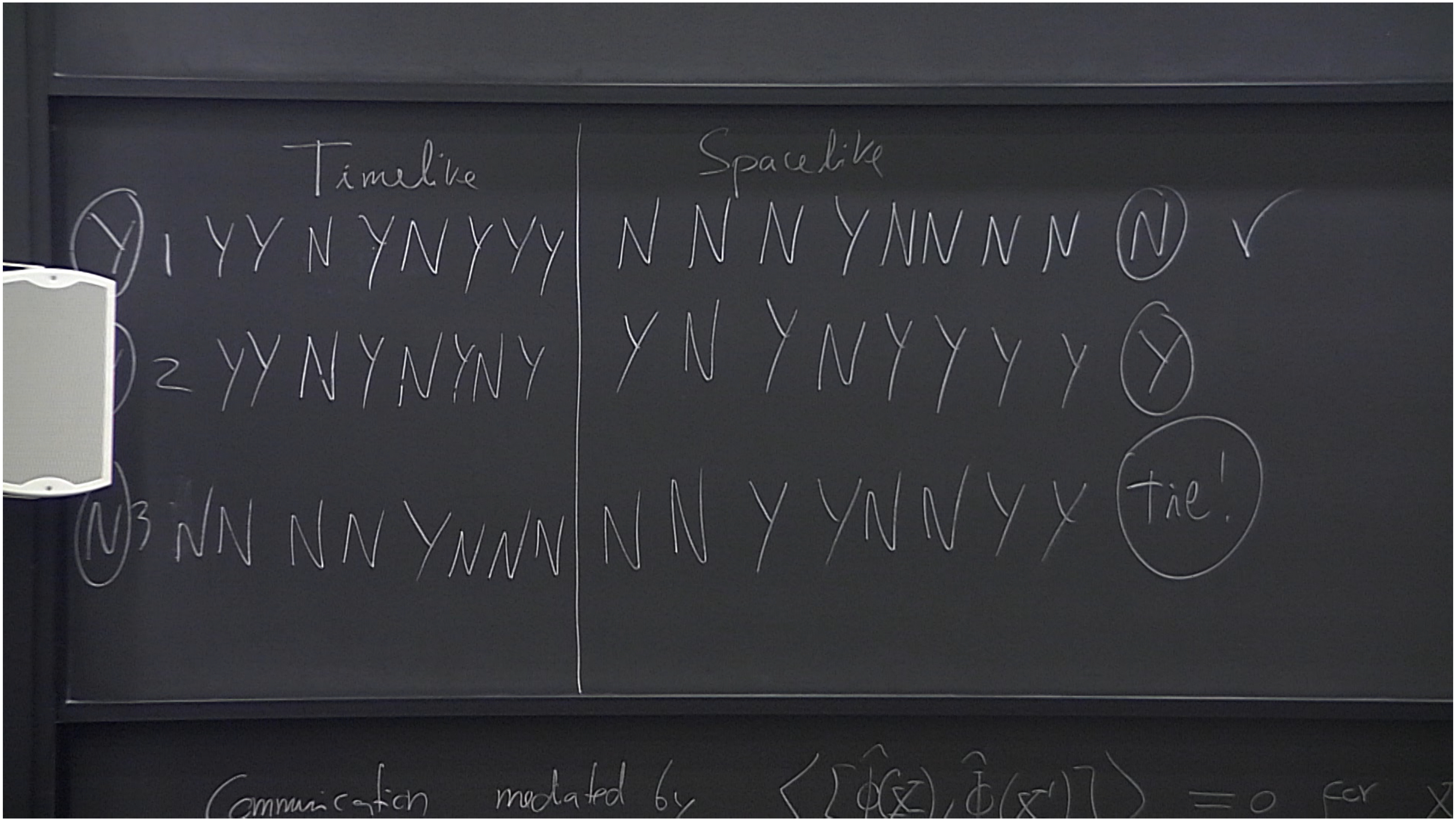
Title: PSI 2018/2019 - Explorations in Quantum Information - Lecture 9

