Title: PSI 2018/2019 - Explorations in Quantum Information - Lecture 7

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Collection: PSI 2018/2019 - Explorations in Quantum Information (Martin-Martinez)

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(pull-back of) Wightman (two-point) function: $W(z, z') =$

$\hat{\phi}$ is

KMS (of temp $T = \frac{1}{\beta}$) w.r.t ∂_z (four-velocity associated to trajectory $(t(z), \vec{x}(z))$) $\Longrightarrow$

- Stationarity: $W(z,z') = W(\Delta z)$
- Holomorphicity of $W(z)$ in the strip $0 \leq \mathrm{Im}\, z \leq$
- Complex anti-periodicity: $W(\Delta z + i\beta) = W$

o-point) function. $W(z,z') = \langle \hat{\phi}(t(z),\vec{x}(z))\, \hat{\phi}(t(z'),\vec{x}(z')) \rangle_{\hat{\rho}}$

- Stationarity: $W(z,z') = W(\Delta z)$
- Holomorphicity of $W(z)$ in the strip $0 \le \operatorname{Im} z \le \beta$
- Complex anti-periodicity: $W(\Delta z + i\beta) = W(-\Delta z)$

Response of a particle detectors to stationary states (equilibria

σ = duration of the interaction

Let us define

$$\chi\left(\frac{z}{\sigma}\right) = \int_{-\infty}^{\infty} d\omega\, \tilde{\chi}(\omega)\, e^{\frac{iz\omega}{\sigma}}$$

$$\chi\left(\frac{z'}{\sigma}\right) = \int_{-\infty}^{\infty} d\omega'\, \tilde{\chi}^*(\omega')\, e^{-\frac{iz\omega}{\sigma}}$$

$$\mathcal{F}^1(\Omega,\sigma) = \frac{1}{\sigma}\int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} dz' \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\omega'\, \tilde{\chi}(\omega)$$

if ρ_ϕ is stationary $\Rightarrow W(z,z') = W$

$$\mathcal{F}^1(\Omega,\sigma) = \frac{1}{2\sigma}$$

...tionary states (equilibration)

$$P^{\pm} = \lambda^2 \sigma \tilde{\mathcal{F}}(\pm\Omega, \sigma) \,;\; \tilde{\mathcal{F}}(\Omega,\sigma) := \frac{1}{\sigma}\int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} dz'\, \chi\Big(\frac{z}{\sigma}\Big) \ldots$$

$$\ldots\sigma) = \frac{1}{\sigma}\int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} dz' \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\omega'\, \tilde{\chi}(\omega)\, \tilde{\chi}^*(\omega')\, W(z,z')\, e^{i\left[\left(\Omega + \frac{\omega}{\sigma}\right)z - \left(\Omega + \frac{\omega'}{\sigma}\right)z'\right]}$$

stationary $\Rightarrow W(z,z') = W(z-z') \rightarrow$ change of var: $\begin{matrix} u = z+z' \\ v = z-z' \end{matrix} \Rightarrow \begin{matrix} z = \frac{u+v}{2} \\ z' = \frac{u-v}{2} \end{matrix}$

$$\ldots = \frac{1}{2\sigma}\int_{-\infty}^{\infty} du \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\omega'\, \tilde{\chi}(\omega)\, \tilde{\chi}^*(\omega')\, W(v)\, e^{\frac{i}{2}\left(\Omega + \frac{\omega}{\sigma}\right)v}\; e^{\frac{i}{2\sigma}(\omega - \omega')u}$$

$$= \lambda^2 \sigma \tilde{\mathcal{F}}(\pm\Omega,\sigma) \; ; \; \tilde{\mathcal{F}}(\Omega,\sigma) := \frac{1}{\sigma}\int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} dz' \, \chi\left(\frac{z}{\sigma}\right)\chi\left(\frac{z'}{\sigma}\right) W(z,z') e^{i\Omega(z-z')} \ldots$$

