Title: PSI 2018/2019 - Explorations in Quantum Information - Lecture 8

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A detector that interacts with a KMS state (w.r.t its proper time) of KMS temperature $T = \frac{1}{\beta}$, as long as it's switched on carefully, thermalizes (catches Boltzmannian populations) to Gibbs temperature $T = \frac{1}{\beta}$ after interacting with the field for a long time.

Detailed balance $R = \frac{P^{+}}{P^{-}}$, $\lim\limits_{\substack{\text{adiabatic} \\ \sigma \to \infty}} R = e^{-\beta\Omega}$

KMS state (w.r.t its proper time) of KMS temperature $T = \frac{1}{\beta}$, as long as
nalizes (catches Boltzmannian populations) to Gibbs temperature $T = \frac{1}{\beta}$ after
long time.

$$\frac{P^+}{P^-} \to \lim_{\substack{\text{adiabatic} \\ \sigma \to \infty}} R = e^{-\beta \Omega}$$

$$W(\tau, \tau') = \mathrm{Tr}\left[\phi(\tau)\, \phi(\tau')\, \hat{\rho} \right]$$

$$W^*(\tau, \tau') = W(\tau', \tau)$$

if $\hat{\rho}_\phi$ is

Fourier Trasf

FT of (1)

$$\int_{-\infty}^{\infty} d\Delta\tau \; W(\Delta\tau$$

Let's define

$$\tilde{C}(\omega) =$$

if $\hat{\rho}_\phi$ is KMS w.r.t $\partial_z \Rightarrow W(z,z') = W(\Delta z)$ and $\underbrace{W(\Delta z + i\beta) = W(-\Delta z)}_{(1)}$, $W(z)$ is holomorphic $0 \leq \mathrm{Im}\, z \leq \beta$

Fourier Transf of $W(\Delta z)$: $\tilde{W}(\omega) := \int_{-\infty}^{\infty} d\Delta z\, W(\Delta z)\, e^{i\omega\Delta z}$

FT of (1)

$$\int_{-\infty}^{\infty} d\Delta z\, W(\Delta z + i\beta)\, e^{i\omega\Delta z} = \underbrace{\int_{-\infty}^{\infty} d\Delta z\, W(-\Delta z)\, e^{i\omega\Delta z}}_{\tilde{W}(-\omega)} \underset{\Delta z' = \Delta z + i\beta}{\Rightarrow} \int_\gamma d\Delta z'\, W(\Delta z')\, e^{i\omega(\Delta z' - i\beta)} = e^{\beta\omega} \overbrace{\int_\gamma d\Delta z'\, W(\Delta z')\, e^{i\omega\Delta z'}} = e^{\beta\omega}\, \tilde{W}(\omega) \Rightarrow \tilde{W}(-\omega) = e^{\beta\omega}\, \tilde{W}(\omega)$$

$$\mathcal{P}(\omega,\beta) := \frac{1}{e^{\beta\omega} - 1}$$

Let's define $C(z,z') := \langle [\hat{\phi}(z), \hat{\phi}(z')] \rangle_{\hat{\rho}} = 2i\, \mathrm{Im}(W(z,z'))$. if $W(z,z') = W(\Delta z) \Rightarrow C(z,z') = C(\Delta z)$

$$\tilde{C}(\omega) = \int_{-\infty}^{\infty} d\Delta z\, C(\Delta z)\, e^{i\omega\Delta z} = \int_{-\infty}^{\infty} d\Delta z \left(W(\Delta z) - \overbrace{W^*(\Delta z)}^{W(-\Delta z)} \right) e^{i\omega\Delta z} = \tilde{W}(\omega) - \tilde{W}(-\omega) \overset{(1)}{\Rightarrow} \tilde{C}(\omega) = \tilde{W}(\omega) - e^{\beta\omega}\, \tilde{W}(\omega) \Rightarrow$$

$$\Rightarrow \boxed{\tilde{W}(\omega,\beta) = -\tilde{C}(\omega,\beta)\, \mathcal{P}(\omega,\beta)}$$

