Title: PSI 2018/2019 - Explorations in Quantum Information - Lecture 5

Speakers: Eduardo Martin-Martinez

Collection: PSI 2018/2019 - Explorations in Quantum Information (Martin-Martinez)

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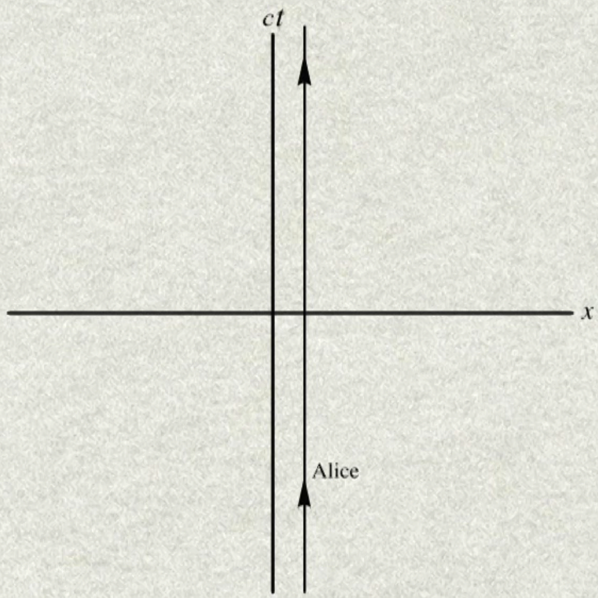
Field QuantiZation Depends on the observer

- Example: different observers of flat spacetime



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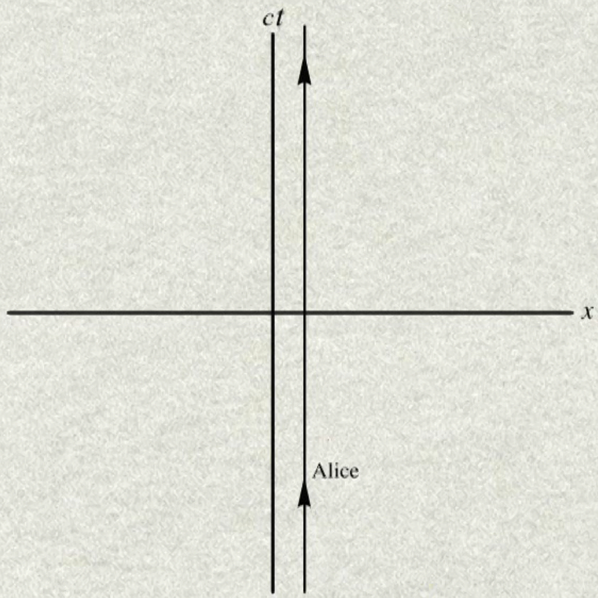

Positive freq. solutions $\nabla^\mu \nabla_\mu u = 0$

$$(x,t) \rightarrow u^{\mathrm{M}}_{\hat{\omega}} \propto e^{-i\hat{\omega}t}$$

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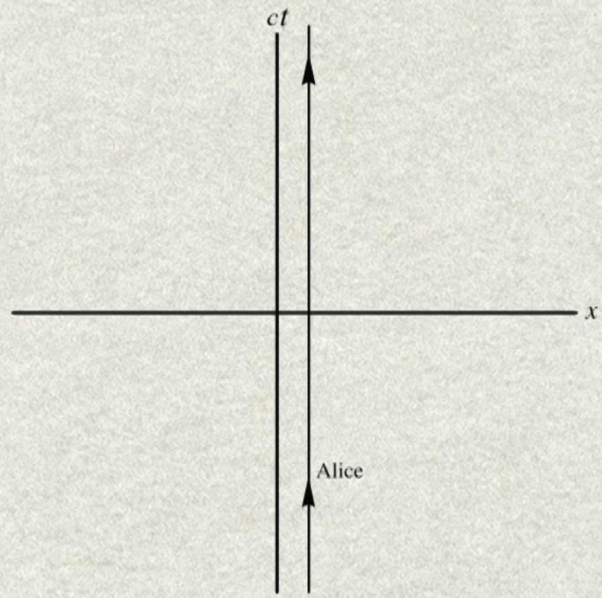

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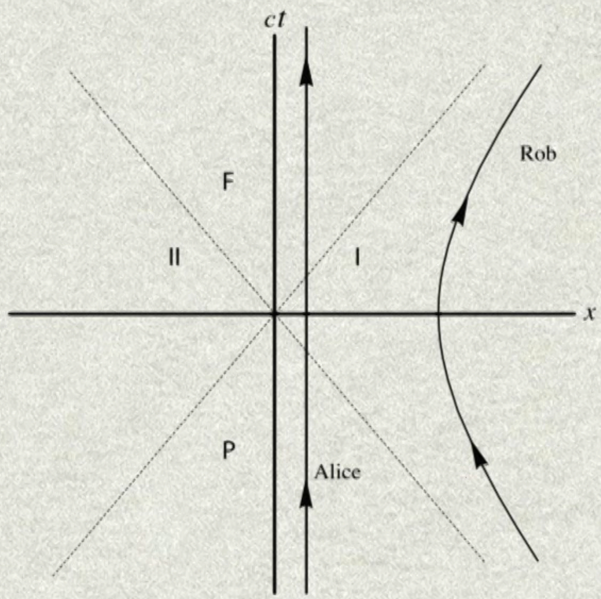

$$ct = \xi \sinh\left(\frac{a\tau}{c}\right), \qquad x = \xi \cosh\left(\frac{a\tau}{c}\right)$$