Speakers: Eduardo Martin-Martinez

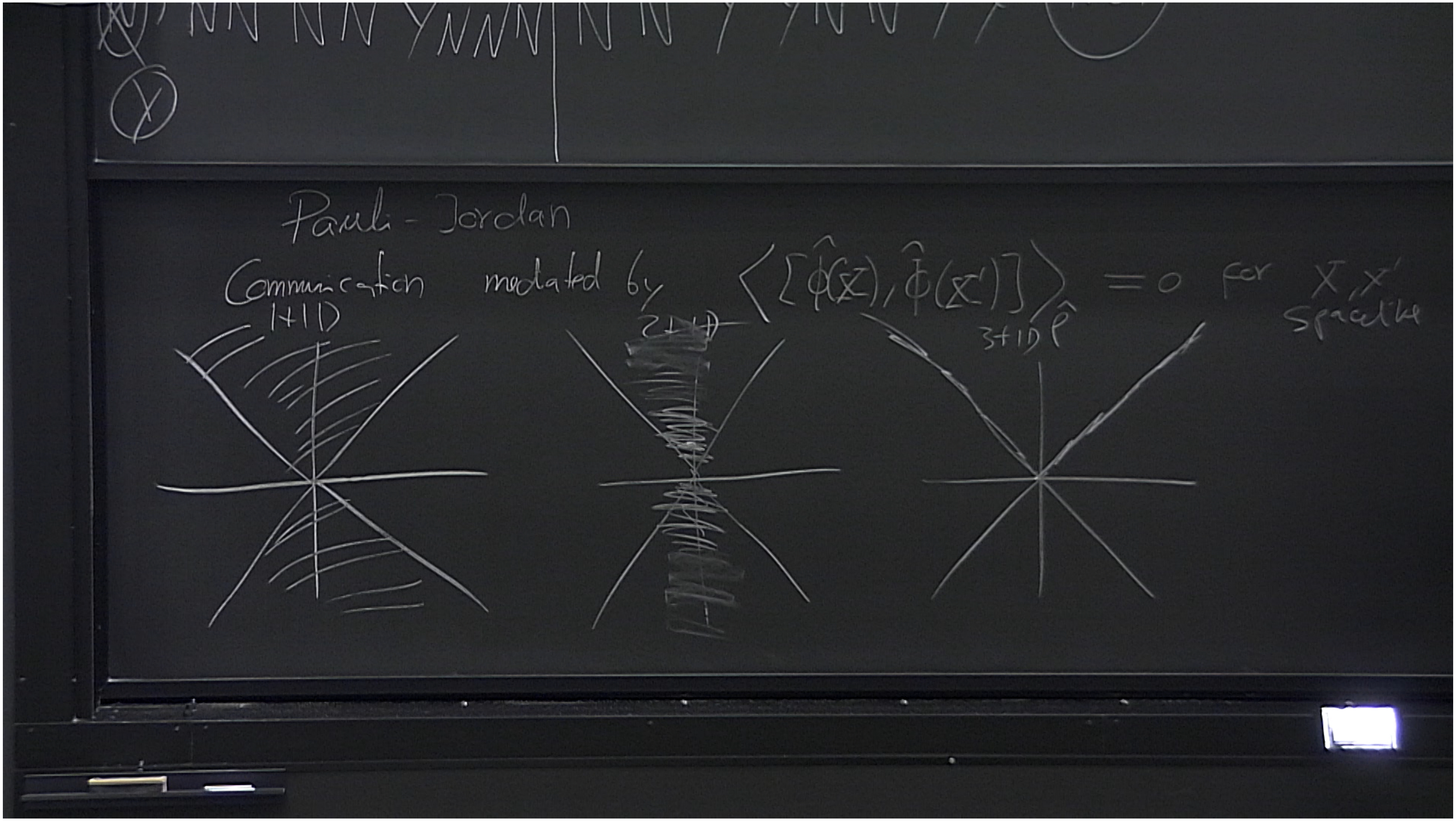
Collection: PSI 2018/2019 - Explorations in Quantum Information (Martin-Martinez)

Date: April 26, 2019 - 9:00 AM

URL: http://pirsa.org/19040060

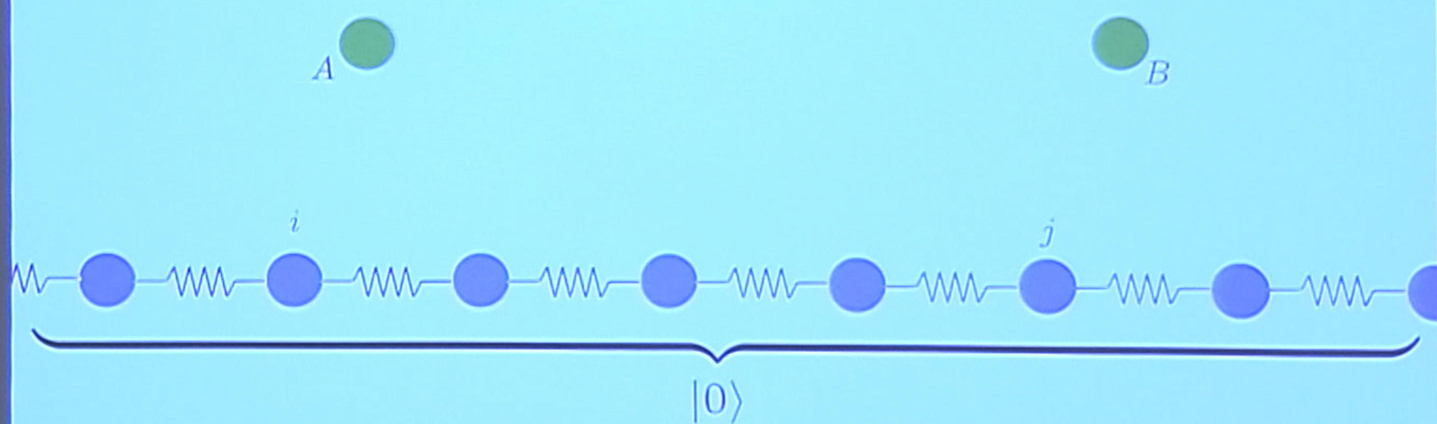

Timelike
Spacelike
Y 1 Y Y N Y N Y Y Y
N N N Y N N N N N
2 Y Y N Y N Y N Y
Y N Y N Y Y Y Y Y Y
N 3 N N N N Y N N N
N N Y Y N N Y Y
Tie!
Communication mediated by

$$\langle \hat{\phi}(\vec{x})\,\hat{\phi}(\vec{x}')\rangle_{\hat{\rho}} - \langle \hat{\phi}(\vec{x})\rangle_{\hat{\rho}}\langle \hat{\phi}(\vec{x}')\rangle_{\hat{\rho}}$$