Does a constantly accelerated detector thermalize to a temperature $T = \frac{a}{2\pi}$, when adiabatically coupled to the field vacuum?

$$\text{the vacuum state of the field is KMS} \begin{cases} \text{w.r.t. } \partial_t \text{ (of inertial observers) of } T = 0 \\ \text{w.r.t. } \partial_\tau \text{ (of const accelerated observers) of } T = \frac{a}{2\pi} \end{cases} !!$$

$$W(\tau,\tau') = \langle 0| \hat{\phi}(t(\tau),\vec{x}(\tau))\, \hat{\phi}(t(\tau'),\vec{x}(\tau'))|0\rangle$$

$$t(\tau) = \frac{1}{a}\sinh(a\tau) \qquad x^2 = x^3 = \dots = x^d = 0$$

$$x^1(\tau) = \frac{1}{a}\left(\cosh(a\tau) - 1\right)$$

$$W(\tau,\tau') = \int \frac{d^d k}{2(2\pi)^d}\, e^{-i\left[|\vec{k}|(t(\tau)-t(\tau')) - \vec{k}\cdot(\vec{x}(\tau)-\vec{x}(\tau'))\right]} \left(e^{-\epsilon|\vec{k}|}\right)$$

$i\epsilon$ prescription
$\Downarrow$
soft UV cutoff

$$W(z,z') = \int \frac{d^{d+1}k}{2(2\pi)^3}\, \Theta(k^0)\, \delta(k^\mu k_\mu)\, e^{i k_\mu \left(X(z) - X(z')\right)^\mu}$$

the trajectory of the detector is timelike

$$(\mathbf{X}-\mathbf{X}')^\mu(\mathbf{X}-\mathbf{X}')_\mu < 0$$

Let's define $\Delta := \sqrt{-(\mathbf{X}-\mathbf{X}')^\mu(\mathbf{X}-\mathbf{X}')_\mu}$

What is $K_\mu(\mathbf{X}-\mathbf{X}')^\mu = K_0\left(t(\tau)-t(\tau')\right) + K_1\left(x^1(\tau)-x^1(\tau')\right)$

$$W(z,z') = \int \frac{d^{d+1}k}{2(2\pi)^3} \Theta(k^0)\, \delta(k^\mu k_\mu)\, e^{i k_\mu (X(z) - X(z'))^\mu}$$

the traj... $(X$ ...

Let's def...

What is k_μ ...

Change of var.

$$k_\mu (X - X')^\mu = -\bar{k}^0 \Delta\, \mathrm{sgn}(t(z) - t(z'))$$

$$\bar{k}^0 := \Delta^{-1} \left[(t(z) - t(z'))k^0 - (x^1(z) - x^1(z'))k^1 \right] \mathrm{sgn}(t(z) - t(z'))$$

$$\bar{k}^1 := \Delta^{-1} \left[-(x^1(z) - x^1(z'))k^0 + (t(z) - t(z'))k^1 \right] \mathrm{sgn}(t(z) - t(z'))$$

$$\bar{k}^i := k^i \quad i > 1$$

$$d^{d+1}\bar{k}\, \Theta(\bar{k}^0)\, \delta(\bar{k}_\mu \bar{k}^\mu) = d^{d+1}k\, \Theta(k^0)\, \delta(k_\mu k^\mu)$$