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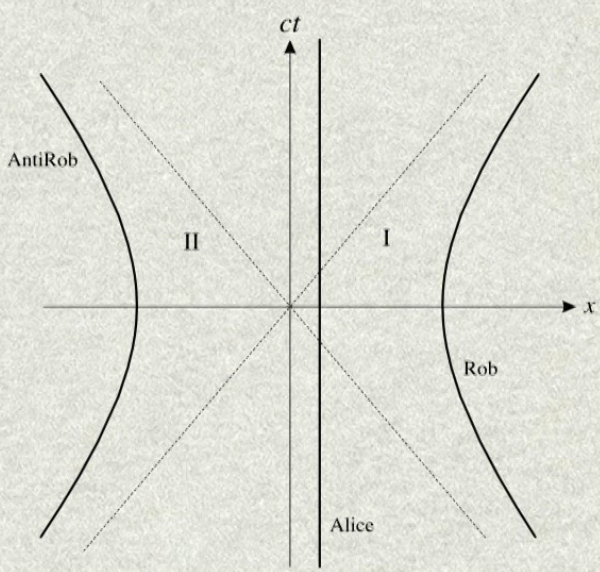

The field can be spanned in either basis

$$\phi = \sum_i \left(a_{\hat{\omega}_i,\mathrm{M}} u^{\mathrm{M}}_{\hat{\omega}_i} + a^{\dagger}_{\hat{\omega}_i,\mathrm{M}} u^{\mathrm{M}*}_{\hat{\omega}_i} \right)$$

or equivalently

$$\phi = \sum_i \left(a_{\omega_i,\mathrm{I}} u^{\mathrm{I}}_{\omega_i} + a^{\dagger}_{\omega_i,\mathrm{I}} u^{\mathrm{I}*}_{\omega_i} + a_{\omega_i,\mathrm{II}} u^{\mathrm{II}}_{\omega_i} + a^{\dagger}_{\omega_i,\mathrm{II}} u^{\mathrm{II}*}_{\omega_i} \right)$$

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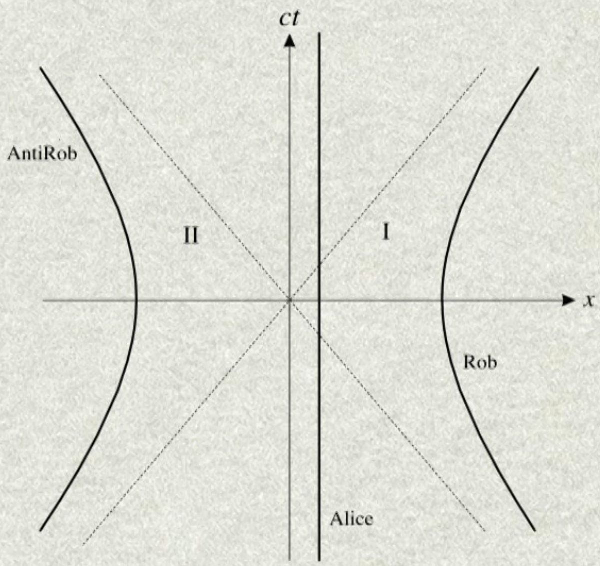

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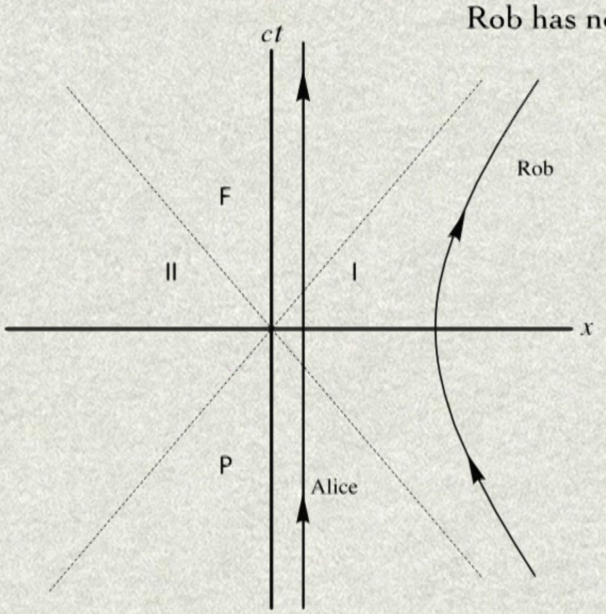
The Importance of The Horizon