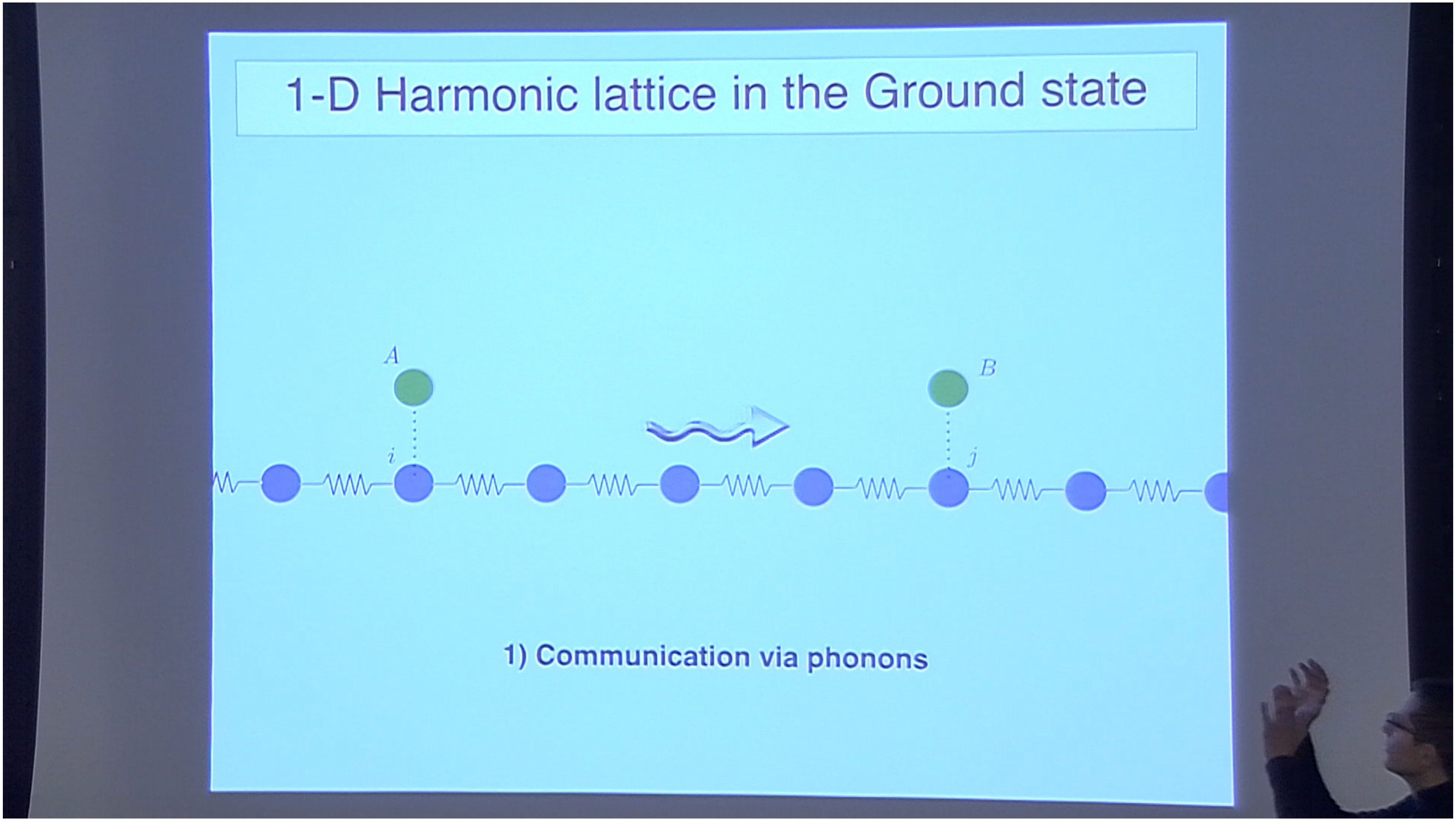

Pauli - Jordan
Communication mediated by $\langle [\hat{\phi}(x), \hat{\phi}(x')] \rangle = 0$ for x, x' spacelike
1+1D
2+1D
3+1D

1-D Harmonic lattice in the Ground state

How do we get two systems entangled by means of local interactions with a lattice in the ground state?

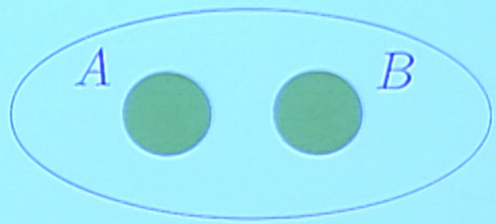


Two possible mechanisms.


1-D Harmonic lattice in the Ground state
A
B
i
j
1) Communication via phonons

1-D Harmonic lattice in the Ground state

1) Communication via phonons



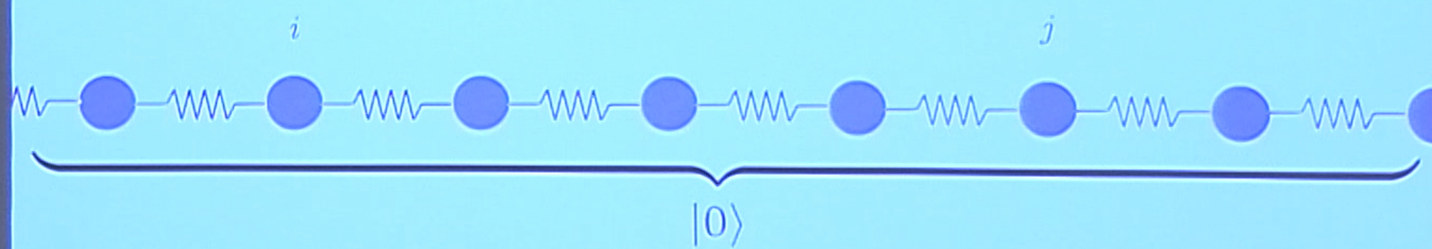
$$\rho_{AB} \neq \sum_i p_i \underset{i}{\rho}_A \otimes \underset{i}{\rho}_B$$

