Title: Probability in many-world theories

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Series: Quantum Foundations

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Abstract: A common criticism directed against many-world theories is that, being deterministic, they cannot make sense of probability. I argue that, on the contrary, deterministic theories with branching provide us the only known coherent definition of objective probability. I illustrate this argument with a toy many-worlds theory known as Kent's universe, and discuss its limitations when applied to the usual Many-Worlds interpretation of quantum mechanics.

I'll also argue that subjective probabilities are unproblematic in the many-worlds setting by showing how the usual decision-theoretical axioms apply there, and finish by showing that together with a proper definition of measurement they suffice to derive the Born rule.

Probability in many-world theories

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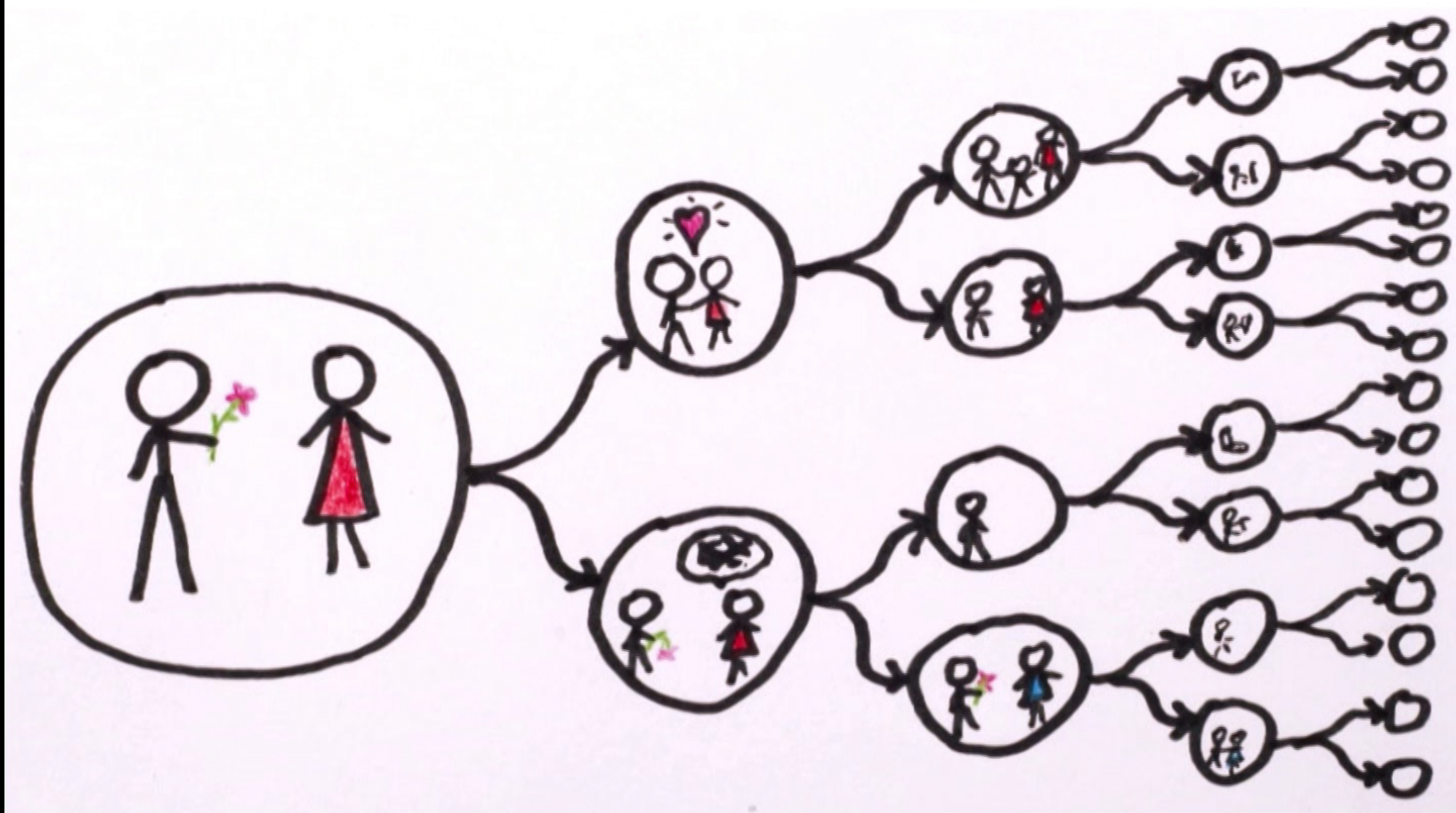
Deterministic branching theories

