

Title: The world as topological insulator

Speakers: David Kaplan

Series: Colloquium

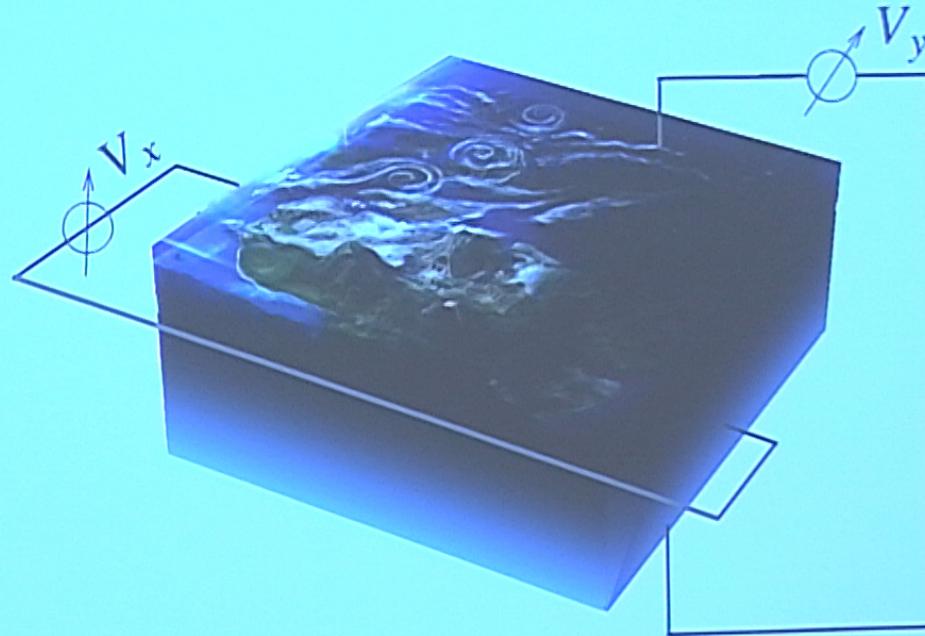
Date: April 17, 2019 - 2:00 PM

URL: <http://pirsa.org/19040050>

Abstract: Over the years, many rich ideas have been exchanged between particle theory and condensed matter theory, such as particle/hole theory, superconductivity and dynamical symmetry breaking, universality and critical phenomena. Here I discuss the interesting case of how the two fields converged independently along different paths on the physics of symmetry-protected topological order: condensed matter physicists motivated by the quantum Hall effect and superconductivity, the particle physicists driven by the desire to understand anomalies and chirality, and to compute QCD and supersymmetry on a lattice -- where the Quantum Hall effect, the Quantum Spin Hall effect and Majorana surface modes have all played a role in practical computations since the 1990s.

&nbs;p;

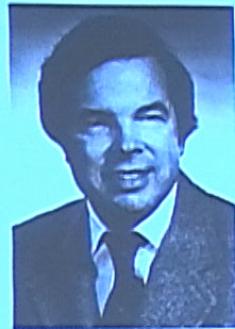
The World as a Topological Insulator



D.B. Kaplan

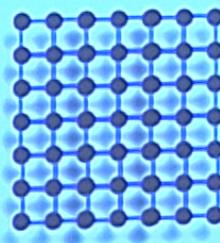
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Ken Wilson:
Relativistic quantum field theory
& condensed matter systems
are closely related

What if I
add a dim.-5
operator?



Try
gallium
arsenide?

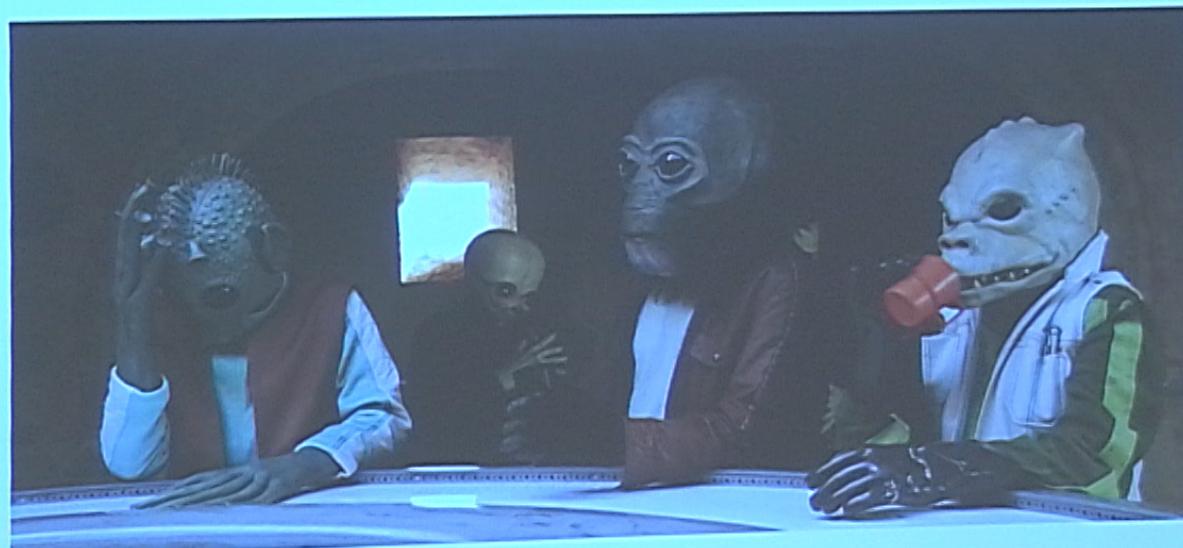


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...even if the particle and condensed matter communities sometimes view each other as this:

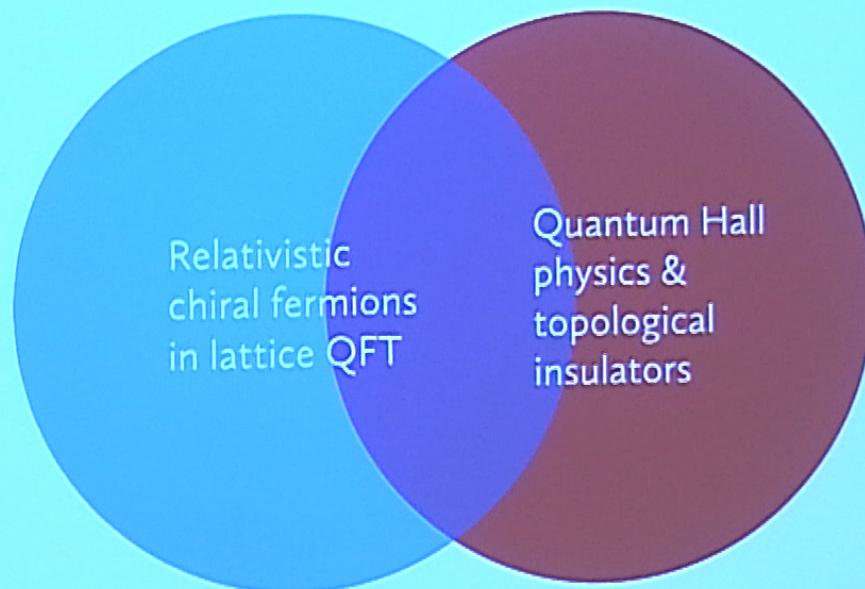


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This talk:

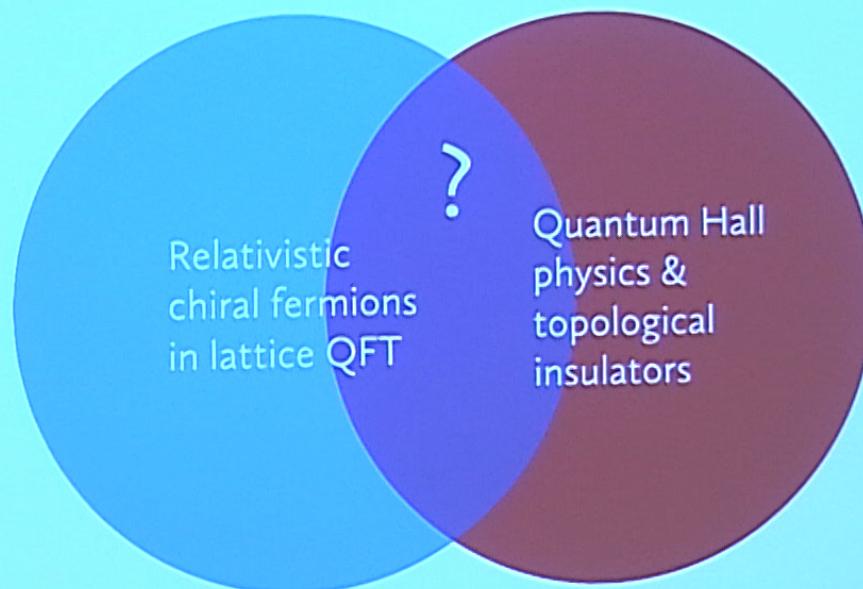


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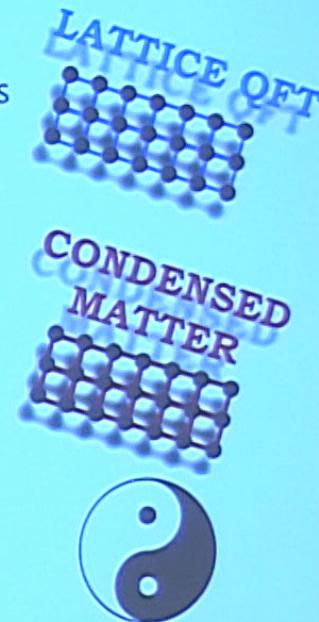


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- Why is chirality on the lattice important?
 - Nielsen-Ninomiya theorem, anomalies, & edge states
 - Domain wall fermions and topology
 - Phases of lattice QCD
-
- Integer Quantum Hall Effect
 - Edge states
 - Topology
-
- Majorana edge states for SUSY & quantum computation
 - Anomaly cancellation for chiral gauge theories & Spin Quantum Hall Effect

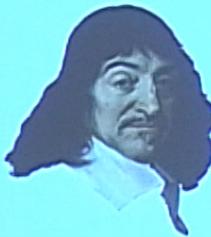


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Chirality



“Any man who, upon looking down at his bare feet, doesn't laugh, has either no sense of symmetry or no sense of humour” (Descartes)



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Chiral symmetry

- exists in even spacetime dimension
- is broken by mass terms
- QED: ensures multiplicative mass renormalization for the electron
- QCD:
 - ◆ multiplicative mass renormalization for the quarks
 - ◆ spontaneous breaking explains light π , K , η
 - ◆ explicit breaking explains π , K , η , baryon masses
 - ◆ anomalous breaking explains η' mass, π^0 , η decay
 - ◆ chiral symmetry controls operator mixing
- $SU(2) \times U(1)$:
 - ◆ chiral gauge theory!
 - ◆ anomalous baryon, lepton violation



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The Euclidian fermion action: $S = \int_{-\pi/a}^{\pi/a} \frac{d^{2k}p}{(2\pi)^4} \bar{\Psi}_{-\mathbf{p}} \tilde{D}(\mathbf{p}) \Psi(\mathbf{p})$

Nielsen-Ninomiya — can't have all four properties:

1. $\tilde{D}(\mathbf{p})$ is a periodic, analytic function of p_μ ; \Leftarrow regulated, local
2. $D(\mathbf{p}) \propto \gamma_\mu p_\mu$ for $a|p_\mu| \ll 1$; \Leftarrow Dirac @ long wavelength
3. $\tilde{D}(\mathbf{p})$ invertible everywhere except $p_\mu = 0$; \Leftarrow No doubling of flavors
4. $\{\Gamma, \tilde{D}(\mathbf{p})\} = 0$. \Leftarrow respects a chiral symmetry
 $\partial_\mu j_5^\mu = \bar{\psi} \{\gamma_5, D\} \psi$

H Nielsen, M Ninomiya, NPB 185 (1981) 20

- $\{\Gamma, \gamma_\mu\} = 0$
- generates chiral symmetry
- defined only in even spacetime dimensions



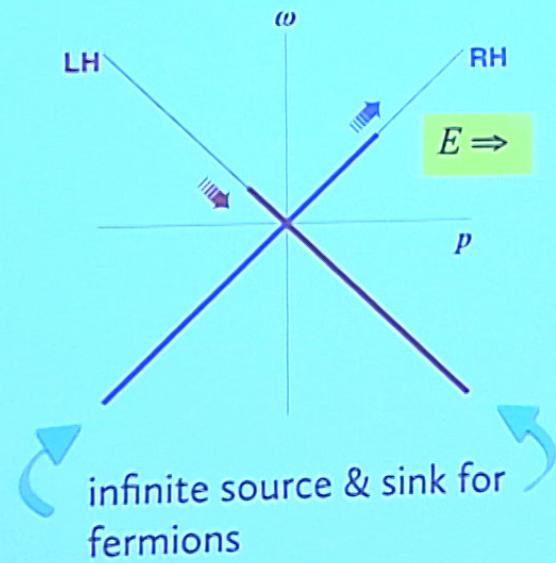
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Nielsen-Ninomiya theorem is a necessary consequence of anomalies:

Massless Dirac fermions in an electric field E , 1+1 dim

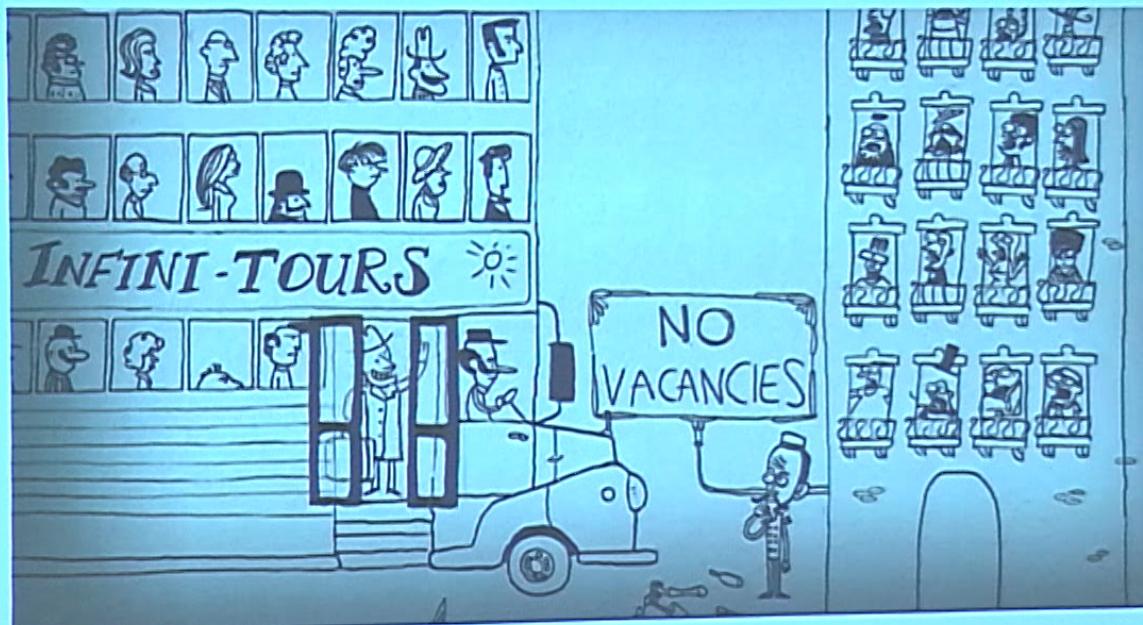


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Anomalies in the continuum an example of the Hilbert Hotel:



There's always room for more in the vacuum... but not on the lattice

Symmetries broken by anomalies in continuum
must be broken explicitly on lattice



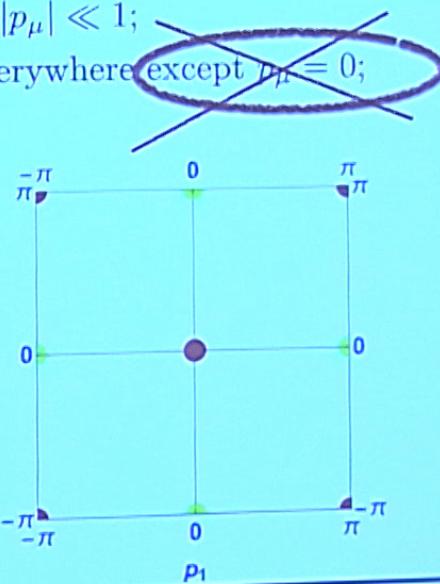
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e.g., naive lattice fermions (Euclidian spacetime):

$$\mathcal{L} = \bar{\psi} \not{D} \psi = \frac{1}{2a} \sum_{\mathbf{n}, \mu} \bar{\psi} \gamma_\mu (\psi_{\mathbf{n}+\hat{\mu}} - \psi_{\mathbf{n}-\hat{\mu}}) \quad \tilde{D}(p)^{-1} = \frac{a}{\sum_\mu (\gamma_\mu \sin ap_\mu)}$$

1. $\tilde{D}(\mathbf{p})$ is a periodic, analytic function of p_μ ;
2. $D(\mathbf{p}) \propto \gamma_\mu p_\mu$ for $a|p_\mu| \ll 1$;
3. $\tilde{D}(\mathbf{p})$ invertible everywhere except $p_\mu = 0$;
4. $\{\Gamma, \tilde{D}(\mathbf{p})\} = 0$.

e.g. d=2 (\triangleright 1+1)



- Poles at every corner of Brillouin zone
- Alternating chiral rotation



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e.g., Wilson fermions:

$$\begin{array}{ccc} \text{Lattice derivatives} & & \text{Lattice covariant derivatives} \\ \swarrow \quad \searrow & & \swarrow \quad \searrow \\ \mathcal{L} \rightarrow \bar{\psi} \left[\not{\partial} + a \frac{R}{2} \partial^2 \right] \psi & \rightarrow \bar{\psi} \left[\not{\partial} + M + a \frac{R}{2} \not{D}^2 \right] \psi \\ \text{violates} & & \text{violates} \\ \text{chiral symmetry} & & \text{chiral symmetry} \end{array}$$

$$\tilde{D}(p)^{-1} = \frac{a}{aM + \sum_{\mu} (\gamma_{\mu} \sin ap_{\mu} + R[\cos ap_{\mu} - 1])}$$

1. $\tilde{D}(p)$ is a periodic, analytic function of p_{μ} ;
2. $D(p) \propto \gamma_{\mu} p_{\mu}$ for $a|p_{\mu}| \ll 1$;
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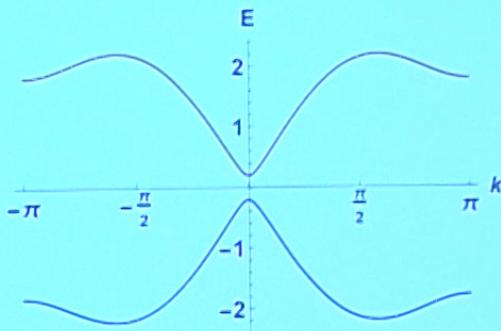
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Wilson fermions:

$$\rightarrow \bar{\psi} \left[i\cancel{D} + M + a \frac{R}{2} D^2 \right] \psi$$

Spectrum of a 1+1 Minkowski Wilson fermion on a 1d spatial lattice



Anomalous chiral symmetries are explicitly broken (eg, $U(1)_A$) ✓

No doublers ✓

Breaks non-anomalous symmetries: $SU(2)_L \times SU(2)_R$ 😢

Requires $O(1/a)$ fine tuning to get to symmetric point (additive mass renormalization) 🙄



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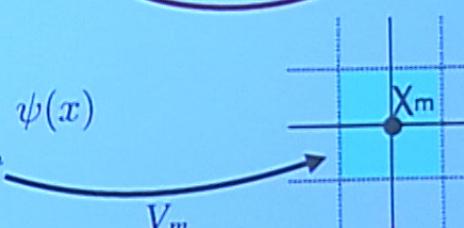
Ginsparg-Wilson (1981)

Minimal breaking of chiral symmetry on the lattice?

- Start with chiral invariant continuum action $S(\bar{\psi}, \psi)$ with chiral invariance up to anomalies
- Perform a block-spin transformation to turn it into an equivalent lattice theory

$$e^{-S_{\text{lat}}(\bar{\chi}, \chi)} = \int_{\bar{\psi}, \psi} e^{-S(\bar{\psi}, \psi)} e^{-\frac{1}{a}(\bar{\psi} - \bar{\chi})_m (\psi - \chi)_m}$$

breaks chiral symmetry

$$\psi_m = \int_{x \in V_m} \psi(x)$$




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Ginsparg-Wilson cont'd

$$S_{\text{lat}}(\bar{\psi}, \psi) \equiv \bar{\chi} D \chi$$

Ginsparg-Wilson equation:

$$\{\Gamma, D\} = a D \Gamma D$$

continuum:

$$\{\gamma_5, \not{D}\} = 0$$

- Looks like chiral symmetry in the $a \gg 0$ limit
- Ensures multiplicative mass renormalization (Lüscher 1998)
- ...but: also correctly reproduces the anomaly in the $a \gg 0$ limit



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chiral Ward identity

$$\partial_\mu j_5^\mu = a D \gamma_5 D = 0 \quad \text{unless ops at same spacetime points}$$

Only problem: GW couldn't solve for a D that obeyed this equation.



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Callan, Harvey NPB 250 (1985) 427

Anomalies and fermion zero modes on strings and domain walls

Explaining Zumino's results relating chiral, parity and non-
Abelian anomalies in different dimensions



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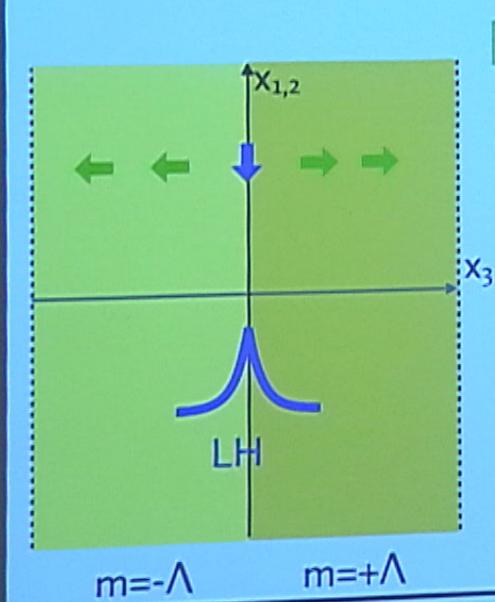
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Callan, Harvey NPB 250 (1985) 427

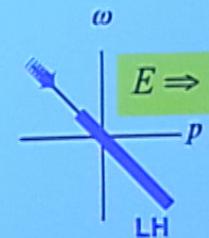
Anomalies and fermion zero modes on strings and domain walls

Explaining Zумino's results relating chiral, parity and non-Abelian anomalies in different dimensions



E ↑

- Dirac fermion in 2+1 w. mass defect
- 1+1 d chiral edge state at defect (DWF)
- Anomalous change of current at defect compensated by Chern Simons current in bulk



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Callan and Harvey (1985): compute 3d (5d) Chern Simons operator...
bulk modes don't decouple entirely!

$$\mathcal{L}_{CS} = \left(\frac{m(x_3)}{|m(x_3)|} + \frac{\Lambda}{|\Lambda|} \right) \mathcal{O}_{CS}$$

$$\mathcal{O}_{CS}^{d=3} = -\frac{e^2}{8\pi} \epsilon_{abc} A_a \partial_b A_c$$

$$\mathcal{O}_{CS}^{d=5} = -\frac{e^3}{48\pi^2} \epsilon_{abcde} A_a \partial_b A_c \partial_d A_e$$



graph in n-dimensions

Differentiate wrt A_3 (A_5) to discover a current between the domain walls:

$$J_3 = -\frac{e^2}{2\pi} (\epsilon(x_3) + 1) \epsilon_{ij} \partial_i A_j$$

nonzero divergence at
 $x_3 = 0$ in presence of E field

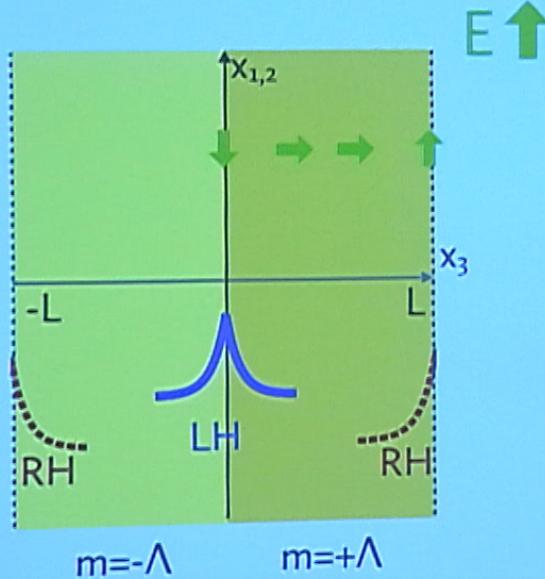


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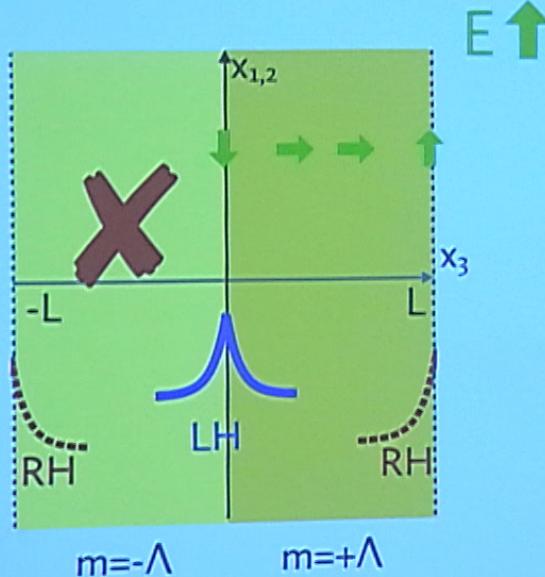
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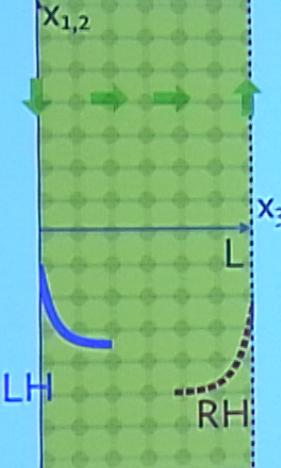


- On a **compact** extra dimension, RH mode at opposite defect
- Total charge is conserved in 3d
- Charge is transported by “Hall current” in the bulk despite gap
- Behavior possible because mass term in d=2+1 violates (i) chirality in d=1+1, (ii) time reversal, (iii) parity

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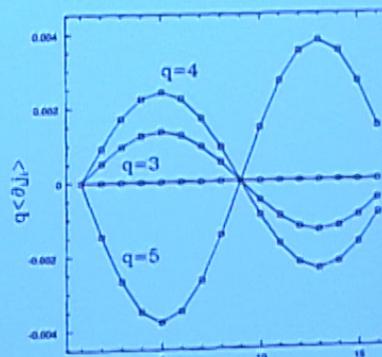
$$M=1/a$$

Suitable construction for **lattice**
since Wilson fermions have same
symmetry as massive bulk fermions
(no chirality)

DBK, PLB 288 (1992) 342
Y Shamir, NPB 406 (1993) 90

Lattice covariant
derivatives

$$\rightarrow \bar{\psi} \left[i \not{D} + M + a \frac{R}{2} D^2 \right] \psi$$



confirmation of expected
CS currents with
 $M=1/a, R=1$
K Jansen, PLB 288 (1992) 348