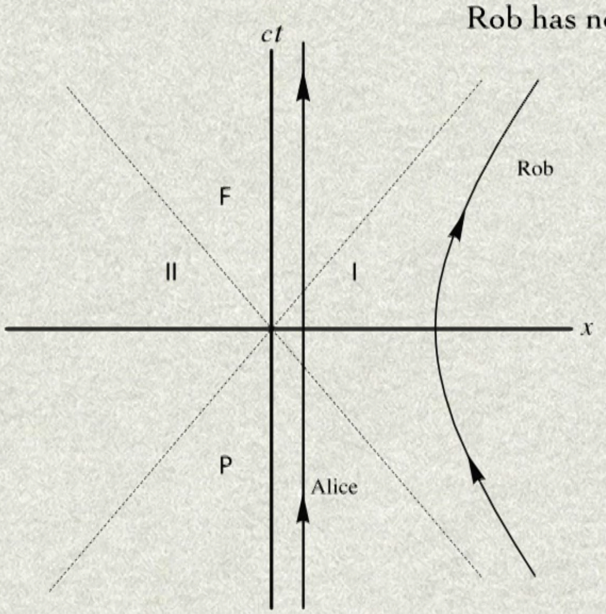
Rob has no access to region II



Example: Minkowskian vacuum. Rob's perspective

$$|0\rangle_M$$

The Importance of The Horizon



$$\tanh r_\omega = \exp\left(-\frac{\pi c\,\omega}{a}\right)$$

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Example: Minkowskian vacuum. Rob's perspective

$$|0\rangle_{\mathrm{M}}$$

First: change of Fock basis

$$|0\rangle_{\mathrm{M}} = \bigotimes_{\omega} \frac{1}{\cosh r_\omega} \sum_{n=0}^{\infty} \tanh^n r_\omega |n\rangle_{\mathrm{I}} |n\rangle_{\mathrm{II}}$$

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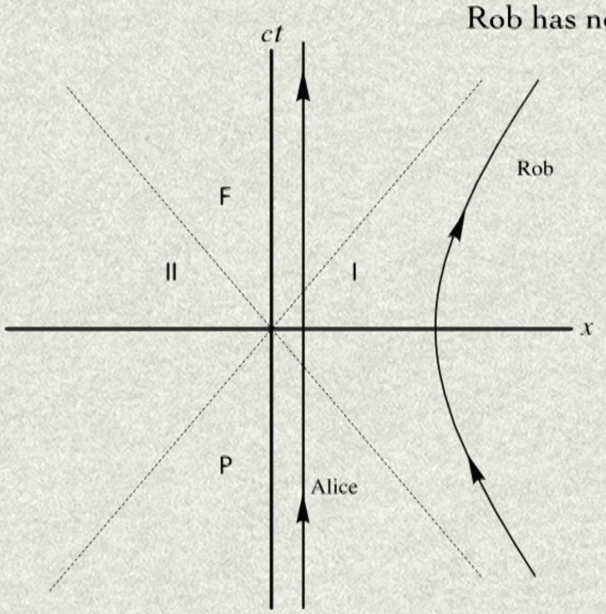

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Second: Trace out the disconnected region

$$\rho_{\mathrm{R},\omega} = \mathrm{Tr}_{\mathrm{II}}\left(|0_\omega\rangle\langle 0_\omega|\right) = \frac{1}{\cosh^2 r_\omega} \sum_n \tanh^{2n} r_\omega |n_\omega\rangle_{\mathrm{I}} \langle n_\omega|_{\mathrm{I}}$$

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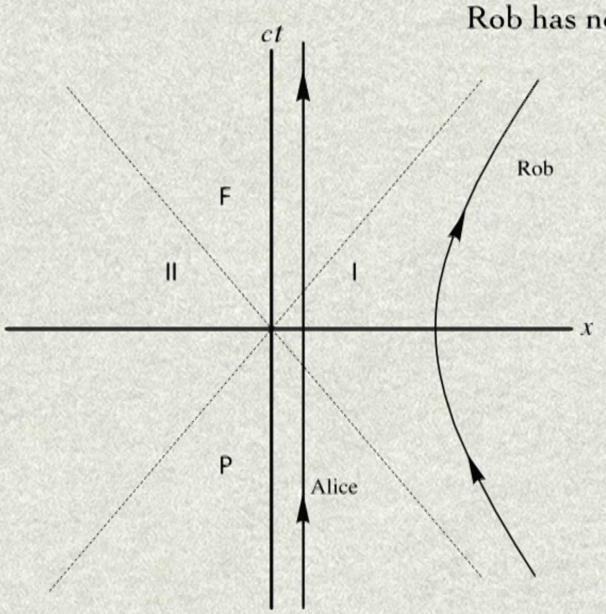

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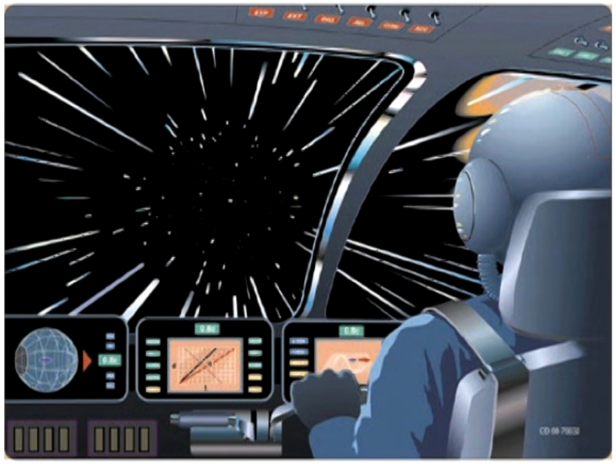
Result: thermal state

$$\langle N_{\omega,\rm R}\rangle = \frac{1}{e^{2\pi c/\omega a}-1} \qquad T_{\rm U} = \frac{\hbar a}{2\pi K_{\rm B}}$$

