Title: Jackiw-Teitelboim gravity and the Schwarzian

Speakers: Thomas Mertens

Series: Quantum Gravity

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Abstract: We derive Schwarzian correlation functions using the BF formulation of Jackiw-Teitelboim gravity, where bilocal operators are interpreted as boundary-anchored Wilson lines in the bulk. Crossing Wilson lines are associated with OTO-correlators and give rise to 6j-symbols. We discuss the semi-classical bulk black hole physics contained within the correlation functions.

Extensions including bulk defects related to the other coadjoint orbits are discussed.

Jackiw-Teitelboim Gravity and the Schwarzian

Thomas Mertens

Ghent University

Based on arXiv:1606.03438 with J. Engelsöy and H. Verlinde
arXiv:1705.08408 with G.J. Turiaci and H. Verlinde
arXiv:1801.09605
arXiv:1804.09834 with H. Lam, G.J. Turiaci and H. Verlinde
arXiv:1806.07765 with A. Blommaert and H. Verschelde
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Outline

Motivation

Compact BF-theory
- Structure
- Boundary-anchored Wilson line correlators
- Diagrammatic Rules

JT Gravity
- Constrained non-compact BF model
- Diagrammatic Rules
- 2-point function and virtual black holes
- OTO 4-point function and shockwaves

Generalizations
- Complex SYK
- Defects
- Bulk Observables

Conclusion

Jackiw-Teitelboim gravity and the Schwarzian

Jackiw-Teitelboim (JT) 2d dilaton gravity

$$S = \frac{1}{16\pi G}\int d^2x\sqrt{-g}\Phi(R+2) + \frac{1}{8\pi G}\int d\tau\sqrt{-\gamma}\Phi_{bdy}K$$

Teitelboim '83, Jackiw '85

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Path integrate over $\Phi \Rightarrow R = -2$: Geometry fixed as AdS_2

$\Rightarrow$ Only boundary term survives: $S = \frac{1}{8\pi G}\int d\tau\sqrt{-\gamma}\Phi_{bdy}K$

Consider boundary curve $(f(\tau), \epsilon\dot{f}(\tau))$ as UV cutoff, carving out a shape from AdS, $f(\tau) =$ time reparametrization

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Using $\Phi_{bdy} = a/(2\epsilon)$, $\quad \sqrt{-\gamma} = 1/\epsilon$, $\quad K = 1 + \epsilon^2\{f,\tau\}$

$$\boxed{\Rightarrow S = -C\int d\tau\,\{f,\tau\}}, \quad C = \frac{a}{16\pi G}$$

Almheiri-Polchinski '15, Jensen '16, Maldacena-Stanford-Yang '16, Engelsoy-TM-Verlinde '16

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Compare to CS / WZW topological duality

Semi-classical regime: $C \to \infty \equiv G, \hbar \to 0$

The Schwarzian theory: goal

Main goal:
Compute all correlation functions of JT gravity / Schwarzian QM:

$$\boxed{\langle \mathcal{O}_{\ell_1}\mathcal{O}_{\ell_2}\ldots\rangle_\beta = \frac{1}{Z}\int_{\mathcal{M}}[\mathcal{D}f]\mathcal{O}_{\ell_1}\mathcal{O}_{\ell_2}\ldots\, e^{C\int_0^\beta d\tau\,\{F,\tau\}}}$$

with $\{F,\tau\} = \{f,\tau\} + \frac{2\pi^2}{\beta^2}f'^2$, $\qquad F \equiv \tan\left(\frac{\pi f(\tau)}{\beta}\right)$

where $\{f,\tau\} = \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2$ the Schwarzian derivative

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$\mathcal{M} = \mathrm{Diff}(S^1)/SL(2,\mathbb{R})$, $\qquad f(\tau+\beta) = f(\tau)+\beta,\ f' \geq 0$

The Schwarzian theory: Operator insertions

Most natural operator insertions are $SL(2,\mathbb{R})$-invariant

- Local operator:

$$\mathcal{O}(\tau) = \{F, \tau\}$$

  is the Hamiltonian operator itself $\Rightarrow$ correlators determined by Ward identities