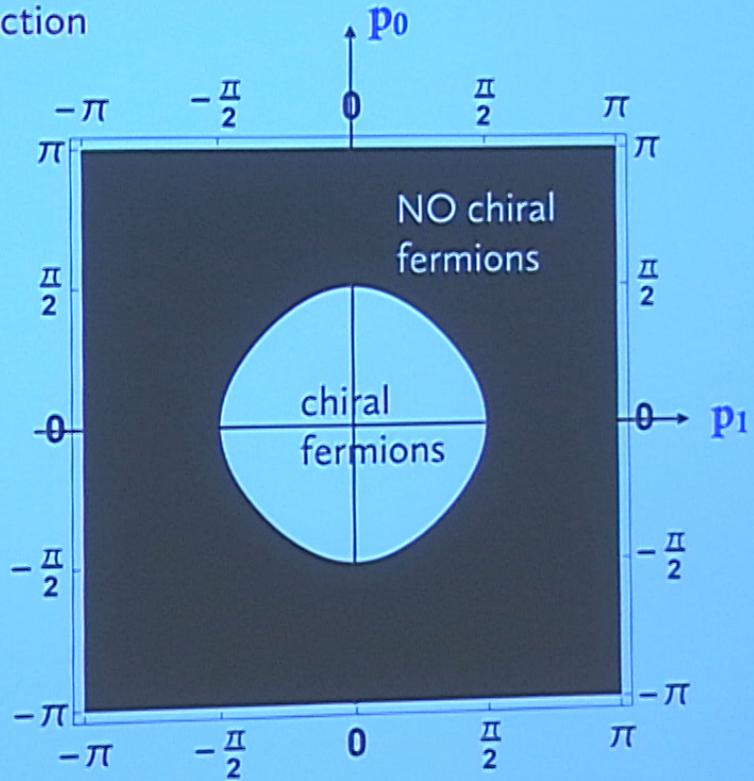


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$d=3$ Euclidian dimensional lattice: chiral edge state, no doublers
 $x_0, x_1 = 2d$ surface
 $x_3 = \text{bulk direction}$



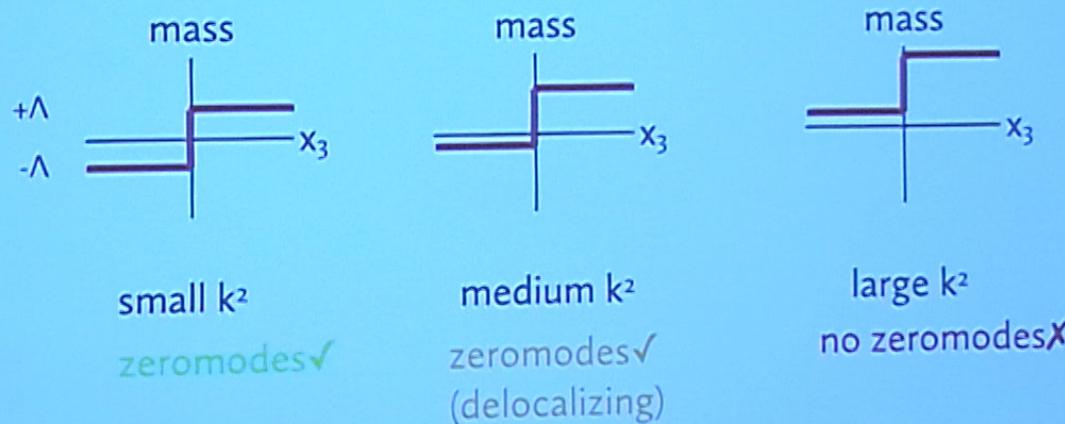
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Effect of the Wilson term $a\bar{\Psi}D^2\Psi$

Looks like a “momentum dependent mass shift” scaling like k^2

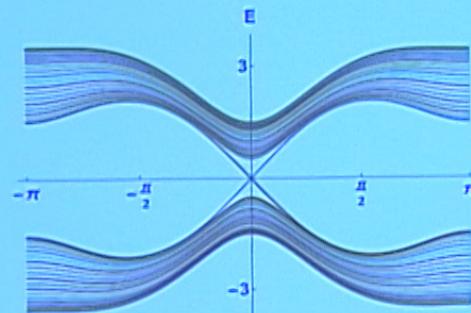


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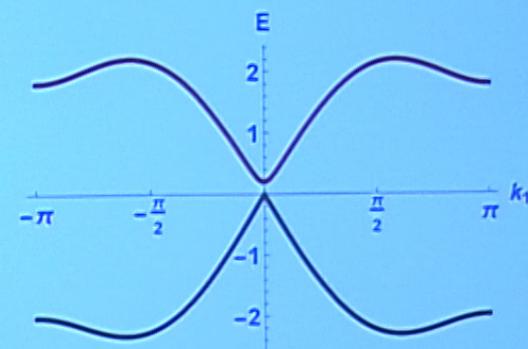
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1988



VS.



4Mb RAM

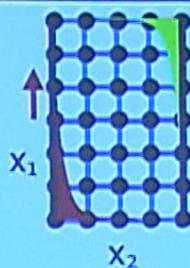


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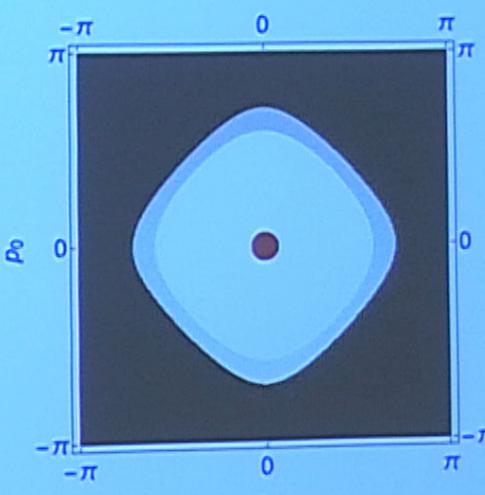
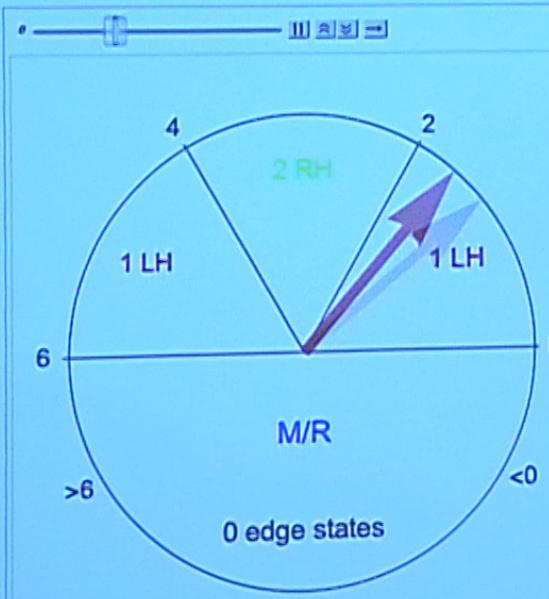
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Dialing
mass/Wilson
coefficient:



Euclidian $\{p_0, p_1\}$
Brillouin zone

Black= no chiral edge modes ✗
White= chiral edge modes ✓



M Schmaltz, PLB 296 (1992) 374

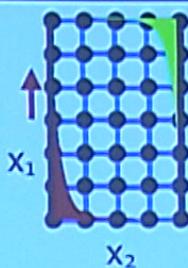


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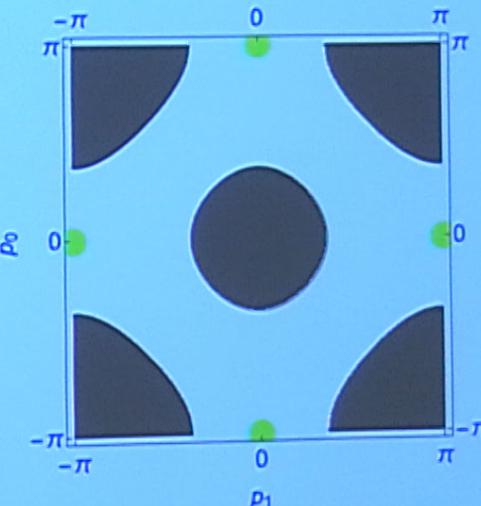
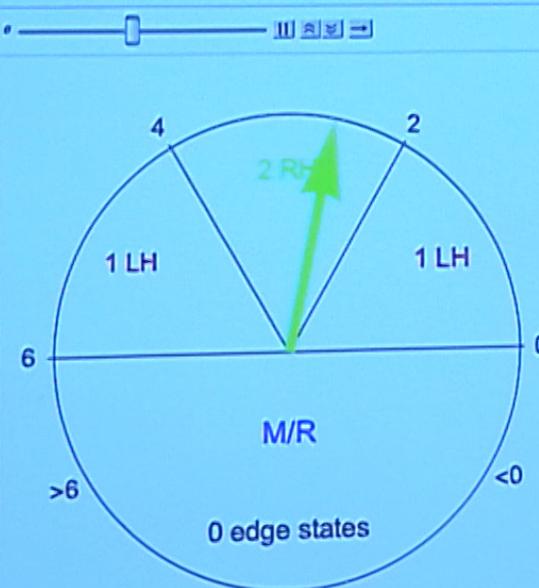
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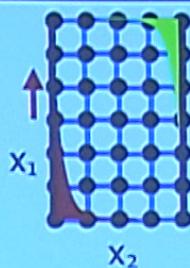


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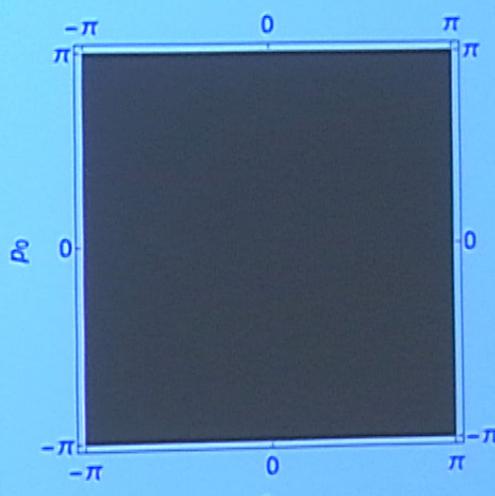
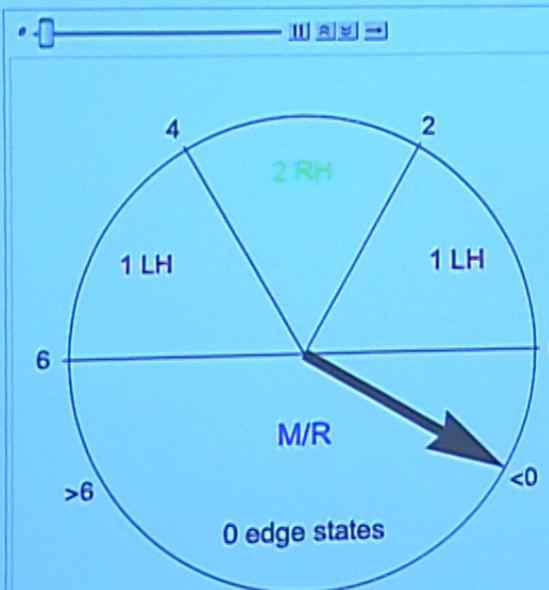
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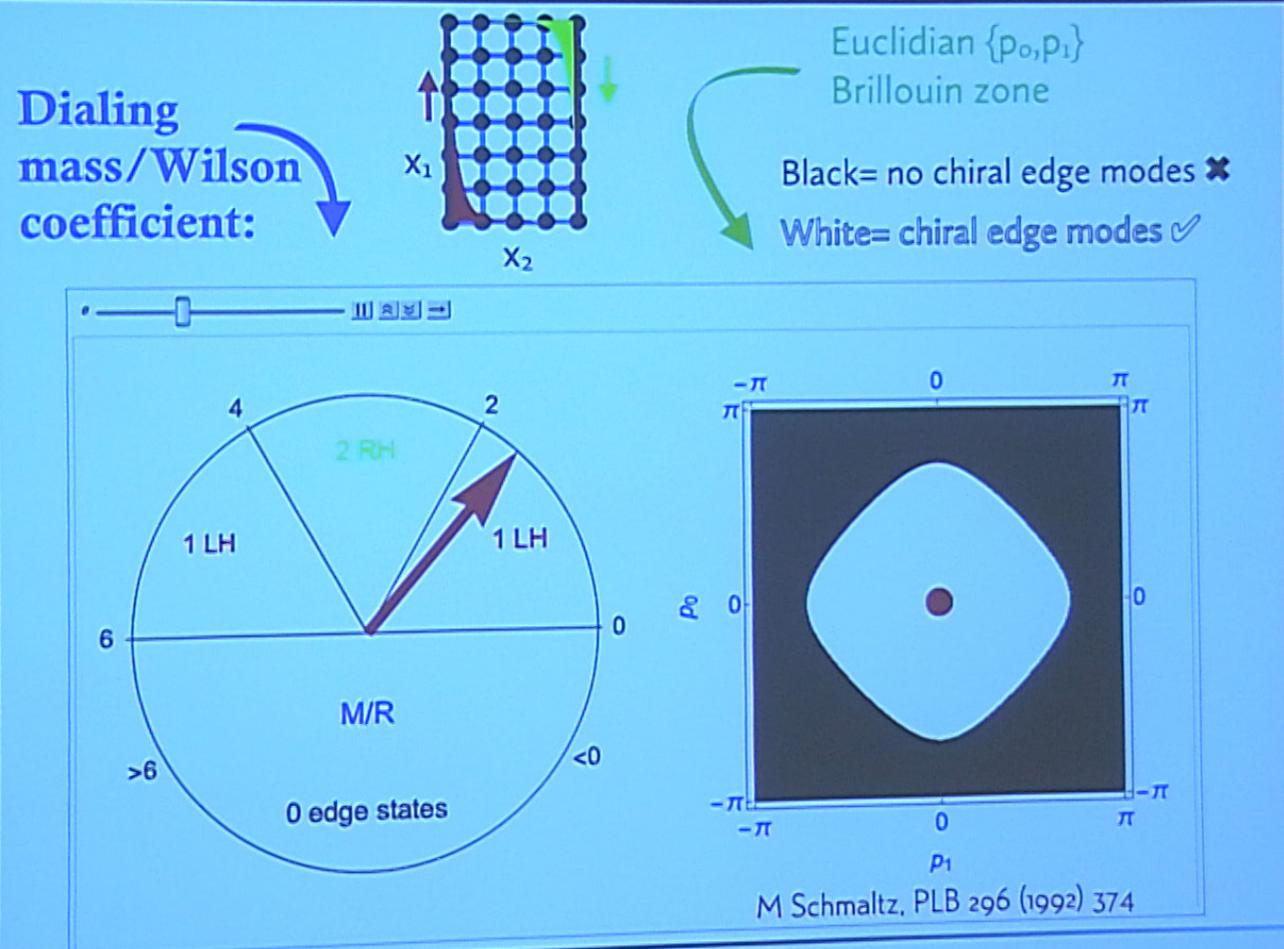
M Schmaltz, PLB 296 (1992) 374



