Title: PSI 2018/2019 - Explorations in Quantum Information - Lecture 3

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EM field - Hydrogen-like atom dipolar coupling

$$H_I = e\int d^3x\, \hat{\vec{d}}(t,\vec{x})\cdot\hat{\vec{E}}(t,\vec{x})$$

$$\hat{\vec{d}} = \vec{F}(\vec{x})\, e^{i\Omega t}\hat{\sigma}^+ + \vec{F}^*(\vec{x})\, e^{-i\Omega t}\hat{\sigma}^- \;;\quad \vec{F}(\vec{x}) = \psi_e(\vec{x})\,\vec{x}\,\psi_g^*(\vec{x})$$

$$\hat{\vec{E}}(t,\vec{x}) = \sum_s \int \frac{d^3k}{\sqrt{(2\pi)^3}}\sqrt{\frac{|\vec{k}|}{2}}\, i\left[\hat{a}^\dagger_{\vec{k},s}\,\vec{\epsilon}^*(\vec{k},s)\, e^{i(|\vec{k}|t-\vec{k}\cdot\vec{x})} - \hat{a}_{\vec{k},s}\,\vec{\epsilon}(\vec{k},s)\, e^{-i\cdots}\right.$$

$\hbar = c = 1$ $\qquad |\vec{k}| = \omega_k$ (

Unruh-DeWitt model (captures all relevant physics when exchange of olar coupling

$$; \;\vec{\vec{F}}(\vec{x}) = \Psi_e(\vec{x})\, \vec{x}\, \Psi_g^*(\vec{x})$$

taking a plane-wave basis

$$^*(\vec{k},s)\, e^{i(|\vec{k}|t - \vec{k}\cdot\vec{x})} - \hat{a}_{\vec{k},s}\, \vec{\epsilon}(\vec{k},s)\, e^{-i(|\vec{k}|t - \vec{k}\cdot\vec{x})} \Big]$$

$\hbar = c = 1$ $\quad |\vec{k}| = \omega_{\vec{k}}$ (massless field)

relevant physics when exchange of angular momentum is not crucial)

Unruh-DeWitt model (captures all relevant physics when exchange

$$\hat{H}_I = \lambda \int d^3x \, \hat{m}(t,\vec{x}) \, \hat{\phi}(t,\vec{x})$$

$$\hat{m}(t,\vec{x}) = F(\vec{x}) \overbrace{\left(\hat{\sigma}^+ e^{i\Omega t} + \hat{\sigma}^- e^{-i\Omega t}\right)}^{\hat{\mu}(t)} = F(\vec{x})\,\hat{\mu}(t)$$

$$\hat{\phi}(t,x) = \int \frac{d^3k}{\sqrt{2(2\pi)^3|\vec{k}|}} \left(\hat{a}^\dagger_{\vec{k}} e^{i(|\vec{k}|t - \vec{k}\cdot\vec{x})} + H.c.\right)$$

$|\vec{k}| = \omega_{\vec{k}}$ (massless field)

when exchange of angular momentum is not crucial)

Point-like UDW 1976-1979

$$F(\vec{x}) = \delta^{(3)}(\vec{x} - \vec{x}_0)$$

$$F(\vec{x})\,\hat{\mu}(t)$$

$$H_I = \lambda\, \mu(t)\, \phi(t, \vec{x})$$

Jaynes-Cummings model: UDW + pointlike + single m

$$\text{J-C}: \mathcal{H}_I = \lambda\left(\hat{\sigma}^+ \hat{a}_k e^{i(\Omega-\omega_k)t} + \hat{\sigma}^- \hat{a}_k^\dagger e^{-i(\Omega-\omega_k)t}\right)$$

UDW under single mode : $\sim \underbrace{\hat{\sigma}^+ \hat{a}_k e^{i(\Omega-\omega_k)t} + \hat{\sigma}^- \hat{a}_k^\dagger e^{-i(\Omega-\omega_k)t}}_{\text{Rotating terms}} + \underbrace{\hat{\sigma}^+ \hat{a}_k^\dagger e^{i(\Omega+\omega_k)t}}_{\text{counter-ro}}$

