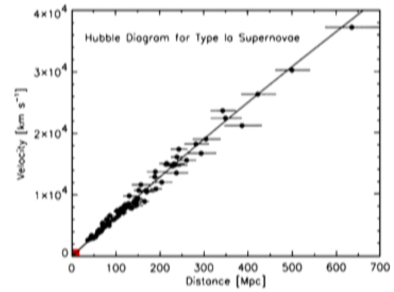
Variety of datasets, field is rapidly evolving.



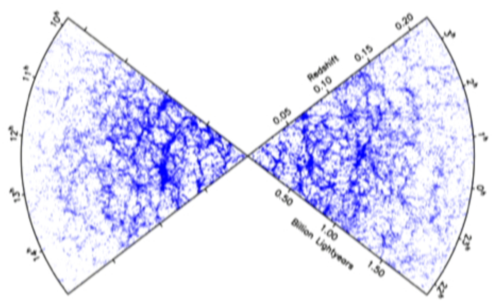
Cosmic microwave background (CMB)



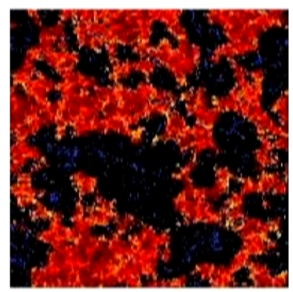
Gravitational lensing



Type Ia supernovae



