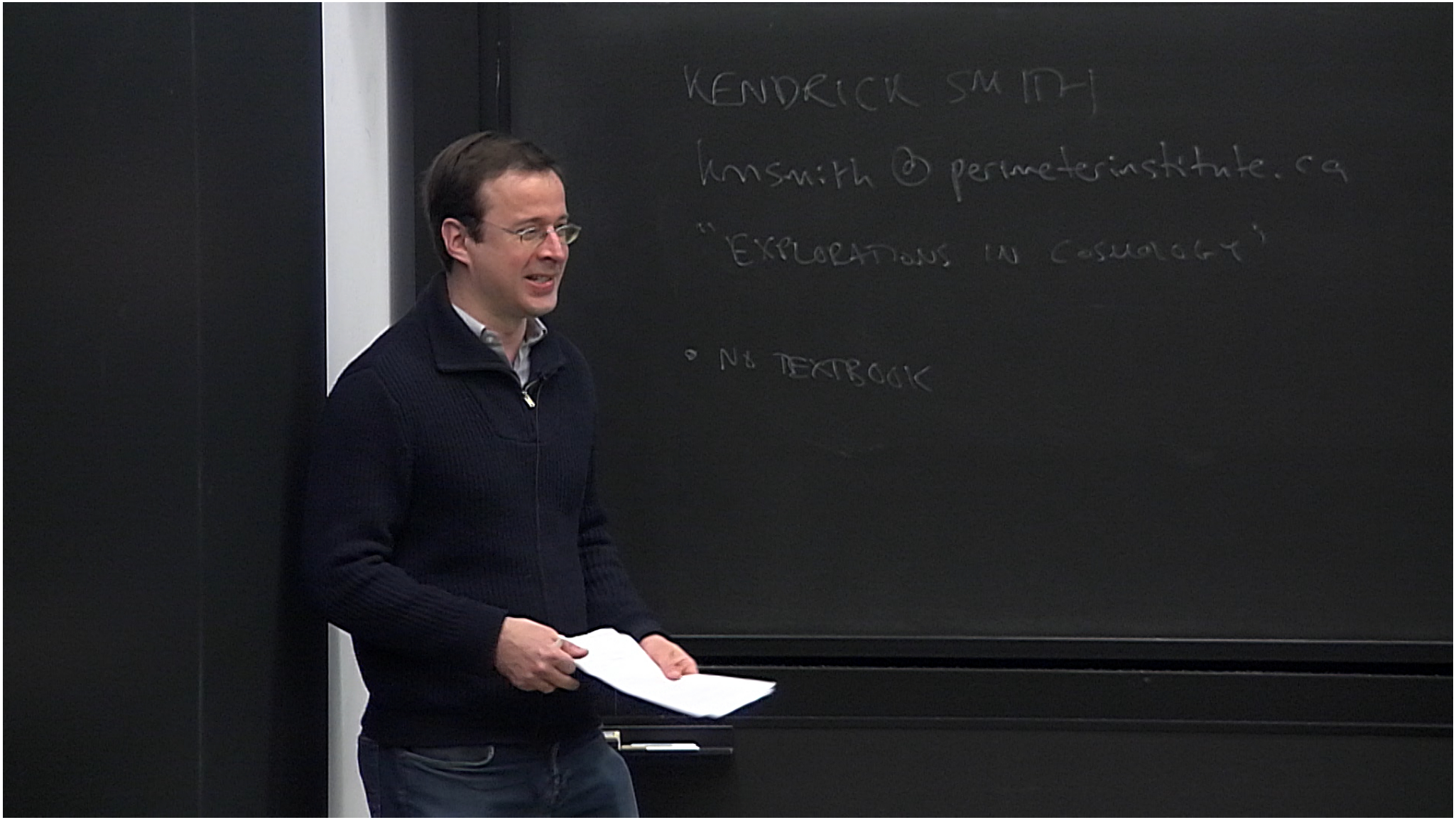
Title: PSI 2018/2019 - Explorations in Cosmology - Lecture 1

