Title: PSI 2018/2019 - Explorations in Quantum Information - Lecture 6

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KMS CONDITION: A state $\hat{\rho}$ is KMS (with inv

translations generated by a Hamiltonian $\hat{H}$ iff for any p

$\hat{A}(z) = e^{i\hat{H}z} A(0)\, e^{-i\hat{H}z}$; $\hat{B}(z) = e^{i\hat{H}z}\hat{B}(0) e^{-i\hat{H}z}$:

i) $\langle \hat{A}(0)\,\hat{B}(z)\rangle_{\hat{\rho}}$, $\langle \hat{B}(z)\hat{A}(0)\rangle_{\hat{\rho}}$ are boundary values of the complex fu

ii) $\langle \hat{A}(0)\hat{B}(z+i\beta)\rangle_{\hat{\rho}} = \langle \hat{B}(z)\hat{A}(0)\rangle_{\hat{\rho}}$

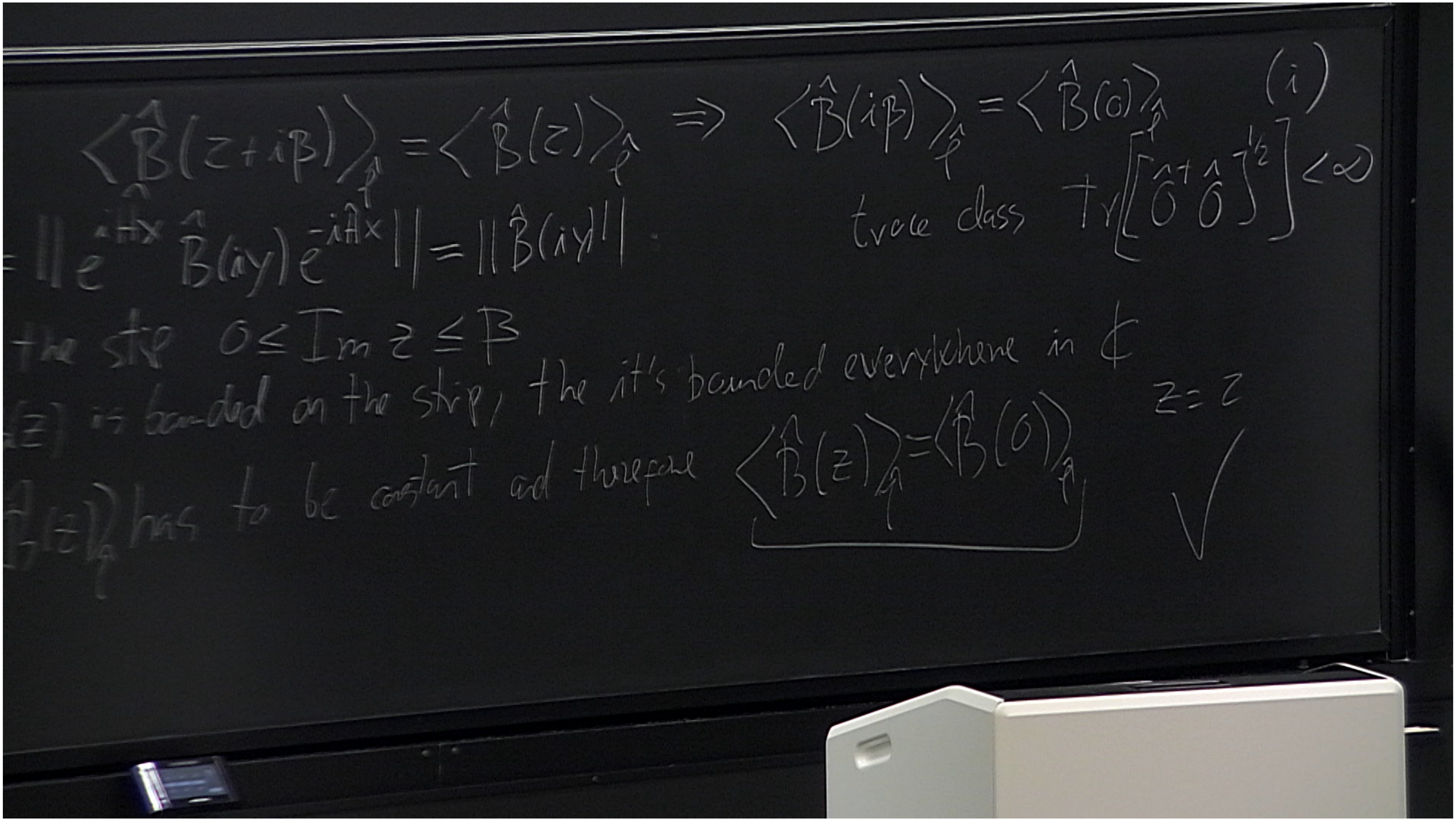
s (with inverse temperature β) w.r.t. the time parameter z parametrizing