$\hat{\sigma}^+ \hat{a}^\dagger : |0\rangle|g\rangle$ ———

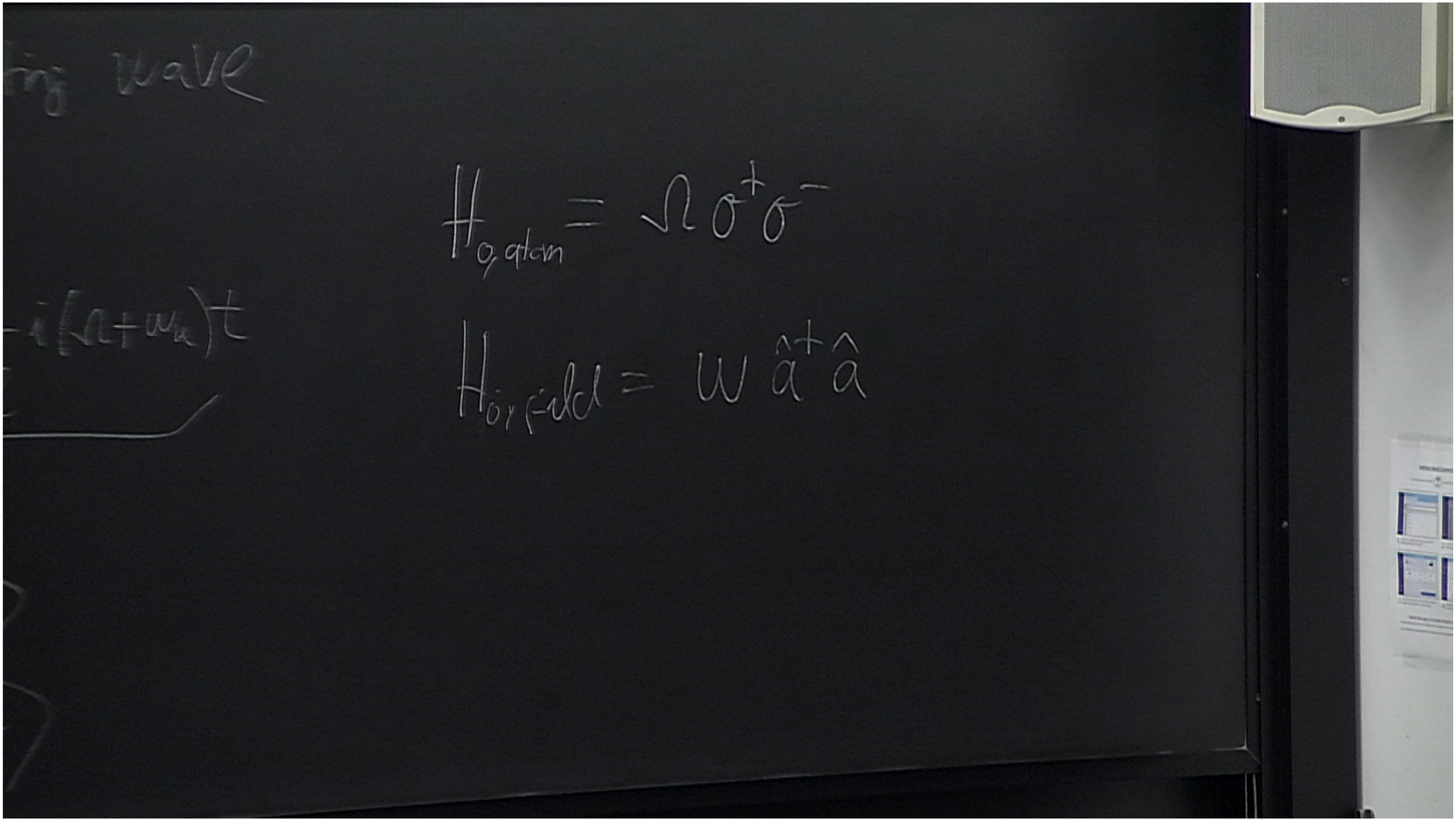
...ings model: UDW + pointlike + single mode + Rotating wave