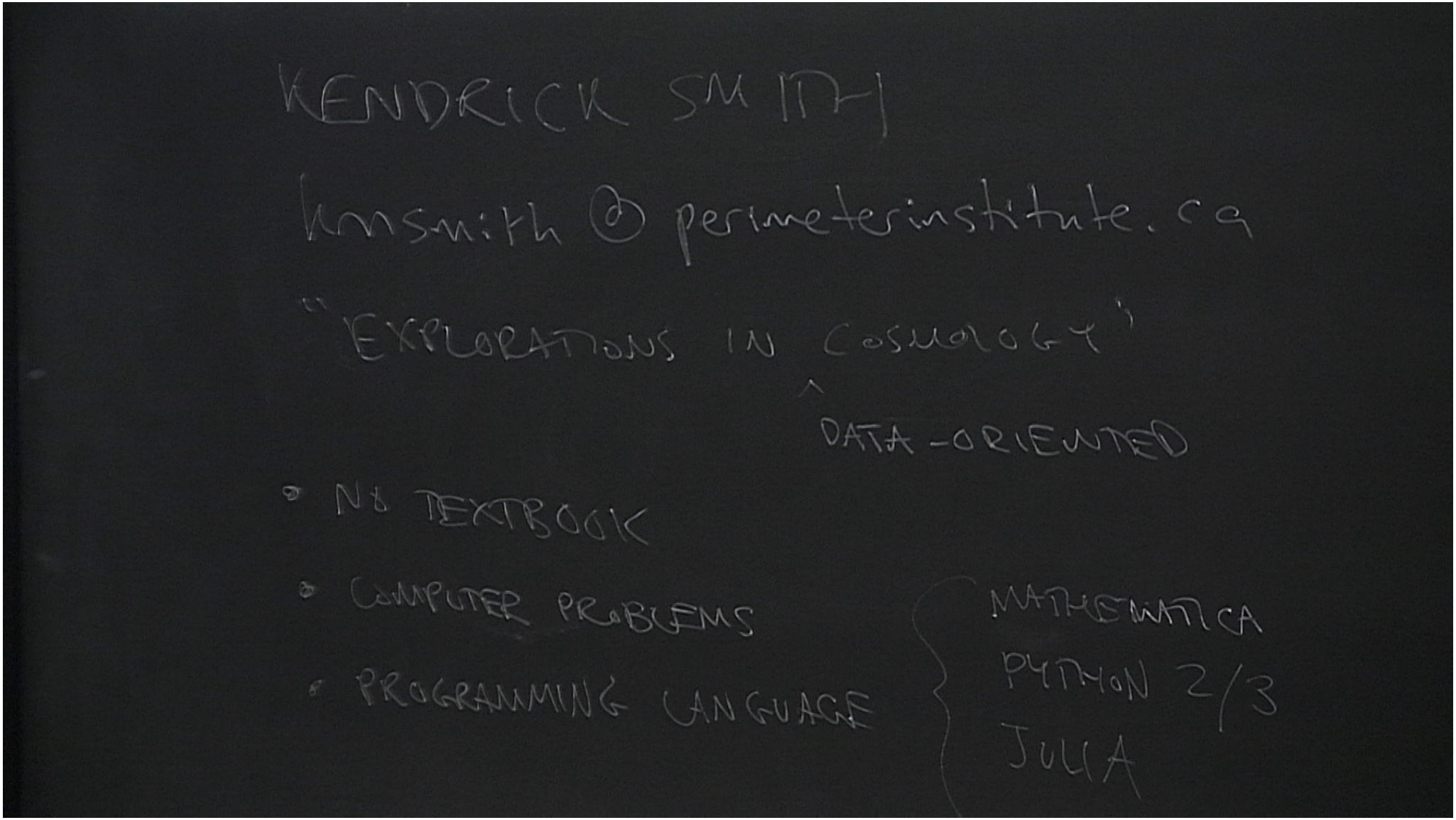
Speakers: Kendrick Smith

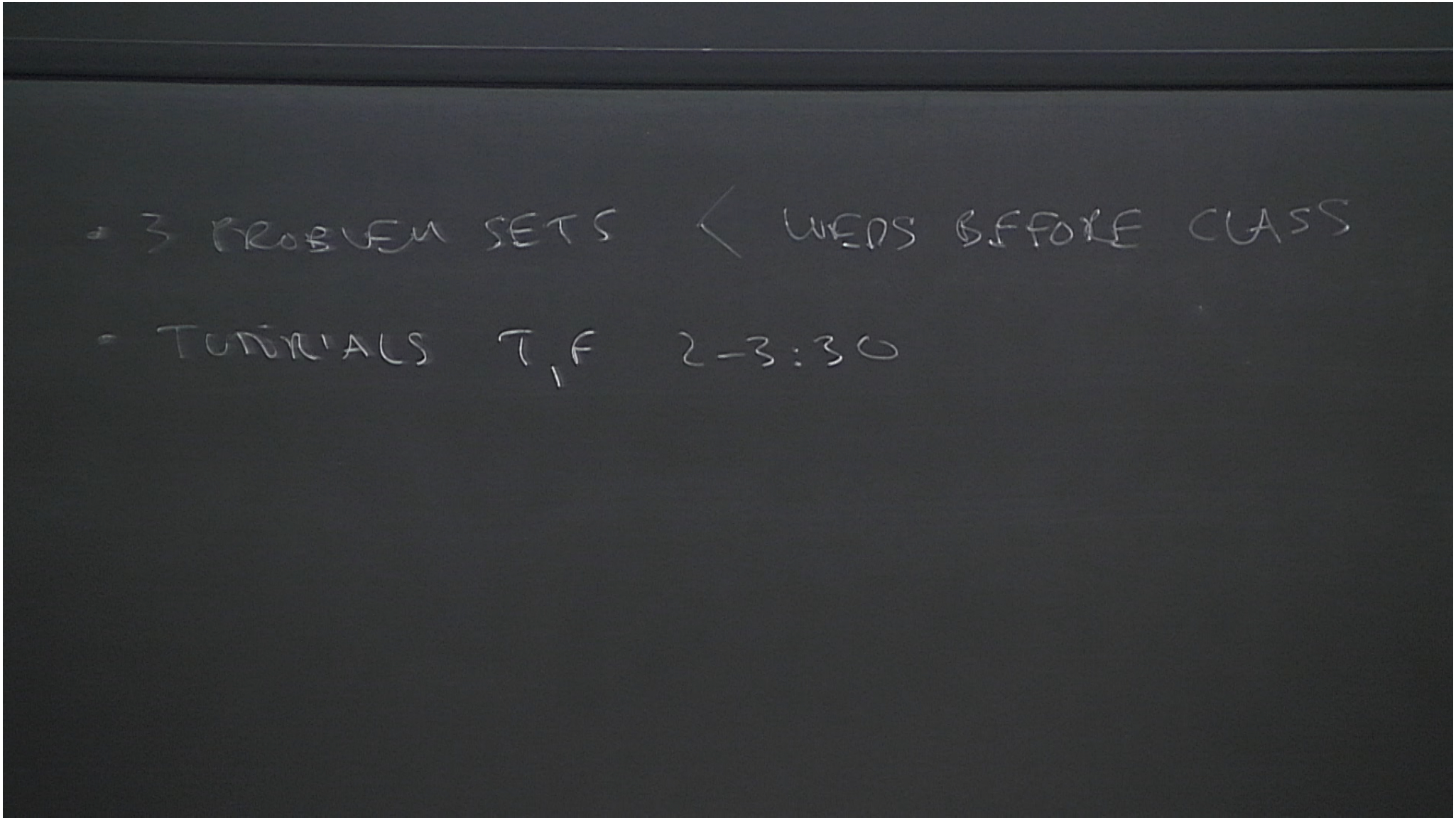
Collection: PSI 2018/2019 - Explorations in Cosmology (Smith)