Galaxy clustering



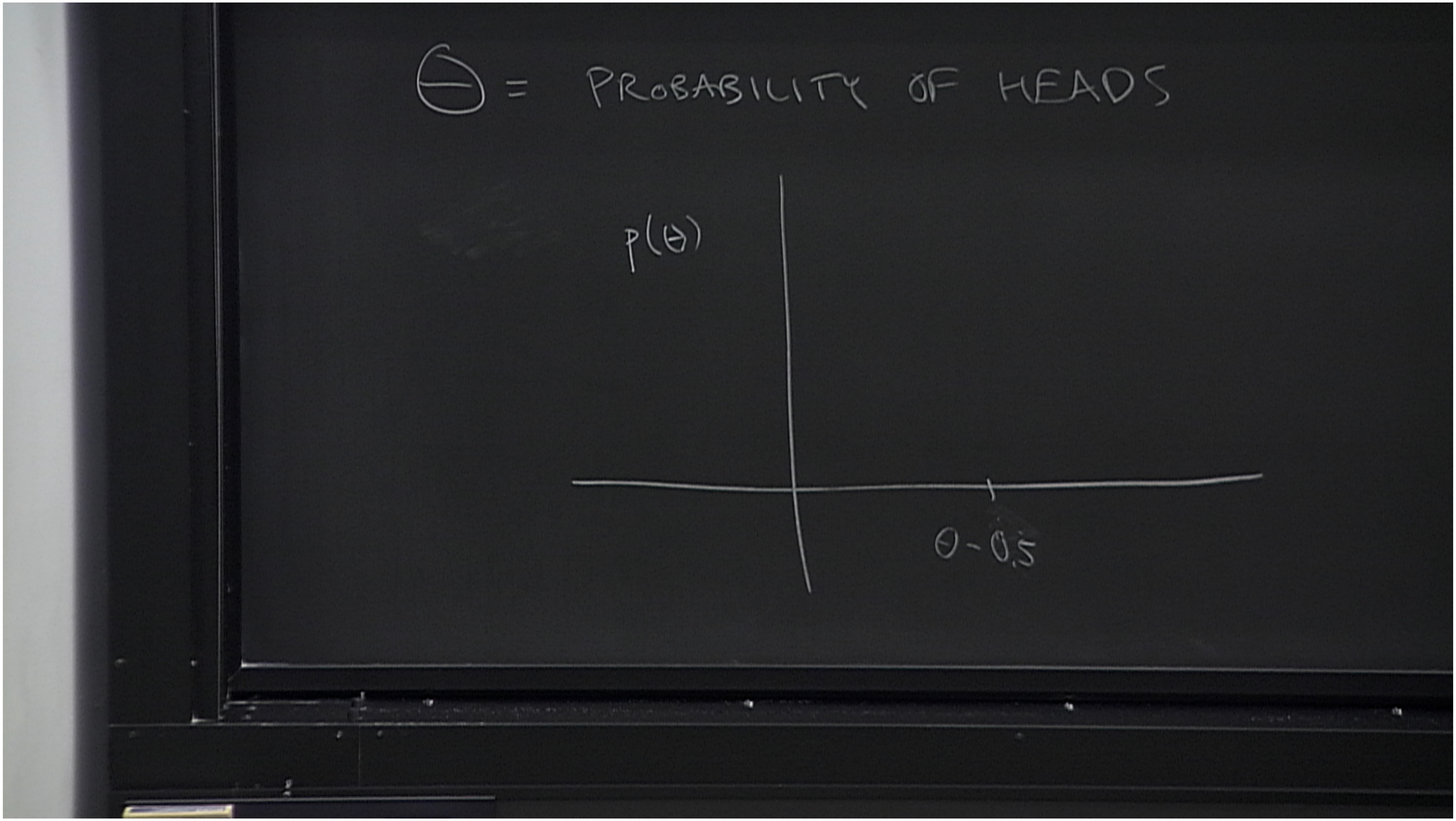
21-cm intensity mapping

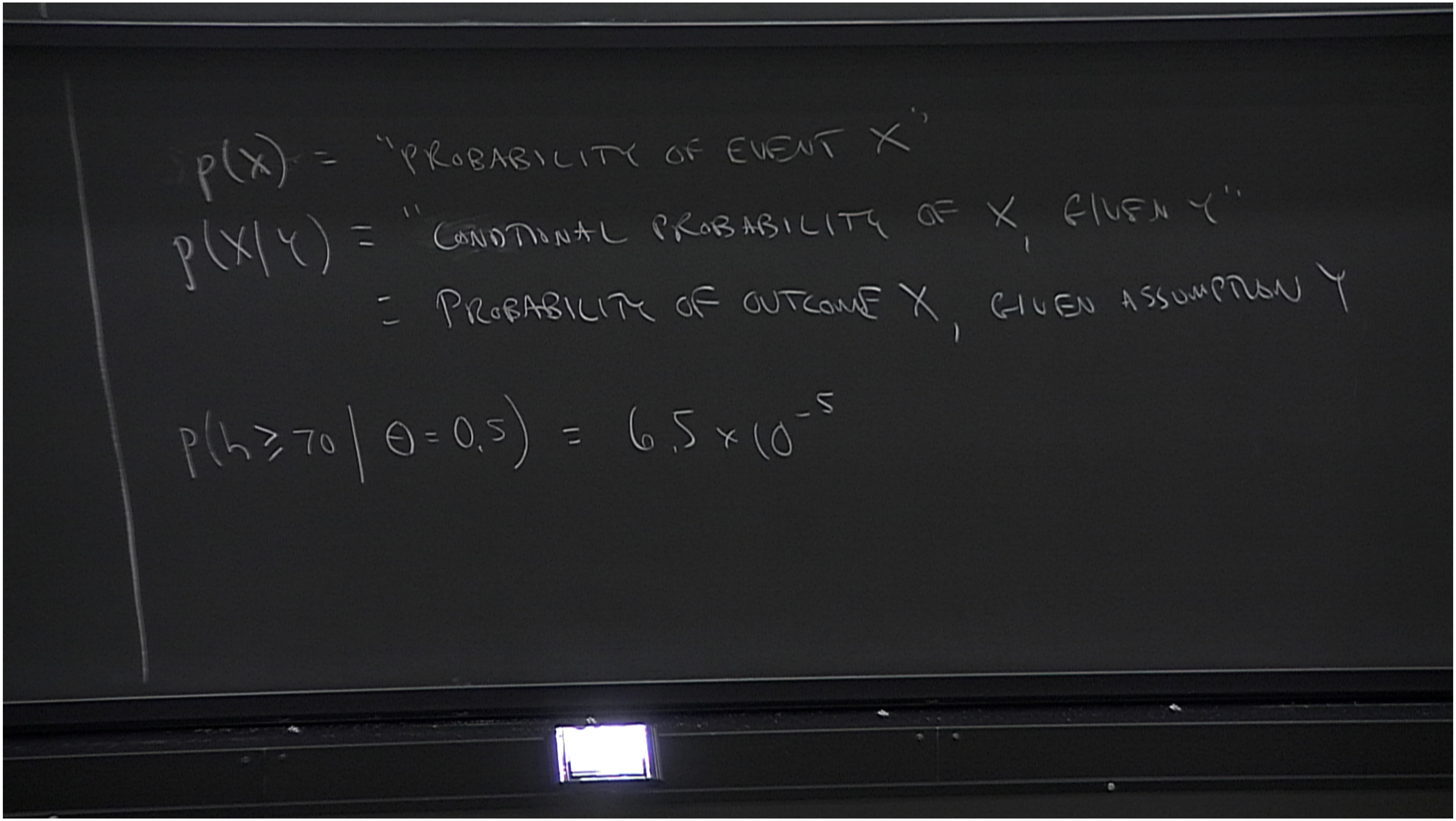
Galaxy cluster abundance

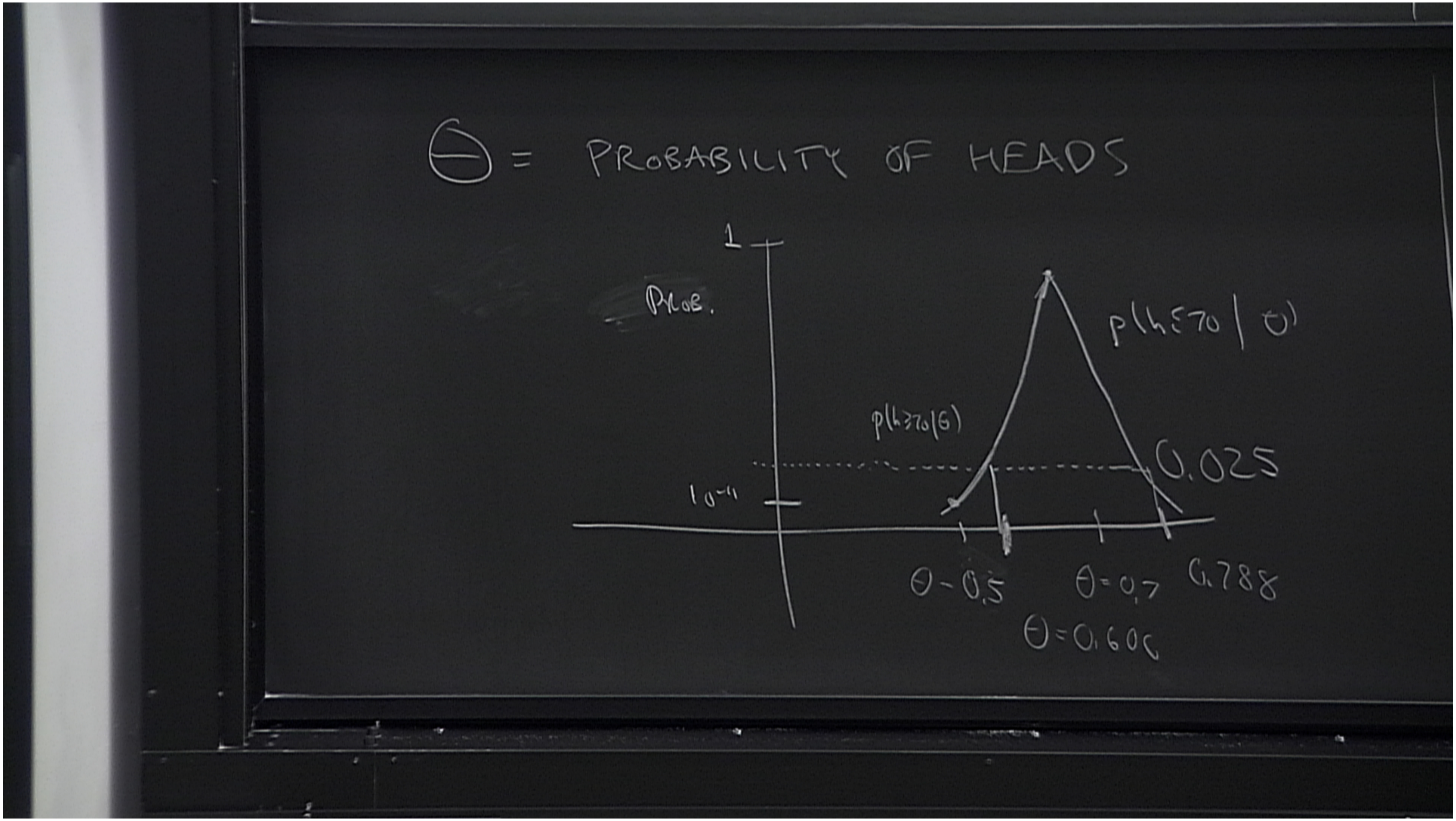
CMB currently dominates constraints on 6-parameter model space