- Bilocal operator:

$$\mathcal{O}_\ell(\tau_1,\tau_2) \equiv \left(\frac{F'(\tau_1)F'(\tau_2)}{(F(\tau_1)-F(\tau_2))^2}\right)^\ell \equiv \left(\frac{f'(\tau_1)f'(\tau_2)}{\frac{\beta}{\pi}\sin^2\frac{\pi}{\beta}[f(\tau_1)-f(\tau_2)]}\right)^\ell$$

  Think of this expression as two-point function $\mathcal{O}_\ell(\tau_1,\tau_2) = \langle \mathcal{O}(\tau_1)\mathcal{O}(\tau_2)\rangle_{\text{CFT}}$ of some 1D 'matter CFT' at finite temperature coupled to the Schwarzian theory

Approaches to Schwarzian QM: an overview

Several approaches to obtain Schwarzian correlators exist (and are being developed):

- 1d Liouville $f' = e^{\phi}$ Bagrets-Altland-Kamenev '16, '17
- 2d Liouville CFT between ZZ-branes TM-Turiaci-Verlinde '17, TM '18
- 2d BF bulk Blommaert-TM-Verschelde '18 $\longrightarrow$ this talk
- Particle in infinite B-field in AdS_2 Yang '18, Kitaev-Suh '18
  $\Rightarrow$ related to representation theory of universal cover of $SL(2,\mathbb{R})$ Iliesiu-Pufu-Verlinde-Wang (In progress)

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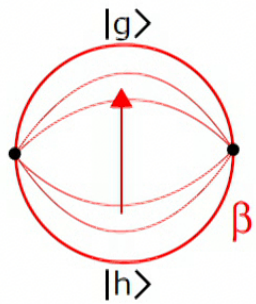
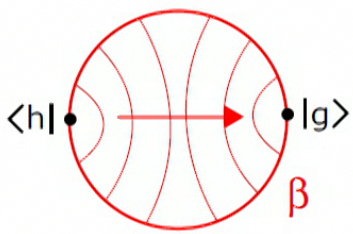
BF partition function

Disk Partition function, with circumference β:

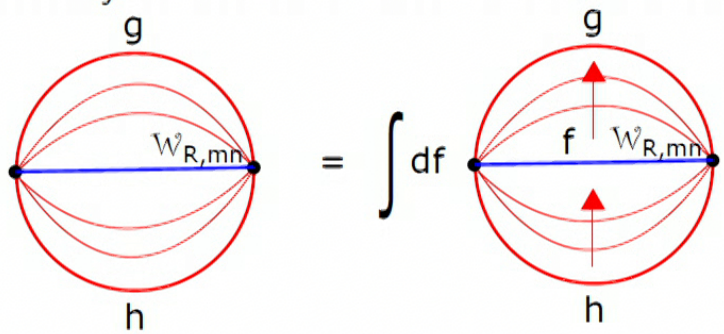
$Z(g,h) = \langle g| \, e^{-\beta H} \, |h\rangle$





BF Wilson line

Including a boundary-anchored Wilson line in the bulk:



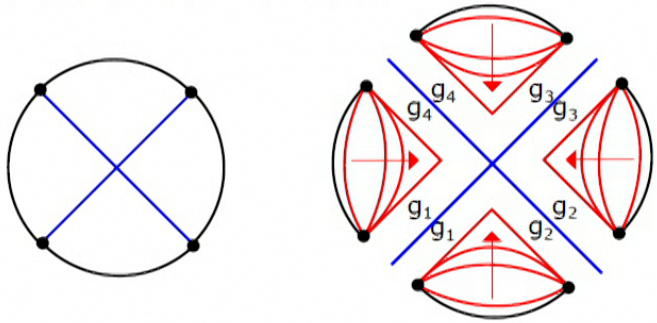
$$\langle g|\, e^{-TH}\mathcal{W}_{R,mn}\,|h\rangle = \int df\, \langle g|\, e^{-T_1 H}\,|f\rangle\, \mathcal{W}_{R,mn}(f)\, \langle f|\, e^{-T_2 H}\,|h\rangle$$

