Title: PSI 2018/2019 - Explorations in Quantum Information - Lecture 4

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Side note. Hamiltonians under time reparametrization:

Given a Hamiltonian ${}^{t}H(t)$ that generates time evolution with respect to $t$, and a time reparametrization $t = t(\tau)$, how does the Hamiltonian change?

What is ${}^{t}H(t(\tau))$? ${}^{t}H(t(\tau))$ the Hamiltonian that generates translations w.r.t. $t$ (as a function of $\tau$)

$$i \frac{d}{dt} |\psi\rangle = {}^{t}\hat{H}(t) |\psi\rangle \rightarrow i \frac{d\tau}{dt} \frac{d}{d\tau} |\psi\rangle = {}^{t}\hat{H}(t(\tau)) |\psi\rangle \Rightarrow i \frac{d}{d\tau} |\psi\rangle = \frac{dt}{d\tau} {}^{t}H(t(\tau)) |\psi\rangle \quad \boxed{{}^{\tau}H(\tau) = \frac{dt}{d\tau} {}^{t}H(t(\tau))}$$

$$i \frac{d}{d\tau} |\psi\rangle = {}^{\tau}H(t(\tau)) |\psi\rangle$$

$$i\frac{d}{dt}|\psi\rangle = {}^{t}\hat{H}(t)\,|\psi\rangle \;\rightarrow\; i\frac{d\tau}{dt}\frac{d}{d\tau}|\psi\rangle = {}^{t}\hat{H}(t(\tau))\,|\psi\rangle \;\Rightarrow\; i\frac{d}{d\tau}|\psi\rangle = \frac{dt}{d\tau}\,{}^{t}H(t(\tau))\,|\psi\rangle \qquad \boxed{{}^{\tau}H(\tau) = \frac{dt}{d\tau}\,{}^{t}H(t(\tau))}$$

$$i\frac{d}{d\tau}|\psi\rangle = {}^{\tau}H(t(\tau))\,|\psi\rangle$$

Time evolution is invariant under time reparametrization. $\hat{U} = \mathcal{T}\exp\left[-i\int_{\mathbb{R}} d\tau\, {}^{\tau}H(\tau)\right] = \mathcal{T}\exp\left[-i\int_{\mathbb{R}} d\tau\, \frac{dt}{d\tau}\,{}^{t}H(t(\tau))\right] = \mathcal{T}\exp\left[-i\int_{\mathbb{R}} dt\, {}^{t}H(t)\right]$

Detectors: free Hamiltonian ${}^{\tau}\hat{H}_{d}(\tau) = \Omega\,\hat{\sigma}^{+}\hat{\sigma}^{-}$ $\quad$ $\Omega$ between $|e\rangle$ and $|g\rangle$ $\quad$ $\tau$ detector's proper time

Fields: free Hamiltonian ${}^{t}\hat{H}_{\phi}(t) = \int_{\mathbb{R}^n} d^n k\; \omega_k\, \hat{a}^{\dagger}_k \hat{a}_k$ $\quad$ $t$ quantization frame time (Lab time)

they couple: ${}^{\tau}\hat{H}_{I} = \lambda\, \chi(\tau)\, \hat{\mu}(\tau)\, \hat{\phi}(t(\tau), \vec{x}(\tau))$

We have a finite-sized detector whose centre of mass moves with a trajectory (in the lab frame) given by $\vec{x}(t)$

$(\tau, \vec{\xi})$ proper frame of the detector $(t, \vec{x})$ Lab frame (inertial)

Fermi-Walker rigidity the distance between $\vec{\xi} = \vec{0}$ and all points of the detector remains constant in the $(\tau, \vec{\xi})$ frame

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$$^{\tau}\hat{H}_I = \lambda \chi(\tau) \int_{\mathbb{R}^n} \mathrm{d}^n\xi \, \underbrace{F(\vec{\xi}) \, \hat{\mu}(\tau)}_{\hat{m}(\tau,\vec{\xi})} \, \hat{\phi}\big(t(\tau, \vec{\xi}), \vec{x}(\tau, \vec{\xi})\big)$$