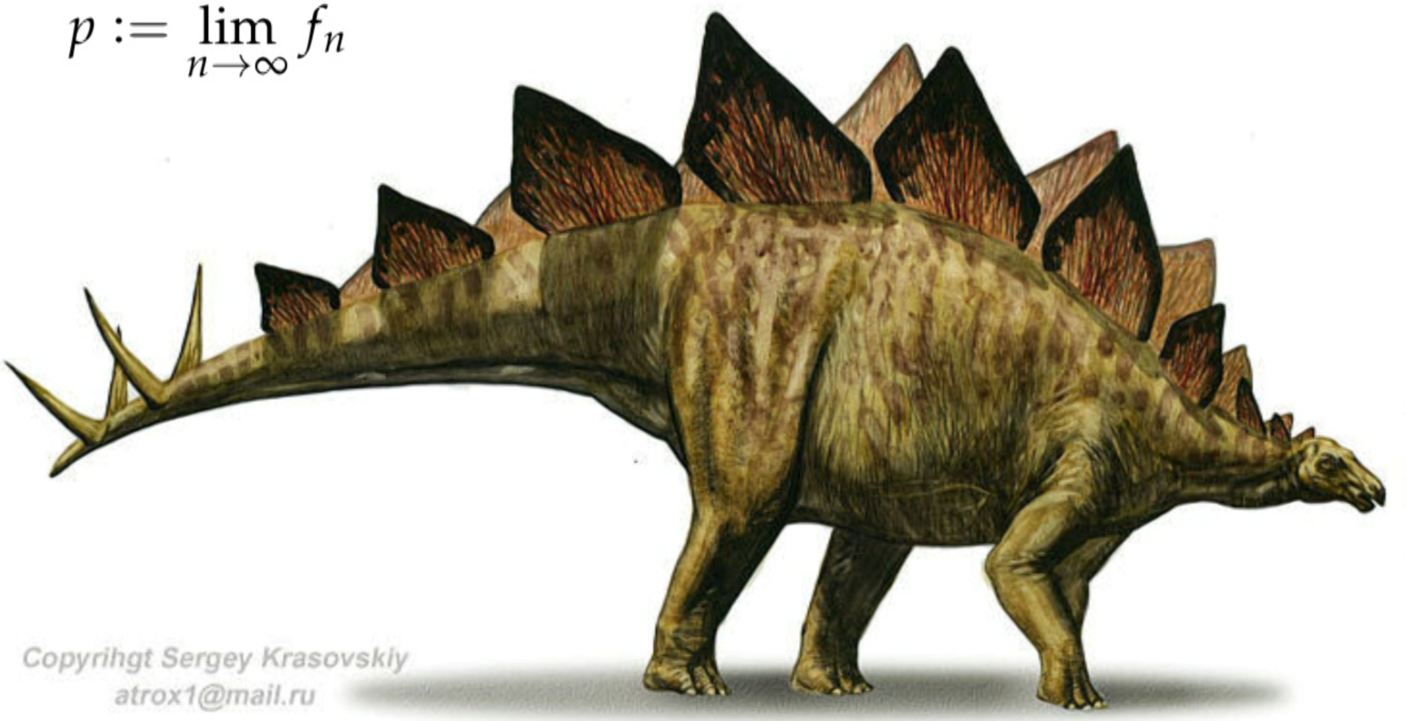


The problem of probability

What does it mean to say that event E happens with objective probability $p = 2/3$?