The Unruh Effect

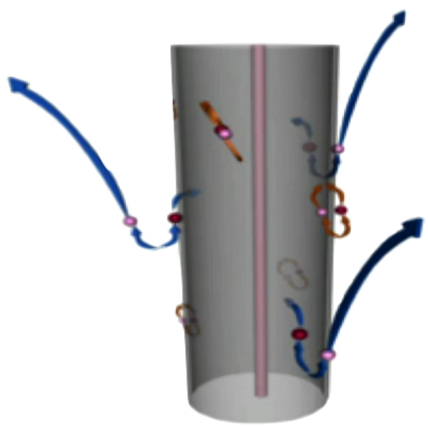
Inertial frame	Accelerated frame
	

- Alice Observes the field vacuum.
- Bob observes a thermal bath of temperature $T_{\mathrm{U}} \propto a$

Entanglement in a Stellar Collapse

Vacuum in the far past evolves into two mode squeezed state between infalling and outgoing modes in the far future

$$|0\rangle \to \bigotimes_{\omega} \frac{1}{\cosh r} \sum \tanh^n r_\omega \, |n_\omega\rangle_{\text{hor}} \, |n_\omega\rangle_{\text{out}}$$



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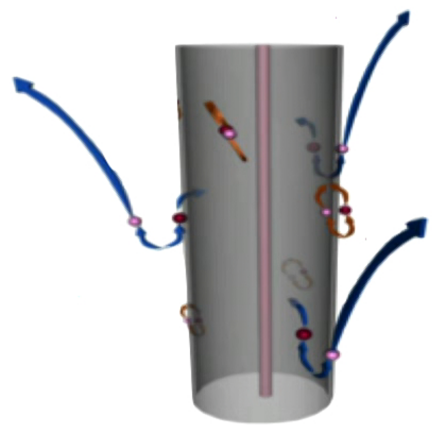
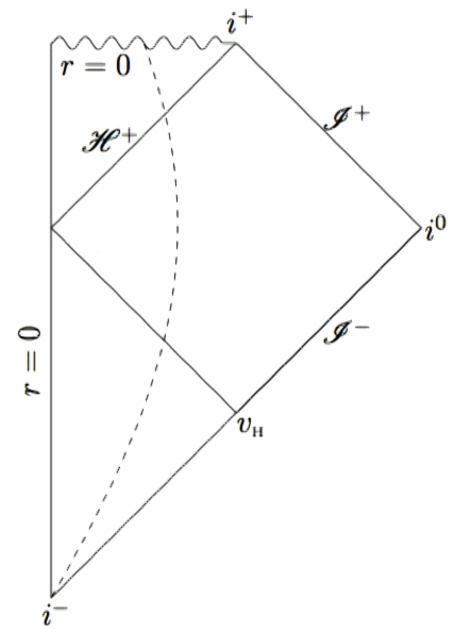


What do we really see if we look at the black hole?

Hawking Radiation

We need to write the annihilation operators of field modes in the asymptotic past in terms of the corresponding creation and annihilation operators defined in terms of modes in the future:

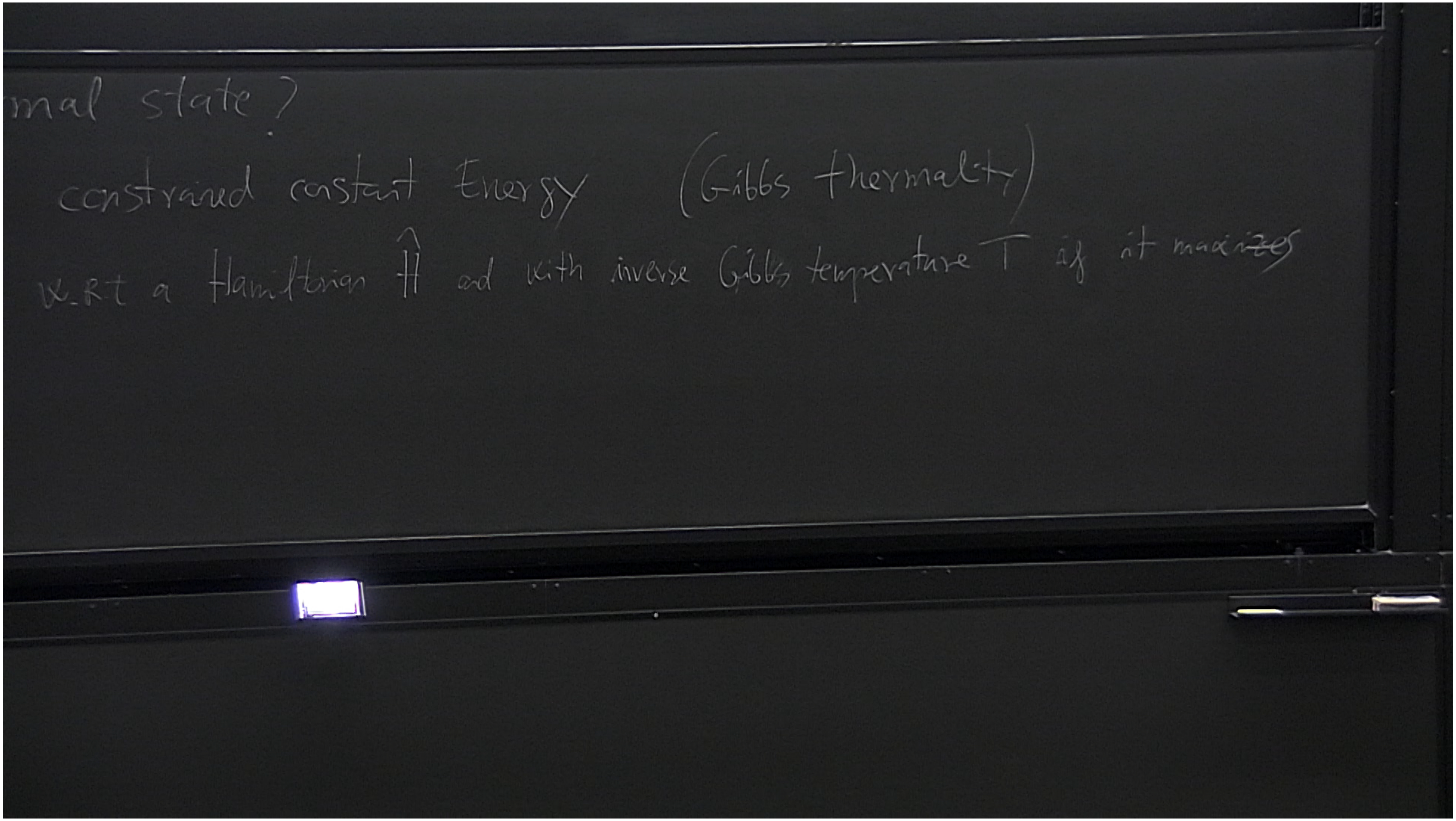


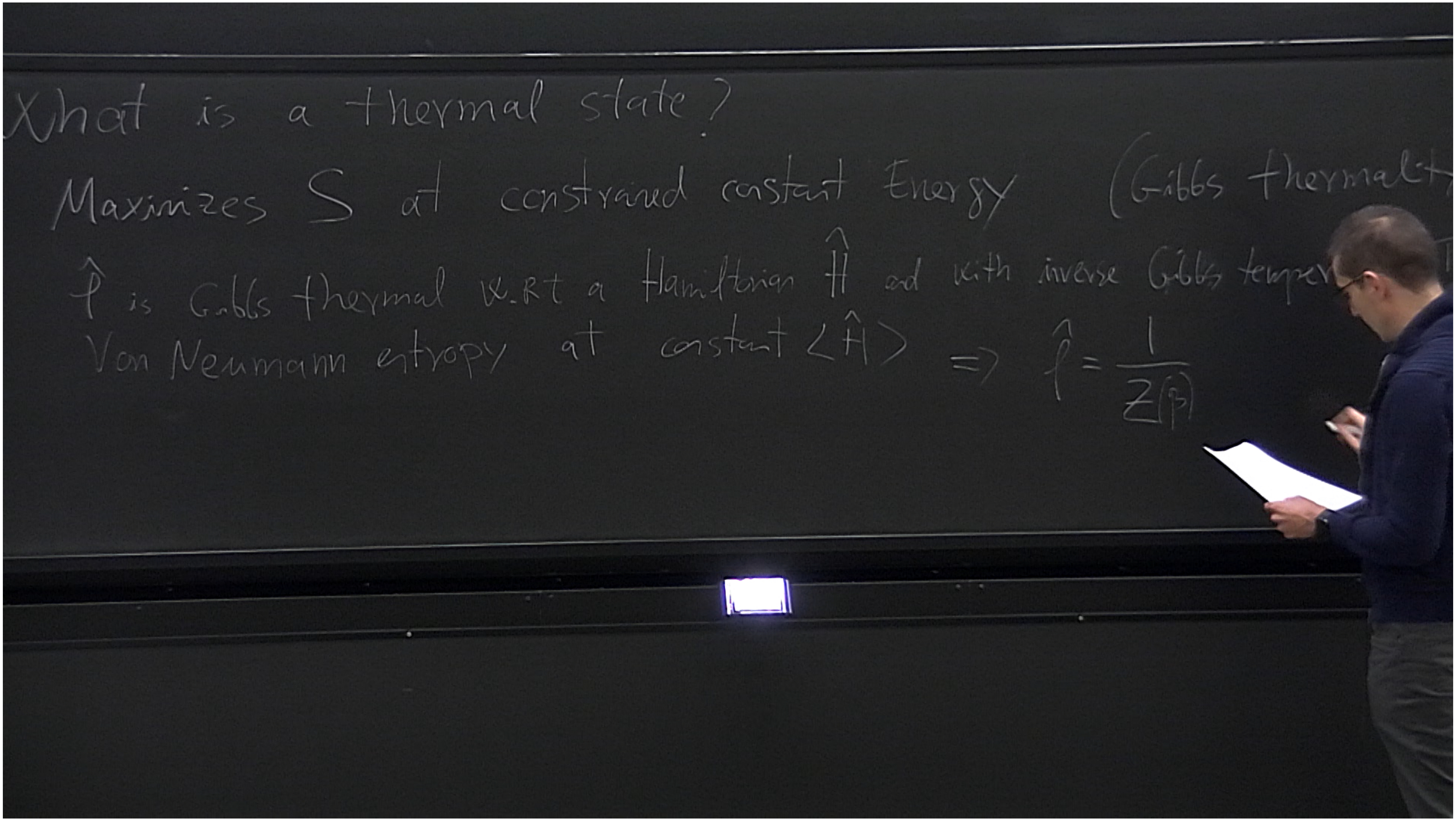
Black holes are not that black

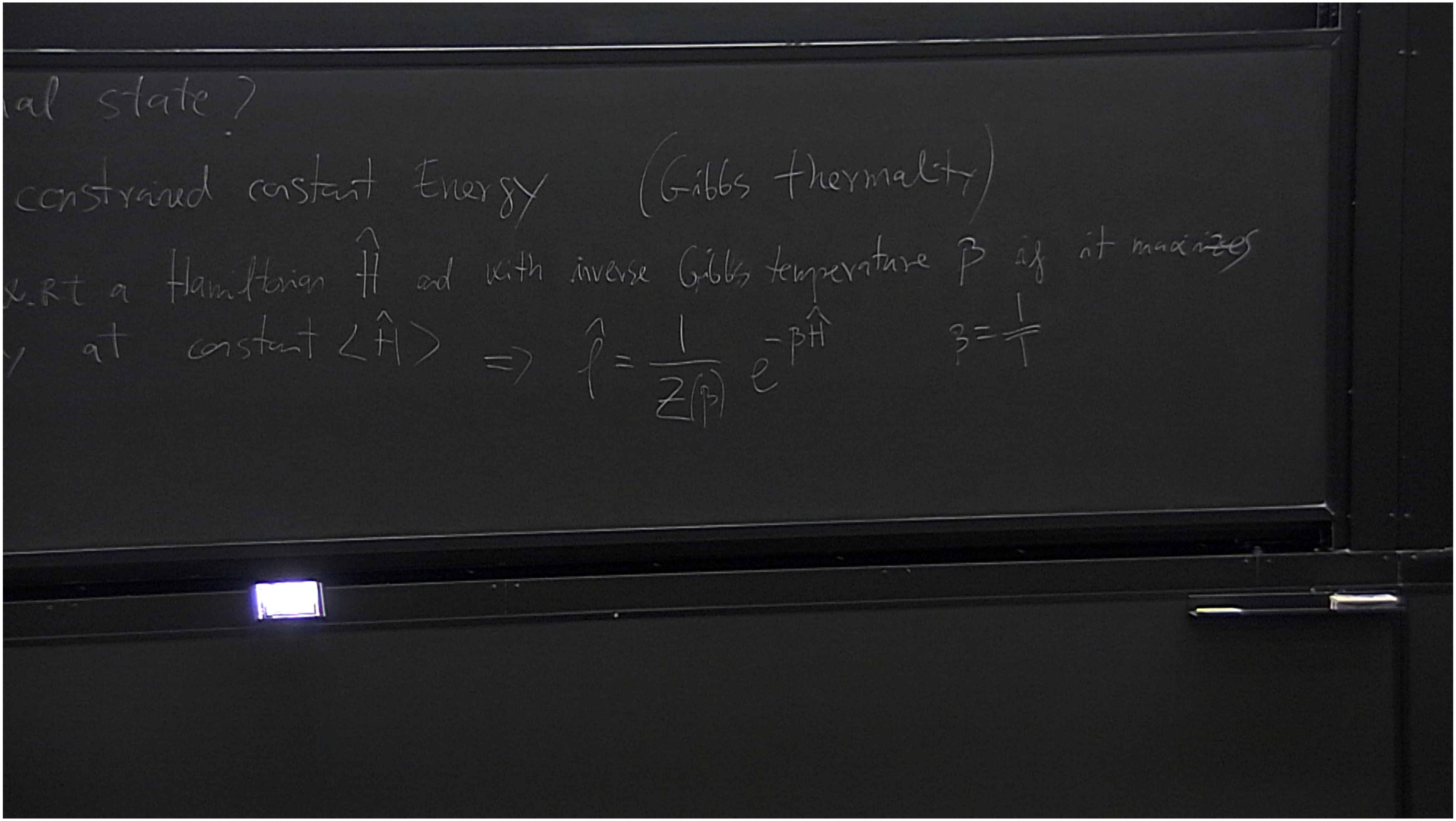
We see outgoing radiation

$$\rho_{\text{out}} = \text{Tr}_{\text{hor}}\left(|0\rangle\langle 0|\right) = \bigotimes_{\omega} \frac{1}{\cosh^2 r} \sum \tanh^{2n} r_{\omega} \, |n_{\omega}\rangle_{\text{out}} \langle n_{\omega}|_{\text{out}}$$

$$\text{Tr}\left(N_{\omega}\rho_{\text{out}}\right) = \frac{1}{e^{\hbar\omega/K_{\text{B}}T_{\text{H}}} - 1} \qquad T_{\text{H}} = \frac{1}{8\pi G}\frac{\hbar c^3}{m K_{\text{B}}}$$