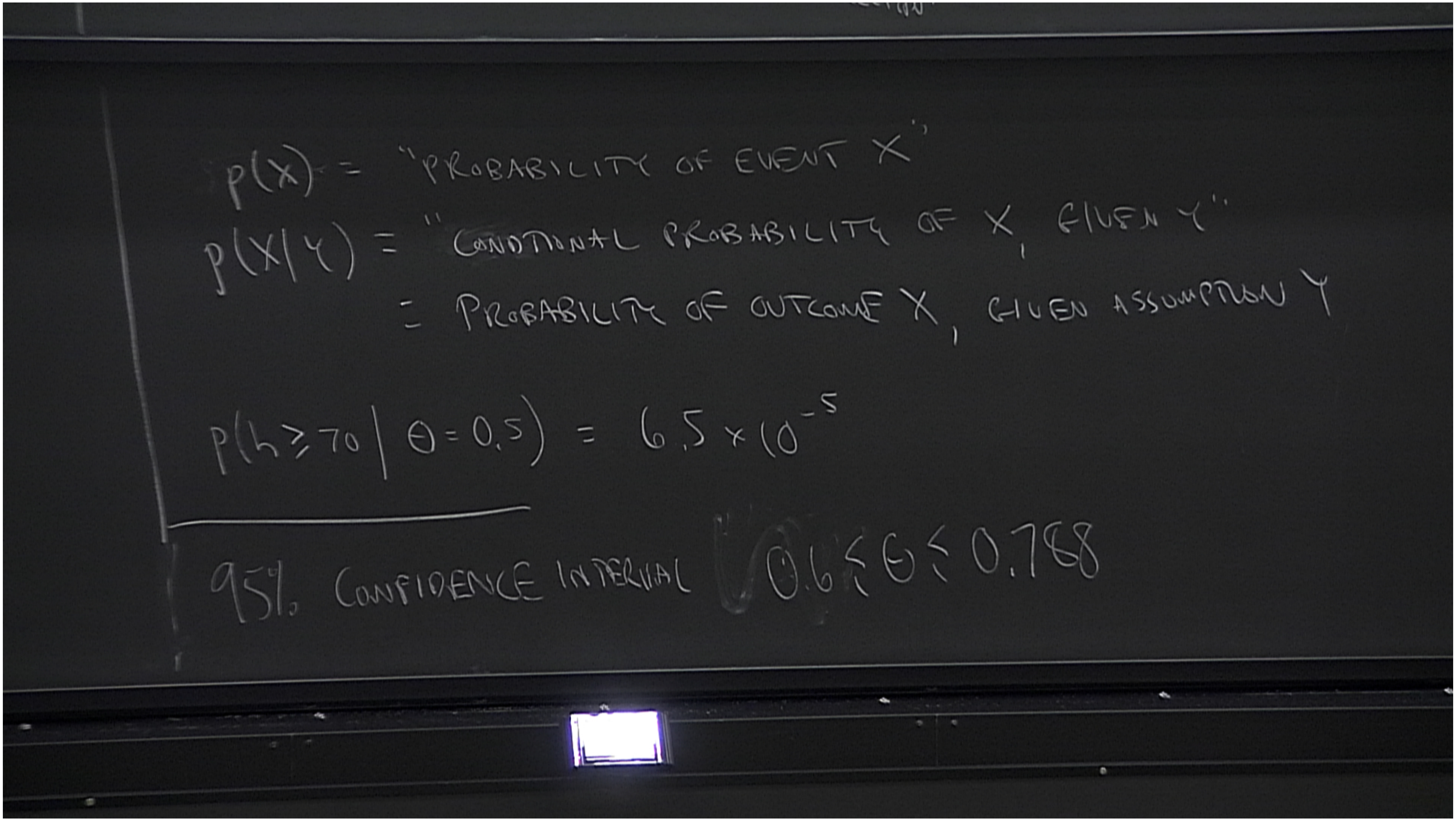
	CMB alone	CMB + baryon acoustic oscillations + type IA supernovae + direct H_0 measurements
DM density $\rho_{c,0}$	0.2618 ± 0.0087	0.2589 ± 0.0063 $(\times\rho_{tot})$
Bary. density $\rho_{b,0}$	0.04884 ± 0.00085	0.04860 ± 0.00070 $(\times\rho_{tot})$
Cosm. constant Λ	2.543 ± 0.071	2.567 ± 0.051 $(\times 10^{-47}\ \mathrm{GeV}^4)$
Amplitude A_ζ	2.130 ± 0.053	2.142 ± 0.049
Spectral index n_s	0.9653 ± 0.0048	0.9667 ± 0.0040
Optical depth τ	0.063 ± 0.014	0.066 ± 0.012