The frequentosaurus

$$p := \lim_{n\to\infty} f_n$$



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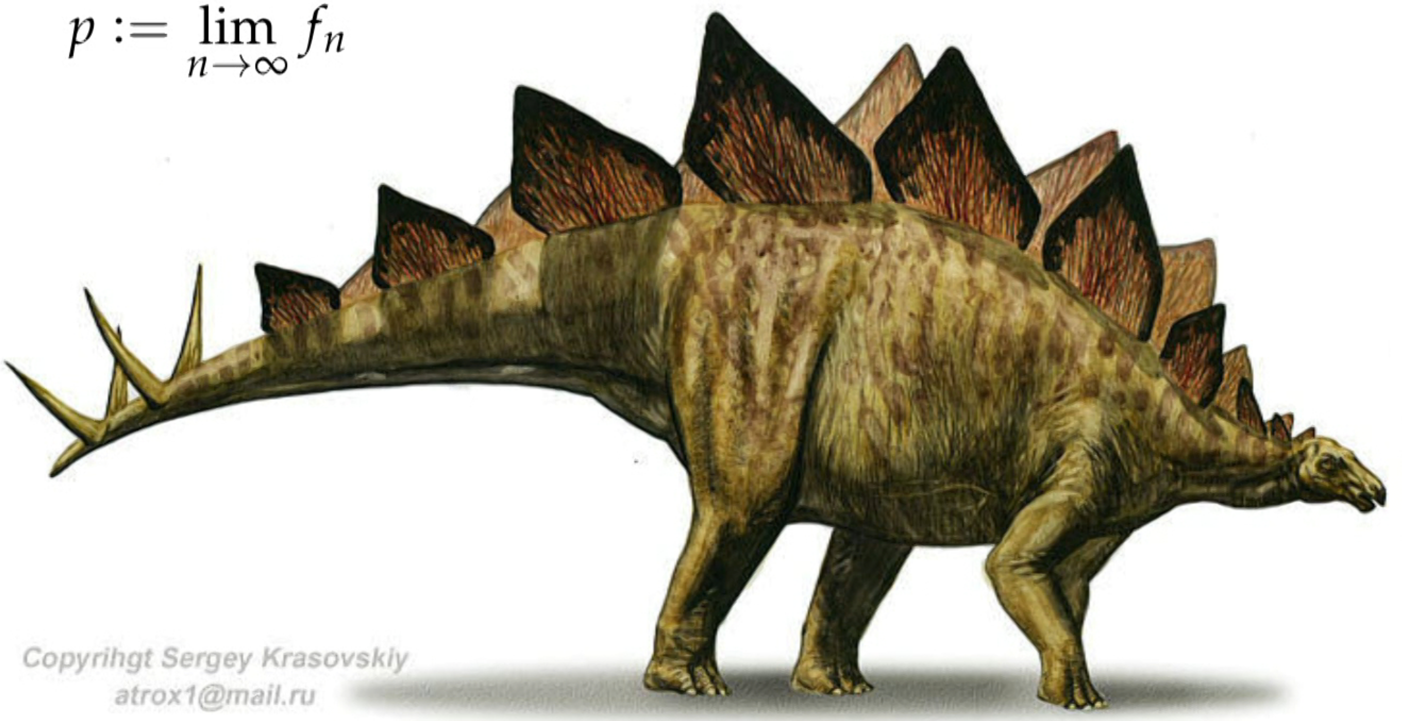