y any pair of (Heisenberg picture) bounded operators

e complex functions $\langle \hat{A}(0)\hat{B}(z)\rangle_{\hat{\rho}}$ and $\langle \hat{B}(z)\hat{A}(0)\rangle_{\hat{\rho}}$ holomorphic in $\begin{cases} 0 \leq \mathrm{Im}\, z \leq \beta \\ -\beta \leq \mathrm{Im}\, z \leq 0 \end{cases}$

today we prove: + KMS $\Longrightarrow$ stationarity

+ KMS $\Longleftrightarrow$ Gibbs when Gibbs is well-defined



$\langle \hat{B}(z+i\beta) \rangle_{\hat{\rho}} = \langle \hat{B}(z) \rangle_{\hat{\rho}} \Rightarrow \langle \hat{B}(i\beta) \rangle_{\hat{\rho}} = \langle \hat{B}(0) \rangle_{\hat{\rho}}$
(i)
trace class $\mathrm{Tr}\left[\left[\hat{O}^\dagger \hat{O}\right]^{1/2}\right] < \infty$
$= \| e^{i\hat{H}x} \hat{B}(iy) e^{-i\hat{H}x} \| = \| \hat{B}(iy) \|$
the strip $0 \leq \mathrm{Im}\, z \leq \beta$
(z) is bounded on the strip, the it's bounded everywhere in $\mathbb{C}$
$z = z$
(z)⟩ has to be constant and therefore $\langle \hat{B}(z) \rangle_{\hat{\rho}} = \langle \hat{B}(0) \rangle_{\hat{\rho}}$

Gibbs $\Rightarrow$ KMS : Let $\hat{\rho}$ be a Gibbs state of inverse t

Gibbs $\Rightarrow$ i) the operator $\hat{A}(0)\hat{B}(z+i\sigma)\hat{\rho} = \underset{Bo}{\hat{A}(0)}\, \underset{TC}{e^{-\sigma\hat{H}}}\, \underset{Bo}{\hat{B}(z)}\, e^{-(}$

(the product of a bounded and a trace class operator is trace class. Re

this means that in $0 < \mathrm{Im}\, z < \beta$ $\langle \hat{A}(0)\hat{B}(z)\rangle_{\rho} = \mathrm{Tr}\left[\hat{A}(0)\hat{B}(z)\hat{\rho}\right]$

Gibbs state of inverse temp β $\quad \hat{\rho} = \frac{1}{Z(\beta)} e^{-\beta \hat{H}}$

$(z + i\sigma)\hat{\rho} = \underbrace{\hat{A}(0)}_{Bo} \underbrace{e^{-\sigma \hat{H}}}_{TC} \underbrace{\hat{B}(z)}_{Bo} \underbrace{e^{-(\beta-\sigma)\hat{H}}}_{TC} / Z(\beta)$ is trace-class for $0 \le \sigma \le \beta$

class operator is trace class. Reed and Simon section VI: Bounded op. Exercises 27