Using well-known group integral:

$$\int df\, R_{1,m_1 n_1}(f) R_{2,m_2 n_2}(f) R_{3,m_3 n_3}(f) = \begin{pmatrix} R_1 & R_2 & R_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \begin{pmatrix} R_1 & R_2 & R_3 \\ n_1 & n_2 & n_3 \end{pmatrix}$$

Crossing Wilson lines

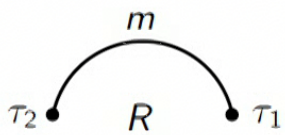
Crossing Wilson lines in the BF disk:

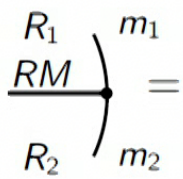


Four disk-regions glued together by integrals over $g_i, i = 1, 2, 3, 4$
Four 3R-integrals $\Rightarrow$ eight 3j-symbols: four external and four internal
Internal 3j's give a 6j-symbol:

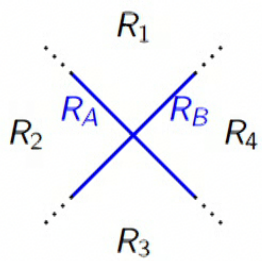
$$\begin{Bmatrix} R_B & R_1 & R_4 \\ R_A & R_3 & R_2 \end{Bmatrix} = \sum_{m_i} \begin{pmatrix} R_1 & R_2 & R_A \\ m_1 & m_2 & m_A \end{pmatrix} \begin{pmatrix} R_2 & R_3 & R_B \\ m_2 & m_3 & m_B \end{pmatrix} \begin{pmatrix} R_3 & R_4 & R_A \\ m_3 & m_4 & m_A \end{pmatrix} \begin{pmatrix} R_4 & R_1 & R_B \\ m_1 & m_2 & m_B \end{pmatrix}$$

Diagrammatic Rules

- Draw a disk with boundary-anchored Wilson lines
  Each region in the disk has a irrep label R, each endpoint of each line an index m
- 

$= \quad e^{-\mathcal{C}_R(\tau_2-\tau_1)}$,


$= \begin{pmatrix} R_1 & R_2 & R \\ m_1 & m_2 & M \end{pmatrix}$



$= \begin{Bmatrix} R_B & R_1 & R_4 \\ R_A & R_3 & R_2 \end{Bmatrix}$

- Sum over all labels

Gravity from SL(2,$\mathbb{R}$): Path-integral approach

Gravity = SL$(2,\mathbb{R})$ group element g with constraint at the boundary:

$$A_\tau|_{r\to\infty} = \chi|_{r\to\infty} = \begin{pmatrix} 0 & T(\tau) \\ 1 & 0 \end{pmatrix} = g\partial_t g^{-1}$$

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- Action:
  Plugging this into the particle on group G action, with $T(t) = \{f, t\}$ field redefinition, leads to Schwarzian action:
  $L = \mathrm{Tr}(g^{-1}\partial_t g)^2 \sim \{f, t\}$

Gravity from SL(2,$\mathbb{R}$): Canonical approach

States:

Peter-Weyl $\Rightarrow$ Hilbert space spanned by Matrix elements:

$R_{k,\mathrm{ii}}(g) = e^{\phi} K_{2ik}\left(e^{\phi}\right)$

- k labels continuous irreps with Casimir $\mathcal{C}_k = k^2 + 1/4$
- Gravitational coset constraints at the boundary of the parabolic type: $J_{\pm}\left|\mathrm{i}_{\pm}\right\rangle = \pm i \left|\mathrm{i}_{\pm}\right\rangle$ (Hamiltonian reduction, Liouville constraints)
- Using Gauss decomposition of group element $g(\phi, \gamma_-, \gamma_+)$

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Role of dim R is Plancherel measure $\rho(k) = k \sinh(2\pi k)$