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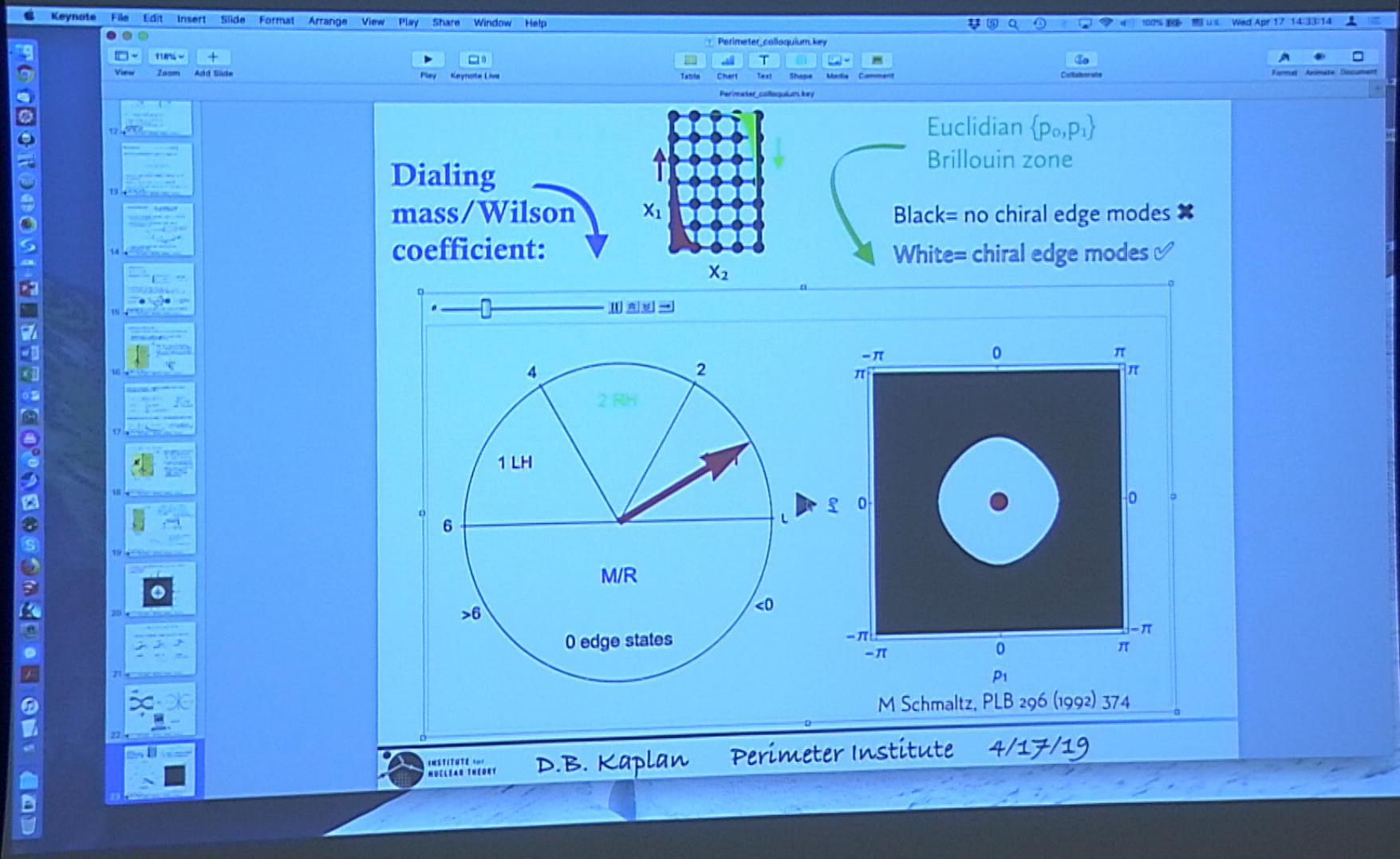
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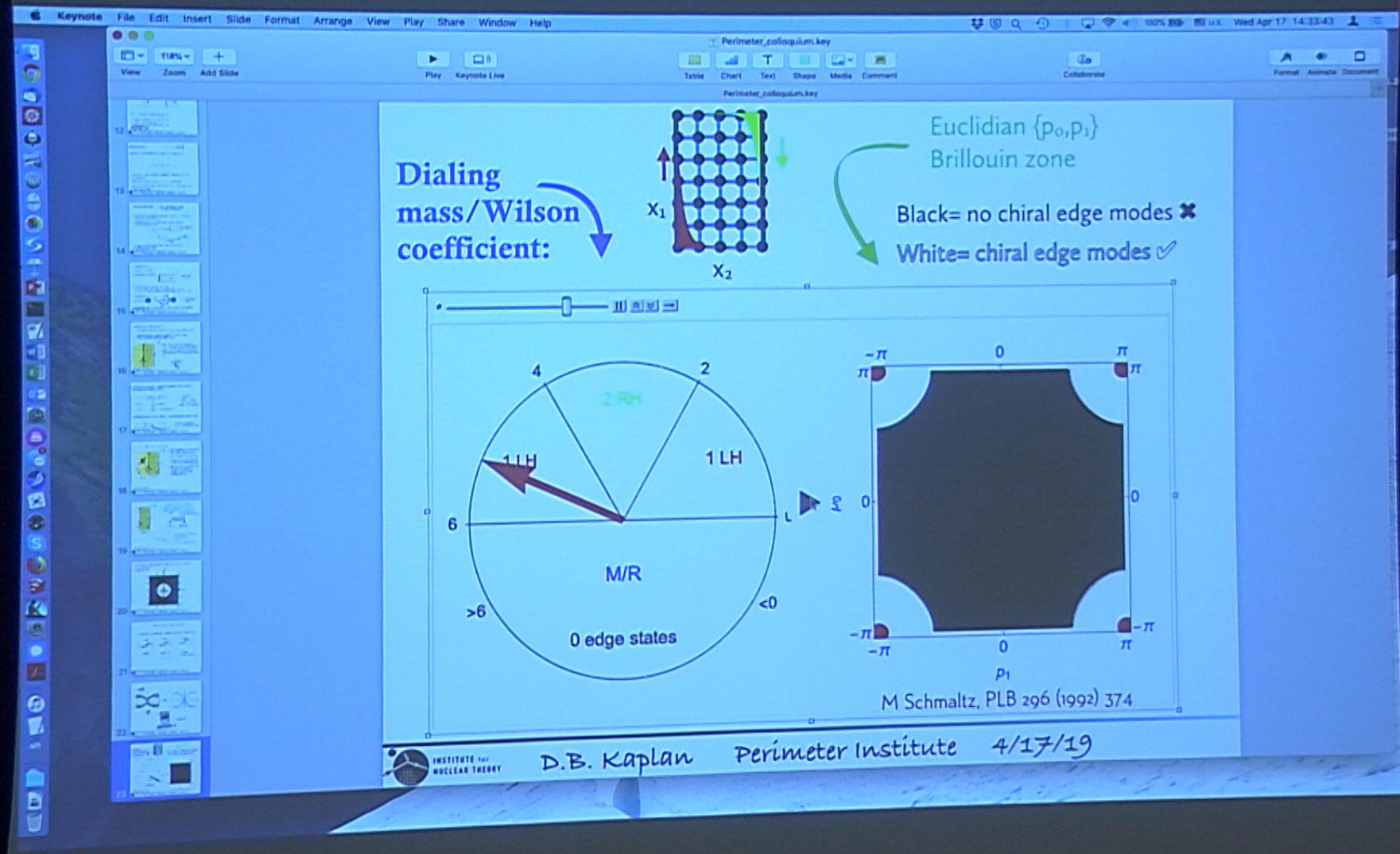