Date: April 15, 2019 - 11:30 AM

URL: http://pirsa.org/19040063


KENDRICK SMITH
krsmith @ perimeterinstitute.ca
"EXPLORATIONS IN COSMOLOGY"
• NO TEXTBOOK

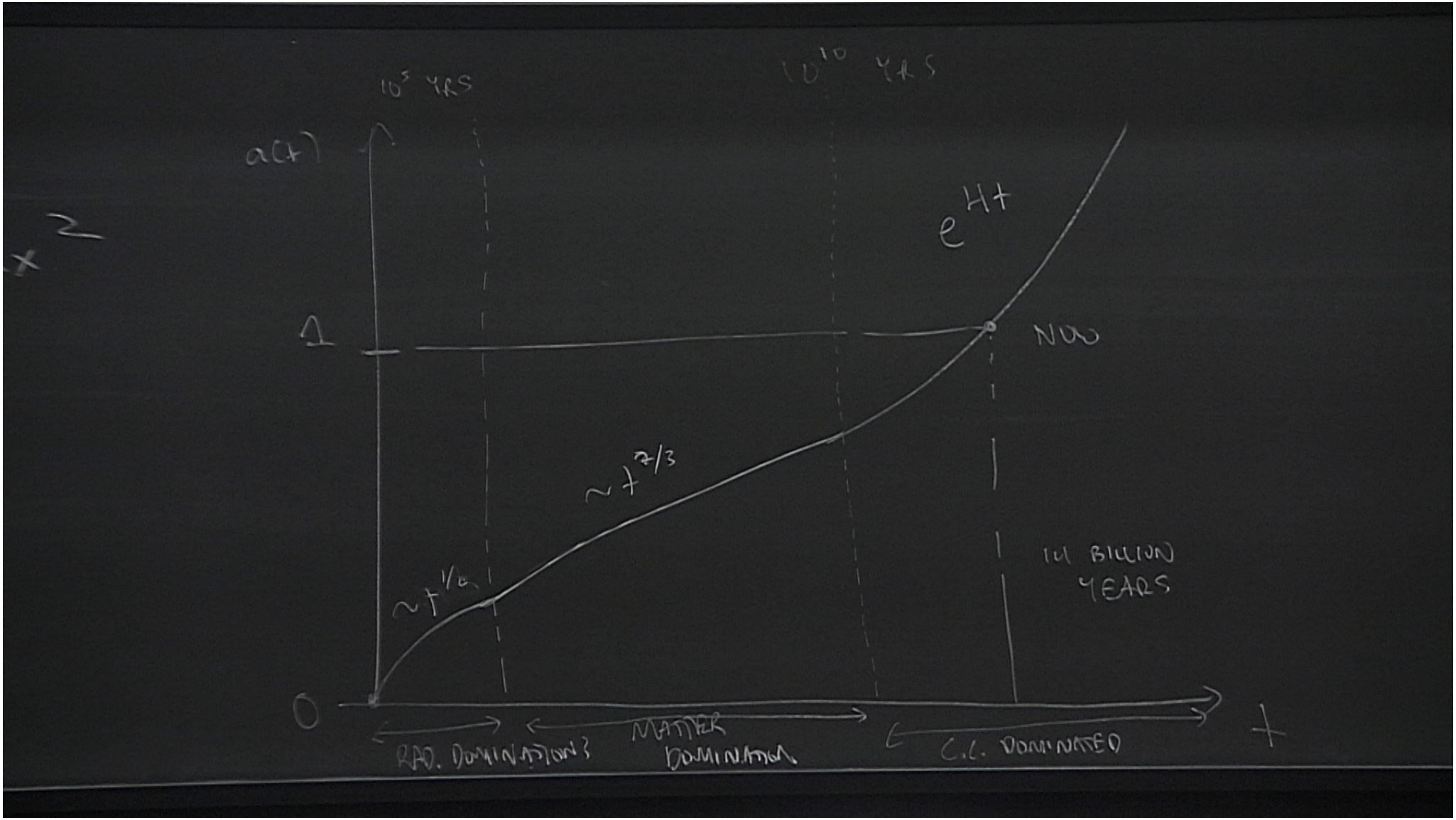

KENDRICK SMITH
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"EXPLORATIONS IN COSMOLOGY"
DATA-ORIENTED
• N& TEXTBOOK
• COMPUTER PROBLEMS
• PROGRAMMING LANGUAGE
MATHEMATICA
PYTHON 2/3
JULIA


- 3 PROBLEM SETS < WEDS BEFORE CLASS
- TUTORIALS T, F 2-3:30

EXPANDING UNIVERSE REVIEW

FRW METRIC: $ds^2 = -dt^2 + a(t)^2 dx^2$

$a(t)$ = "SCALE FACTOR"