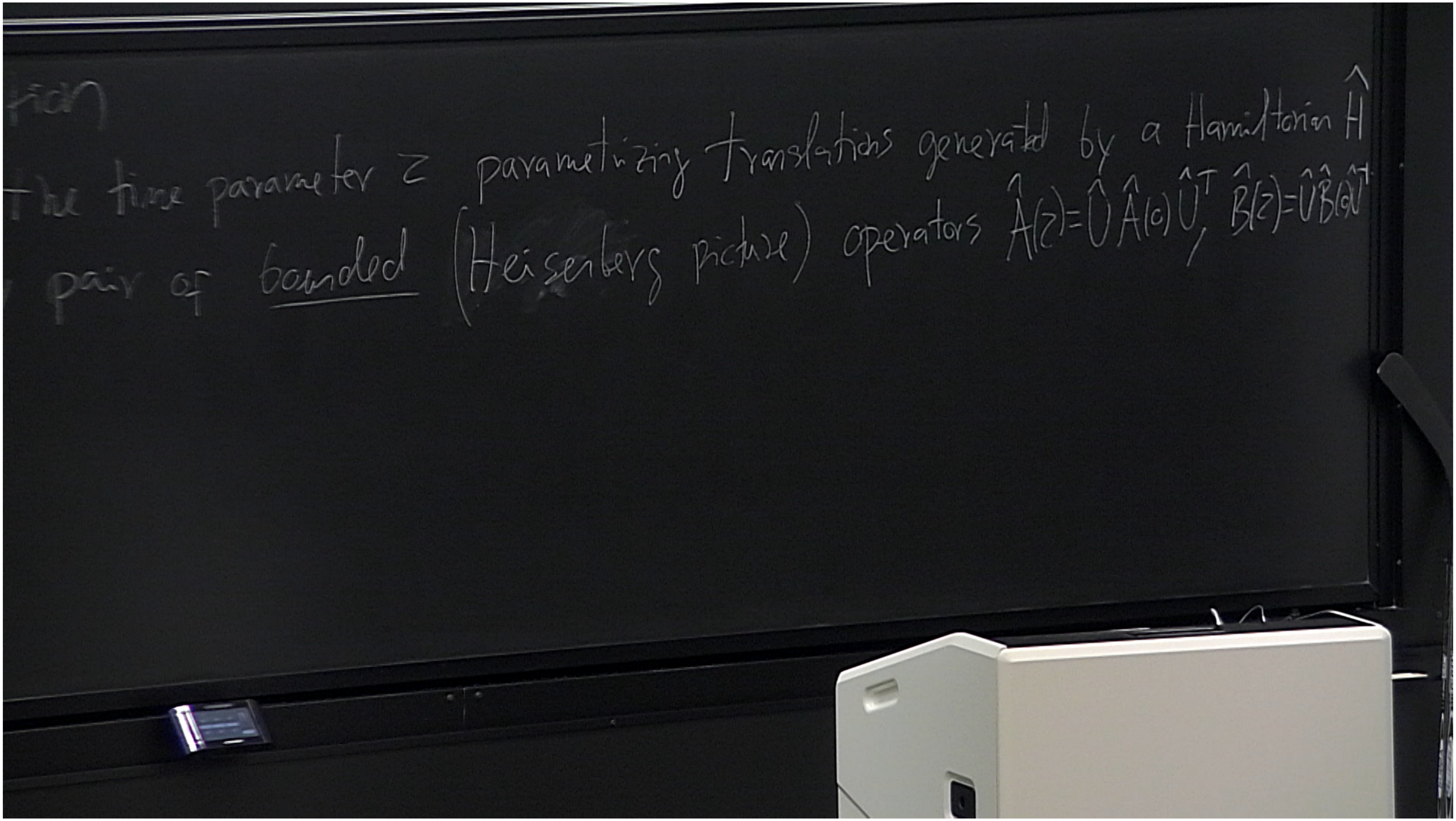

mal state ?
constrained constant Energy (Gibbs thermality)
W.R.T a Hamiltonian Ĥ and with inverse Gibbs temperature T if it maximizes


What is a thermal state?
Maximizes S at constrained constant Energy (Gibbs thermalit
ρ̂ is Gibbs thermal w.r.t a Hamiltonian Ĥ and with inverse Gibbs temper
Von Neumann entropy at constant ⟨Ĥ⟩ ⇒ ρ̂ = 1/Z(β)


al state?
constrained constant Energy (Gibbs thermality)
w.r.t a Hamiltonian $\hat{H}$ and with inverse Gibbs temperature β if it maximizes
y at constant $\langle \hat{H} \rangle \Rightarrow \hat{\rho} = \frac{1}{Z(\beta)} e^{-\beta \hat{H}}$ $\beta = \frac{1}{T}$

Kubo-Martin-Schwinger (KMS) condition

A state $\hat{\rho}$ is (KMS) thermal with respect to the time p


tion)
the time parameter τ parametrizing translations generated by a Hamiltonian Ĥ
pair of bounded (Heisenberg picture) operators Â(τ)=Û Â(0) Û†, B̂(τ)=Û B̂(0) Û†