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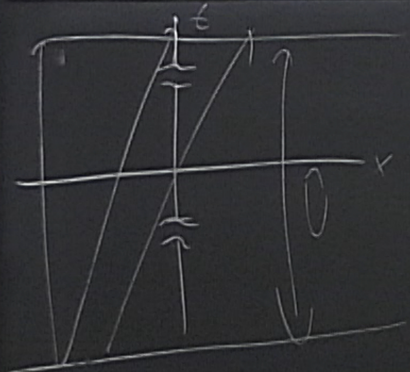
$(\tau,\vec{\xi})$ proper frame of the detector $\qquad (t,\vec{x})$ Lab frame (inertial)

- Fermi-Walker rigidity: the distance between $\vec{\xi}=\vec{0}$ and all points of the detector remains constant in the $(\tau,\vec{\xi})$ frame

$$ {}^{\tau}\hat{H}_I = \lambda\chi(\tau)\int_{\mathbb{R}^n} d^n\xi\, \underbrace{F(\vec{\xi})\,\hat{\mu}(\tau)}_{\hat{\mu}(\tau,\vec{\xi})}\,\hat{\phi}\big(t(\tau,\vec{\xi}),\vec{x}(\tau,\vec{\xi})\big) $$



$$ {}^{\tau}\hat{H} = \int d^n\xi\, {}^{\tau}\hat{h}(\vec{\xi}), \quad {}^{\tau}\hat{h}(\vec{\xi}) = \lambda\chi(\tau)\,\hat{\mu}(\tau)\,F(\vec{\xi})\,\phi\big(t(\tau,\vec{\xi}),\vec{x}(\tau,\vec{\xi})\big) $$

Ham density lab

Ham density detector

$$ \hat{U} = \mathcal{T}\exp\left(-i\int_{\mathbb{R}} d\tau\, {}^{\tau}\hat{H}(\tau)\right) = \mathcal{T}\exp\left(-i\int_{\mathbb{R}} d\tau\int_{\mathbb{R}^n} d^n\xi\, {}^{\tau}\hat{h}(\vec{\xi})\right) = \mathcal{T}\exp\left(-i\int_{\mathbb{R}} dt\int d^n x\, {}^{t}\hat{h}(\vec{x})\right) $$

$$ {}^{t}\hat{H} = \int_{\mathbb{R}^n} d^n x\, {}^{t}\hat{h}(\vec{x}) = \lambda\int_{\mathbb{R}} d^n x\, \chi(\tau(t,\vec{x}))\, F(\vec{\xi}(t,\vec{x}))\, \hat{\mu}(\tau(t,\vec{x}))\, \phi(t,\vec{x}) \left|\frac{\partial(\tau,\vec{\xi})}{\partial(t,\vec{x})}\right| $$

$$ {}^{t}\hat{H} = \int_{\mathbb{R}^n} d^n x \; {}^{t}h(\vec{x}) = \lambda \int_{\mathbb{R}} d^n x \; \chi(\tau(t,\vec{x})) \, F(\vec{\xi}(t,\vec{x})) \, \hat{\mu}(\tau(t,\vec{x})) \, \phi(t,\vec{x}) \left| \frac{\partial(\tau,\vec{\xi})}{\partial(t,\vec{x})} \right| $$

Trans. prob $|g\rangle \underset{-\Omega}{\overset{\Omega}{\rightleftarrows}} |e\rangle$ for an arbitrary state of the field $\hat{\rho}_\phi$ and pointlike detector. $\hat{\rho}_0 = |g\rangle\langle g| \otimes \hat{\rho}_\phi$

$$ {}^{\tau}\hat{H} = \lambda\, \chi\!\left(\frac{\tau}{\sigma}\right) \int d^n\xi \, F(\vec{\xi}) \, \hat{\mu}(\tau) \, \phi(t(\tau,\vec{\xi}), \vec{x}(\tau,\vec{\xi})) $$

pointlike: $F(\vec{\xi}) = \delta^{(n)}(\vec{\xi}) \Rightarrow {}^{\tau}\hat{H}_I = \lambda \chi\!\left(\frac{\tau}{\sigma}\right) \hat{\mu}(\tau) \, \phi\big(\underbrace{t(\tau,\vec{0})}_{t(\tau)}, \underbrace{\vec{x}(\tau,\vec{0})}_{\vec{x}(\tau)}\big)$