$\langle \hat{A}(0)\hat{B}(z)\rangle_\rho = \mathrm{Tr}[\hat{A}(0)\hat{B}(z)\hat{\rho}] = f(z)$ is a well-defined function of z

$(z = \tau + i\sigma)$ $\partial_\tau f = -i\partial_\sigma f \Rightarrow \langle \hat{A}(0)\hat{B}(z)\rangle_\rho$ is holomorphic in the strip ✓

$$\hat{\rho} = \frac{1}{Z(\beta)} e^{-\beta \hat{H}}$$

$\ldots(-\sigma)\hat{H} / Z(\beta)$ is trace-class for $0 \le \sigma \le \beta$

and Simon section VI: Bounded op. Exercises 27 and 28)

is a well-defined function of z

$\langle \ldots \hat{B}(z) \rangle_\rho$ is holomorphic in the strip $\checkmark$

Gibbs $\Rightarrow$ ii) . $\hat{B}(z) = e^{i\hat{H}z}\,\hat{B}(0)\,e^{-i\hat{H}z}$ extending z to the comp

$\langle \hat{A}(0)\,\hat{B}(z+i\beta)\rangle_{\hat{\rho}} = tr\left[\hat{A}(0)\, e^{-\beta H} B(z) e^{\beta H} e^{-\beta H}\right]/Z(\beta) = Tr\left[\hat{B}(z)\right.$

KMS $\Rightarrow$ Gibbs : let's assume that $\hat{\rho}$ is KMS , ii) at $z=0 \Rightarrow$

then $[\hat{B}(0), e^{\beta\hat{H}}\hat{\rho}] = 0$ for any Bounded $\hat{B}$ so $e^{\beta \hat{H}}\hat{\rho} \propto \mathbb{1}$

to the complex plane $\hat{B}(z+i\beta) = e^{i\hat{H}(z+i\beta)}\,\hat{B}(0)\,e^{-i\hat{H}(z+i\beta)} =$

$$= \mathrm{Tr}\left[\hat{B}(z)\hat{A}(0)\, e^{-\beta H}/Z(\beta)\right] = \mathrm{Tr}\left[\hat{B}(z)\hat{A}(0)\hat{\rho}\right] = \underline{\langle \hat{B}(z)\hat{A}(0)\rangle_{\hat{\rho}}} \quad \checkmark$$

i) at $z=0 \Rightarrow \mathrm{tr}\left(\hat{A}(0)\, e^{-\beta H}\left[\hat{B}(0), e^{\beta H}\rho\right]\right) = 0$ for any bounded op $\hat{A}$

$$e^{\beta H}\hat{\rho} \propto \mathbb{1} \Rightarrow \hat{\rho} = \frac{1}{Z(\beta)}\, e^{\beta H} \quad \checkmark$$

$W_{\hat\rho}(z,z') := \langle \hat{A}(z)\, \hat{A}(z') \rangle_{\hat\rho}$ then

KMS $\Rightarrow$

1. Stationarity: $W_{\hat\rho}(z,z') = W_{\hat\rho}(0,\Delta z) \equiv W_{\hat\rho}(\Delta z$
2. holomorphicity of $W_{\hat\rho}(z)$ in the upper strip $0 \leq$
3. Complex anti-periodicity (of period β) $W_{\hat\rho}(\Delta z + i\beta) =$

in a QFT a state $\hat\rho$ is KMS (w.r.t. z of inv. temp β) satisfies ①,

$W_{\hat\rho}(X(z), X(z')) := \langle \hat\phi(t(z), \vec{x}(z))\, \hat\phi(t(z'), \vec{x}(z')) \rangle_{\hat\rho}$

then

$(0, \Delta z) \equiv W_\beta(\Delta z)$ with $\Delta z = z' - z$

upper strip $0 \le \mathrm{Im}\, z \le \beta$

$W_\beta(\Delta z + i\beta) = W_\beta(-\Delta z)$

temp β)

$\Phi(z')\rangle_\beta$ satisfies 1, 2, 3

$\Phi(z)$ is the worldline generated by ∂_z