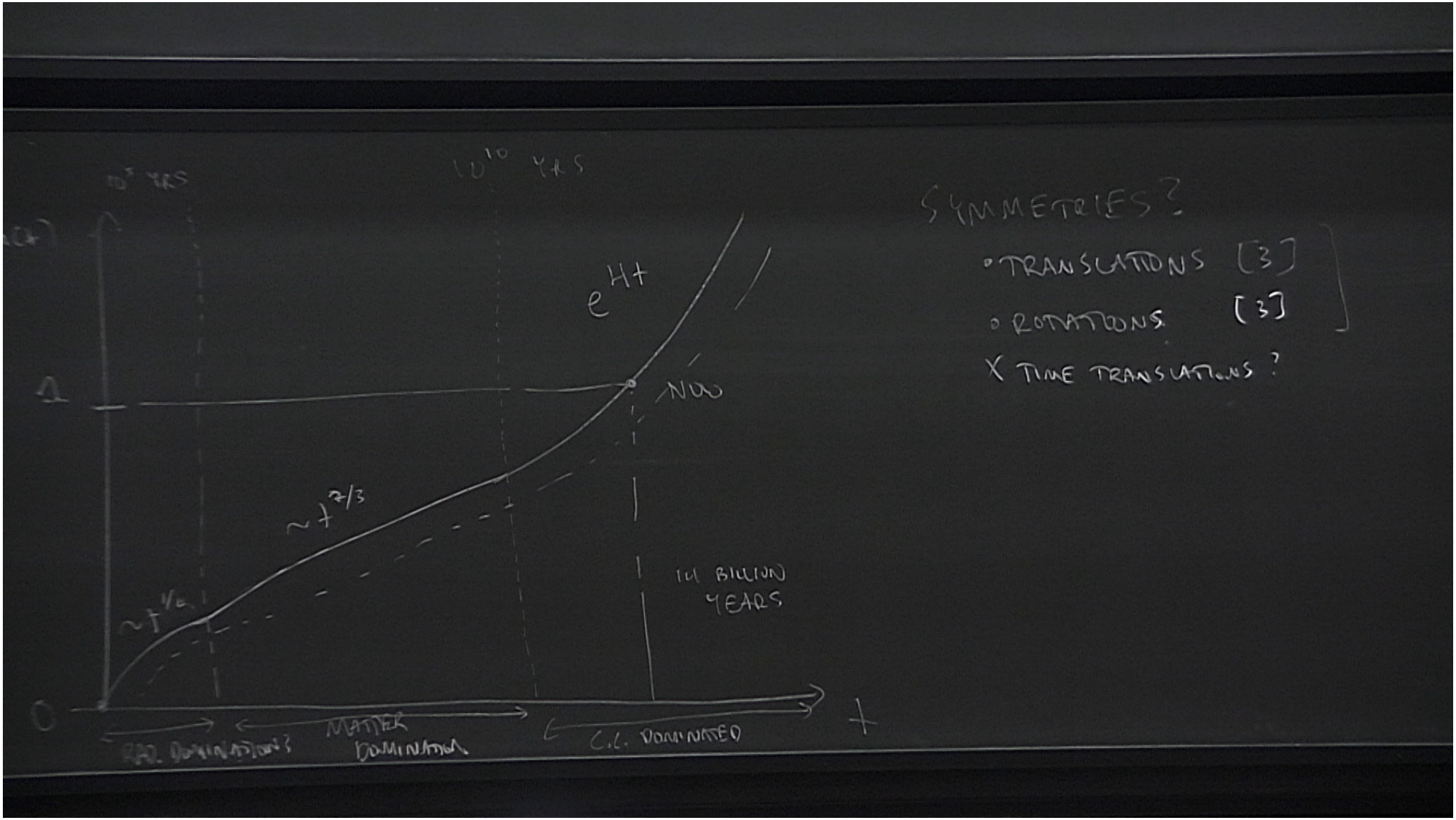

10^5 YRS
10^{10} YRS
a(t)
e^{Ht}
1
NOW
$\sim t^{2/3}$
$\sim t^{1/2}$
14 BILLION YEARS
0
RAD. DOMINATION
MATTER DOMINATION
C.C. DOMINATED
t

EXPANDING UNIVERSE REVIEW

FRW METRIC: $ds^2 = -dt^2 + a(t)^2 dx^2$

$a(t)$ = "SCALE FACTOR"

[SPATIAL CURVATURE $k = 0$]

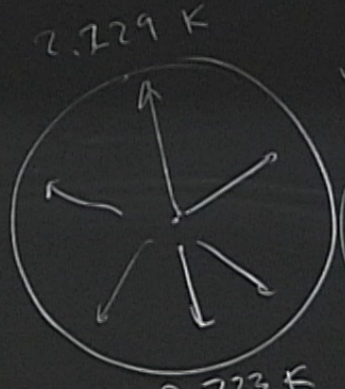

10^5 YRS
10^{10} YRS
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NOW
$\sim t^{2/3}$
$\sim t^{1/2}$
14 BILLION YEARS
0
MATTER DOMINATED
C.C. DOMINATED
t
SYMMETRIES?
• TRANSLATIONS [3]
• ROTATIONS [3]
X TIME TRANSLATIONS ?

SYMMETRIES?

- TRANSLATIONS [3]
- ROTATIONS [3]

X TIME TRANSLATIONS ?

X BOOSTS ?



"MONOPOLE"
2.726 K

"DIPOLE"
3 mK

COSMOLOGICAL
100 MK

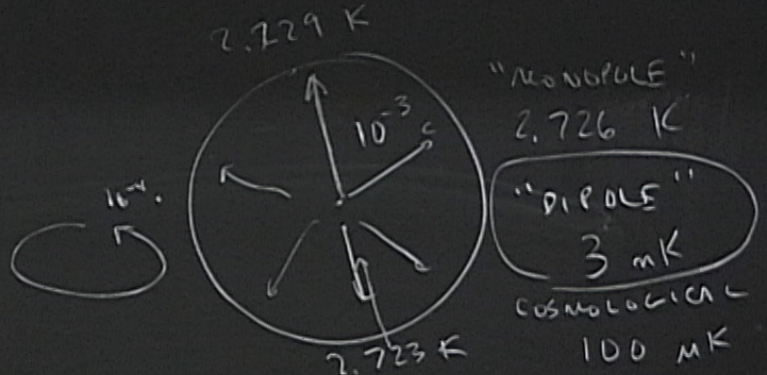
$\Longleftrightarrow$ UNIVERSE HAS A PREFERRED REST UNIVERSE

SYMMETRIES?