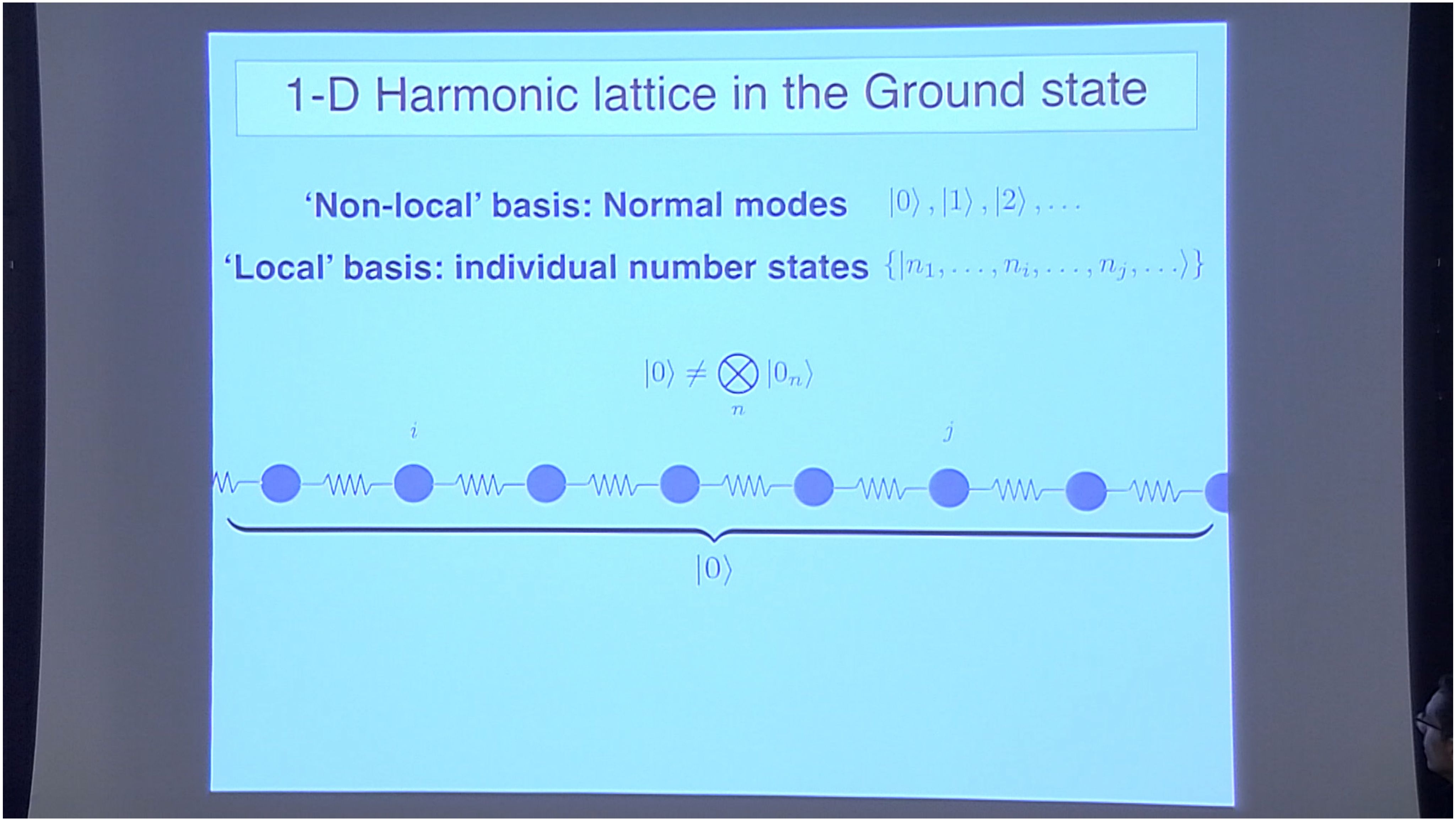
Limited by the speed of 'sound'

1-D Harmonic lattice in the Ground state

There's another possibility:

Take advantage of pre-existent entanglement



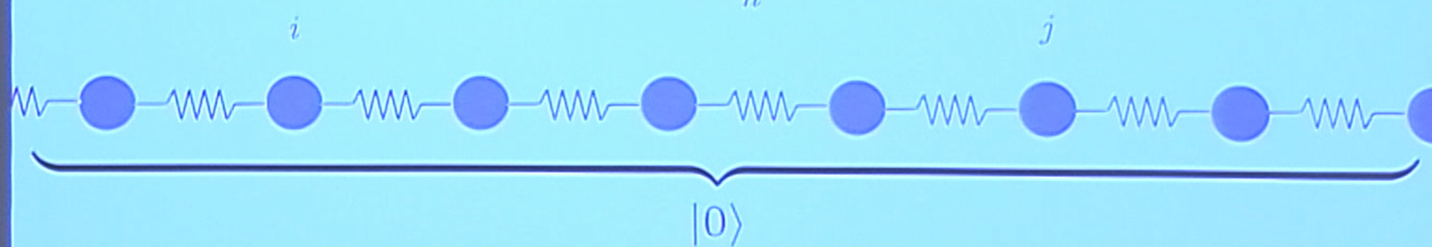

1-D Harmonic lattice in the Ground state
'Non-local' basis: Normal modes $|0\rangle, |1\rangle, |2\rangle, \ldots$
'Local' basis: individual number states $\{|n_1, \ldots, n_i, \ldots, n_j, \ldots\rangle\}$
$|0\rangle \neq \bigotimes_n |0_n\rangle$
i
j
$|0\rangle$