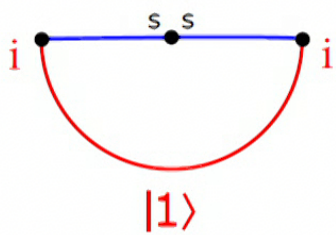
Operators:

Discrete irrep matrix elements for Wilson lines: $R_{\ell,00}(\phi) = e^{2\ell\phi}$

$$\begin{pmatrix} k_1 & \ell & k_2 \\ \mathrm{i} & 0 & \mathrm{i} \end{pmatrix}^2 = \int_{-\infty}^{+\infty} d\phi\, e^{2\ell\phi} K_{2ik_1}(e^{\phi}) K_{2ik_2}(e^{\phi}) = \frac{\Gamma(\ell \pm ik_1 \pm ik_2)}{\Gamma(2\ell)}$$

Application: TFD and Factorization

Thermofield double state by evolving from $|g = \mathbf{1}\rangle$ state for $\beta/2$ (semidisk):



$$|\mathrm{TFD}\rangle = \int_0^{+\infty} dk\, \sqrt{k \sinh 2\pi k}\, e^{-\frac{\beta}{2}k^2}\, |k, \mathrm{i}, \mathrm{i}\rangle\,, \qquad \langle \mathrm{TFD}\,|\mathrm{TFD}\rangle = Z$$

$|k, \mathrm{i}, \mathrm{i}\rangle$ is wormhole state

Can be factorized using $R^k_{\mathrm{ii}}(g_1 \cdot g_2) = \int ds\, R^k_{\mathrm{is}}(g_1) R^k_{\mathrm{si}}(g_2)$

$$\Rightarrow |\mathrm{TFD}\rangle = \int_0^{+\infty} dk \int_{-\infty}^{+\infty} ds\, e^{-\frac{\beta}{2}k^2}\, |k, s, \mathrm{i}\rangle \otimes |k, s, \mathrm{i}\rangle$$

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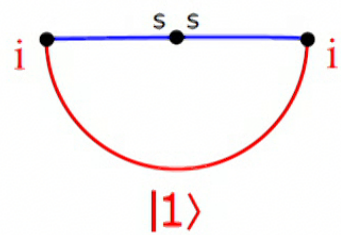

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Full $\mathrm{SL}(2, \mathbb{R})$ group on dividing surface, edge sector $|k, s\rangle$ lives on dividing surface (black hole horizon)

$|k, s, \mathrm{i}\rangle$ is one-sided black hole state

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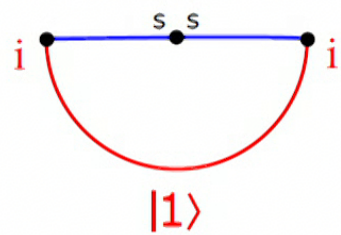

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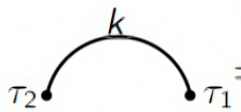
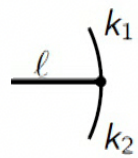
Gravitational constraints *only* on holographic boundary

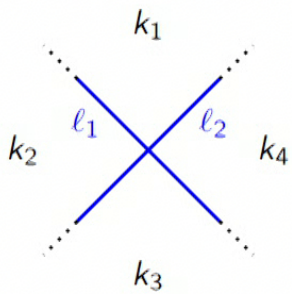
Diagrammatic decomposition

Rules:

- 

$= e^{-k^2(\tau_2-\tau_1)}$, 

$= \gamma_\ell(k_1, k_2) = \sqrt{\frac{\Gamma(\ell \pm ik_1 \pm ik_2)}{\Gamma(2\ell)}}$

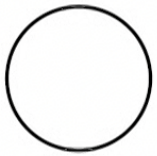
- 

$= \begin{Bmatrix} k_1 & \ell_1 & k_2 \\ k_3 & \ell_2 & k_4 \end{Bmatrix}.$

- Integrate over intermediate momenta k_i with measure $d\mu(k) = dk^2 \sinh(2\pi k)$