The law of large numbers

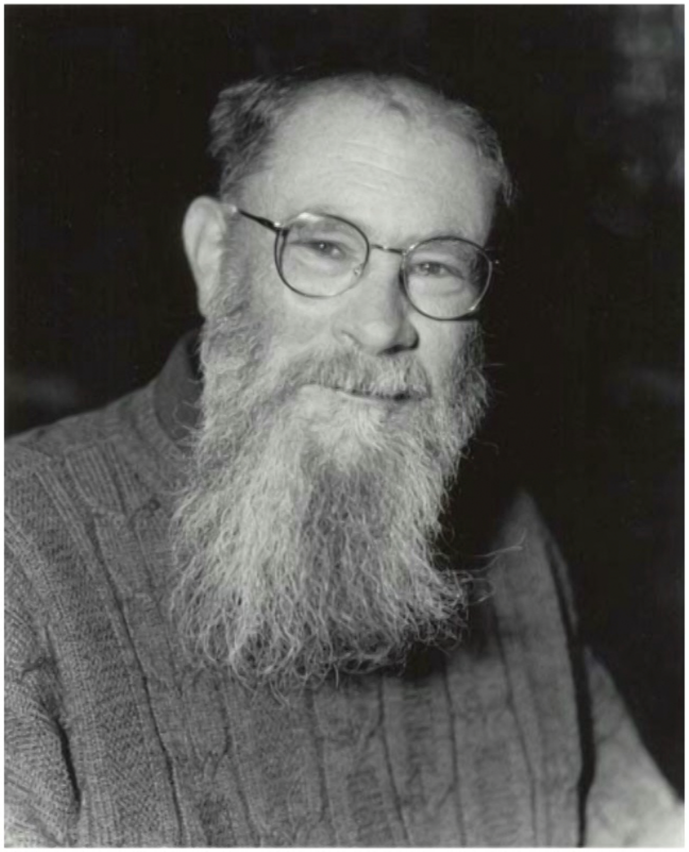
If some event happens with objective probability p, then after n trials the objective probability that the frequency f_n deviates more than ε from p is bounded by

$$\Pr\Big(|f_n - p| \geq \varepsilon\Big) \leq 2e^{-2n\varepsilon^2}$$

The Bayesian mystics

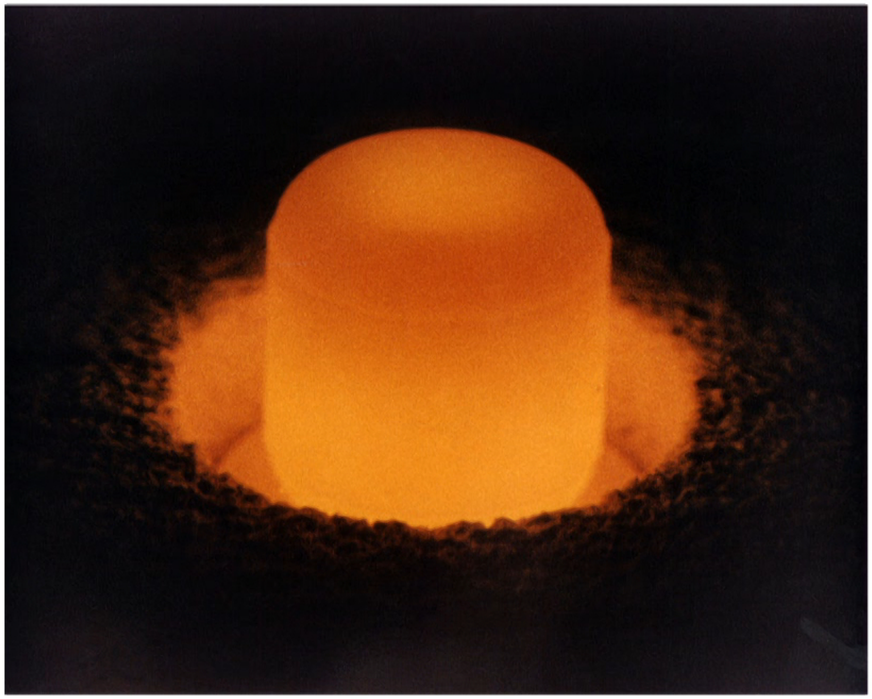
Probability is a degree of belief.