1-D Harmonic lattice in the Ground state

'Non-local' basis: Normal modes $|0\rangle, |1\rangle, |2\rangle, \ldots$

'Local' basis: individual number states $\{|n_1, \ldots, n_i, \ldots, n_j, \ldots\rangle\}$

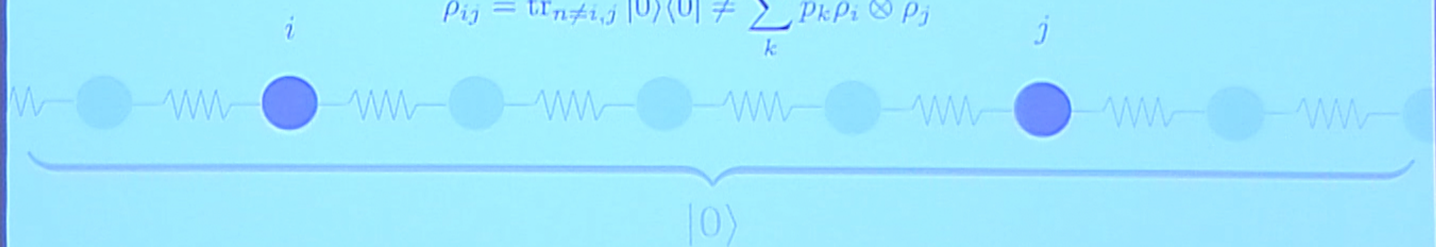
$$|0\rangle \neq \bigotimes_n |0_n\rangle$$



1-D Harmonic lattice

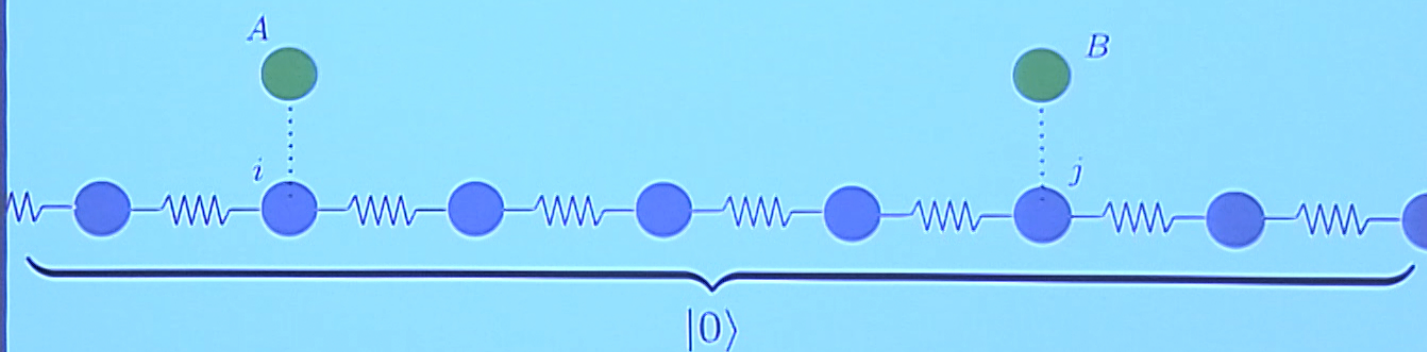
$$|0\rangle \neq \bigotimes_n |0_n\rangle$$

$$\rho_{ij} = \mathrm{tr}_{n\neq i,j}\, |0\rangle\langle 0| \neq \sum_k p_k \rho_i \otimes \rho_j$$



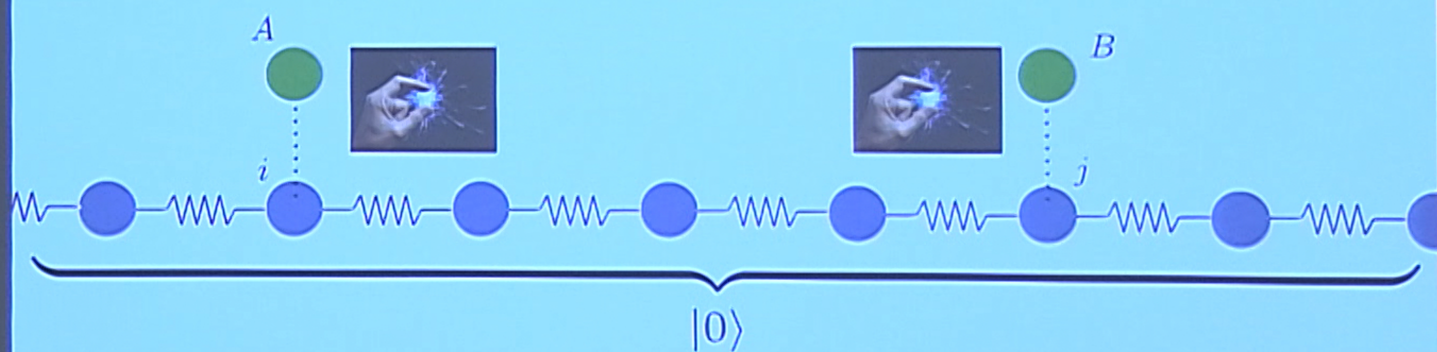
1-D Harmonic lattice in the Ground state