$\tilde{\mathcal{W}}(-\omega)$

Let's define $C(z,z') := \left\langle \left[\hat{\phi}(z), \hat{\phi}(z')\right] \right\rangle_{\hat{\rho}} = 2i\,\mathrm{Im}\left(\mathcal{W}(z,z')\right)$. if $\mathcal{W}(z,z') = \mathcal{W}(\Delta z) \Rightarrow C(z,z') = C($

$$\tilde{C}(\omega) = \int_{-\infty}^{\infty} d\Delta z\, C(\Delta z)\, e^{i\omega\Delta z} = \int_{-\infty}^{\infty} d\Delta z \left(\mathcal{W}(\Delta z) - \overbrace{\mathcal{W}^*(\Delta z)}^{\mathcal{W}(-\Delta z)}\right) e^{i\omega\Delta z} = \tilde{\mathcal{W}}(\omega) - \tilde{\mathcal{W}}(-\omega) \overset{(1)}{\Rightarrow} \tilde{C}(\omega) = \tilde{\mathcal{W}}($$

$\Rightarrow \boxed{\tilde{\mathcal{W}}(\omega)}$

$$\mathcal{W}(z,z') = \frac{(4\pi)^{-d/2}}{\Gamma\left(\frac{d}{2}\right)} \int_0^\infty d|\vec{k}|\; k^{d-2}\, e^{-i|\vec{k}|\Delta\,\mathrm{sgn}(t(z)-t(z'))} = \frac{\Gamma\left(\frac{d-1}{2}\right)}{4\pi^{\frac{d+1}{2}}} \left(\Delta\,\mathrm{sgn}(t(z)-t(z'))\right)$$

$$\Delta\,\mathrm{sgn}(t(z)-t(z')) = 2a^{-1}\sinh\left(\frac{a}{2}(z-z')\right) \longleftarrow \sinh(x) - \sinh(y) = 2\cosh\frac{x+y}{2}\sinh\frac{x-y}{2}$$

$$\cosh(x) - \cosh(y) = 2\sinh\left(\frac{x+y}{2}\right)\sinh\left(\frac{x-y}{2}\right)$$

$$\mathcal{W}(\Delta z) = \underbrace{\frac{\Gamma\left(\frac{d-1}{2}\right)}{2\pi^{\frac{d+1}{2}}}}_{G_d} \frac{1}{a}\sinh\left(\frac{a}{2}\Delta z\right) \quad \checkmark \text{ stationary}$$

$$\Delta\,\mathrm{sgn}(t(\tau)-t(\tau')) = 2a^{-1}\sinh\left(\frac{a}{2}(\tau-\tau')\right) \longleftarrow \sinh(x)-\sinh(y) = 2\cosh\frac{x+y}{2}\sinh\frac{x-y}{2}$$

$$\cosh(x)-\cosh(y) = 2\sinh\left(\frac{x+y}{2}\right)\sinh\left(\frac{x-y}{2}\right)$$

$$W(\Delta\tau) = \underbrace{\frac{\Gamma\left(\frac{d-1}{2}\right)}{2\pi^{\frac{d+1}{2}}}}_{G_d}\frac{1}{a}\sinh\left(\frac{a}{2}\Delta\tau\right) \quad \checkmark \text{ stationary}$$