The Principal Principle



$$\mathrm{Pr}_{\mathrm{o}}(E) = \mathrm{Pr}_{\mathrm{s}}(E|HT)$$

The half-life of plutonium-238 is 88 years



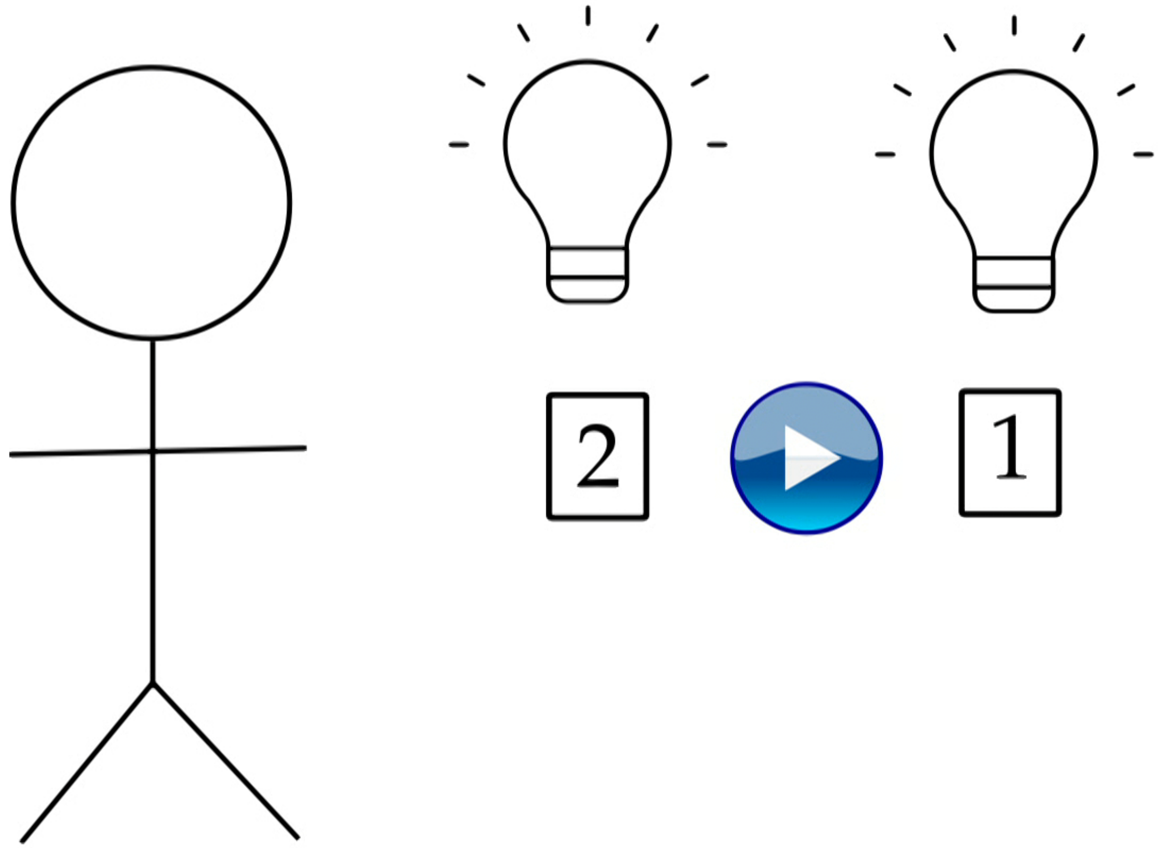
Desiderata for objective probability

1 – Agent-independent.

