

Title: PSI 2018/2019 - Explorations in Quantum Information - Lecture 2

Speakers: Eduardo Martin-Martinez

Collection: PSI 2018/2019 - Explorations in Quantum Information (Martin-Martinez)

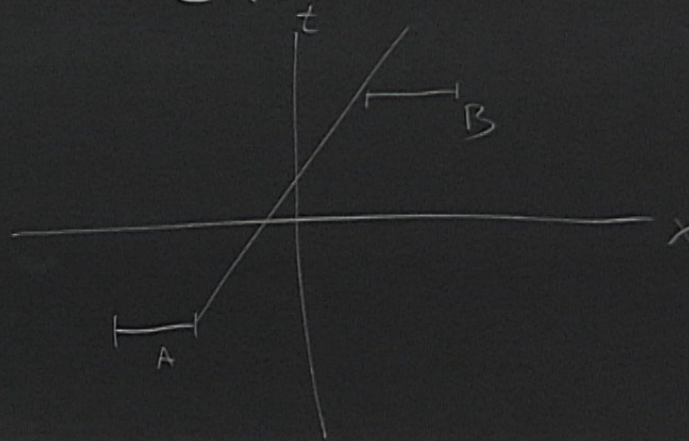
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URL: <http://pirsa.org/19040052>

R. D. Sorkin arXiv: gr-qc 9302018 (1993)

F. Dowker arXiv: 1111.2308 (2011)

D.M.T. Benincasa et al. CQG 31, 075007 (2014)



07 (2014)

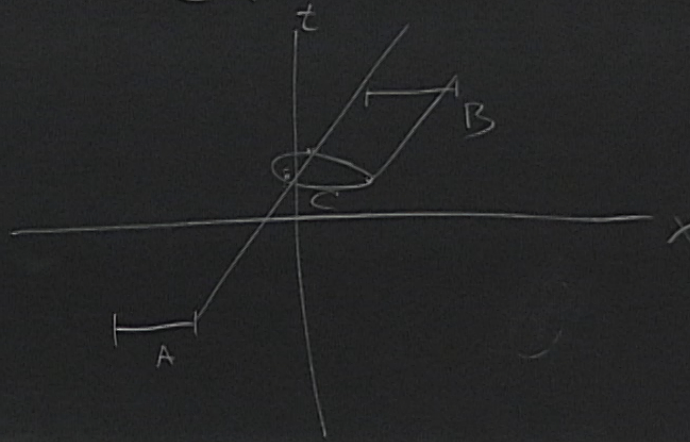
Particle detectors:

- a) Localized quantum systems
- b) Couple to quantum fields
- c) Non-relativistic (in their internal energy levels)
- d) We can measure

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Par
a) L
b) C
c) N
d) We

The light-matter interaction from first principles

An electron in a Hydrogen-like atom: $\hat{H} = \frac{\hat{\vec{p}}^2}{2m} + eV(\hat{\vec{x}})$ (orbitals, etc)

electron couples to the EM field. $\hat{H} = \frac{(\hat{\vec{p}} - e\vec{A}(t, \hat{\vec{x}}))^2}{2m} + eU(t, \hat{\vec{x}}) + V(\hat{\vec{x}})$

$$\hat{H} = \underbrace{\frac{\hat{\vec{p}}^2}{2m} + eV(\hat{\vec{x}})}_{\hat{H}_0} - \underbrace{\frac{e}{2m} \left(\hat{\vec{p}} \cdot \vec{A}(t, \hat{\vec{x}}) + \vec{A}(t, \hat{\vec{x}}) \cdot \hat{\vec{p}} \right)}_{\hat{H}_I} + eU(t, \hat{\vec{x}}) + \frac{e^2}{2m} [\vec{A}(t, \hat{\vec{x}})]^2$$

$$\hat{H} = \frac{\vec{p}^2}{2m} + eV(\vec{x}) - \frac{e}{2m} \left(\vec{p} \cdot \vec{A}(t, \vec{x}) + \vec{A}(t, \vec{x}) \cdot \vec{p} \right) + eU(t, \vec{x}) + \frac{e^2}{2m} [\vec{A}(t, \vec{x})]^2$$

$\hat{H}_0$
 $\hat{H}_I$

Pick the Coulomb gauge, $U(t, \vec{x}) = 0$, $[\vec{A}(t, \vec{x}), \vec{p}] = 0 \Rightarrow \hat{H} = \hat{H}_0 - \frac{e}{m} \vec{p} \cdot \vec{A}(t, \vec{x}) + \frac{e^2}{2m} [\vec{A}(t, \vec{x})]^2$ (neglect)

Mania Goepfert-Meyer, write H in terms of $\vec{E}$, $\vec{B}$. Pick Poincaré gauge and carry out a multipole expansion, and keep the dipole terms

$$\hat{H} = \hat{H}_0 + e \vec{x} \cdot \vec{E}(t, \vec{x})$$

A gauge transformation $|\psi\rangle \rightarrow e^{ie\chi(t, \vec{x})} |\psi\rangle$

$$\begin{aligned}
 \hat{U}_I &= e^{i\hat{H}_0 t} \hat{U}_I e^{-i\hat{H}_0 t} = e^{\vec{\hat{x}} \cdot \vec{E}(\vec{x})} = \sum_i \sum_j \langle j | \vec{\hat{x}} \cdot \vec{E}(t, \vec{x}) | i \rangle e^{i\Omega_{ij}t} |j\rangle \langle i| = \\
 &= e \sum_{i,j} \int d^3x \int d^3x' \underbrace{\langle j | \vec{x} \rangle}_{\psi_j^*(\vec{x})} \underbrace{\langle \vec{x} | \vec{x}' \rangle}_{\delta^{(3)}(\vec{x} - \vec{x}')} \underbrace{\vec{E}(t, \vec{x}) | \vec{x}' \rangle}_{\psi_i(\vec{x}')} \underbrace{\langle \vec{x}' | i \rangle}_{\psi_i(\vec{x}')} e^{i\Omega_{ij}t} |j\rangle \langle i| \\
 &= e \sum_{i \neq j} \int d^3x \psi_j^*(\vec{x}) \vec{x} \psi_i(\vec{x}) \cdot \vec{E}(t, \vec{x}) e^{i\Omega_{ij}t} |j\rangle \langle i|
 \end{aligned}$$

$$\begin{aligned}
 q &\rightarrow \bar{q} \\
 \Delta n &= \pm 1 \\
 \Delta l &= \pm 1, 0
 \end{aligned}$$

Restricting to two levels:

$$i = g, e$$

$$j = g, e$$

$$\hat{\sigma}^{+2} = \hat{\sigma}^{-2} = 0$$

$$|e\rangle\langle g| = \hat{\sigma}^+$$

$$|g\rangle\langle e| = \hat{\sigma}^-$$

$$\vec{F}(\vec{x}) = \psi_e(\vec{x}) \vec{x} \psi_g^*(\vec{x})$$

Smearing vector

$$H_I = e \int d\vec{x} \left(\vec{F}(\vec{x}) e^{i\vec{x}\cdot\vec{p}} + \vec{F}^*(\vec{x}) e^{-i\vec{x}\cdot\vec{p}} \right) \cdot \vec{E}(t, \vec{x})$$

$$\hat{d}(t, \vec{x})$$

$$H_I = e \int d\vec{x} \hat{d}(t, \vec{x}) \cdot \vec{E}(t, \vec{x})$$