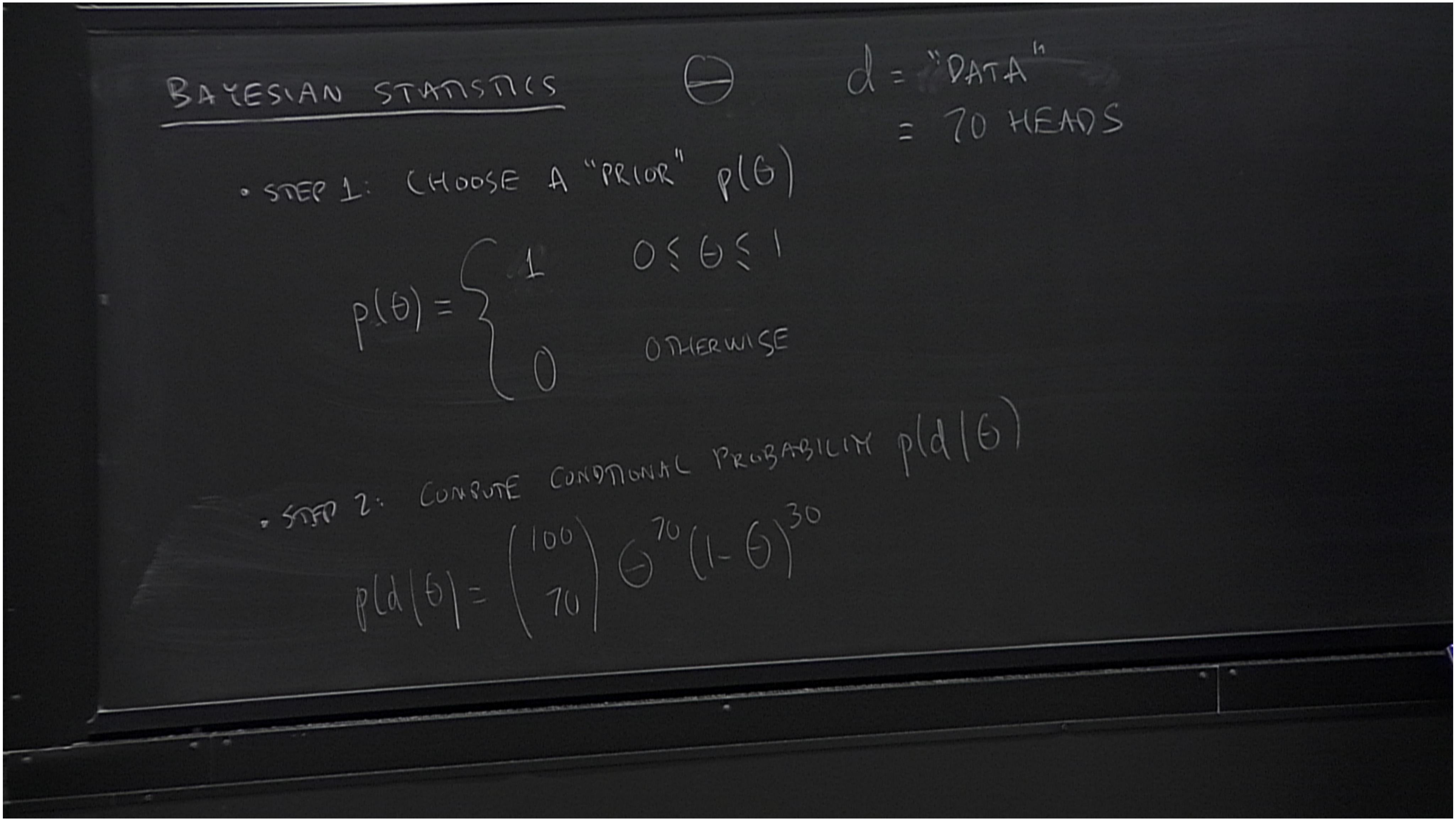
Not true in extensions of standard model! Non-CMB datasets are important when more parameters are added. However, since CMB is "center stage", I'll tend to concentrate on it in lectures.

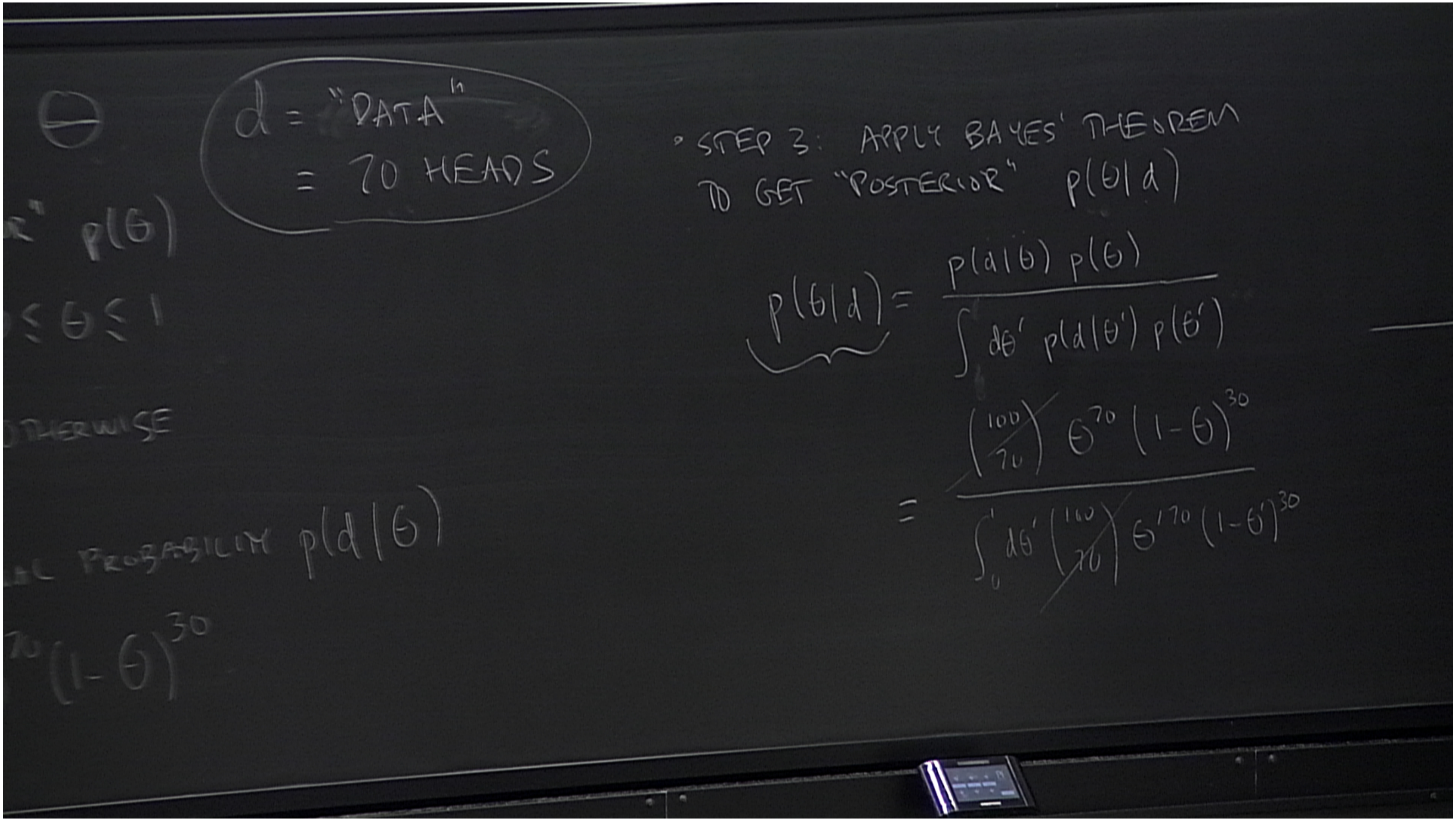

Θ = PROBABILITY OF HEADS
p(Θ)
Θ ~ 0.5


p(X) = "PROBABILITY OF EVENT X"
p(X|Y) = "CONDITIONAL PROBABILITY OF X, GIVEN Y"
= PROBABILITY OF OUTCOME X, GIVEN ASSUMPTION Y
p(h ≥ 70 | θ = 0.5) = 6.5 × 10^-5