$$= \lambda\left(\hat{\sigma}^+ \hat{a}_k e^{i(\Omega-\omega_k)t} + \hat{\sigma}^- \hat{a}_k^\dagger e^{-i(\Omega-\omega_k)t}\right)$$

$$\underbrace{\hat{a}_k e^{i(\Omega-\omega_k)t} + \hat{\sigma}^- a_k^\dagger e^{-i(\Omega-\omega_k)t}}_{\text{Rotating terms}} + \underbrace{\hat{\sigma}^+ \hat{a}_k^\dagger e^{i(\Omega+\omega_k)t} + \hat{\sigma}^- \hat{a}_k e^{-i(\Omega+\omega_k)t}}_{\text{counter-rotating terms}}$$

$$\hat{\sigma}^+ \hat{a}^\dagger : \overset{0}{|0\rangle|g\rangle} \longrightarrow \overset{\hbar\omega + \hbar\Omega}{|1_k\rangle|e\rangle}$$

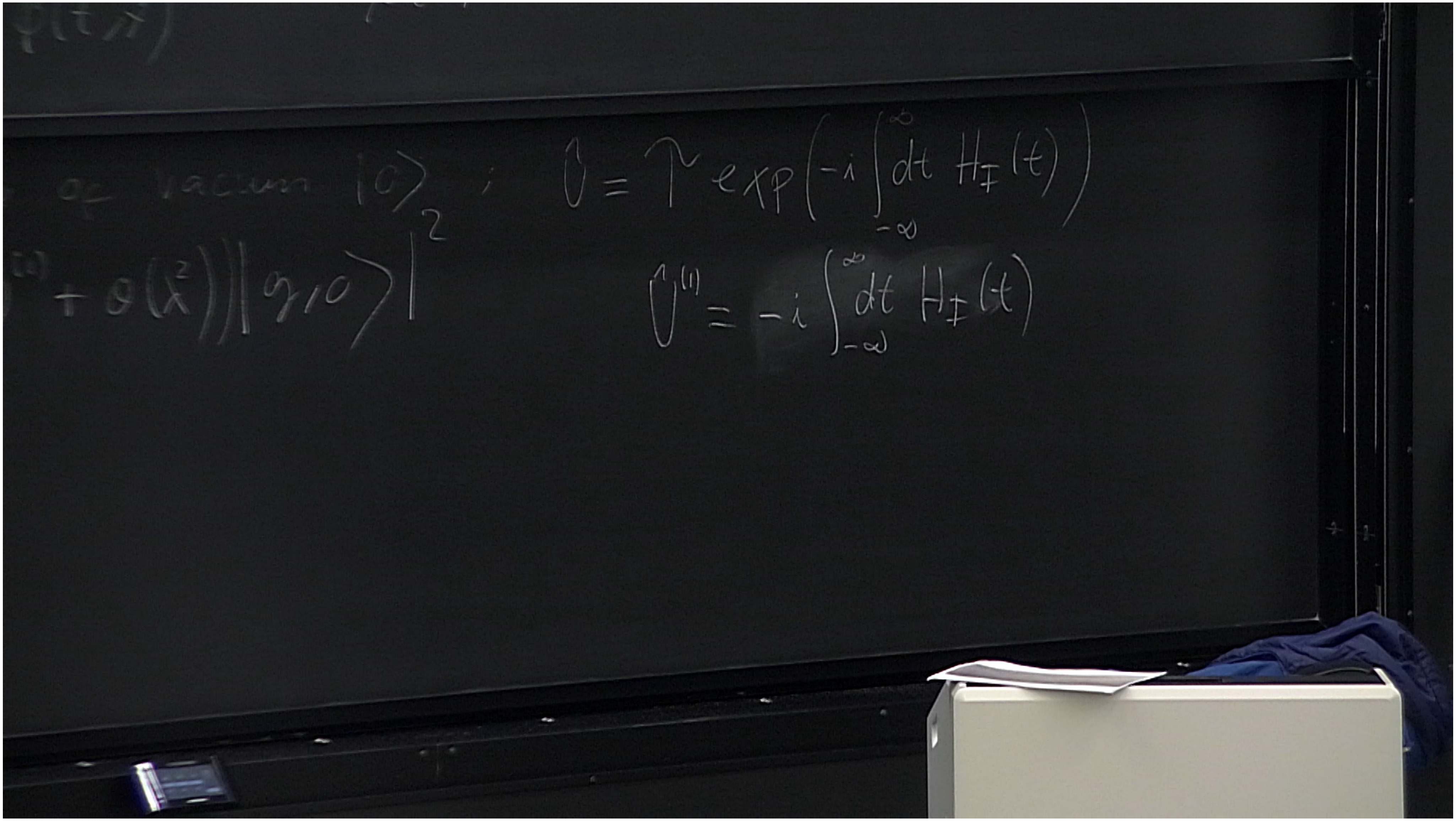
$$\hat{\sigma}^- \hat{a}^\dagger : \underset{0+\hbar\Omega}{|0\rangle|e\rangle} \longrightarrow \underset{\hbar\omega + 0}{|1_k\rangle|g\rangle}$$