Note: non-perturbative expression in Schwarzian coupling C

JT Partition function and 2-point function

$Z(\beta) =$  $= \int_0^{+\infty} dk^2 \sinh(2\pi k) e^{-\beta k^2} = \left(\frac{\pi}{\beta}\right)^{3/2} e^{\pi^2/\beta}$

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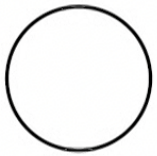
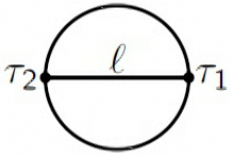
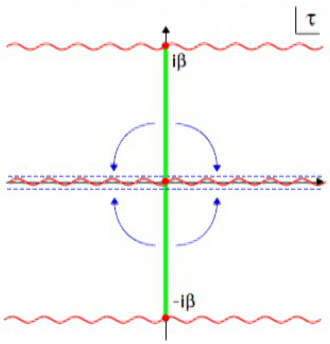
$Z(\beta) =$  $= \int_0^{+\infty} dk^2 \sinh(2\pi k) e^{-\beta k^2} = \left(\frac{\pi}{\beta}\right)^{3/2} e^{\pi^2/\beta}$

$\langle \mathcal{O}_\ell(\tau_1, \tau_2) \rangle = G_\ell^\beta(\tau_1, \tau_2) =$ 

$$G_\ell^\beta(\tau_1, \tau_2) = \frac{1}{Z(\beta)} \int d\mu(k_1) d\mu(k_2)\, e^{-\tau k_1^2 - (\beta - \tau) k_2^2} \frac{\Gamma\left(\ell \pm i(k_1 \pm k_2)\right)}{2\sqrt{\pi}\Gamma(2\ell)}$$

Application: Real-time and thermofield double

Divergence as $\tau_{12} \to 0$, actually branch cut when complexifying
$\Rightarrow$ Two real-time ($\tau \to i\tau$) correlators $G^{\pm}$



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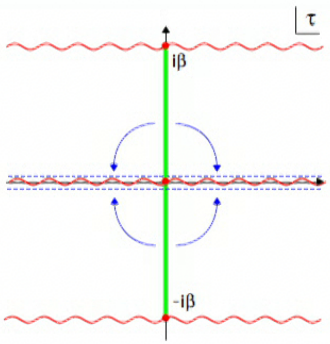
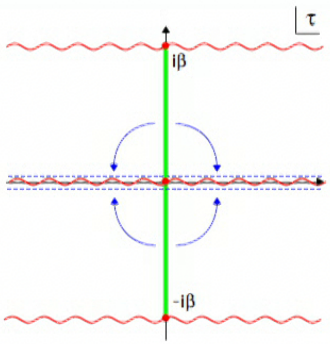


$$G_\ell^{\pm} = \frac{1}{Z(\beta)(2C)^{2\ell}} \int d\mu(k_1) d\mu(k_2) e^{\pm i \frac{\tau}{2C}(k_1^2 - k_2^2) - \frac{\beta}{2C} k_2^2} \frac{\Gamma\left(\ell \pm i(k_1 \pm k_2)\right)}{2\sqrt{\pi}\Gamma(2\ell)}$$

Application: Real-time and thermofield double

Divergence as $\tau_{12} \to 0$, actually branch cut when complexifying
$\Rightarrow$ Two real-time ($\tau \to i\tau$) correlators $G^{\pm}$



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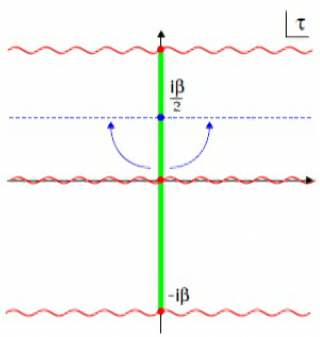
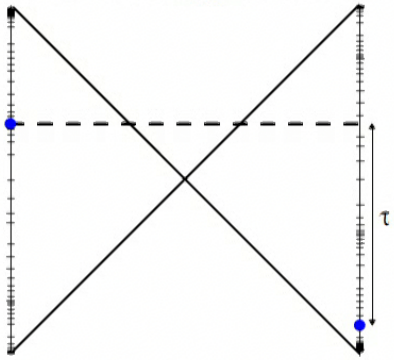
Rotate one of the times by $\beta/2$ = TFD configuration:





Application: Zero-temperature and semi-classics

Zero-temperature limit: $\beta \to \infty$ ($k_2 \approx 0$) agrees with

Bagrets-Altland-Kamenev '16:

$$G_\ell^\infty(\tau_1, \tau_2) \sim \int dk^2 \sinh(2\pi k)\, e^{-\frac{\tau}{2C} k^2} \frac{\Gamma^2(\ell+ik)\Gamma^2(\ell-ik)}{\Gamma(2\ell)}$$

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Semiclassics for heavy bilocal operator $\ell \sim C \to \infty$

- Take limit directly in expression
- Solve classical Schwarzian EOM sourced by heavy bilocal:
  $S_0 = -C \int dt\, \{f, \tau\} - \ell \ln \frac{\dot{f}_1 \dot{f}_2}{(f_1 - f_2)^2}$

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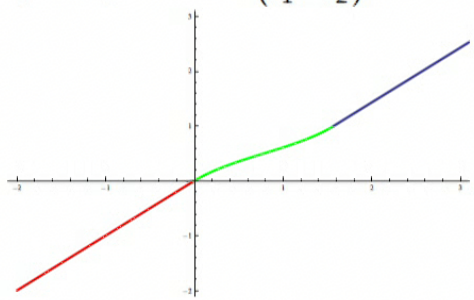
Solution:



Shapiro time delay due to mass $\sim \ell$ in the bulk

$\Rightarrow$ matches, also at finite temp

Application: Semiclassics for light operators

Semi-classical regime $C \to \infty$, $\ell \ll C$: $k_1 \sim k_2 \gg 1$
Redefine $k_1^2 = M + \omega$, $k_2^2 = M$

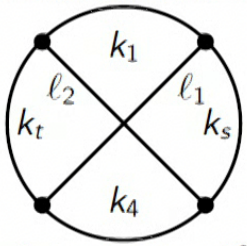
$$G_\ell^\pm \sim \int_0^\infty dM e^{2\pi\sqrt{M} - \frac{\beta}{2C}M} \int \frac{d\omega}{2\pi} e^{\pm i \frac{\tau}{2C}\omega + \pi \frac{\omega}{2\sqrt{M}}} \frac{\Gamma(\ell \pm i \frac{\omega}{2\sqrt{M}})}{\Gamma(2\ell)} (2\sqrt{M})^{2\ell-1}$$

Interpretation:

- M-integral has saddle: $M_0 = 4\pi^2 C^2/\beta^2$, the JT black hole $E(T)$-relation
  Remaining ω-integral is done explicitly to yield:
  $G_\ell^{\pm,cl}(\tau_1, \tau_2) = \left(\frac{\pi}{\beta \sinh \frac{\pi}{\beta}\tau_{12}}\right)^{2\ell}$

OTO four-point correlator

Schwarzian out-of-time ordered (OTO) 4-point function:



$$\int d\mu(k_i) e^{-k_1^2(\tau_3-\tau_1)-k_t^2(\tau_3-\tau_2)-k_4^2(\tau_4-\tau_2)-k_s^2(\beta-\tau_4+\tau_1)}$$

$$\times \gamma_{\ell_1}(k_1,k_s)\gamma_{\ell_2}(k_s,k_4)\gamma_{\ell_1}(k_4,k_t)\gamma_{\ell_2}(k_t,k_1) \times \left\{ \begin{array}{ccc} \ell_1 & k_1 & k_s \\ \ell_2 & k_4 & k_t \end{array} \right\}$$

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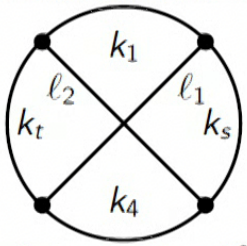