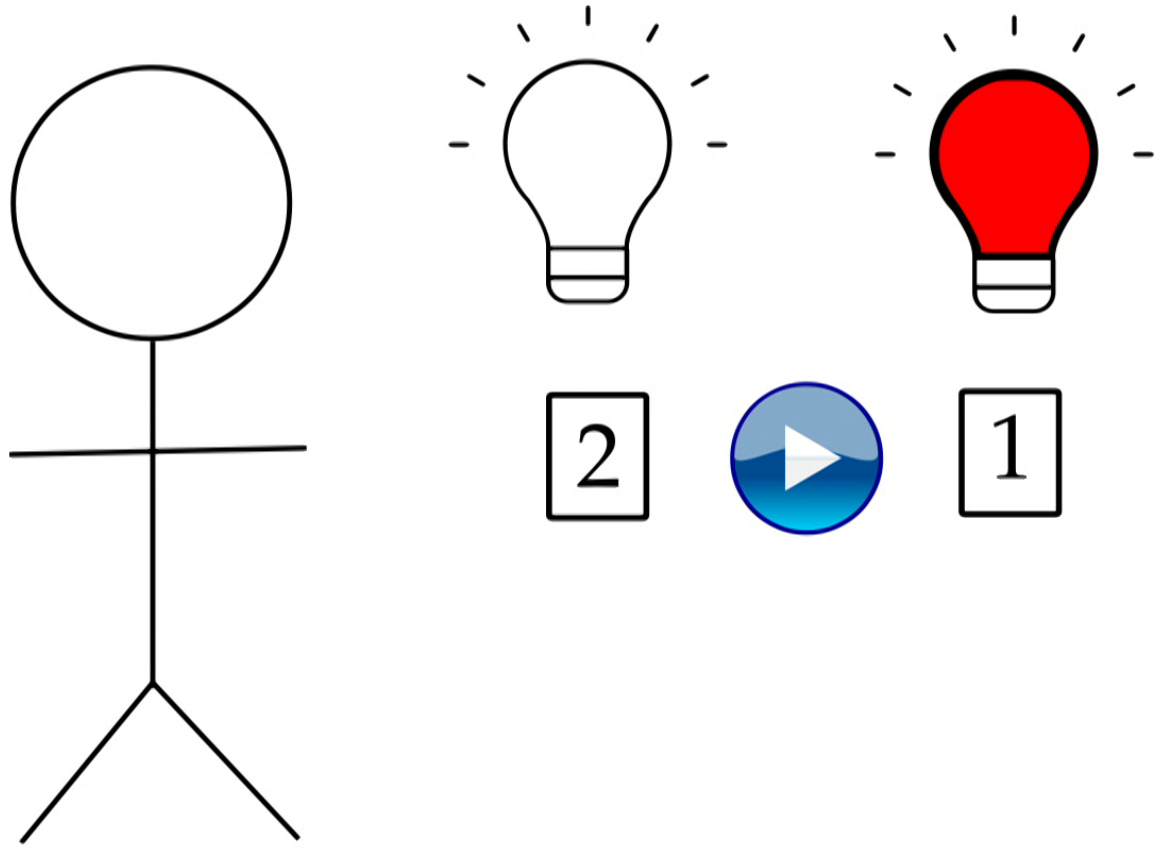
2 – Respect law of large numbers.