wave
$H_{0,atom} = \Omega \sigma^+ \sigma^-$
$H_{0,field} = \omega \hat{a}^+ \hat{a}$

$$\hat{H}_I = \lambda \underline{\chi(t)} \int d^3x \, \hat{m}(t,\vec{x}) \, \phi(t,\vec{x}) \qquad \chi(t)$$

Consider a transition $|g\rangle \longrightarrow |e\rangle$ in the presence of vacuum $|0\rangle$;

$$P^{(1)}_{|g\rangle\to|e\rangle} = \sum_{out} |\langle e, out | \hat{U} | g, 0\rangle|^2 = \sum_{out} |\langle e, out | \left(\mathbb{1} + \hat{U}^{(1)} + \mathcal{O}(\lambda^2)\right) | g, 0\rangle$$


of vacuum $|0\rangle$;
$\hat{U} = \mathcal{T} \exp\left(-i \int_{-\infty}^{\infty} dt \, H_I(t)\right)$
$+ \mathcal{O}(\lambda^2))|g,0\rangle|^2$
$\hat{U}^{(1)} = -i \int_{-\infty}^{\infty} dt \, H_I(t)$

$$H_I = \lambda \underline{\chi(t)} \int dx \, m(t,x) \, \phi(t,x)$$

Consider a transition $|g\rangle \longrightarrow |e\rangle$ in the presence of

$$P^{(1)}_{|g\rangle \to |e\rangle} = \sum_{out} \left| \langle e, out | \hat{U} | g, 0 \rangle \right|^2 = \sum_{out} \left| \langle e, out | \left(\cancel{\mathbb{1}} + \hat{U}^{(1)} + \mathcal{O} \right.\right.$$

$$= \sum_{out} \langle g, 0 | \hat{U}^{(1)\dagger} | e, out \rangle \langle e, out | \hat{U}^{(1)} | g, 0 \rangle$$