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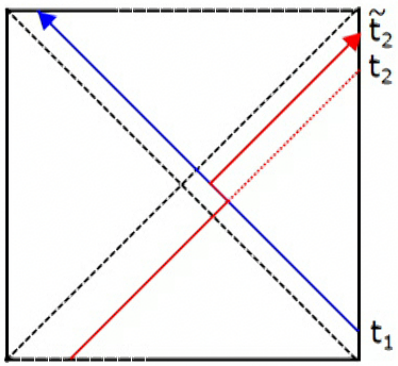
$$\left\{ \begin{array}{ccc} \ell_1 & k_2 & k_s \\ \ell_3 & k_4 & k_t \end{array} \right\} = \sqrt{\frac{\Gamma(\ell_1+ik_2\pm ik_s)\Gamma(\ell_3-ik_2\pm ik_t)\Gamma(\ell_1-ik_4\pm ik_t)\Gamma(\ell_3+ik_4\pm ik_s)}{\Gamma(\ell_1-ik_2\pm ik_s)\Gamma(\ell_3+ik_2\pm ik_t)\Gamma(\ell_1+ik_4\pm ik_t)\Gamma(\ell_3-ik_4\pm ik_s)}}$$

$$\times \int_{-i\infty}^{i\infty} \frac{du}{2\pi i} \frac{\Gamma(u)\Gamma(u-2ik_s)\Gamma(u+ik_{2+4-s+t})\Gamma(u-ik_{s+t-2-4})\Gamma(\ell_1+ik_{s-2}-u)\Gamma(\ell_3+ik_{s-4}-u)}{\Gamma(u+\ell_1-ik_{s-2})\Gamma(u+\ell_3-ik_{s-4})}$$

Application: Shockwaves from semiclassics



Time delay:

$$\tilde{t}_2 - t_2 \sim e^{\lambda_M (t_2 - t_1)}, \quad \lambda_M = \frac{2\pi}{\beta_M}$$

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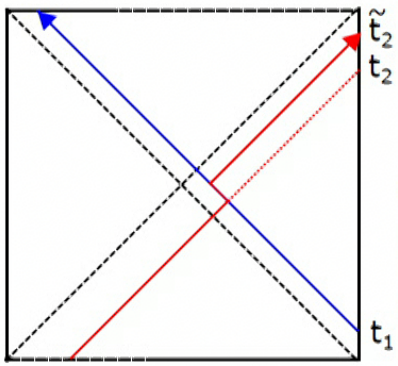

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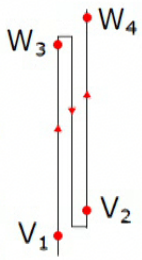


Shenker-Stanford: OTO-correlator $\langle V_1 W_3 V_2 W_4 \rangle$ in boundary theory captures this behavior Shenker-Stanford '15

$\langle V_1 W_3 V_2 W_4 \rangle$, written using shockwaves in the AdS_2 bulk as

$$\int_0^{+\infty} dq_+ \int_0^{+\infty} dp_- \Psi_1^*(q_+) \Phi_3^*(p_-) \mathcal{S}(p_-, q_+) \Psi_2(q_+) \Phi_4(p_-)$$

- Ψ, Φ = Kruskal wavefunctions = bulk-to-boundary propagators
- $\mathcal{S} = \exp\left(\frac{i\beta}{4\pi C} p_- q_+\right)$ the Dray-'t Hooft shockwave $\mathcal{S}$-matrix

Application: Shockwaves from the exact OTO correlator

Large C limit of complete OTO 4-point function, with light ℓ, gives full eikonal shockwave expressions Lam-Mertens-Turiaci-Verlinde '18

$$\left\{ \begin{matrix} \ell_1 & k_2 & k_s \\ \ell_3 & k_4 & k_t \end{matrix} \right\} \sim \int_{-i\infty}^{i\infty} \frac{du}{2\pi i} \frac{\Gamma(u)\Gamma(u-\ldots)\Gamma(u+\ldots)\Gamma(u-\ldots)\Gamma(\ldots-u)\Gamma(\ldots-u)}{\Gamma(u+\ldots)\Gamma(u+\ldots)}$$