2) Swapping ground state entanglement



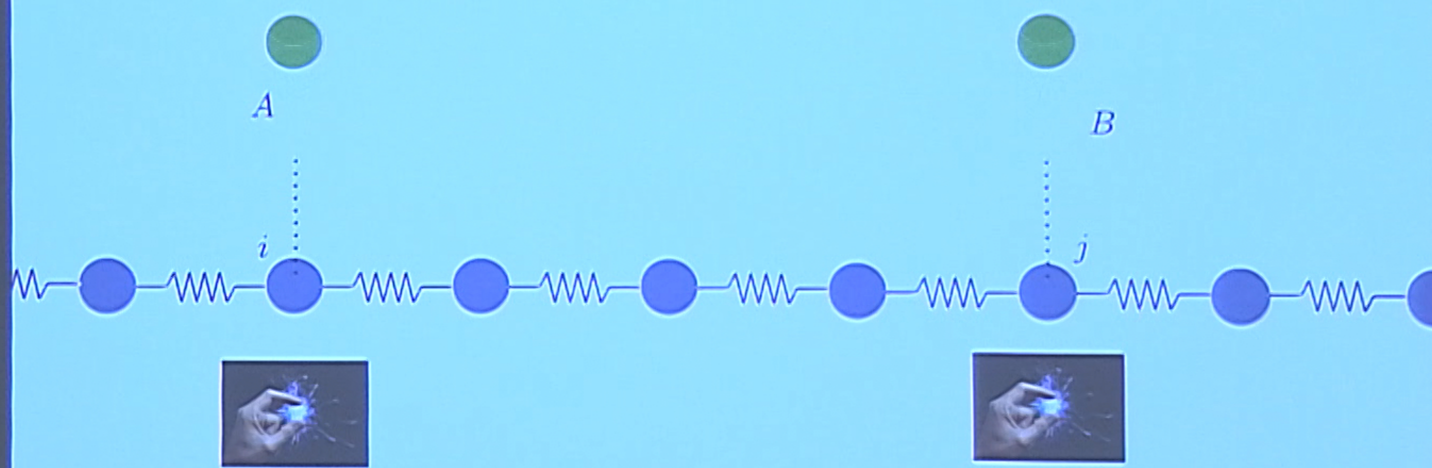
1-D Harmonic lattice in the Ground state

2) Swapping ground state entanglement



1-D Harmonic lattice in the Ground state

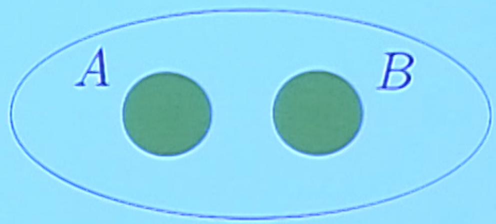
Local coupling to the vacuum: Observed fluctuations are correlated



2) Swapping ground state entanglement

1-D Harmonic lattice in the Ground state

2) Swapping ground state entanglement



$$\rho_{AB} \neq \sum_i p_i \underset{i}{\rho}_A \otimes \underset{i}{\rho}_B$$

NOT Limited by the speed of 'sound'

We have two particle detectors in the ground state coupled

$$\hat{\rho}_0 = \hat{\rho}_{A,0} \otimes \hat{\rho}_{B,0} \otimes \hat{\rho}_{\phi,0} = |g_A\rangle\langle g_A| \otimes |g_B\rangle\langle g_B| \otimes |0\rangle\langle 0|$$

$$H_I =$$

$$\hat{\rho}_T = \hat{U} \hat{\rho}_0 \hat{U}^\dagger, \quad \hat{U} = \mathcal{T} \exp\left(-i \int_{-\infty}^{\infty} dt\, \hat{H}_I(t)\right)$$

$$\hat{U} = \mathbb{1} + \hat{U}^{(1)} + \hat{U}^{(2)} + \mathcal{O}(\lambda^3)$$

$$\hat{\rho}_T = \left(\mathbb{1} + \hat{U}^{(1)} + \hat{U}^{(2)} + \mathcal{O}(\lambda^3)\right) \hat{\rho}_0 \left(\mathbb{1} + U^{(1)\dagger} + U^{(2)\dagger} + \mathcal{O}(\lambda^3)\right)$$

the ground state coupled to the vacuum of a scalar field $\hat{\Phi}$

$$ {}_{AB}| \otimes |0\rangle\langle 0| \qquad H_I = \sum_{\nu \in \{A,B\}} \lambda_\nu \chi_\nu(t) \int d^n x \, F_\nu(\vec{x} - \vec{x}_\nu) \, \hat{\mu}_\nu(t) \, \hat{\phi}(t, \vec{x}) $$

$$ \hat{H}_I(t) \Big) $$

$$ \cancel{(\mathbb{1} + U^{(1)} + U^{(2)}) |\psi_0\rangle \to (\quad) |\psi_0\rangle\langle\psi_0| (\quad)^\dagger} $$