- TRANSLATIONS [3]
- ROTATIONS [3]

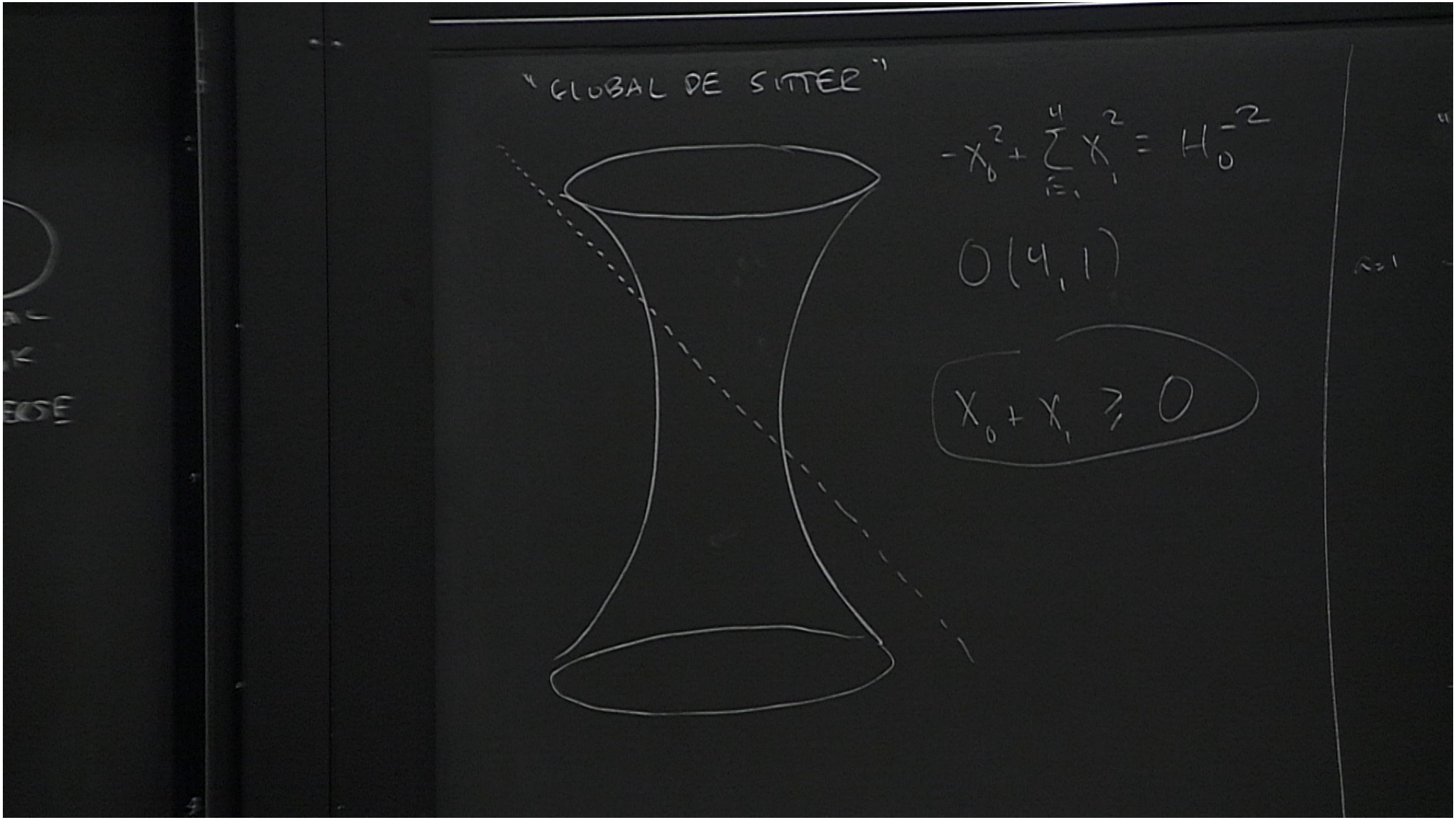
X TIME TRANSLATIONS ?

X BOOSTS? $\Longleftrightarrow$ UNIVERSE HAS A PREFERRED REST UNIVERSE

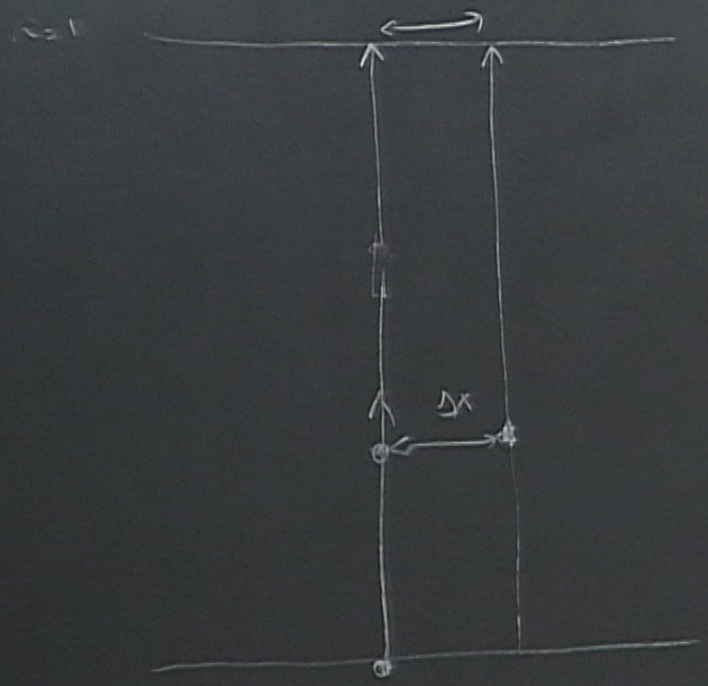


FOR FUN: SPECIAL CASES W/ MORE SYMMETRY?