Θ = PROBABILITY OF HEADS
1
PROB.
0.025
θ = 0.5
θ = 0.7
0.788
θ = 0.606


p(x) = "PROBABILITY OF EVENT X"
p(X|Y) = "CONDITIONAL PROBABILITY OF X, GIVEN Y"
= PROBABILITY OF OUTCOME X, GIVEN ASSUMPTION Y
p(h ≥ 70 | θ = 0.5) = 6.5 × 10^-5
95% CONFIDENCE INTERVAL 0.6 ≤ θ ≤ 0.788


BAYESIAN STATISTICS
Θ
d = "DATA"
= 70 HEADS
• STEP 1: CHOOSE A "PRIOR" $p(\Theta)$
$p(\Theta) = \begin{cases} 1 & 0 \leq \Theta \leq 1 \\ 0 & \text{OTHERWISE} \end{cases}$
• STEP 2: COMPUTE CONDITIONAL PROBABILITY $p(d|\Theta)$
$p(d|\Theta) = \binom{100}{70} \Theta^{70} (1-\Theta)^{30}$


d = "DATA"
= 70 HEADS
p(θ)
≤ θ ≤ 1
OTHERWISE
PROBABILITY p(d|θ)
70 (1-θ)^30
• STEP 3: APPLY BAYES' THEOREM
TO GET "POSTERIOR" p(θ|d)
$p(\theta|d) = \frac{p(d|\theta)\,p(\theta)}{\int d\theta'\,p(d|\theta')\,p(\theta')}$
$= \frac{\binom{100}{70}\theta^{70}(1-\theta)^{30}}{\int_0^1 d\theta'\binom{100}{70}\theta'^{70}(1-\theta')^{30}}$

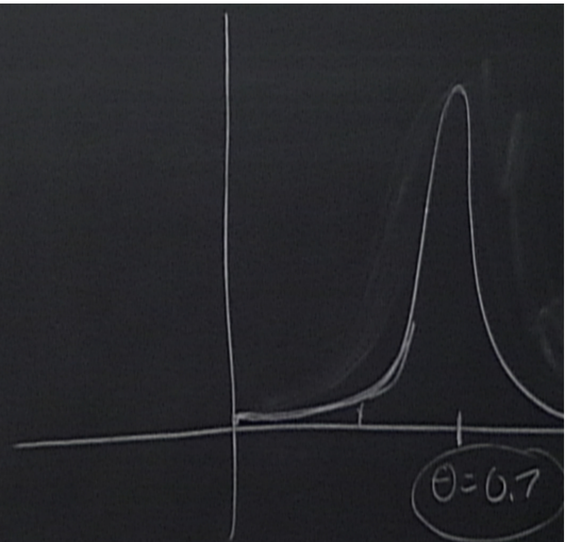
"DATA"
70 HEADS

• STEP 3: APPLY BAYES' THEOREM TO GET "POSTERIOR" $p(\theta|d)$

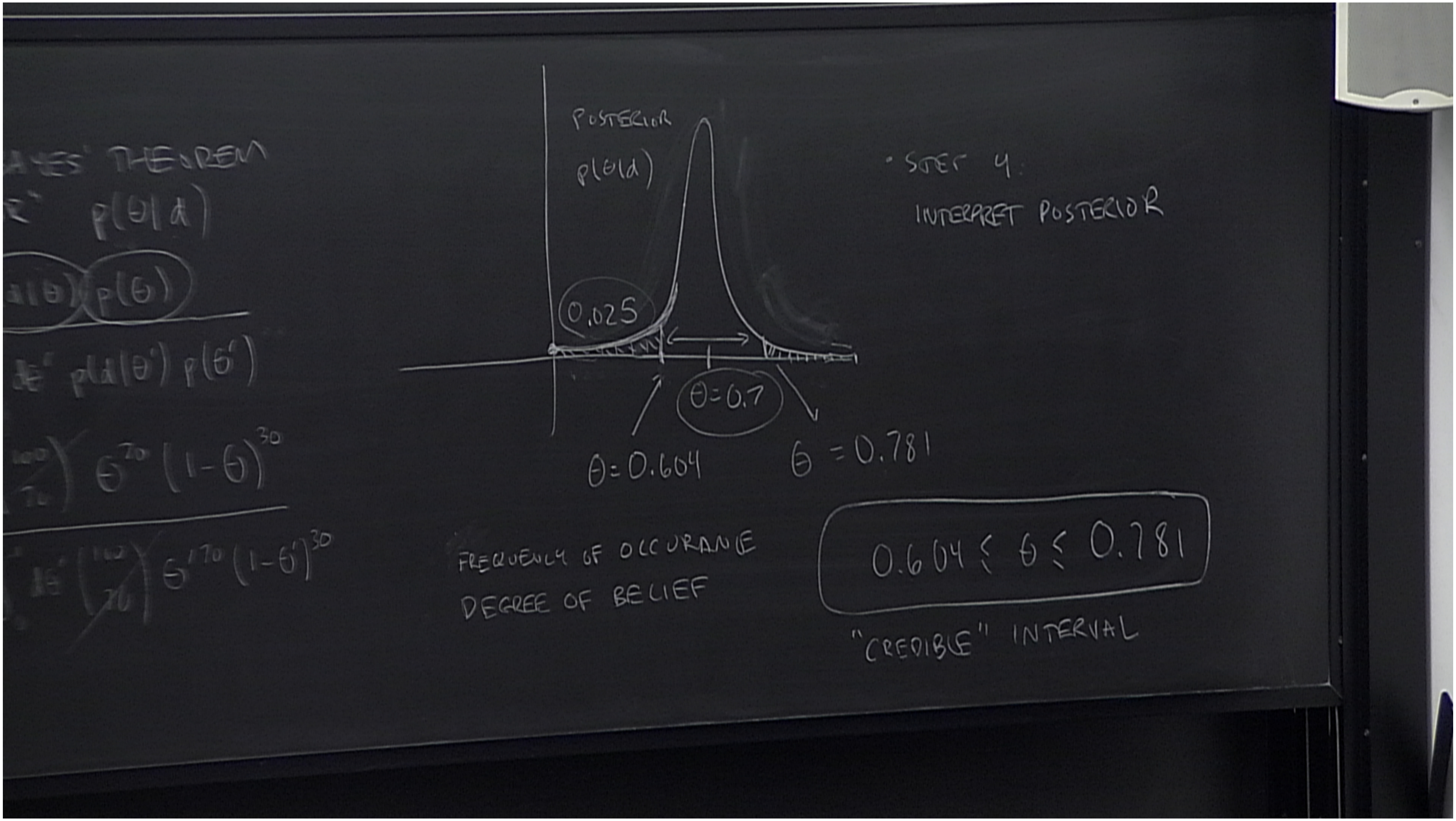
$$p(\theta|d) = \frac{p(d|\theta)\,p(\theta)}{\int d\theta'\, p(d|\theta')\, p(\theta')}$$