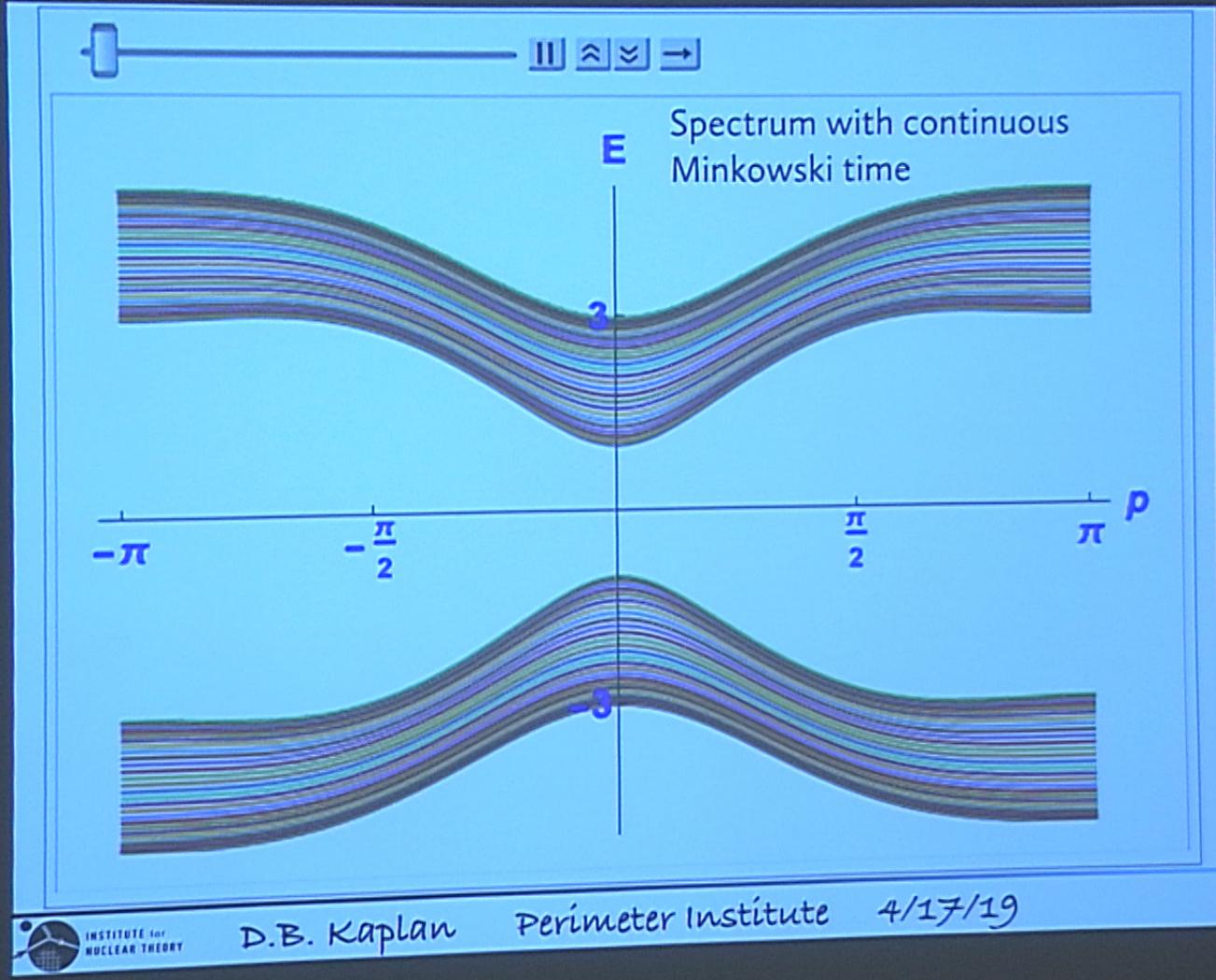
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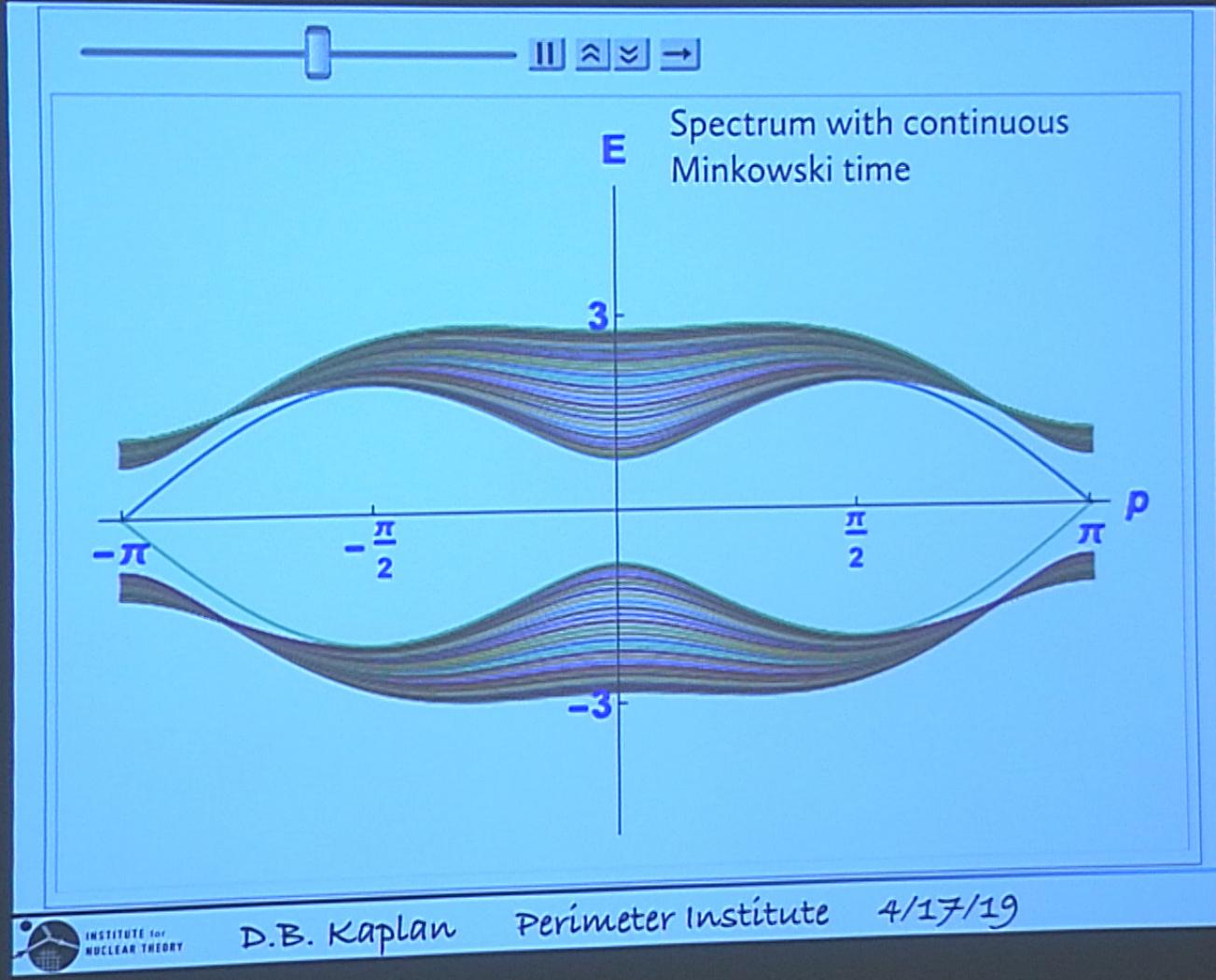
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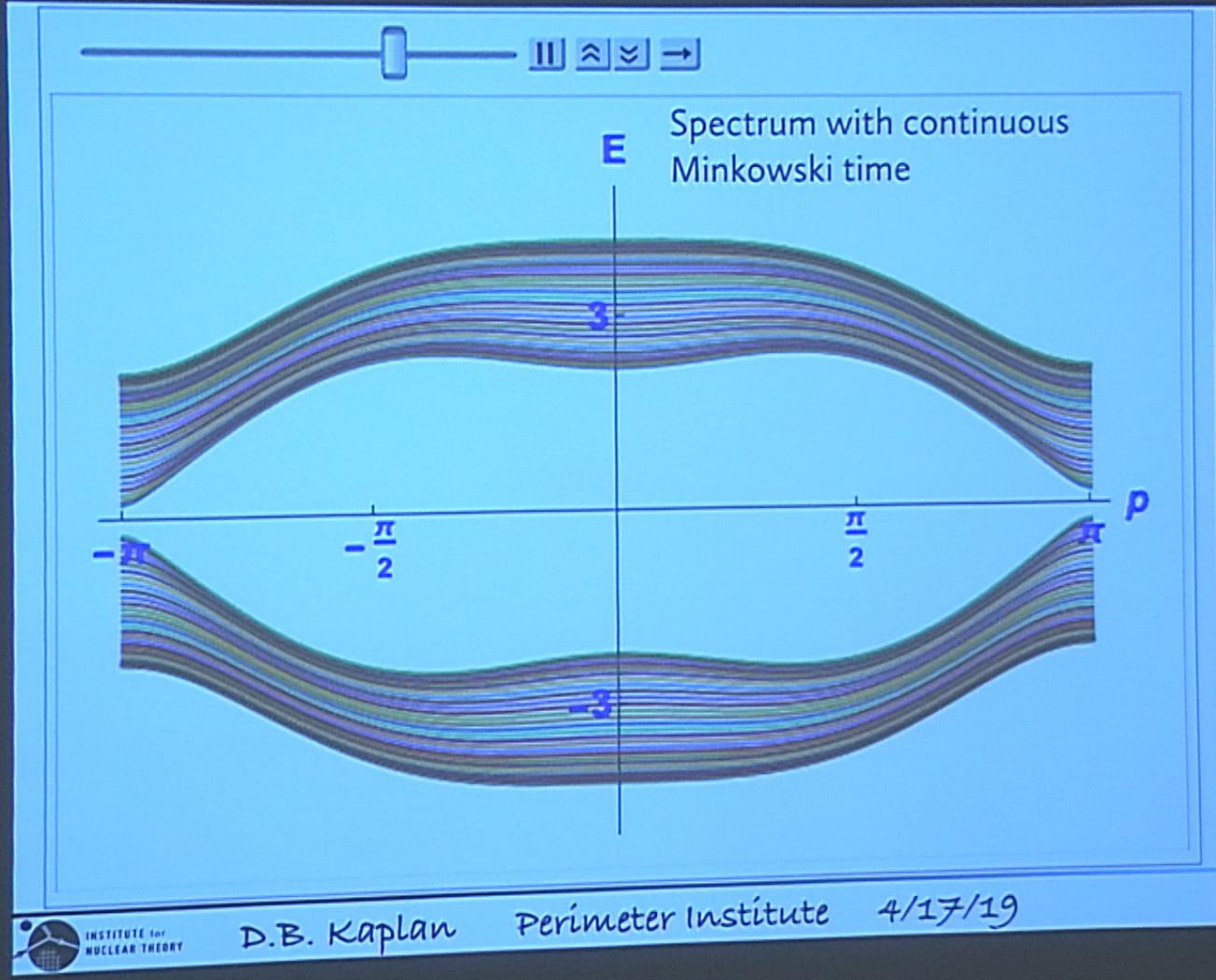
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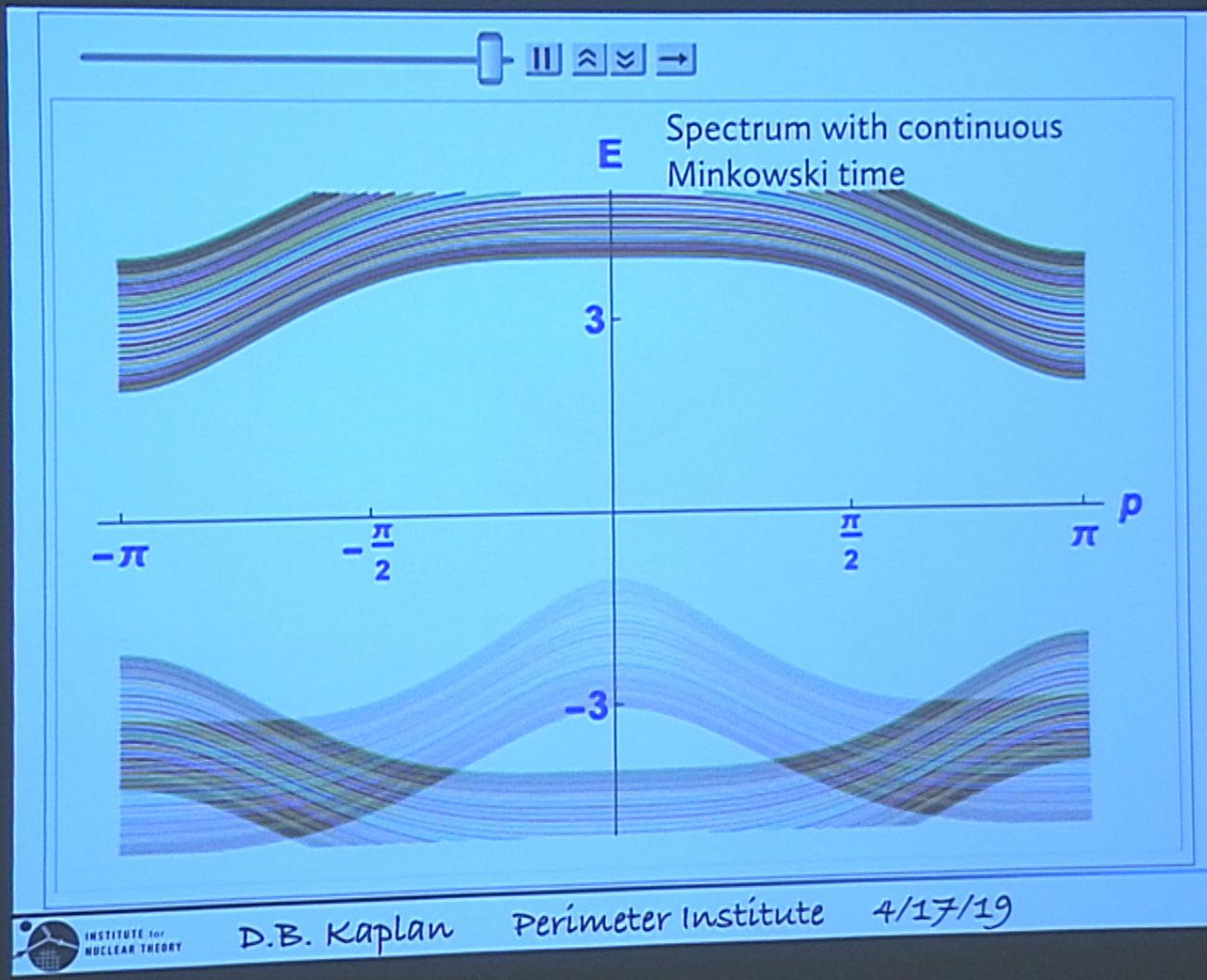