$\int dx \, m(t,x) \, \phi(t,x))$

in the presence of vacuum $|0\rangle$

Pointlike detector $\hat{U} = \mathcal{T} \exp($

$$\sum_{out} \left| \langle e, out | \left(\mathbb{1} + \hat{U}^{(1)} + \mathcal{O}(\lambda^2) \right) | g, 0 \rangle \right|^2$$

$$\hat{U}^{(1)} = -i$$

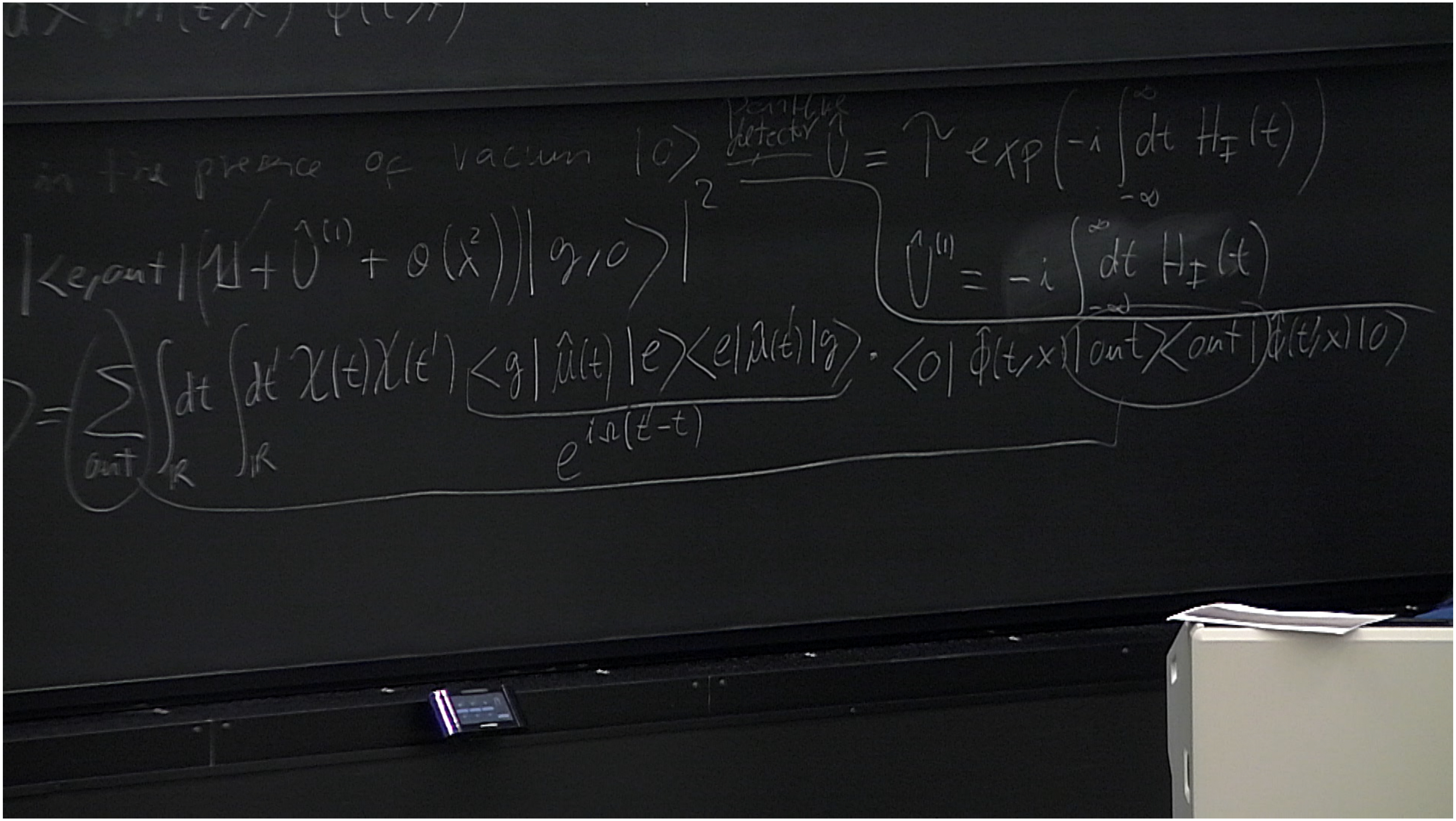
$$0\rangle = \sum_{out} \int_{\mathbb{R}} dt \int_{\mathbb{R}} dt' \, \chi(t) \chi(t') \langle g|$$

vacuum $|0\rangle$ pointlike detector $\hat{U} = T \exp\left(-i\int_{-\infty}^{\infty} dt \, H_I(t)\right)$