- MINKOWSKI [a = const.] TIME TRANSLATIONS + BOOSTS [10]
- "DE SITTER" [$a = e^{Ht} \Leftrightarrow H$ const. $\Leftrightarrow \dot{H} = 0$] [10]


"GLOBAL DE SITTER"
$-X_0^2 + \sum_{i=1}^{4} X_i^2 = H_0^{-2}$
$O(4,1)$
$X_0 + X_1 \geq 0$

"COMOVING OBSERVER" = AT REST IN FRW COORDINATES



$$(\Delta x)_{phys} = a(t)\,(\Delta x)_{coord}$$

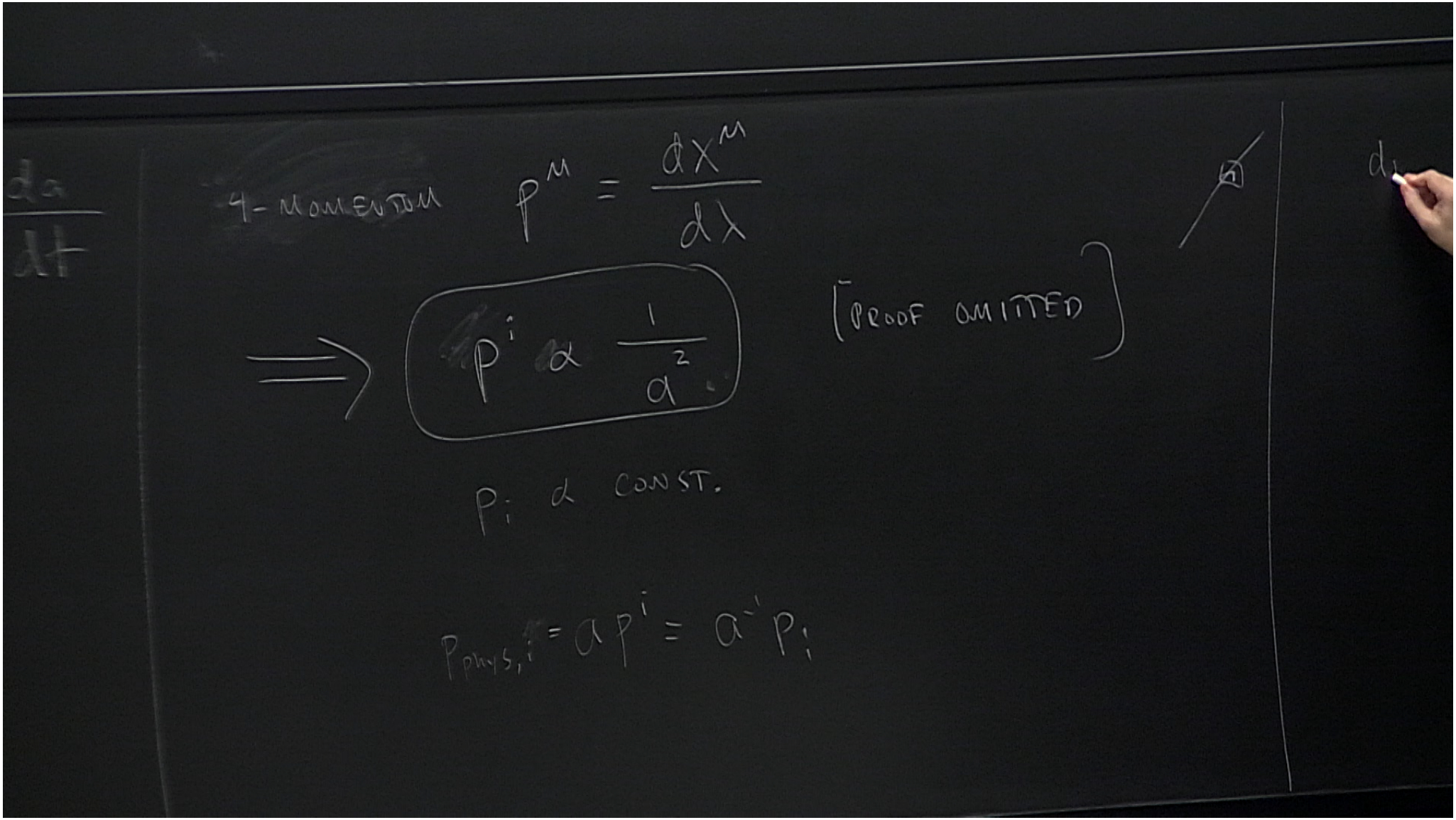
HUBBLE EXPANSION RATE $H(t) = \frac{1}{a(t)} \frac{da}{dt}$