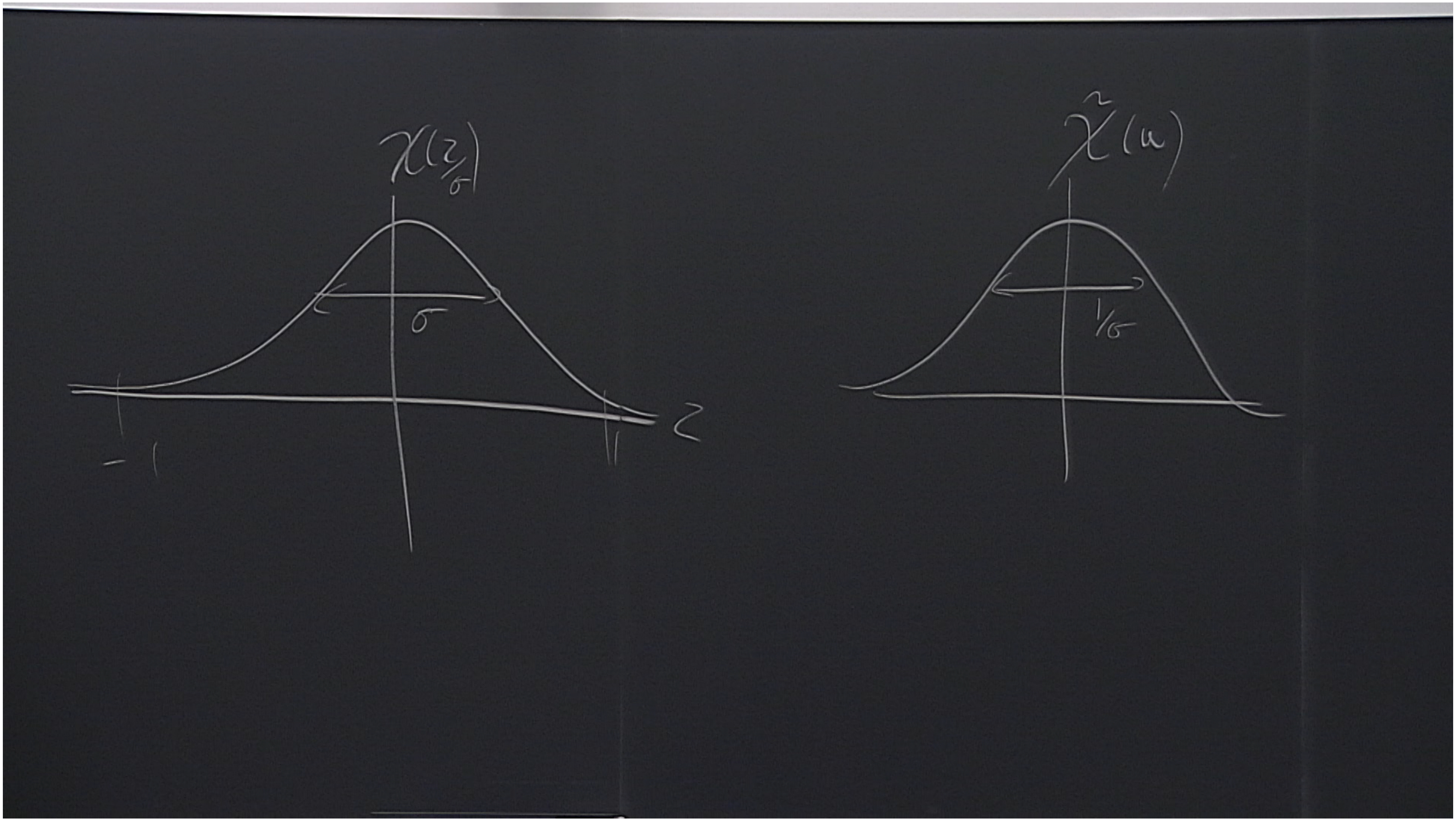
$$(z,z') \, e^{i\left[\left(\Omega+\frac{\omega}{\sigma}\right)z - \left(\Omega+\frac{\omega'}{\sigma}\right)z'\right]}$$

$\rightarrow$ change of var: $\begin{matrix} u = z + z' \\ v = z - z' \end{matrix} \Rightarrow \begin{matrix} z = \frac{u+v}{2} \\ z' = \frac{u-v}{2} \end{matrix}$

$$(v) \, e^{\frac{i}{2}\left(\Omega+\frac{\omega}{\sigma}\right)v} \quad e^{\frac{i}{2\sigma}(\omega-\omega')u}$$

$$\left(\ldots + \frac{w}{\sigma}\right) v \qquad e^{\frac{i}{2\sigma}(\omega - \omega')u}$$

$$\int_{-\infty}^{\infty} du \; e^{\frac{i}{2\sigma}(\omega-\omega')u} = 2\pi\,\delta\!\left(\frac{1}{2\sigma}(\omega-\omega')\right) = 4\pi\sigma\,\delta(\omega-\omega')$$