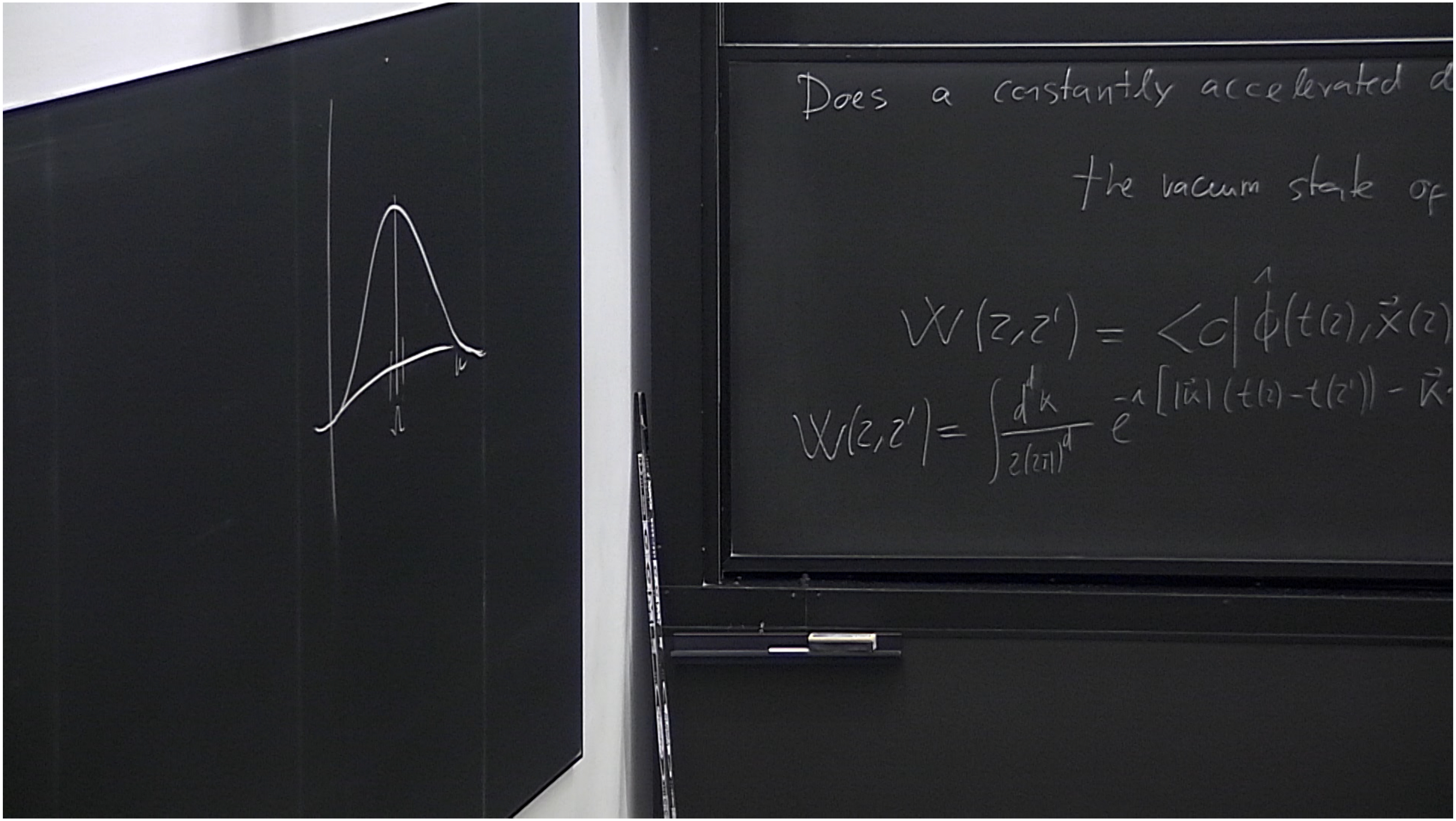
$$W\left(\Delta\tau + i\frac{2\pi}{a}\right) = G_d\frac{1}{a}\sinh\left(\frac{a}{2}\Delta\tau + i\pi\right)$$

$$= G_d\frac{1}{a}\sinh\left(\frac{a}{2}(-\Delta\tau)\right) = W(-\Delta\tau)$$ Complex anti-periodicity ✓

$W(\Delta\tau)$ is holomorphic everywhere!

$$\sinh[x+i\pi] = \frac{e^{x}(e^{i\pi}) - e^{-x}(e^{-i\pi})}{2} = -\sinh[x] = \sinh[-x]$$

$|0\rangle$ is KMS w.r.t. ∂_τ of a constantly accelerated observer with KMS Temperature $T = \frac{a}{2\pi} \Rightarrow \boxed{T_U = \frac{\hbar a}{2\pi k_B c}}$ Unruh temp.


Does a constantly accelerated d
the vacuum state of
W(τ,τ') = <0| φ̂(t(τ), x⃗(τ)
W(τ,τ') = ∫ d^d k / 2(2π)^d e^{-i[|k⃗|(t(τ)-t(τ')) - k⃗·