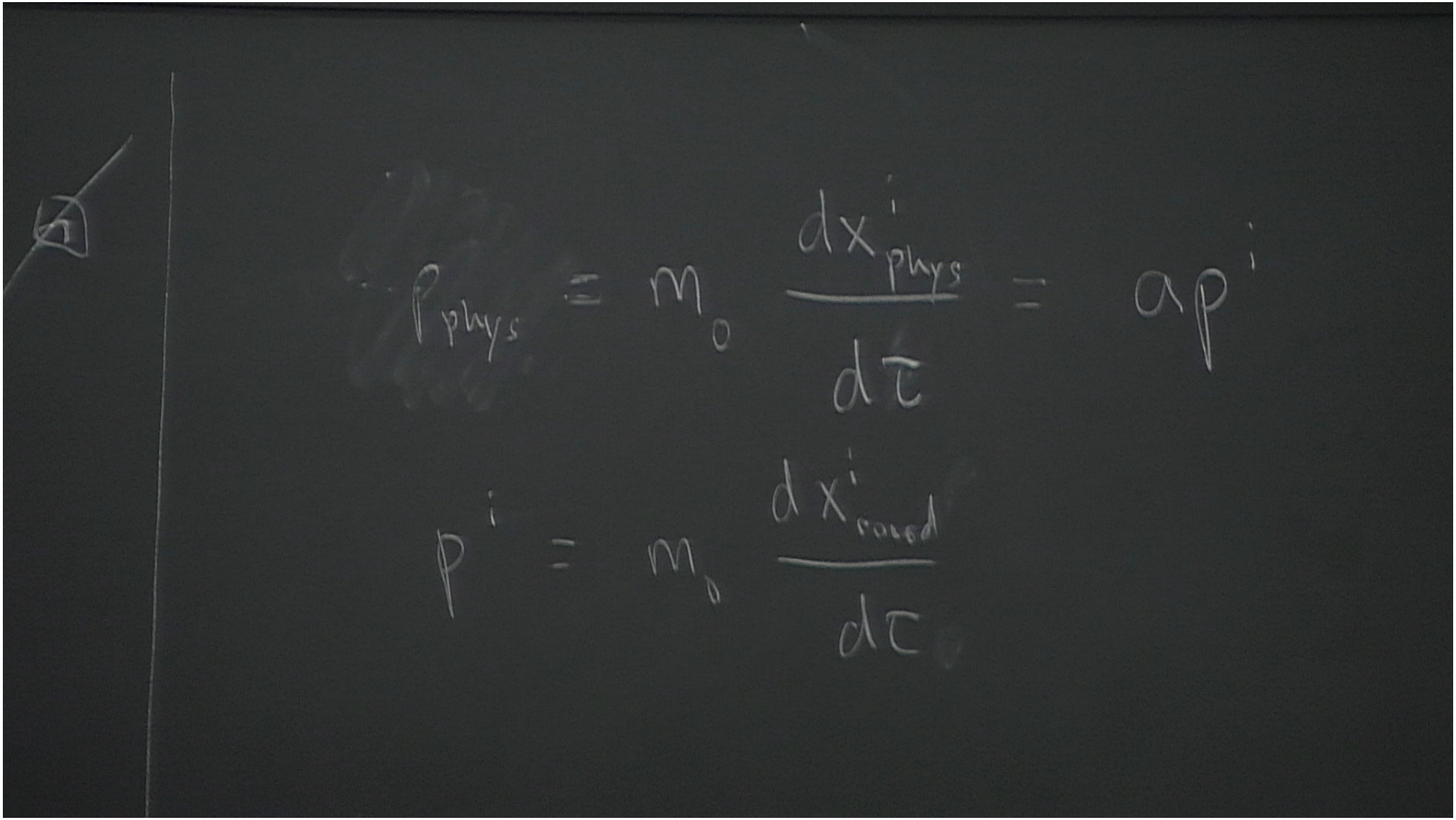
GEODESICS IN FRW

$$X^\mu = X^\mu(\lambda)$$

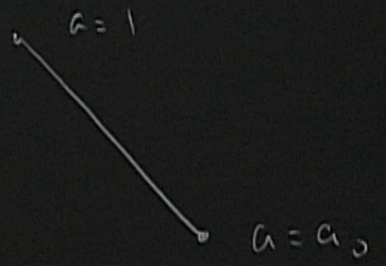
$$\frac{d^2X^\mu}{d\lambda} + \Gamma^\mu_{\nu\rho} \frac{dX^\nu}{d\lambda} \frac{dX^\rho}{d\lambda}$$

$$\Gamma^0_{ij} = a^2 H \delta_{ij} \qquad \Gamma^j_{0i} = H \delta^j_i$$

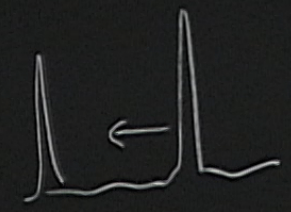

4-MOMENTUM $p^\mu = \frac{dx^\mu}{d\lambda}$
$\Rightarrow$ $p^i \propto \frac{1}{a^2}$
[PROOF OMITTED]
$p_i \propto$ CONST.
$p_{phys,i} = a p^i = a^{-1} p_i$


$p_{phys} = m_0 \frac{dx^i_{phys}}{d\tau} = ap^i$
$p^i = m_0 \frac{dx^i_{coord}}{d\tau}$

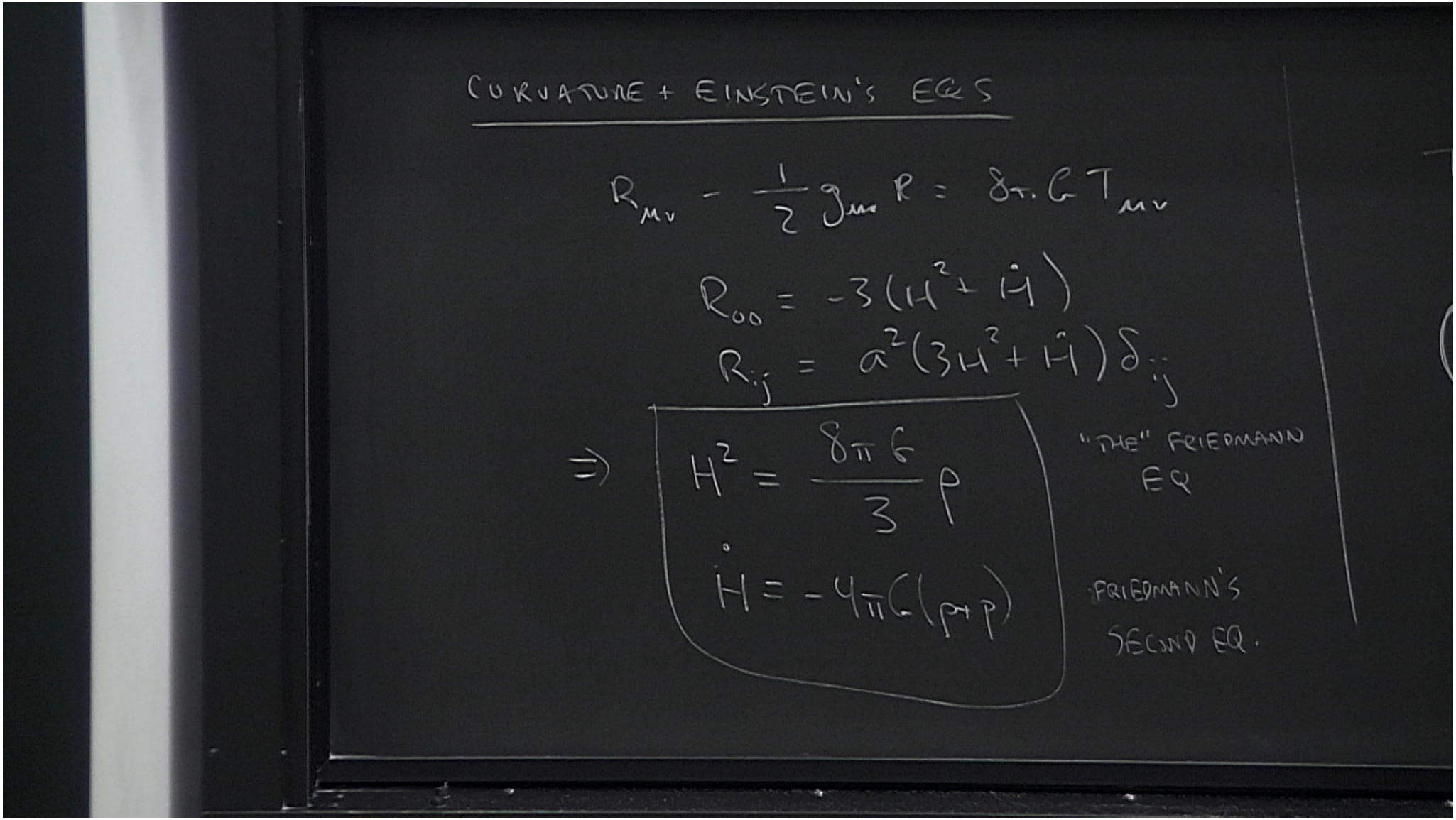
$$p_{phys} = \frac{m_0 v}{\sqrt{1 - v^2}}$$