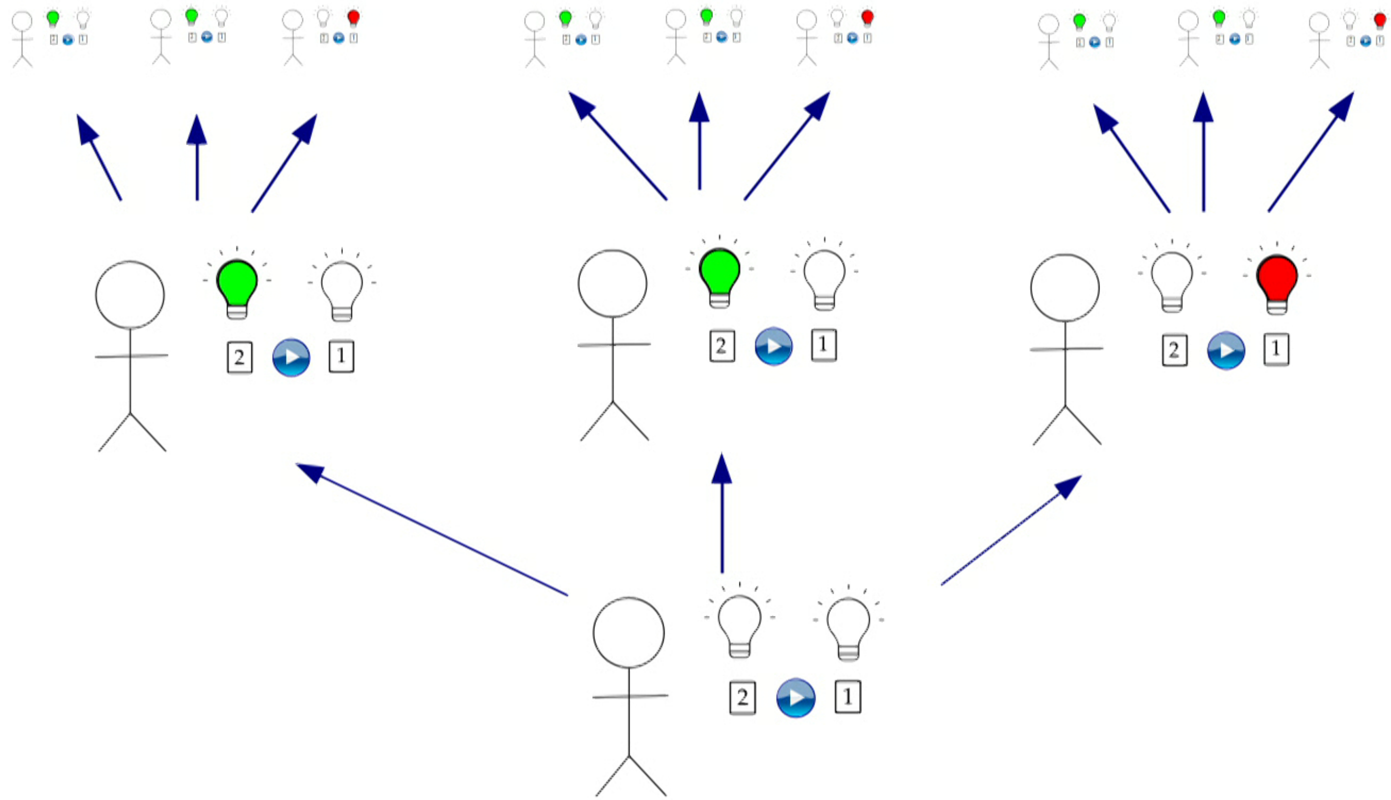
3 – Respect Principal Principle

Kent's universe



Kent's universe





2
1
2
1
2
1
2
1

Relative frequencies

$$\text{observers that see } k \text{ greens} = \binom{n}{k} 2^k$$

$$\text{total observers} = 3^n$$

After 10 000 trials, there are $\approx 1.63 \times 10^{4771}$ observers of which $\approx 1.58 \times 10^{4771}$ observe relative frequencies

$$\in \left(\frac{2}{3} - \frac{1}{100}, \frac{2}{3} + \frac{1}{100} \right)$$

Relative frequencies

$$\#(k, N) = \binom{N}{k} n_G^k n_R^{N-k}$$

$$\frac{\#(k, N)}{(n_G + n_R)^N} = \binom{N}{k} \left(\frac{n_G}{n_G + n_R}\right)^k \left(\frac{n_R}{n_G + n_R}\right)^{N-k}$$

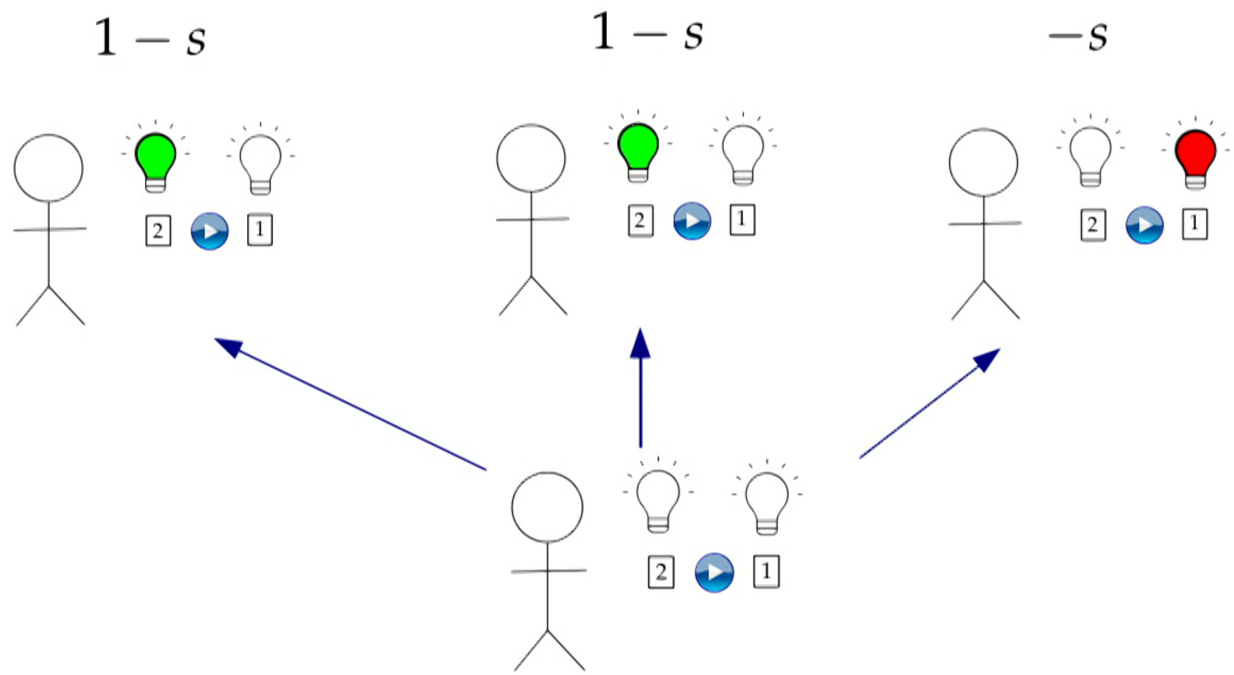
$$\mu(k, N) = \binom{N}{k} \mu_G^k (1 - \mu_G)^{N-k}$$