$$ P = \lambda^2 \int_{\mathbb{R}} d\tau \int_{\mathbb{R}} d\tau' \, \chi\!\left(\frac{\tau}{\sigma}\right) \chi\!\left(\frac{\tau'}{\sigma}\right) \overbrace{\langle e|\hat{\mu}(\tau)|g\rangle\langle g|\hat{\mu}(\tau')|e\rangle}^{e^{\pm i\Omega(\tau-\tau')}} \overbrace{\mathrm{tr}_\phi\left(\hat{\rho}_\phi \, \phi(t(\tau'), x(\tau')) \, \phi(t(\tau), \vec{x}(\tau))\right)}^{W(t(\tau'), x(\tau'), t(\tau), x(\tau)) = W(\tau', \tau)} = \boxed{\lambda^2 \int d\tau \int d\tau' \, \chi\!\left(\frac{\tau}{\sigma}\right) \chi\!\left(\frac{\tau'}{\sigma}\right) e^{\mp i\Omega(\tau-\tau')} \, W_\phi(\tau', \tau)} $$

$$i\frac{d}{dt}|\psi\rangle = \hat H(t)|\psi\rangle \;\to\; i\frac{dz}{dt}\frac{d}{dz}|\psi\rangle = \hat H(t(z))|\psi\rangle \;\Rightarrow\; i\frac{d}{dz}|\psi\rangle = \frac{dt}{dz}\hat H(t(z))|\psi\rangle \qquad \tilde H(z) = \frac{dt}{dz}\hat H$$

$$i\frac{d}{dz}|\psi\rangle = \tilde{H}(t(z))|\psi\rangle$$

Wightman "function" $\; W(x,x') := \langle \hat\phi(x)\hat\phi(x')\rangle_{\hat\rho} = \mathrm{tr}\left[\hat\rho\,\phi(x)\,\phi(x')\right] \Rightarrow W(x',x) = W^*(x,x')$

Pauli-Lubanski

$$\mathrm{Im}\,W(x,x') = \frac{1}{2i}\left(W(x,x') - W^*(x,x')\right) = \frac{1}{2i}\mathrm{tr}\left[\hat\rho\left(\hat\phi(x)\hat\phi(x') - \hat\phi(x')\hat\phi(x)\right)\right] = \frac{1}{2i}\left\langle\left[\phi(x),\phi(x')\right]\right\rangle_{\hat\rho}$$

$$\mathrm{Re}\,W(x,x') = \frac{1}{2}\left\langle\left\{\hat\phi(x),\hat\phi(x')\right\}\right\rangle_{\hat\rho} \quad \text{Hadamard}$$

$$i\frac{d}{dt}|\psi\rangle = \hat{H}(t)|\psi\rangle \longrightarrow i\frac{d\tau}{dt}\frac{d}{d\tau}|\psi\rangle = \hat{H}(t(\tau))|\psi\rangle \Rightarrow i\frac{d}{d\tau}|\psi\rangle = \frac{dt}{d\tau}\hat{H}(t(\tau))|\psi\rangle \qquad \boxed{\hat{H}(\tau) = \frac{dt}{d\tau}\hat{H}(\ldots}$$

$$i\frac{d}{d\tau}|\psi\rangle = {}^{\tau}\hat{H}(t(\tau))|\psi\rangle$$

Wightman "function" $W(x,x') := \langle \hat{\phi}(x)\hat{\phi}(x')\rangle_{\hat{\rho}} = \mathrm{tr}\left[\hat{\rho}\,\phi(x)\,\phi(x')\right] \Rightarrow W(x',x) = W^{*}(x,x')$

Pauli-Jordan

$$\mathrm{Im}\, W(x,x') = \frac{1}{2i}\left(W(x,x') - W^{*}(x,x')\right) = \frac{1}{2i}\mathrm{tr}\left[\hat{\rho}\left(\hat{\phi}(x)\hat{\phi}(x') - \hat{\phi}(x')\hat{\phi}(x)\right)\right] = \frac{1}{2i}\left\langle\left[\phi(x),\phi(x')\right]\right\rangle_{\hat{\rho}}$$

$$\mathrm{Re}\, W(x,x') = \frac{1}{2}\left\langle\left\{\hat{\phi}(x),\hat{\phi}(x')\right\}\right\rangle_{\hat{\rho}} \quad \text{Hadamard}$$

$\boxed{\text{arXiv: 1803.01867 PRD (2019)}}$