$$ = \underbrace{\hat{\rho}_0}_{\lambda^0} + \underbrace{\hat{U}^{(1)} \hat{\rho}_0 + \hat{\rho}_0 U^{(1)\dagger}}_{\lambda^1} + \underbrace{U^{(1)} \rho_0 U^{(1)\dagger} + \hat{U}^{(2)} \hat{\rho}_0 + \hat{\rho}_0 U^{(2)\dagger}}_{\lambda^2} + \mathcal{O}(\lambda^3) $$

$$\ldots(t') \qquad \text{We want } \hat{\rho}_{AB} = tr_\phi\left(\hat{U}\hat{\rho}_0\hat{U}^\dagger\right) = \hat{\rho}_{AB,0} + \hat{\rho}^{(1)}_{AB} + \hat{\rho}^{(2)}_{AB} + \mathcal{O}(\lambda^4)$$

$$\ldots\lambda_\eta\Big[\int_{-\infty}^{\infty}dt\int_{-\infty}^{\infty}dt'\,\chi_\nu(t')\chi_\eta(t)\,\hat{\mu}_\nu(t')\hat{\rho}_{0,AB}\hat{\mu}_\eta(t)\,W(t,\vec{x}_\nu,t',\vec{x}_\eta) -$$

$$\chi_\nu(t)\chi_\eta(t')\Big(\hat{\mu}_\nu(t)\hat{\mu}_\eta(t')\hat{\rho}_{AB,0}\,W(t,\vec{x}_\nu,t',\vec{x}_\eta) + \hat{\rho}_{AB,0}\,\hat{\mu}_\eta(t')\mu_\nu(t)\,W(t',\vec{x}_\eta,t,\vec{x}_\nu)\Big)\Big]$$

$$F(\vec{x}-\vec{x}_\nu)F(\vec{x}'-\vec{x}_\eta)\underbrace{Tr_\phi\left(\hat{\phi}(t,\vec{x})\phi(t',\vec{x}')\hat{\rho}_\phi\right)}_{W_\phi(t,\vec{x},t',\vec{x}')}$$

$$\hat{U}^{(1)} = -i\int_{-\infty}^{\infty} dt\, \hat{H}_I(t) \quad ; \quad \hat{U}^{(2)} = -\int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt'\, \hat{H}(t)\,\hat{H}(t')$$

$$\hat{\rho}^{(1)}_{AB} = \mathrm{tr}_\phi\left(\hat{U}^{(1)}\hat{\rho}_0 + \rho_0 U^{(1)\dagger}\right) = 0$$

$$\hat{\rho}^{(2)}_{AB} = \sum_{\nu,\eta} \lambda_\nu \lambda_\eta \Big[\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt'\, \chi_\nu(t') \ldots$$

$$- \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt'\, \chi_\nu(t)\,\chi_\eta(t') \Big(\hat{\mu}_\nu(t)\,\hat{\mu}_\eta \ldots$$

$$W(t, \vec{x}_\nu, t', \vec{x}_\eta) = \int d^d x \int d^d x'\, F(\vec{x} - \vec{x}_\nu)\, F(\vec{x}' - \ldots$$

We want $\hat{\rho}_{AB} = \mathrm{tr}_\phi\left(\hat{U}\hat{\rho}_0\hat{U}^\dagger\right) = \hat{\rho}_{AB,0} + \hat{\rho}^{(1)}_{AB} + \hat{\rho}^{(2)}_{AB} + \mathcal{O}(\lambda^4)$

$$\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \, \chi_\nu(t')\chi_\eta(t)\, \hat{\mu}_\nu(t')\,\hat{\rho}_{0,AB}\,\hat{\mu}_\eta(t)\, W(t,\vec{x}_\eta,t',\vec{x}_\nu) -$$

$$\chi_\eta(t')\Big(\hat{\mu}_\nu(t)\hat{\mu}_\eta(t')\hat{\rho}_{AB,0}\, W(t,\vec{x}_\nu,t',\vec{x}_\eta) + \hat{\rho}_{AB,0}\,\hat{\mu}_\eta(t')\mu_\nu(t)\, W(t',\vec{x}_\eta,t,\vec{x}_\nu)\Big)\Big]$$

$$(\vec{x}-\vec{x}_\nu)\,F(\vec{x}'-\vec{x}_\eta)\,\underbrace{\mathrm{Tr}_\phi\left(\hat{\phi}(t,\vec{x})\,\phi(t,\vec{x}')\,\hat{\rho}_\phi\right)}_{W_\phi(t,\vec{x},t',\vec{x}')}$$

$$\rho_{AB} = \mathrm{tr}_\phi \left[U \rho_0 + \rho_0 U \right] = 0$$

$$\rho^{(2)}_{AB} = \sum_{\nu,\eta} \lambda_\nu \lambda_\eta \Big[\int_{-\infty} dt \int_{-\infty} dt' \, \chi_\nu(t) \ldots$$

$$- \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \, \chi_\nu(t) \chi_\eta(t') \big(\hat{\mu}_\nu(t) \hat{\mu} \ldots$$