Objective probabilities

If some event happens in a proportion of worlds μ_G, then after n trials the proportion of worlds where the frequency f_n deviates more than ε from μ_G is bounded by

$$\mu\left(|f_n - \mu_G| \geq \varepsilon\right) \leq 2e^{-2n\varepsilon^2}$$

Quick and dirty decision theory



$$(1-s)+(1-s)+(-s)\geq 0$$

$$s\leq\frac{2}{3}$$

Desiderata for objective probability

1 – Agent-independent.

2 – Respect law of large numbers.

3 – Respect Principal Principle

Objective probabilities

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$$s\leq \frac{2}{3}$$

$$\Lambda(3,3) = \Lambda(G,G,G)$$

$$\Lambda(2,3) = \Lambda(G,G,R) + \Lambda(G,R,G) + \Lambda(R,G,G)$$

$$\Lambda(1,3) = \Lambda(G,R,R) + \Lambda(R,G,R) + \Lambda(R,R,G)$$

$$\Lambda(0,3) = \Lambda(R,R,R)$$

$$\Lambda(3,3) = \Lambda_G^3$$

$$\Lambda(2,3) = 3\Lambda_G^2\Lambda_R$$

$$\Lambda(1,3) = 3\Lambda_G\Lambda_R^2$$

$$\Lambda(0,3) = \Lambda_R^3$$

$$\Lambda(k, N) = \binom{N}{k} \Lambda_G^k \Lambda_R^{N-k}$$

$$\frac{\Lambda(k, N)}{(\Lambda_G + \Lambda_R)^N} = \binom{N}{k} \left(\frac{\Lambda_G}{\Lambda_G + \Lambda_R} \right)^k \left(\frac{\Lambda_R}{\Lambda_G + \Lambda_R} \right)^{N-k}$$

$$\lambda(k, N) = \binom{N}{k} \lambda_G^k (1 - \lambda_G)^{N-k}$$

In any many-worlds theory where the measure of worlds is multiplicative

$$\lambda\Big(|f_n - \lambda_G| \geq \varepsilon\Big) \leq 2e^{-2n\varepsilon^2}$$

$$\Lambda(w_i) := \| |w_i\rangle \|_2^2$$

$$\Lambda(w_{01}) = \| |w_{01}\rangle \|_2^2 = |\alpha|^2 |\beta|^2$$

$$\Lambda(w_0) = \| |w_0\rangle \|_2^2 = |\alpha|^2 \qquad \Lambda(w_1) = \| |w_1\rangle \|_2^2 = |\beta|^2$$

$$\Lambda(w_{01}) = \Lambda(w_0)\Lambda(w_1)$$

But

$$\Lambda(w_i) := \||w_i\rangle\|_p^p$$

$$\Lambda(w_{01}) = \||w_{01}\rangle\|_p^p = |\alpha|^p|\beta|^p$$

$$\Lambda(w_0) = \||w_0\rangle\|_p^p = |\alpha|^p \qquad \Lambda(w_1) = \||w_1\rangle\|_p^p = |\beta|^p$$

$$\Lambda(w_{01}) = \Lambda(w_0)\Lambda(w_1)$$