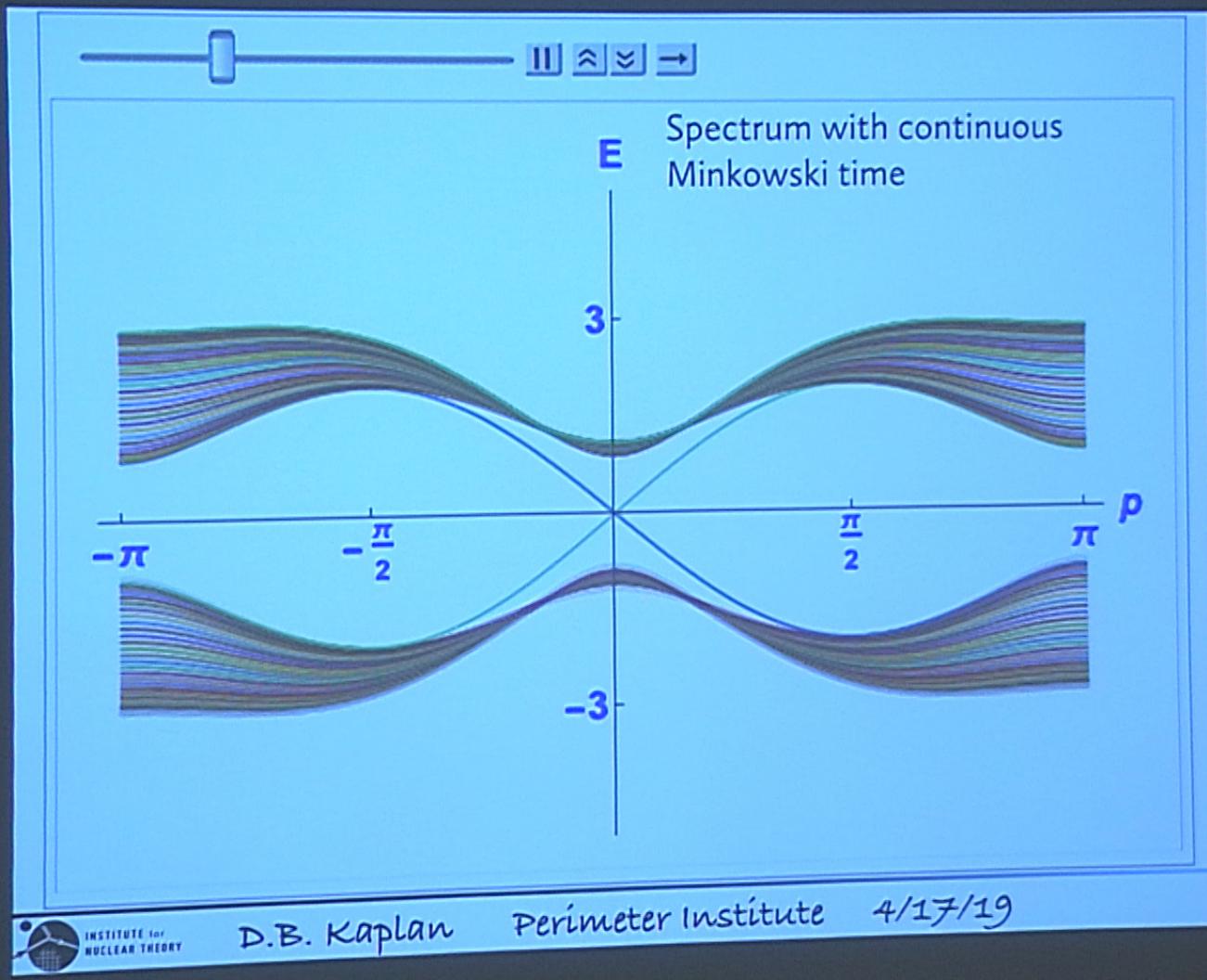


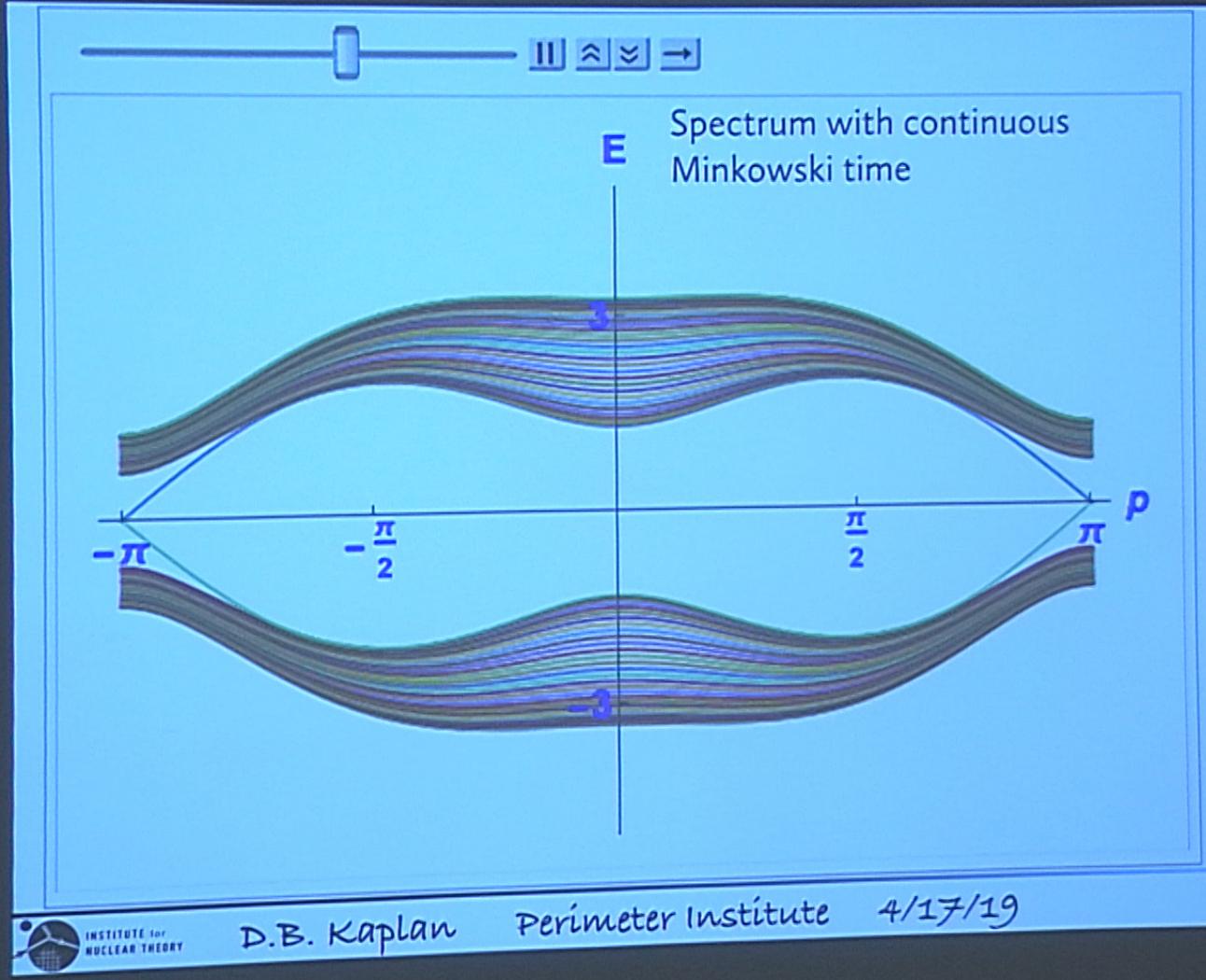


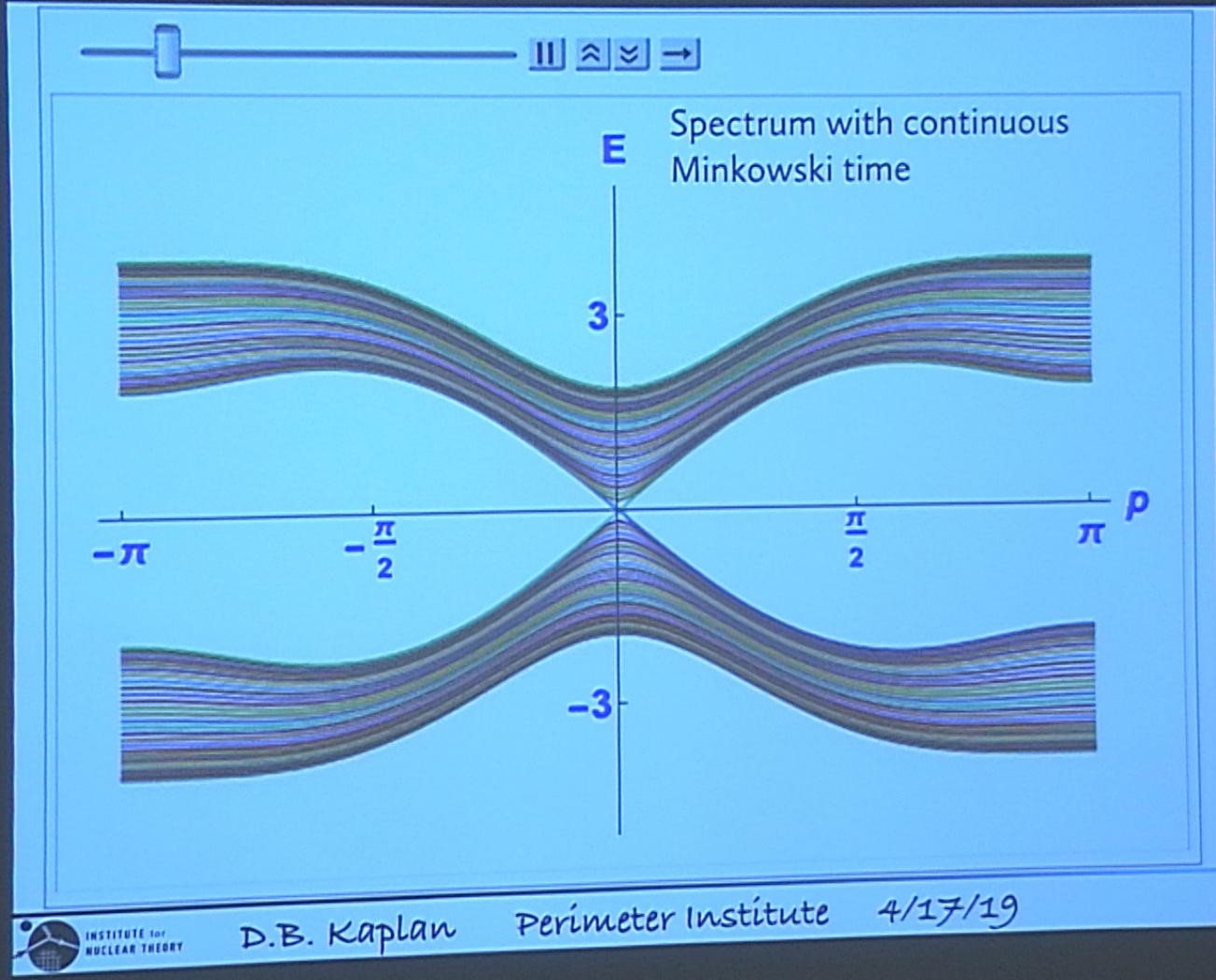


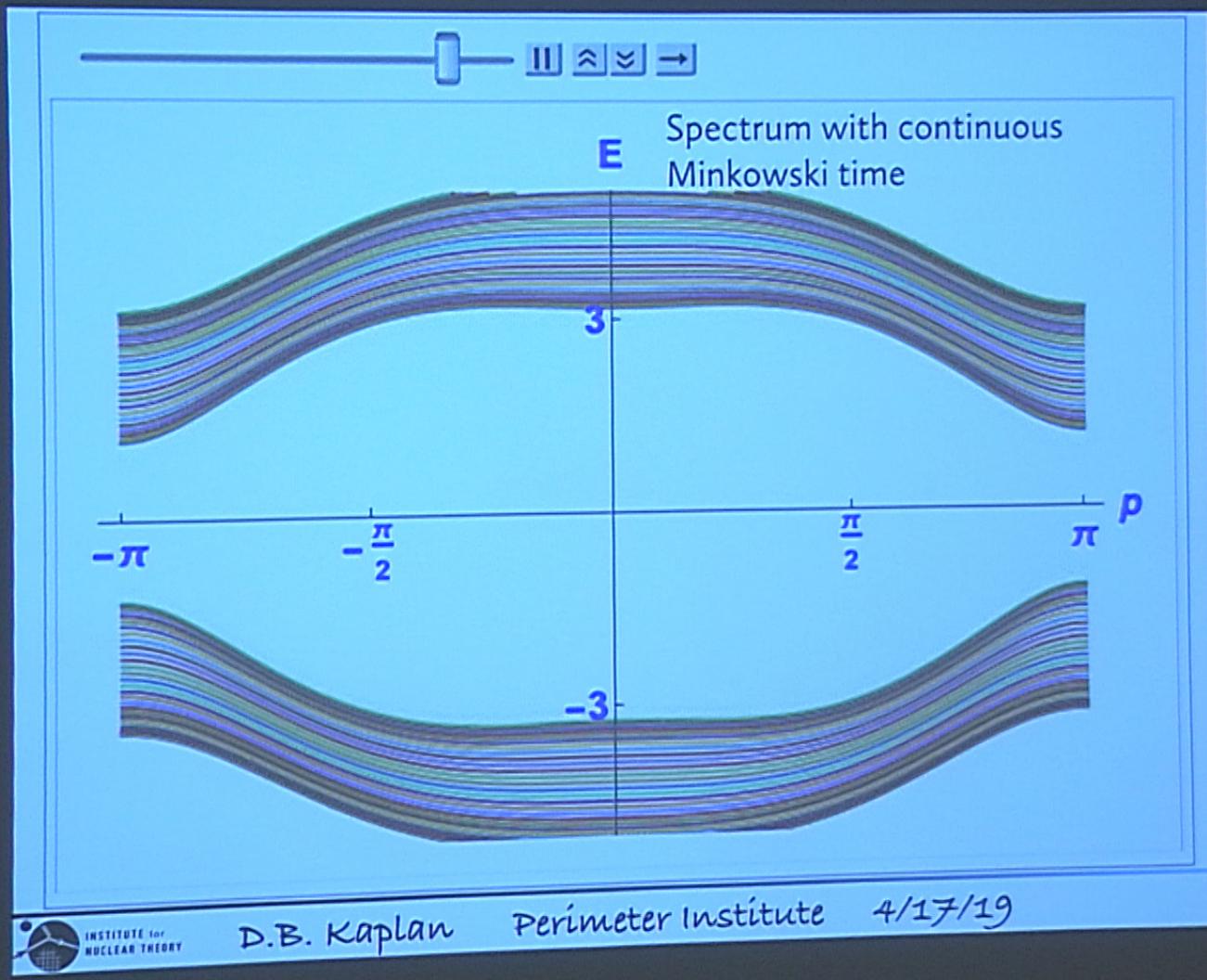












NOT expected from Callan-Harvey work in continuum.

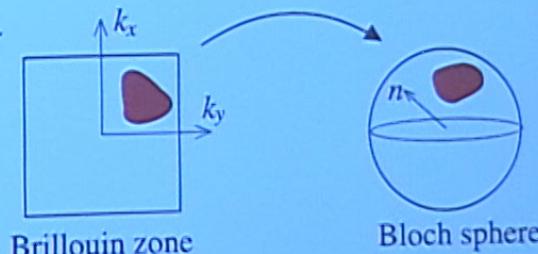
- # & chirality of edge states changes discontinuously with M/R
- ...so coefficient of CS term must change discontinuously
- ...but CS coefficient is result of 1-loop Feynman diagram with messy vertices & propagators...??

Using Ward identity, can show CS coefficient in $d=2n+1$

$$c_n = \frac{(-i)^n \epsilon_{\mu_1 \dots \mu_{2n+1}}}{(n+1)(2n+1)!} \int \frac{d^{2n+1}p}{(2\pi)^{2n+1}} \text{Tr}\{ [S(p) \partial_{\mu_1} S(p)^{-1}] \dots [S(p) \partial_{\mu_{2n+1}} S(p)^{-1}] \}$$

where $S(p)$ = fermion lattice propagator

C_n = topological winding number



M. Golterman, K Jansen, DBK, PLB 201 (1993) 219

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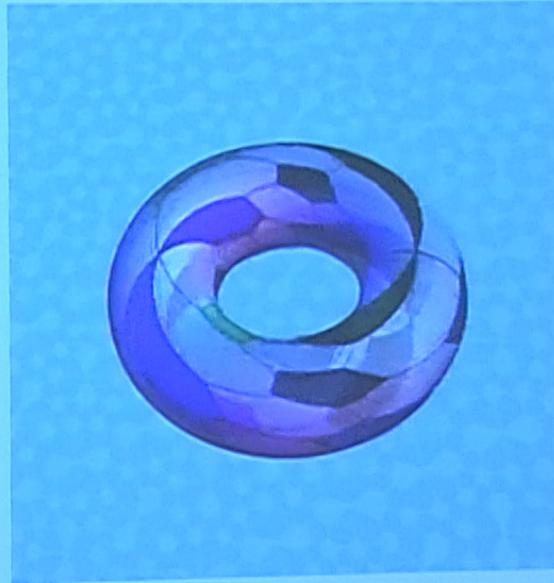


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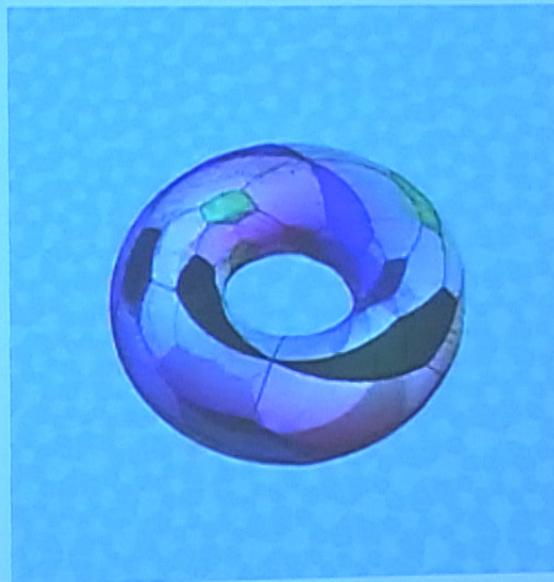
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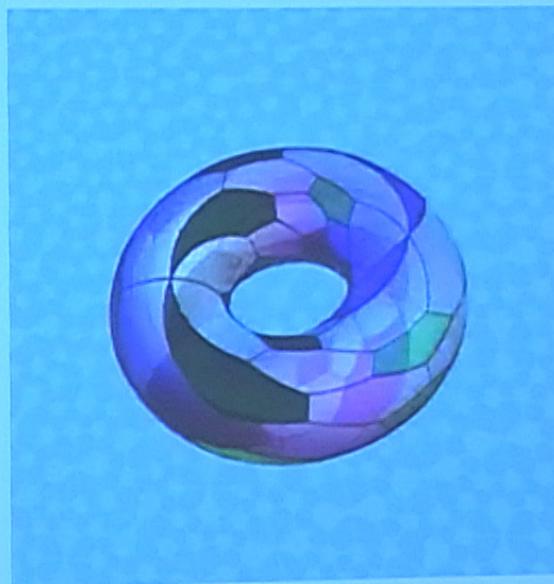
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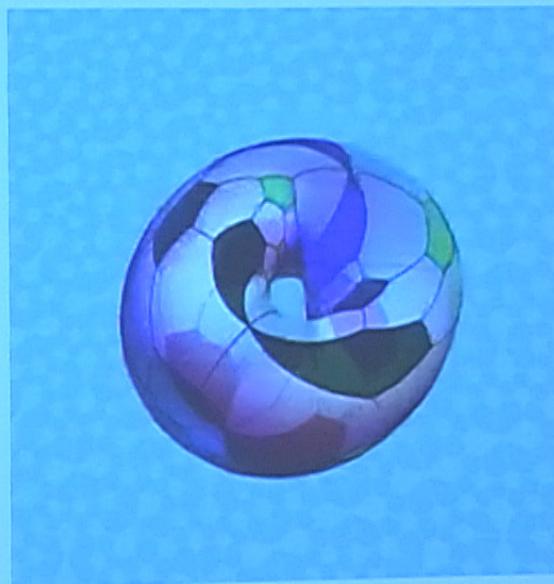
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credit: <http://www.mathematicaguidebooks.org/soccer/>