$\theta(\lambda^2))|g,0\rangle\Big|^2$

$\hat{U}^{(1)} = -i\int_{-\infty}^{\infty} dt \, H_I(t)$

$(t')\langle g|\hat{\mu}(t)|e\rangle\langle e|\hat{\mu}(t)|g\rangle \cdot \langle 0|\hat{\phi}(t,x)|out\rangle\langle out|\hat{\phi}(t,x)|0\rangle$


in the presence of vacuum $|0\rangle$
point-like detector
$\hat{U} = \mathcal{T} \exp\left(-i \int_{-\infty}^{\infty} dt \, H_I(t)\right)$
$\hat{U}^{(1)} = -i \int_{-\infty}^{\infty} dt \, H_I(t)$
$\left| \langle e, \text{out} | \left(\mathbb{1} + \hat{U}^{(1)} + O(\lambda^2) \right) | g, 0 \rangle \right|^2$
$= \sum_{\text{out}} \int_{\mathbb{R}} dt \int_{\mathbb{R}} dt' \, \chi(t) \chi(t') \langle g | \hat{\mu}(t) | e \rangle \langle e | \hat{\mu}(t') | g \rangle \cdot \langle 0 | \hat{\phi}(t,x) | \text{out} \rangle \langle \text{out} | \hat{\phi}(t',x) | 0 \rangle$
$e^{i\Omega(t'-t)}$

$$P^{(c)}_{|g\rangle\to|e\rangle} = \sum_{out} \left|\langle e, out|\hat{U}|g,0\rangle\right|^2 = \sum_{out} \left|\langle e, out|(\mathbb{1} + \hat{U}^{(1)} + \mathcal{O}(\lambda^2)\right.$$