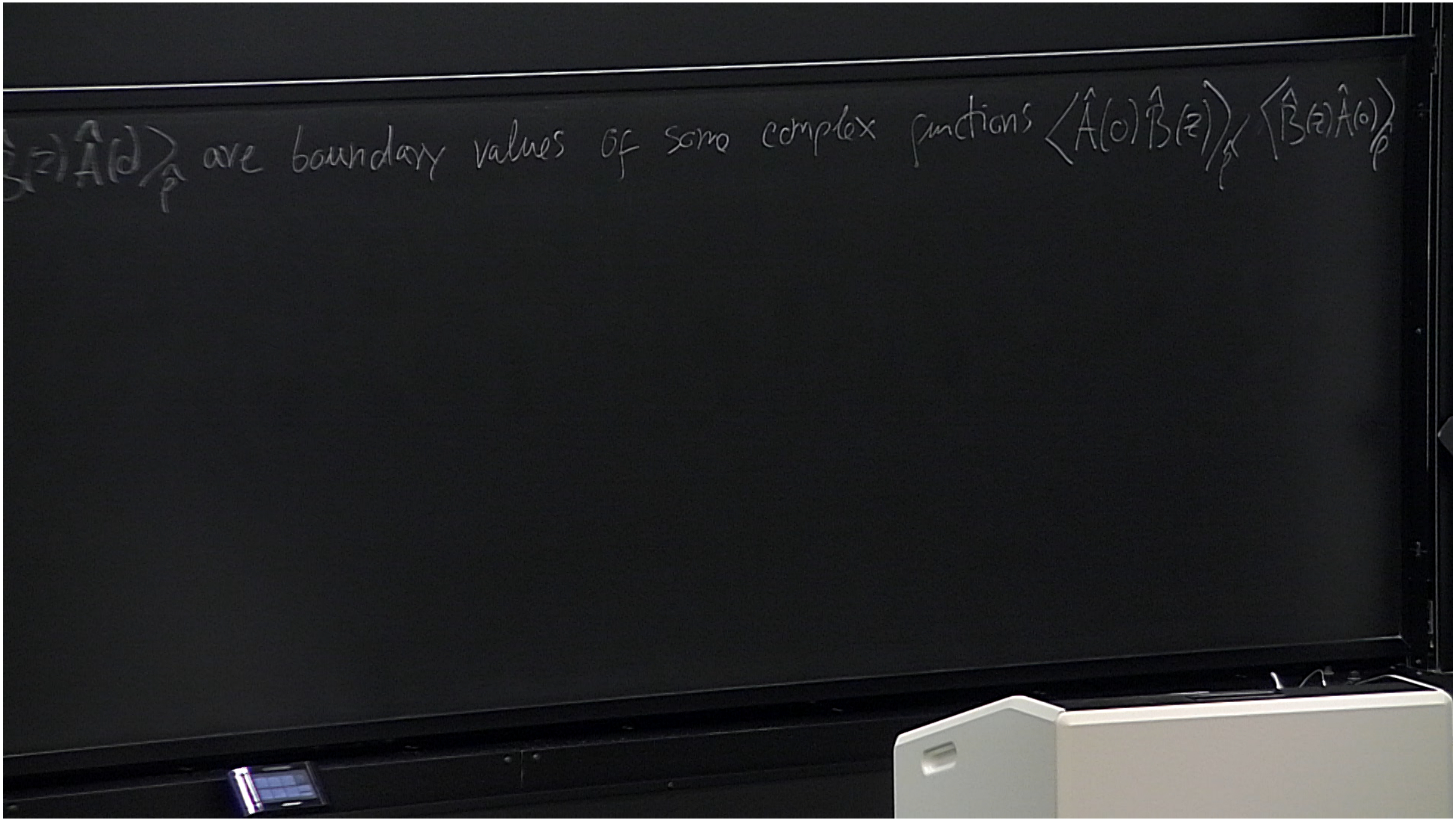
Kubo-Martin-Schwinger (KMS) condition

A state $\hat{\rho}$ is (KMS) thermal with respect to the time p

iff it satisfies the following conditions for any pair of

(where $\hat{U} = e^{iHz}$)

i) The expectation values $\langle \hat{A}(c)\hat{B}(z)\rangle_{\hat{\rho}}$ and $\langle \hat{B}(z)\hat{A}(c)\rangle_{\hat{\rho}}$ are b


$\hat{B}(z)\hat{A}(0)\rangle_{\hat\rho}$ are boundary values of some complex functions $\langle\hat{A}(0)\hat{B}(z)\rangle_{\hat\rho}$, $\langle\hat{B}(z)\hat{A}(0)\rangle_{\hat\rho}$

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holomorphic in the complex plane strips $0 < \mathrm{Im}\, z < \beta$,

ii) the following complex antiperiodicity (of period β) is sat

$$\langle \hat{A}(0)\hat{B}(z+i\beta)\rangle_{\hat{\rho}} = \langle \hat{B}(z)\hat{A}(0)\rangle_{\hat{\rho}}$$

When Gibbs thermality is well-defined   KMS thermal