$p_{phys} \sim v$

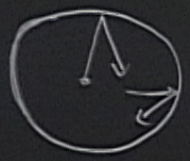


$a_0 v_0 \qquad v_0$


CURVATURE + EINSTEIN'S EQS
$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$
$R_{00} = -3(H^2 + \dot{H})$
$R_{ij} = a^2(3H^2 + \dot{H})\delta_{ij}$
$\Rightarrow$
$H^2 = \frac{8\pi G}{3}\rho$
$\dot{H} = -4\pi G(\rho + p)$
"THE" FRIEDMANN EQ
FRIEDMANN'S SECOND EQ.

RAD. DOMINATION? DOMINATION C.C. DOMINATED

$$T_{\mu\nu} = \begin{pmatrix} \rho(t) & 0 \\ 0 & a^2 p(t)\delta_{ij} \end{pmatrix}$$



$\rho(t)$ = PHYSICAL ENERGY DENSITY

$p(t)$ = PHYSICAL PRESSURE

CONTINUITY EQ $\quad \nabla_\mu T^{\mu\nu} = 0$

$$\frac{d\log\rho}{d\log a} = -3(1+w)$$

$w(t)$ = "EQUATION OF STATE"

$$= \frac{\dot{\rho}(t)}{\rho(t)}$$

TO CLOSE EQS, NEED TO KNOW SOMETHING ABOUT $\rho(t)$

"CONSTANT-w UNIVERSE": $w = \text{CONST.}$

$$\rho(t) \propto a^{-3(1+w)}$$

PROBLEM SET

$$\Rightarrow a = a(t)$$

SPECIAL CASES: $w =$

- $\frac{1}{3}$ — RADIATION DOMINATED — $\rho \sim a^{-4}$
- 0 — MATTER DOMINATED — $\rho \sim a^{-3}$
- -1 — COSMOLOGICAL CONSTANT — $\rho \sim a^{0}$

$\nabla_\mu T^{\mu\nu} = 0$

ΛCDM MODEL: $\rho(a) = \rho_{r0}\, a^{-4} + \rho_{m0}\, a^{-3} + \Lambda$

$\rho(a) = \left(\frac{1}{3}\right)\rho_{r0}\, a^{-4} + (0)\,\rho_{m0}\, a^{-3} + (-1)\Lambda$

$(\cdot)_0$ = "EVALUATED AT $a = 0$"

WEIRD COSMOLOGISTS' PARAMETRIZATION

"NATURAL" PARAMETERIZATION: $\rho_{r0}, \rho_{m0}, \Lambda$

"COSMOLOGISTS PARAMETERIZATION": $h, \Omega_r, \Omega_m, \Omega_\Lambda$

$$\Omega_i = \frac{\rho_{i0}}{\rho_{tot,0}}$$

$$H_0 = 100\, h\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$$

$$V = H_0 D$$

WEIRD COSMOLOGISTS' PARAMETRIZATION

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$$\Omega_i = \frac{\rho_{i0}}{\rho_{tot,0}}$$

$$V = H_0 D$$

$$H_0 = 100\, h\ \text{km s}^{-1}\ \text{Mpc}^{-1}$$

$30\, h^{-1}$ Mpc

REID COSMOLOGISTS' PARAMETRIZATION

"NATURAL" PARAMETERIZATION: $\rho_{r0}, \rho_{m0}, \Lambda$

"COSMOLOGISTS PARAMETERIZATION": $h, \Omega_r, \Omega_m, \Omega_\Lambda$

$$\Omega_i = \frac{\rho_{i0}}{\rho_{tot,0}}$$

$$H_0 = 100\, h\ \text{km s}^{-1}\ \text{Mpc}^{-1}$$

$$V = H_0 D$$

$h \sim 0.67$ [CMB]

$h \sim 0.73$ [SUPERNOVAE]

$z \sim 0.01$

$$D = \frac{z}{H} = 30\, h^{-1}\ \text{Mpc}$$

$$30\, h^{-1}\ \text{Mpc} \quad (\Omega_i h^2) \sim$$