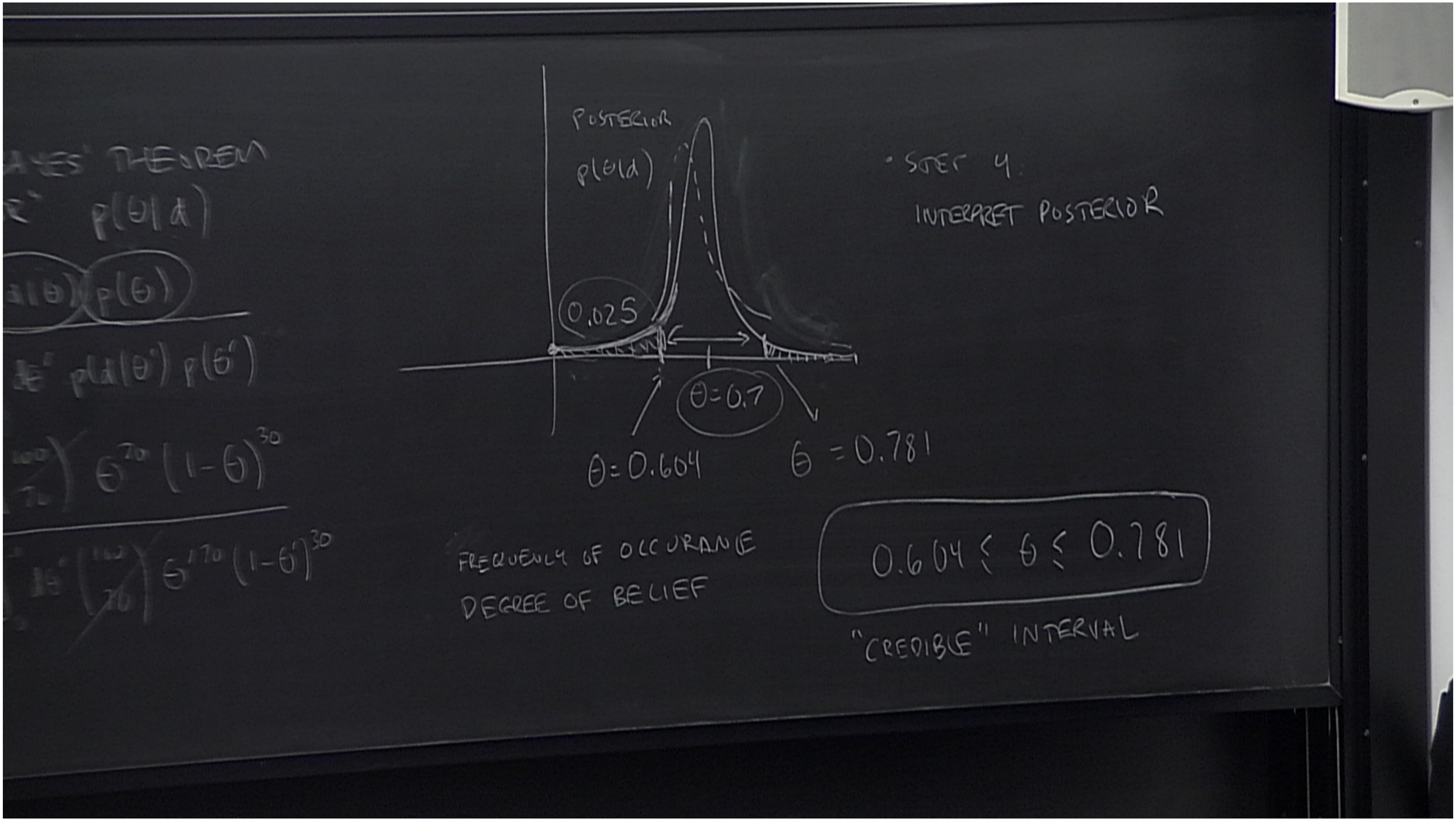
$$= \frac{\binom{100}{70}\theta^{70}(1-\theta)^{30}}{\int_0^1 d\theta' \binom{100}{70}\theta'^{70}(1-\theta')^{30}}$$