$$\hat{\rho}_0 = \hat{\rho}_{AB} \otimes \hat{\rho}_\phi$$

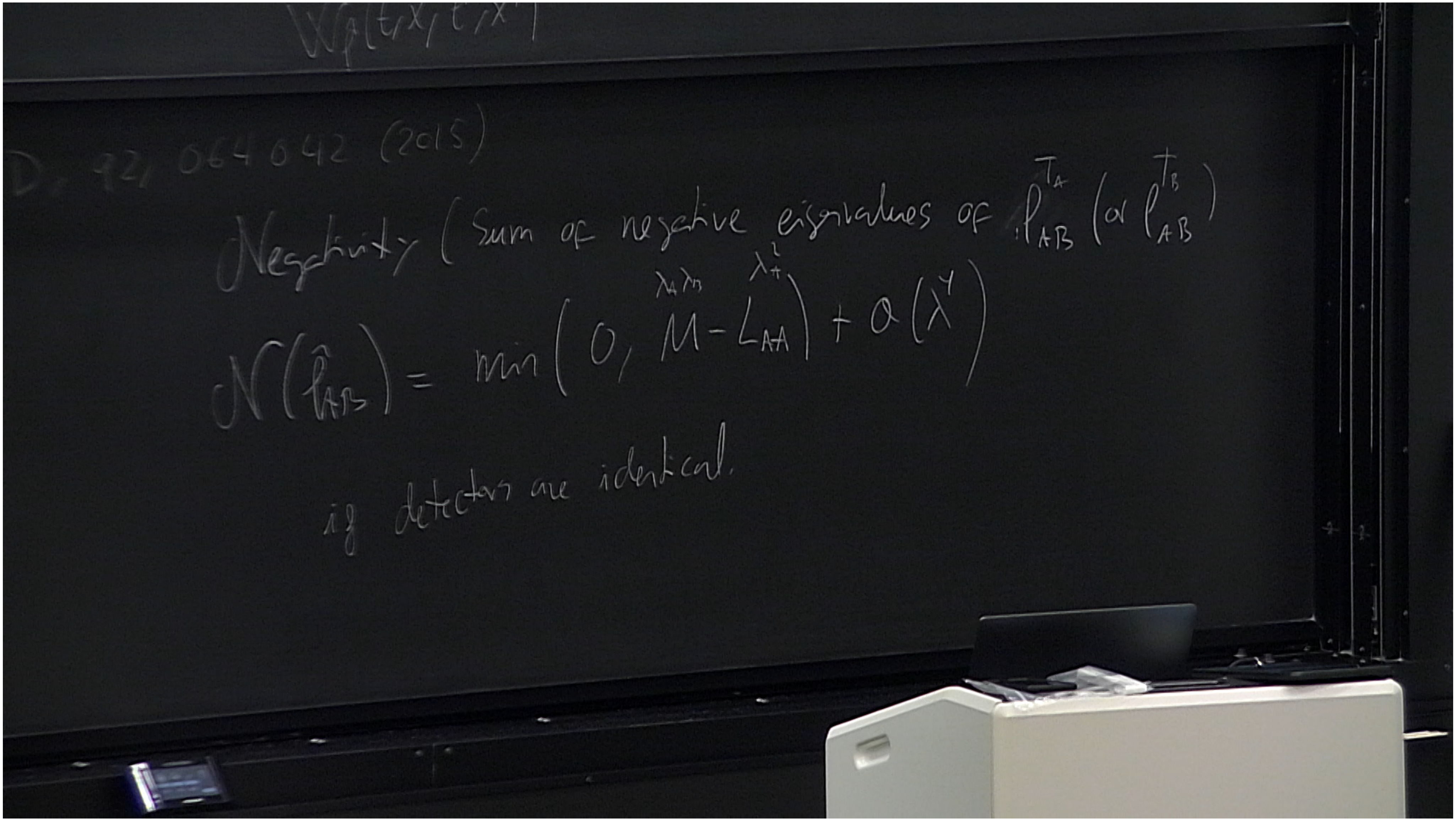
$$W(t, \vec{x}_\nu, t', \vec{x}_\eta) = \int d^d x \int d^d x' \, F(\vec{x} - \vec{x}_\nu) F(\vec{x}' - \ldots$$

$\hat{\rho}_\phi = |0\rangle\langle 0|$ $\quad$ $\hat{\rho}_{AB} = |g_A\rangle\langle g_A| \otimes |g_B\rangle\langle g_B|$

PRD, 92

$$\hat{\rho}_{AB} = \begin{pmatrix} 1-\mathcal{L}_{AA}-\mathcal{L}_{BB} & & & \mathcal{M} \\ & \mathcal{L}_{AA} & \mathcal{L}_{AB} & \\ & \mathcal{L}_{AB}^{*} & \mathcal{L}_{BB} & \\ \mathcal{M}^{*} & & & 0 \end{pmatrix} + \mathcal{O}(\lambda_\nu^4)$$

$$\{ |g_A\rangle \otimes |g_B\rangle,\ |e_A\rangle \otimes |g_B\rangle,\ |g_A\rangle \otimes |e_B\rangle,\ |e_A\rangle \otimes |e_B\rangle \}$$


D 92, 064042 (2015)
Negativity (sum of negative eigenvalues of $\rho_{AB}^{T_A}$ (or $\rho_{AB}^{T_B}$)
$\mathcal{N}(\hat{\rho}_{AB}) = \min\left(0, \overset{\lambda_A\lambda_B}{M} - \overset{\lambda_+^2}{L_{AA}}\right) + O(\lambda^4)$
if detectors are identical.

arXiv: 1506.03681

m of negative eigenvalues of $\rho_{AB}^{T_A}$ (or $\rho_{AB}^{T_B}$)

$$\text{in}\left(0,\ \overset{\lambda_A \lambda_B}{M} - \overset{\lambda_+^2}{L_{AA}}\right) + O(\lambda^4)$$

identical.