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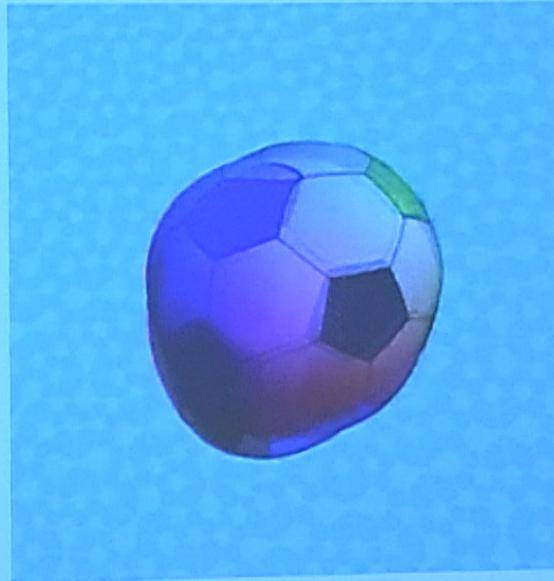
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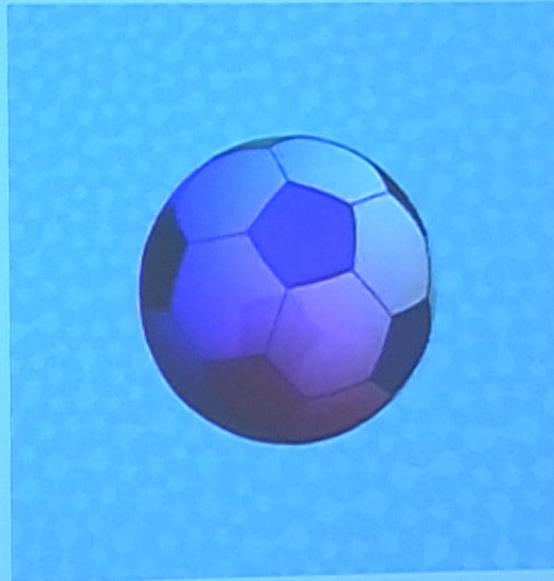
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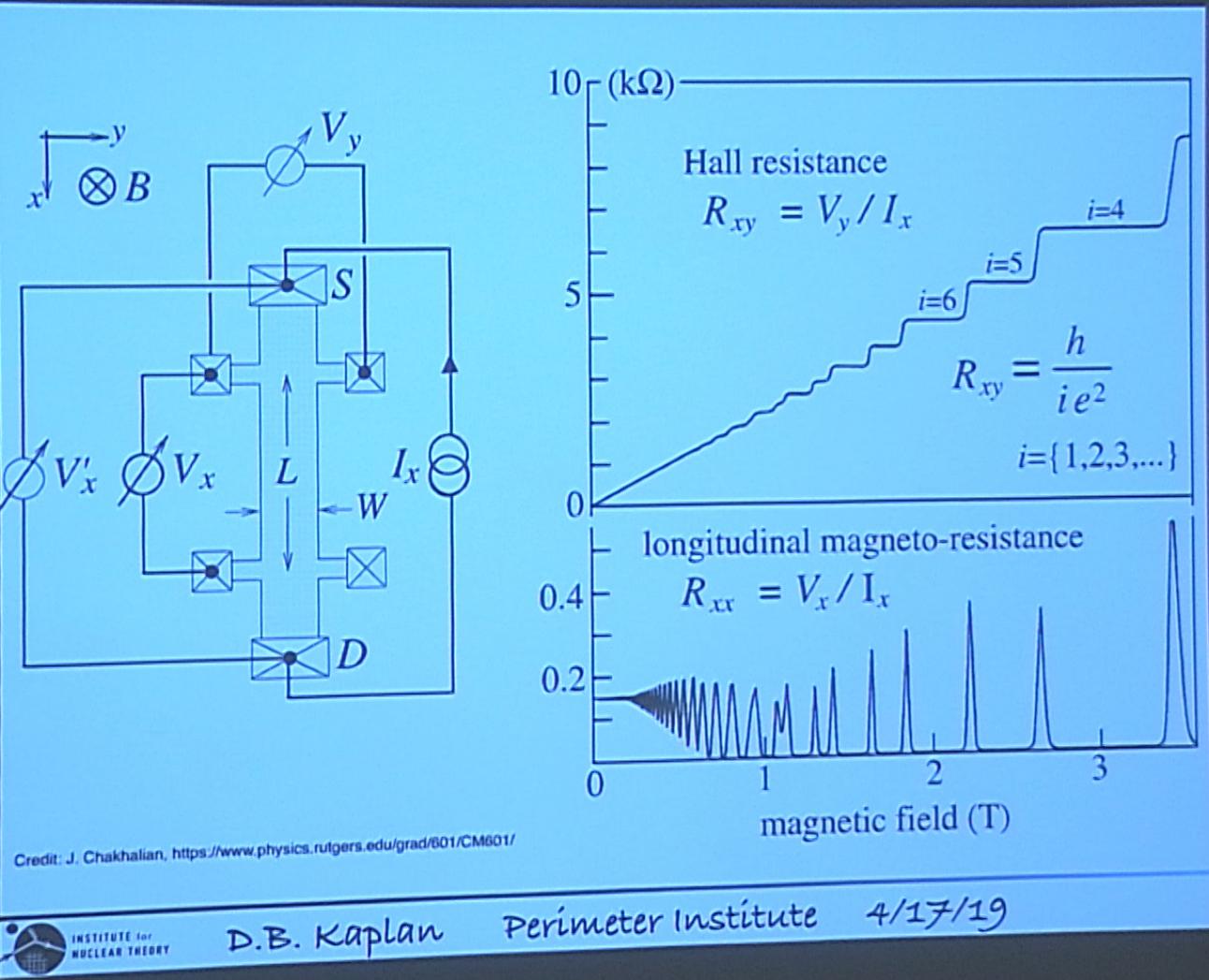
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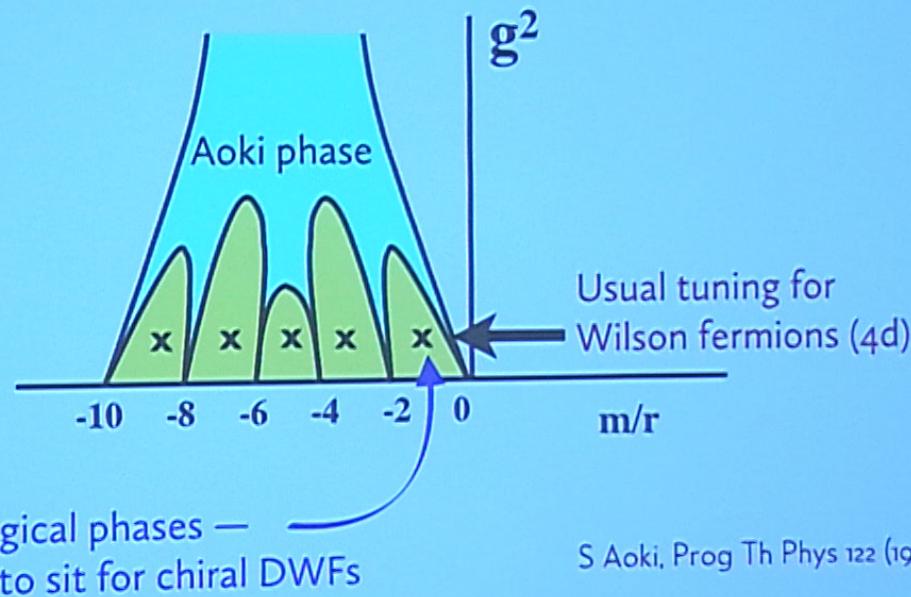
credit: <http://www.mathematicaguidebooks.org/soccer/>



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Phase diagram for QCD with Wilson fermions in 5d Euclidian spacetime



S Aoki, Prog Th Phys 122 (1996) 179



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$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{xx} \end{pmatrix}$$

$$\rho = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{xx} \end{pmatrix} = \sigma^{-1} = \frac{1}{\sigma_{xx}^2 + \sigma_{xy}^2} \begin{pmatrix} \sigma_{xx} & -\sigma_{xy} \\ \sigma_{xy} & \sigma_{xx} \end{pmatrix}$$

$$J_i = \sigma_{ij} E_j$$

$$R_{xx} = 0 , \quad R_{xy} = \frac{2\pi\hbar}{e^2} \frac{1}{\nu} \quad \text{implies:}$$

$$\sigma_{xx} = \rho_{xx} = 0$$

Insulator that dissipates no energy

$$\sigma_{xy} = -\rho_{xy}^{-1} = \frac{e^2}{2\pi\hbar} \nu \quad E_x \text{ causes quantized } J_y$$



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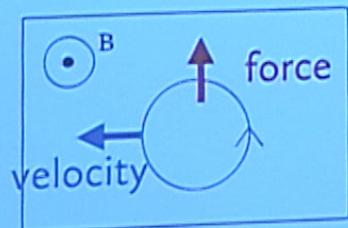
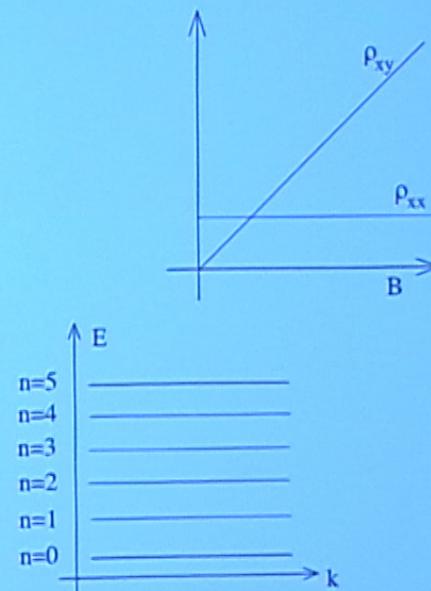
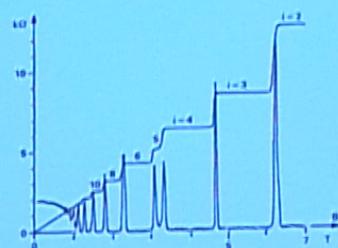
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Classical prediction from Drude model:

$$\rho_{xx} = \frac{m}{ne^2\tau}, \quad \rho_{xy} = \frac{B}{ne}$$

n = electron density, τ = collision time



Density of orbits in Landau level = B/Φ_0 , $\Phi_0 = 2\pi\hbar/e$

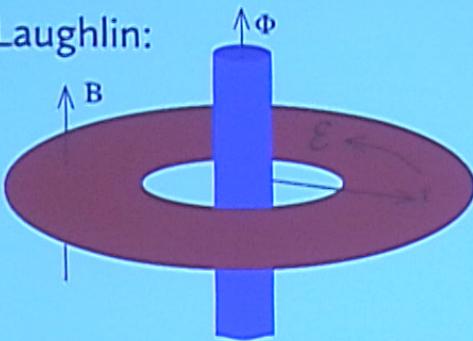
Figure credits: D Tong, ArXiv 1606.06687



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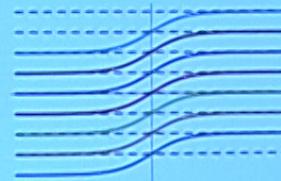
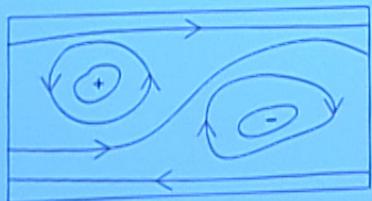
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Laughlin:

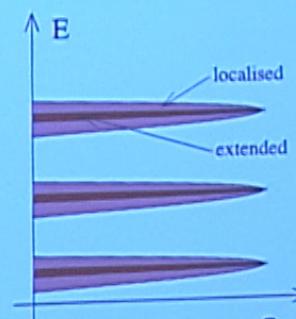
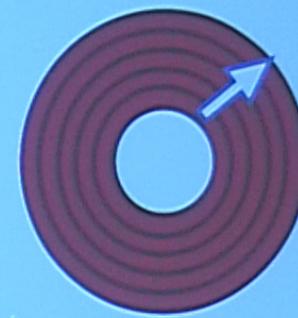


spectral flow as $\Phi \rightarrow \Phi + \Phi_0$

Disorder causes states to be delocalized



One electron per Landau level migrates to outer surface



Only extended states participate...