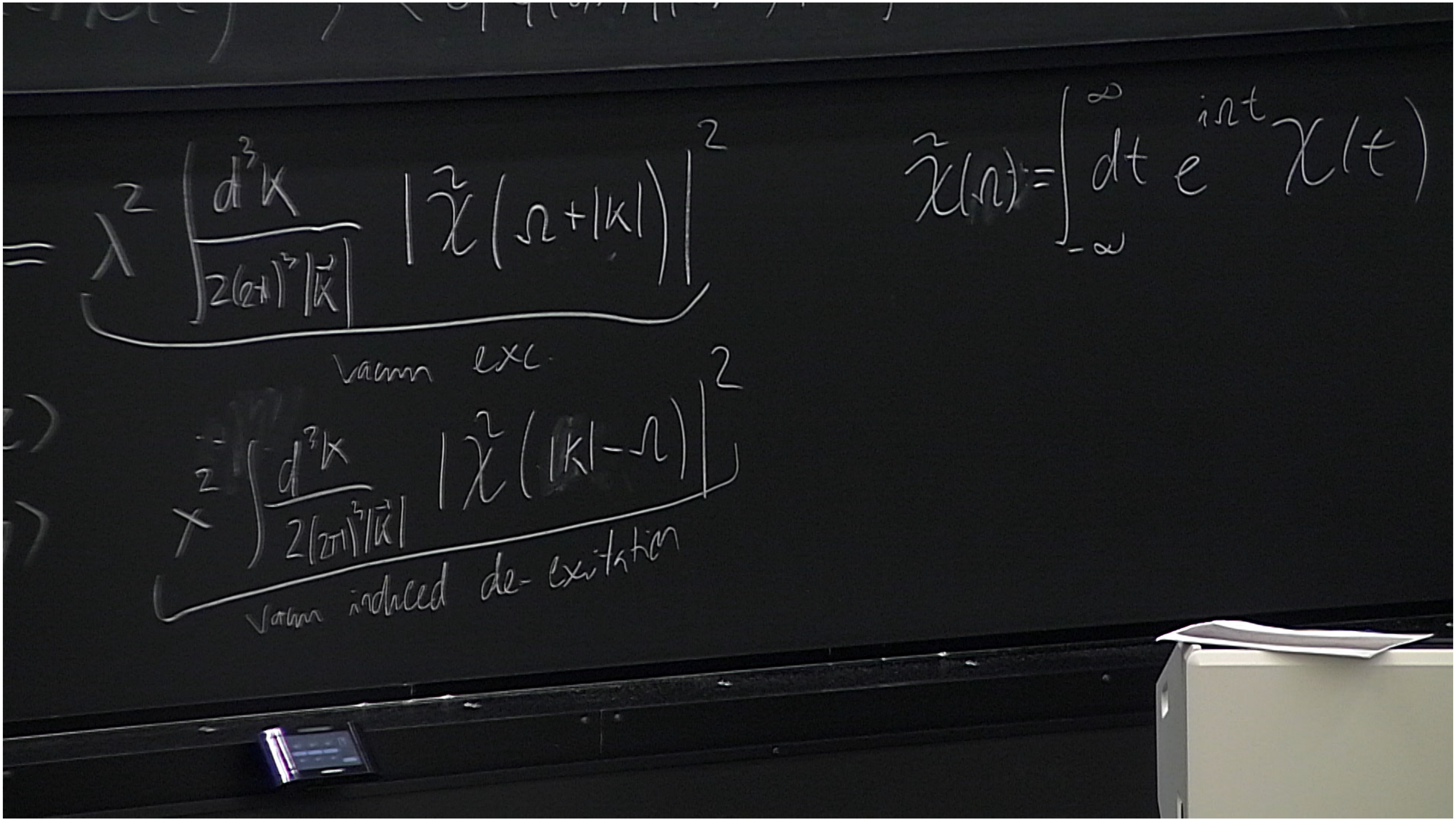
$$= \sum_{out} \langle g,0|\hat{U}^{(1)\dagger}|e,out\rangle\langle e,out|\hat{U}^{(1)}|g,0\rangle = \lambda^2 \sum_{out} \int_{\mathbb{R}} dt \int_{\mathbb{R}} dt'\, \chi(t)\chi(t')$$

$$= \lambda^2 \int dt \int dt'\, e^{i\Omega(t'-t)} \underbrace{\langle 0|\hat{\phi}(t,\vec{x})\,\phi(t',\vec{x})|0\rangle}_{\text{Wightman function}} \chi(t)\chi(t')$$

$$\langle g|\hat{\mu}(t)|e\rangle\langle e|\hat{\mu}(t')|g\rangle \cdot \langle 0|\hat{\phi}(t,x)|out\rangle\langle out|\hat{\phi}(t',x)|0\rangle$$

$$e^{i\Omega(t'-t)}$$

$$\langle 0|\phi(t,\vec{x})\phi(t',\vec{x}')|0\rangle = \int d^3k\, e^{i(|k|t'-\vec{k}\cdot\vec{x}')}\, e^{-i(|\vec{k}|t-\vec{k}\cdot\vec{x})}\frac{1}{\sqrt{2(2\pi)^3|\vec{k}|}}$$


$= \underbrace{\lambda^2 \int \frac{d^3k}{2(2\pi)^3|\vec{k}|} \left| \tilde{\chi}(\Omega + |k|) \right|^2}$
vacuum exc.
$\underbrace{\lambda^2 \int \frac{d^3k}{2(2\pi)^3|\vec{k}|} \left| \tilde{\chi}(|k| - \Omega) \right|^2}$
vacuum induced de-excitation
$\tilde{\chi}(\Omega) = \int_{-\infty}^{\infty} dt \, e^{i\Omega t} \chi(t)$