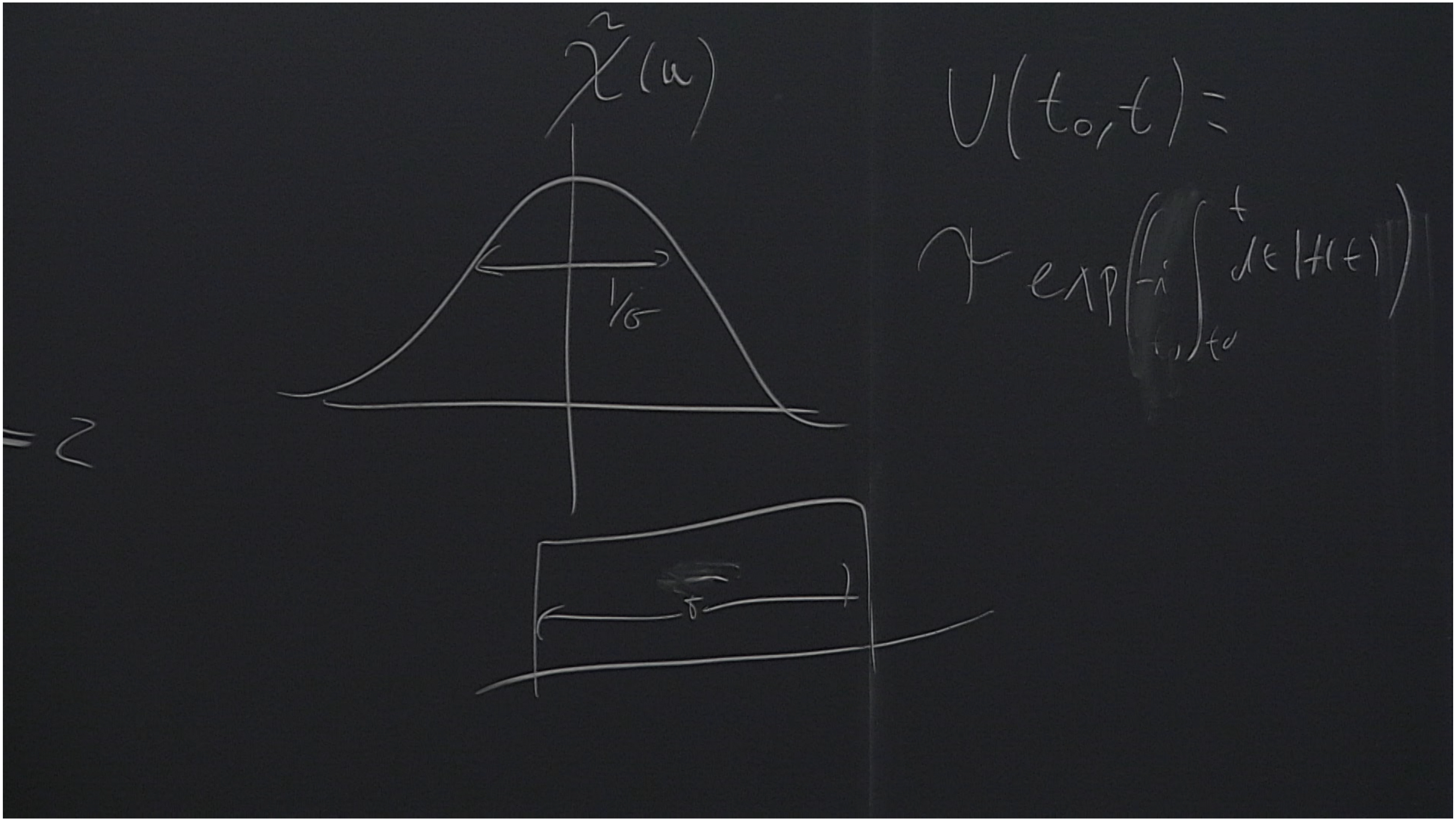


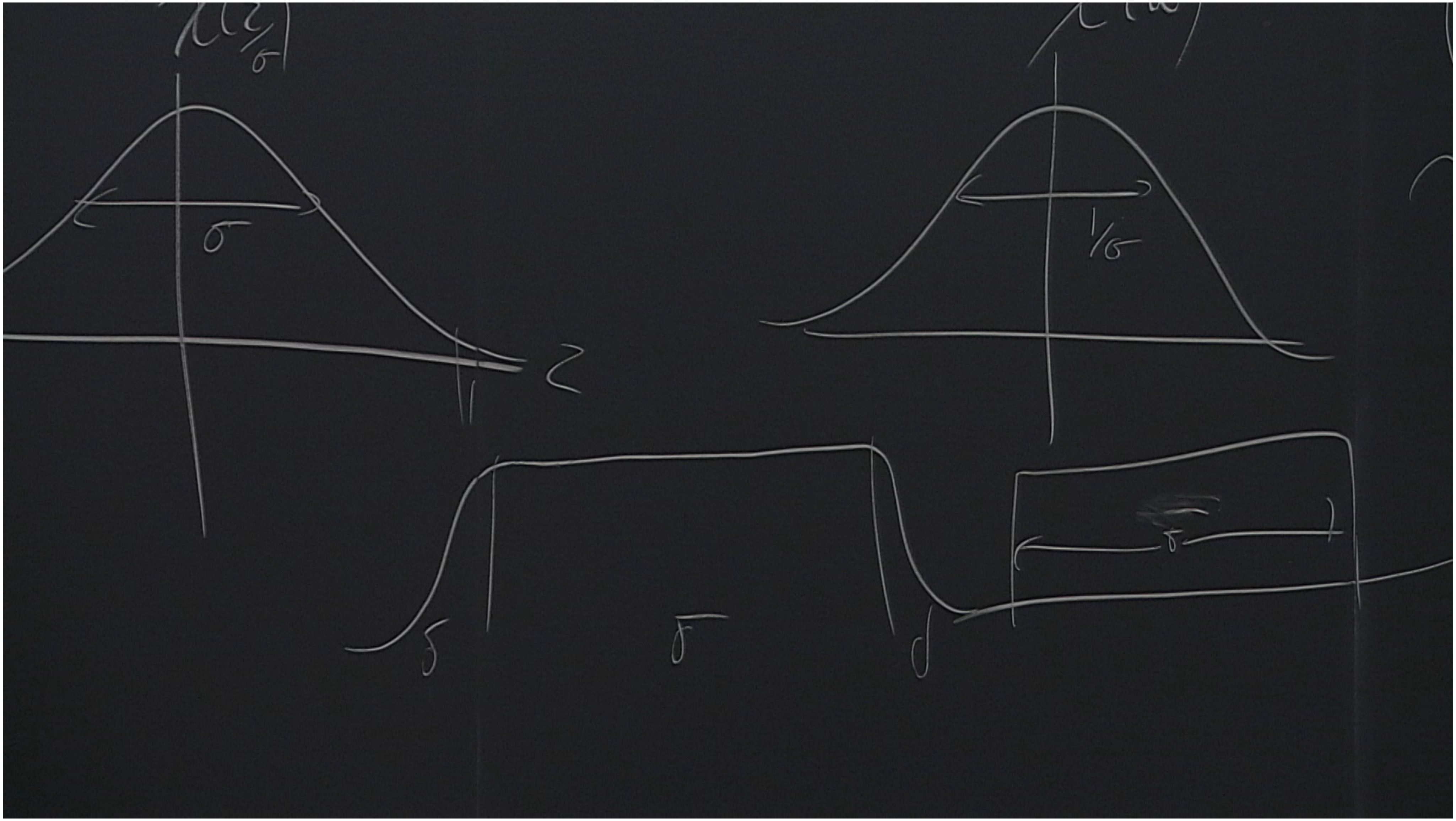
$\chi(z/\sigma)$
σ
-1
1
z
$\tilde{\chi}(u)$
$1/\sigma$

$$\int_{-\infty}^{\infty} d\omega \, |\tilde{\chi}(\omega)|^2 \, \tilde{W}\left(\Omega + \frac{\omega}{\sigma}\right)$$

suppressed for $|\omega| \gg \frac{1}{\sigma}$

$$\approx 2\pi \int_{-\infty}^{\infty} d\omega \, |\tilde{\chi}(\omega)|^2 \, \tilde{W}(\Omega) = 2\pi \, \tilde{W}(\Omega) \, \overbrace{\|\tilde{\chi}(\omega)\|_2}^{1/2\pi} = \tilde{W}(\Omega)$$


$\tilde{\chi}(\omega)$
$1/\sigma$
$U(t_0,t) =$
$T \exp\left(-i\int_{t_0}^{t} dt\, H(t)\right)$
$= 2$
τ


σ
1/σ
σ

if $\hat{\rho}$ is KMS $\Rightarrow$ $W(\Delta\tau + i\beta) = W(-\Delta\tau)$ $\xrightarrow{\text{Fourier}}$

$$\text{ad.}\lim_{\sigma\to\infty} \frac{P^+}{P^-} = \text{ad.}\lim_{\sigma\to\infty} \frac{\mathcal{F}(\Omega,\sigma)}{\mathcal{F}(-\Omega,\sigma)} = \frac{\tilde{W}(\Omega)}{\tilde{W}(-\Omega)} = e^{\beta\omega}$$

detailed ba