Figure credits: D Tong, ArXiv 1606.06687

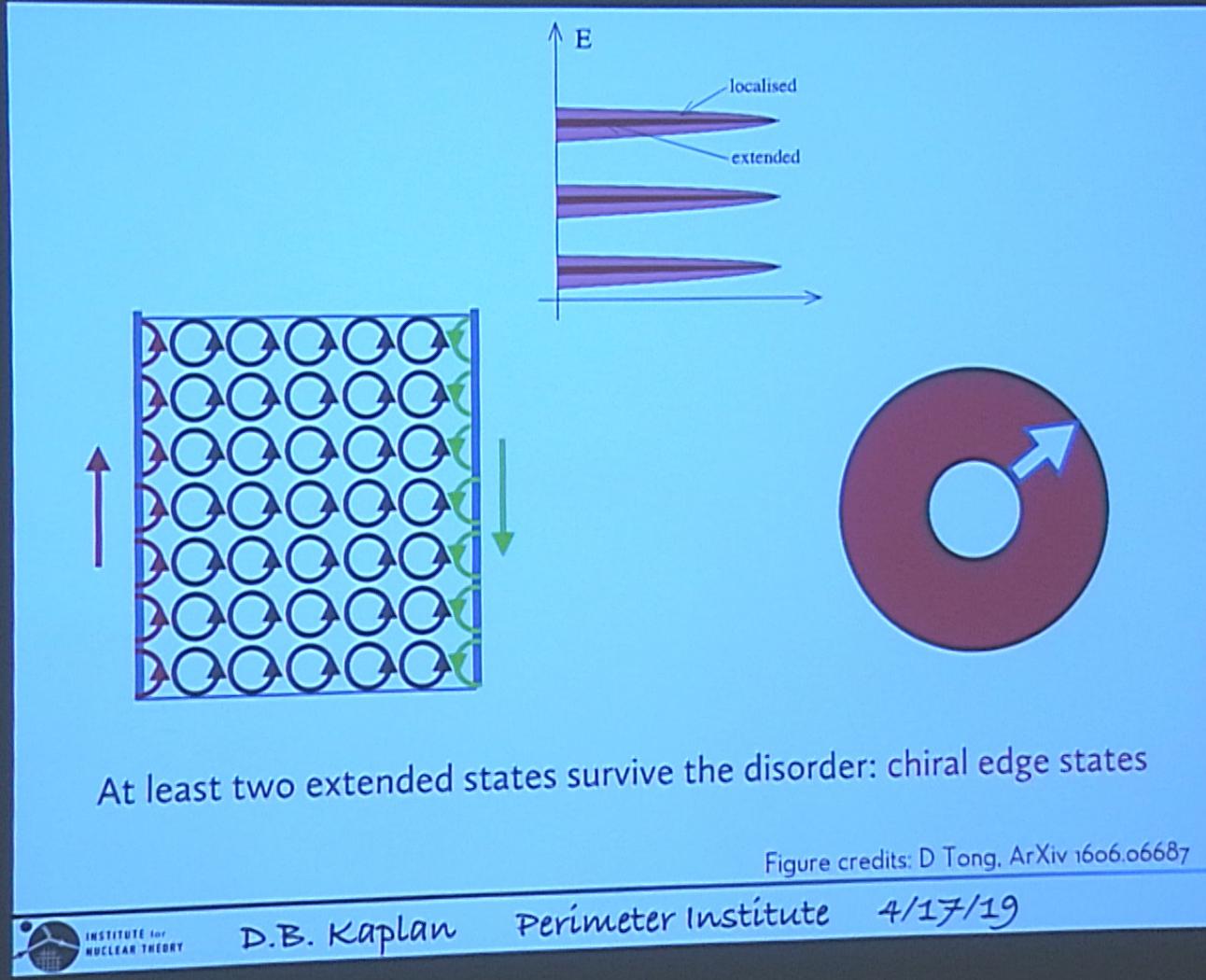


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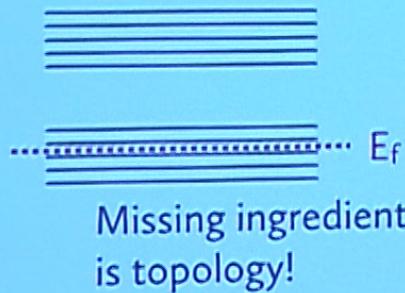
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TKNN: Thouless, Kohomoto,
Nightingale, den Nijs, PRL 49, 405 (1982)

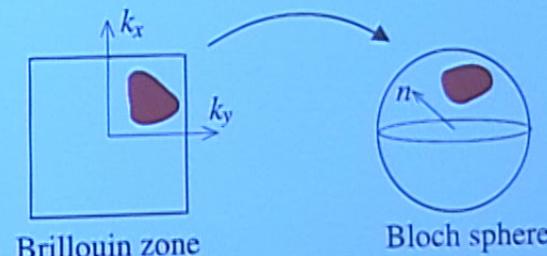
What happens if Landau level splits up
into sub-levels? (eg, on a lattice)



same physics we found for
Chern Simons term for DWF
(but w/o Landau levels)



They showed that σ_{xy} as defined by
current-current correlator (Kubo)
was a topological winding number



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Majorana edge states:

N=1 supersymmetric gauge theory in 3+1:

- Only fields are A_μ , $\lambda =$ gauge field + Majorana gluino
- Only relevant SUSY breaking operator is a Majorana mass
- Majorana mass is forbidden by chiral symmetry

Lattice theory with massless Majorana edge states constructed,
used for simulations

DBK, M Schmaltz hep-lat/0002030
M. Endres PRD 79 (2009) 094503

Error correction for quantum computers:

- particle/hole states can serve as qubits, but are subject to phase errors
$$\exp(i \theta a^\dagger a) |1\rangle \rightarrow e^{i\theta} |1\rangle, \quad \exp(i \theta a^\dagger a) |0\rangle \rightarrow |0\rangle$$
- Create Majorana states $c_1 = a^\dagger + a$, $c_2 = -i(a - a^\dagger)$ located at distant sites n_1, n_2
- $a^\dagger a = 2i c_1 c_2 =$ nonlocal (protected against local noise)
- Lattice model constructed for Majorana edge states

A. Kitaev, cond-mat/0010440



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Quantum Spin Hall Effect

A goal of DWF program was to regulate chiral gauge theories.

Step 1: anomaly-free chiral fermions on each wall

Anomaly cancellation = no gauged CS current in bulk, only chiral flavor currents



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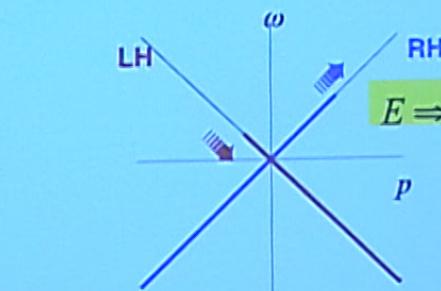
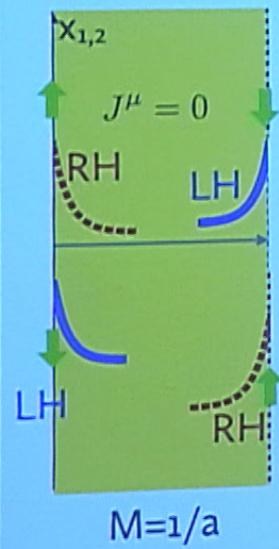
Quantum Spin Hall Effect

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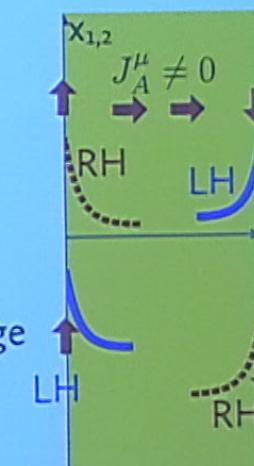
Anomaly cancellation = no gauged CS current in bulk, only chiral flavor currents

Example in continuum: 2 bulk fermions, opposite sign mass



- massless Dirac fermion at each edge
- no charge current in bulk
- ...but $U(1)_A$ current in bulk

DBK, PLB 288 (1992) 342, K Jansen, PLB 288 (1992) 348



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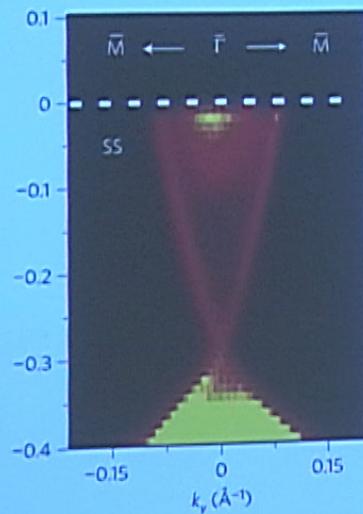
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This is a relativistic version of the Spin Quantum Hall Effect

CL Kane, EJ Mele, PRL 95 (2005) 226801



Dirac cone observed for edge state

Y Xia et al, Nat Phys 5 (2009) 398



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Relativistic lattice field theory	Condensed matter
QCD Domain Wall Fermions	Integer Quantum Hall Effect w/o Landau Levels
SUSY N=1 DWF	Majorana edge states
Models with gauge anomaly cancellation	Quantum Spin Hall Effect
?	Fractional Quantum Hall Effect
EFT for edge states: the overlap operator	?
Chiral gauge theories?	Partially gapped SQHE?



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Overlap operator: the EFT for edge states

H Neuberger & R Narayanan (1992-1998):

Can one explicitly determine the effective theory of only the edge states?

Yes!

$$\mathcal{L}_{\text{edge}} = \bar{\chi} D \chi \quad \text{No extra dimension now}$$

$$D = 1 + \gamma_5 \epsilon(H) = 1 + \gamma_5 \frac{H}{\sqrt{H^2}}$$

$$H \sim \gamma_5 (\not{D} + M + D^2) = H^\dagger$$

H is the Wilson fermion Hamiltonian from d+1 dimensions, treating bulk direction as time

D = “overlap operator” — it is the solution to the Ginsparg Wilson eq!

$$\{\gamma_5, D\} = a D \gamma_5 D$$

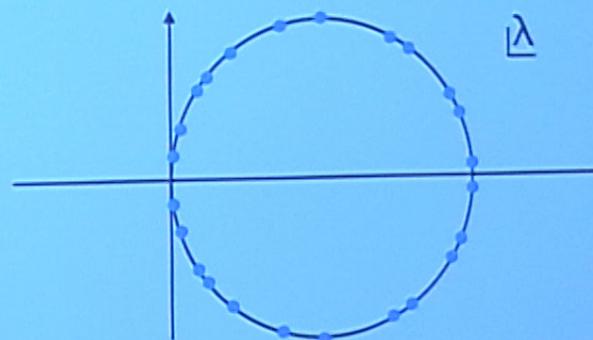
H. Neuberger, PLB 417 (1998) 141



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$$D = 1 + \gamma_5 \epsilon(H) = 1 + \gamma_5 \frac{H}{\sqrt{H^2}}$$

Eigenvalues of overlap operator D



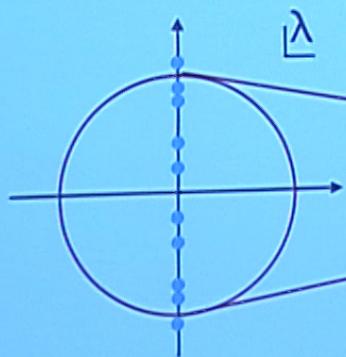
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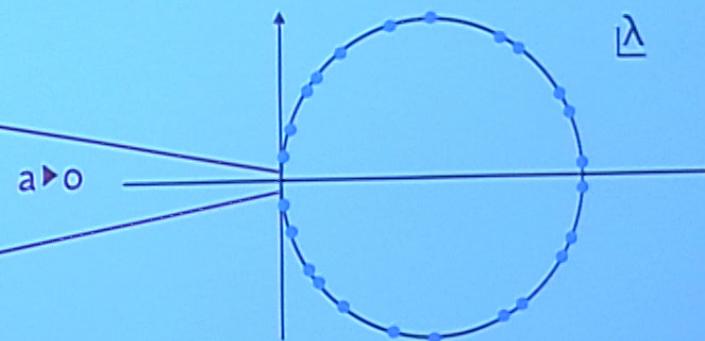
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$$D = 1 + \gamma_5 \epsilon(H) = 1 + \gamma_5 \frac{H}{\sqrt{H^2}}$$

Continuum eigenvalues of \not{D}



Eigenvalues of overlap operator D



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Chiral gauge theories

The standard model is a chiral gauge theory w/o a nonperturbative regulator

More exotic chiral gauge theories have composite massless fermions, can realize Calabi-Yau manifolds...

We would like to formulate lattice XGT!

Problem: lattice produces vector fermions, can't use Wilson trick to eliminate mirrors w/o breaking gauge symmetry

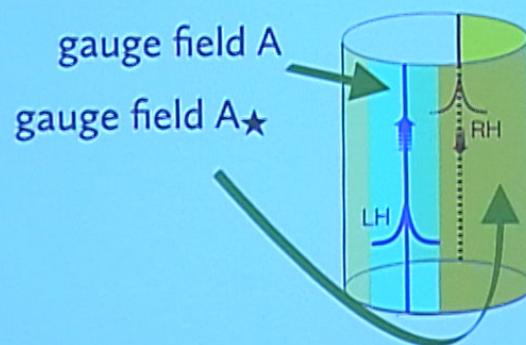
- Break gauge invariance and fine tune? M Golterman, Y Shamir, PRD70 (2004) 094506

Domain wall fermions physically separate fermions from mirrors in extra dimension (the SQHE setup)....can we use that?

- Make the mirror fermions invisible
- Make the mirror fermions heavy (gapped)



Make the mirror fermions invisible?



$A^* = 0$ doesn't work...mirrors reappear

A^* such that RH fermions have infinitely soft form factors might work...

D Grabowska, DBK,
PRL 116 (2016) 211602;
PRD 94 (2016) 114504

...but may give nonlocal,
nonperturbative effects



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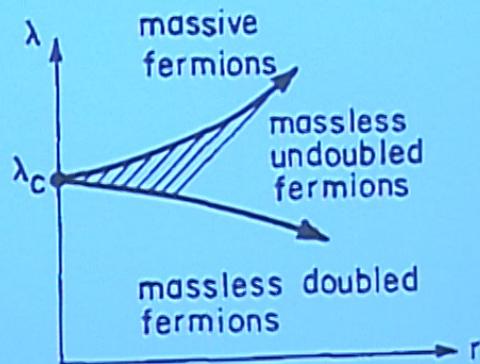
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Make the mirror fermions heavy (gapped)?

Pre- DWF proposal:

Give the mirror fermions strong (high dimension) interactions so that they form composites that can get massive

E Eichten, J Preskill, NPB 268 (1986) 179



Hoped for phase diagram...

...but desired phase
not realized

M Golterman, D Petcher, E Rivas,
NPB 395 (1993) 596



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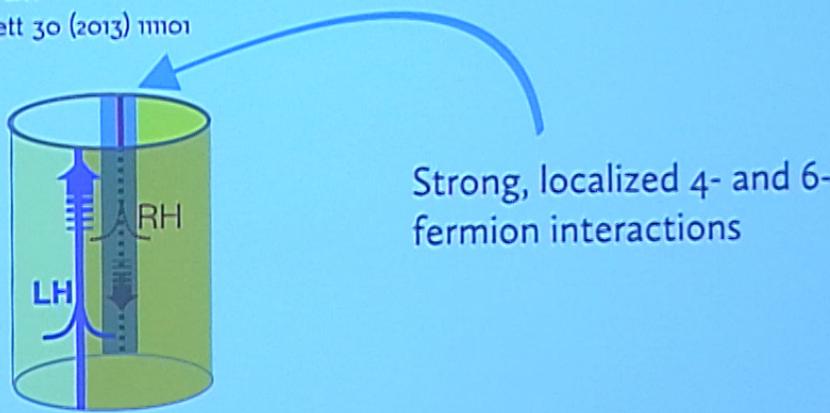
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Make the mirror fermions heavy (gapped)?

DWF proposal:

Introduce strong generalized Wilson terms like Eichten-Preskill for mirrors
on far domain wall

X-G Wen, Chin Phys Lett 30 (2013) 11101



Strong, localized 4- and 6-
fermion interactions



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Conclusions

Relativistic lattice field theory	Condensed matter
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EFT for edge states: the overlap operator	?
Chiral gauge theories?	Partially gapped SQHE?
?	?



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