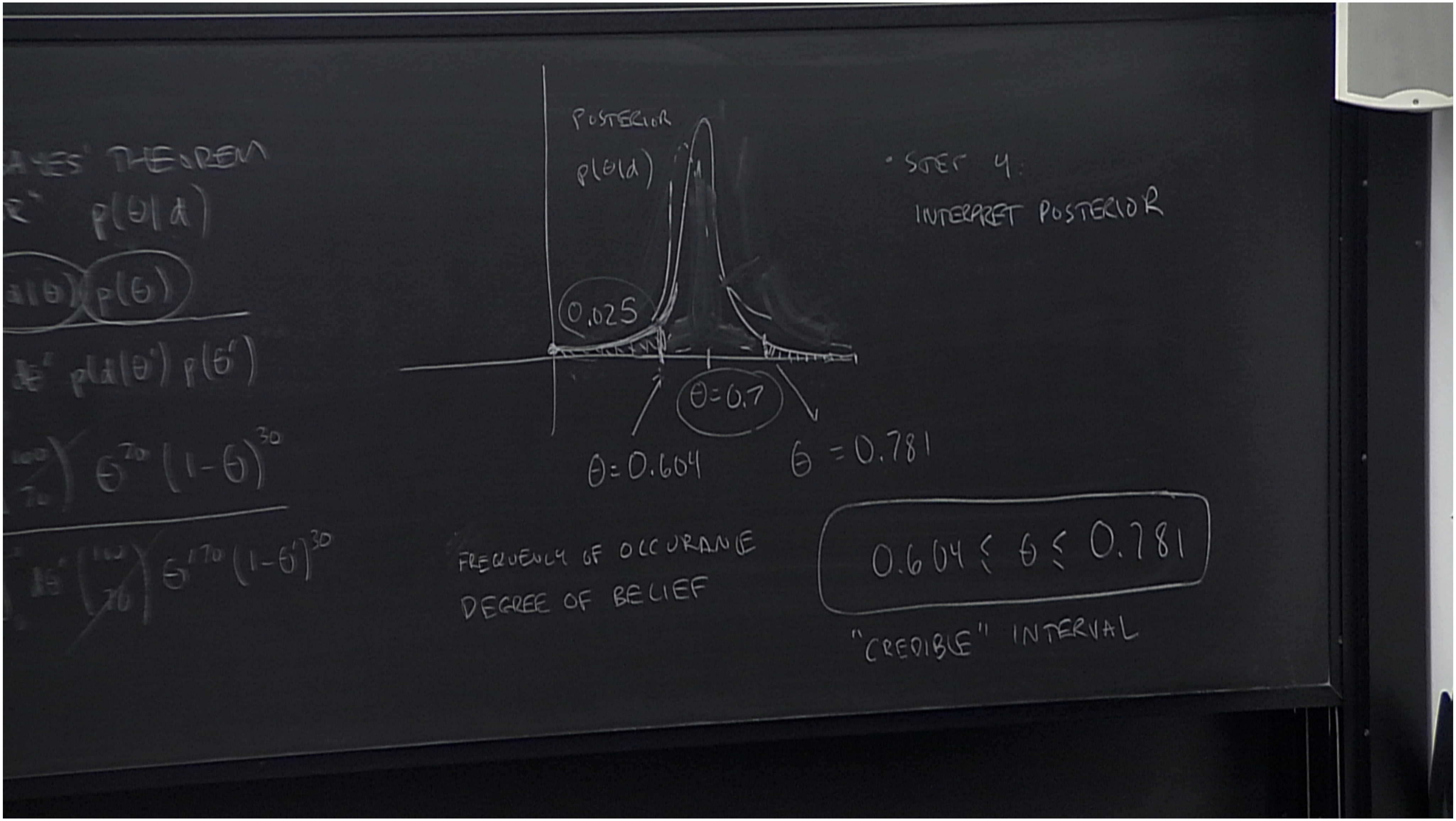
$(d|\theta)$

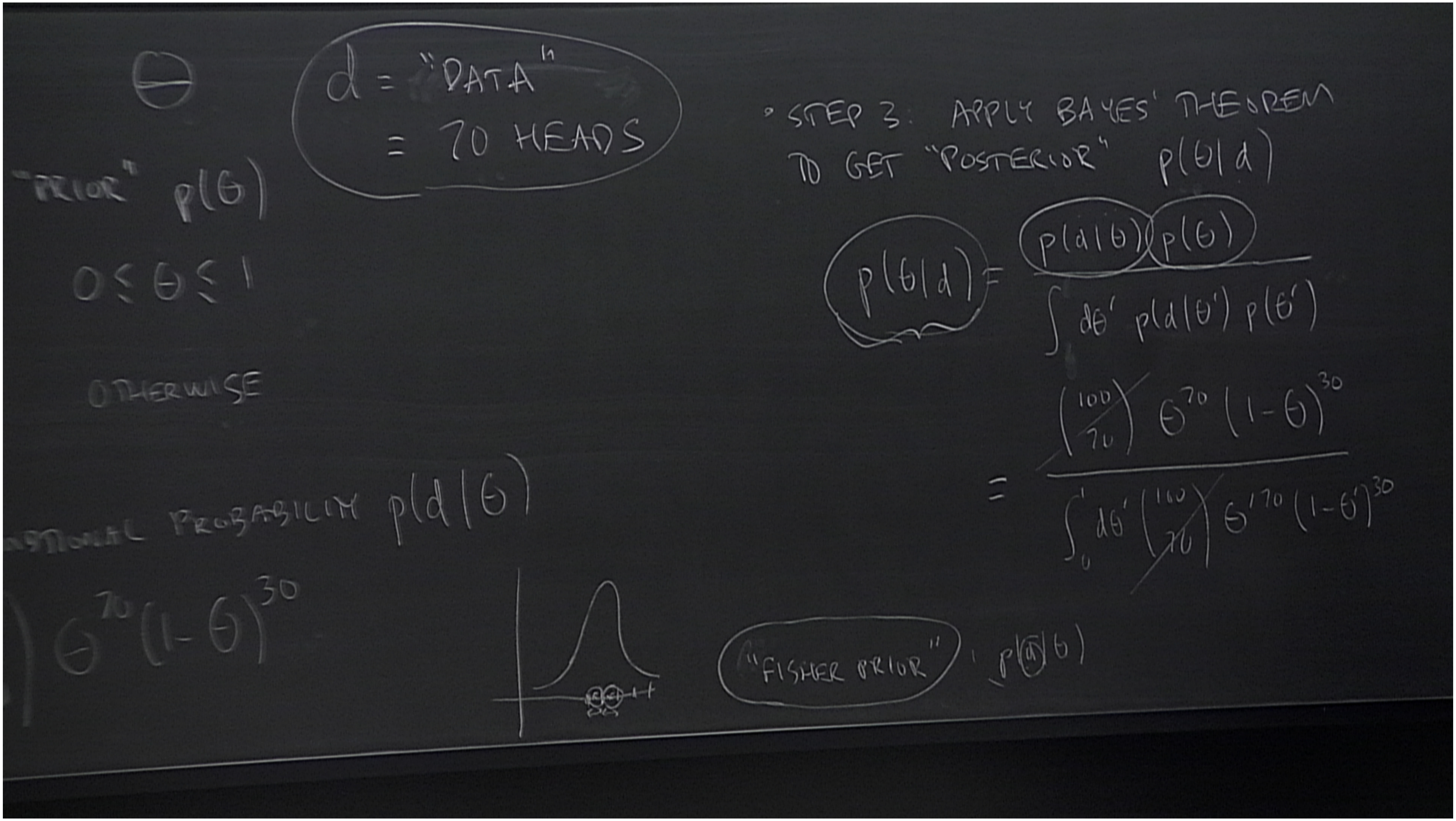


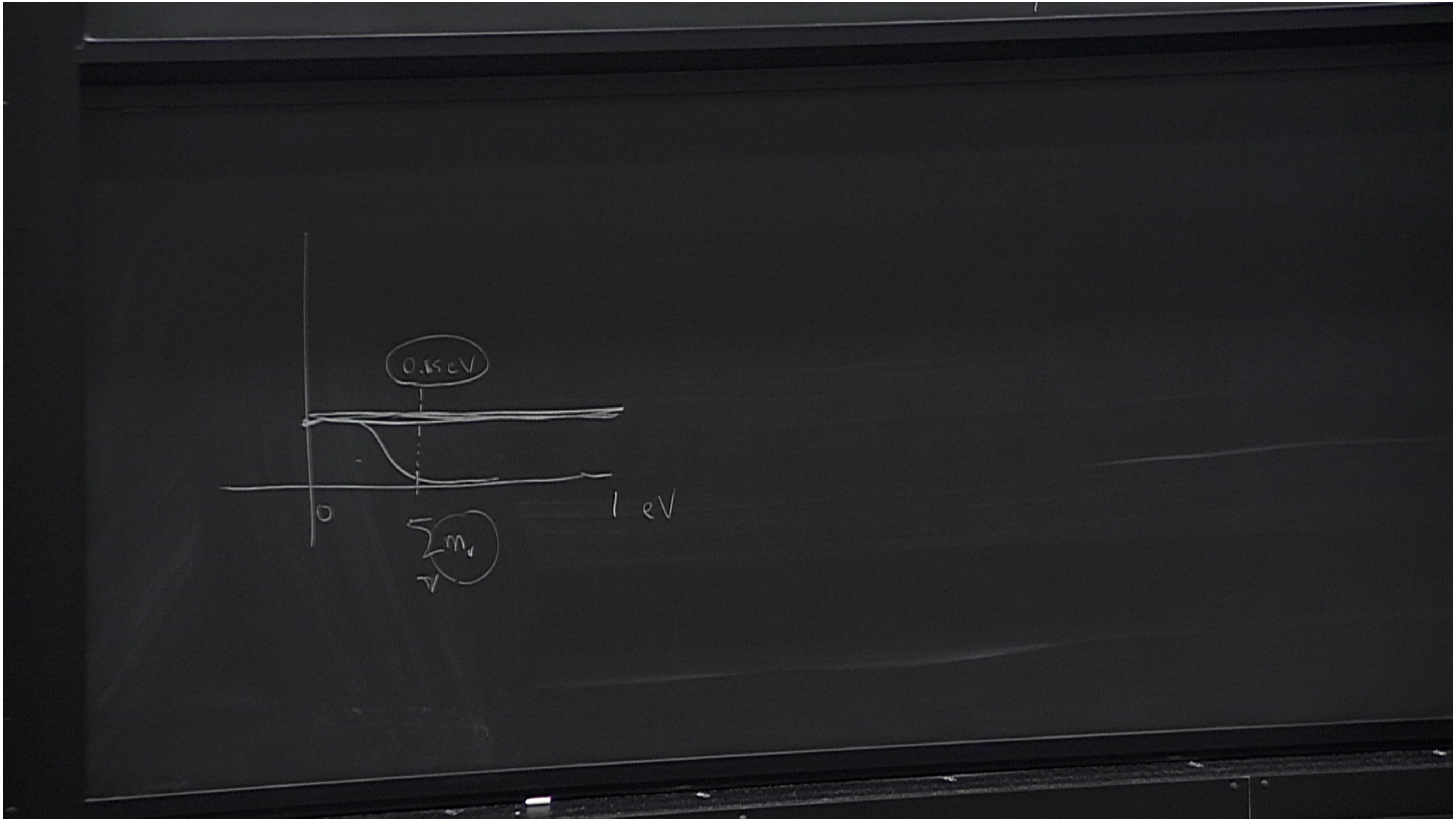
FREQUENCY OF OCCURANCE
DEGREE OF BELIEF


POSTERIOR
p(θ|d)
0.025
θ=0.7
θ=0.604
θ=0.781
STEP 4:
INTERPRET POSTERIOR
FREQUENCY OF OCCURANCE
DEGREE OF BELIEF
0.604 ≤ θ ≤ 0.781
"CREDIBLE" INTERVAL


POSTERIOR
p(θ|d)
0.025
θ=0.7
θ=0.604
θ=0.781
• STEP 4:
INTERPRET POSTERIOR
FREQUENCY OF OCCURANCE
DEGREE OF BELIEF
0.604 ≤ θ ≤ 0.781
"CREDIBLE" INTERVAL


POSTERIOR
p(θ|d)
0.025
θ=0.7
θ=0.604
θ=0.781
STEP 4:
INTERPRET POSTERIOR
FREQUENCY OF OCCURANCE
DEGREE OF BELIEF
0.604 ≤ θ ≤ 0.781
"CREDIBLE" INTERVAL


d = "DATA"
= 70 HEADS
"PRIOR" p(θ)
0 ≤ θ ≤ 1
OTHERWISE
PROBABILITY p(d|θ)
θ^70 (1-θ)^30
• STEP 3: APPLY BAYES' THEOREM
TO GET "POSTERIOR" p(θ|d)
p(θ|d) = p(d|θ) p(θ) / ∫ dθ' p(d|θ') p(θ')
= (100 choose 70) θ^70 (1-θ)^30 / ∫_0^1 dθ' (100 choose 70) θ'^70 (1-θ')^30
"FISHER PRIOR" : p(d|θ)


0.15eV
0
1 eV