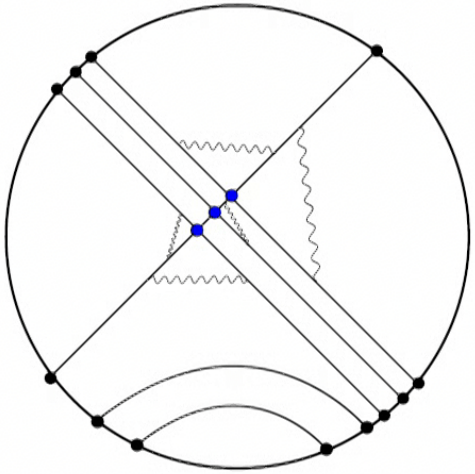
- Deform previous contour to the left $\Rightarrow$ 2 poles dominate
- Define: $(M \gg \alpha, \beta, \omega \gg \ell_1, \ell_3)$
  $k_2^2 = M, \quad k_s^2 = M + \alpha, \quad k_4^2 = M + \omega, \quad k_t^2 = M + \beta$

$\Rightarrow$ maximal Lyapunov behavior in agreement with JT gravity

Exact match $\Rightarrow$ Derived full shockwave in semiclassical regime!

Application: OTO higher-point functions

In the semi-classical regime ($C \to \infty$):
The (Euclidean) Schwarzian diagram = interpreted as real-time shockwave diagram



Only gravitational self-interactions that survive are 2-to-2 shockwaves:

- Consecutive and independent, allowing a factorization of the amplitude
- Uncrossed lines completely factorize

Matches fully with convolving Kruskal wavefunctions with Dray-'t Hooft $\mathcal{S}$-matrix

Extension: Complex SYK and Chemical Potentials

Complex SYK is described at low energies by the action:

Davison-Fu-Georges-Gu-Jensen-Sachdev '16

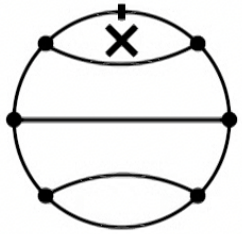
$$S = -C \int_0^\beta dt \left\{ \tan \frac{\pi}{\beta} f(t), t \right\} - \frac{K}{2} \int_0^\beta dt \left(\Lambda'(t) - i\mu f'(t) \right)^2 ,$$

Solution: Chaturvedi-Gu-Song-Yu '18, TM-Turiaci '19

Shifting $\tilde{\Lambda}(\tau) = \Lambda(\tau) - i\mu f(\tau)$ decouples both sectors

Schwarzian correlator $\times$ U(1) correlator with chemical potential defect

E.g. $\langle G_1 G_2 G_3 \rangle_{\beta,\mu} \equiv \frac{1}{Z} \mathrm{Tr} \left[e^{-\beta H} e^{i\mu\beta} G_1 G_2 G_3 \right] =$



Extension: Defects and Orbits

One can add defects to the JT disk diagram TM-Turiaci '19
Similar to 3d gravity, except defects are in spacelike direction
From 3d Chern-Simons: vertical Wilson lines, classified by Virasoro coadjoint orbits constant representative

- Elliptic: Conical deficits in bulk, Conical ensemble
  Insertion of $D_{U(1)_\theta}(k) \equiv \frac{\cosh(2\pi\theta k)}{k\sinh(2\pi k)}$
  Within JT path-integral by inserting $e^{\alpha\Phi(x)}$ operator
  Can be used to get $\langle A \equiv \Phi_h \rangle$ in full quantum theory

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- Hyperbolic: Macroscopic tube with two boundaries, global AdS_2
  Insertion of $D_{U(1)_\lambda}(k) \equiv \frac{\cos(2\pi\lambda k)}{k\sinh(2\pi k)}$
  Required for gluing disks together to reach different topology

Blommaert-TM-Verschelde '18, Saad-Shenker-Stanford '19

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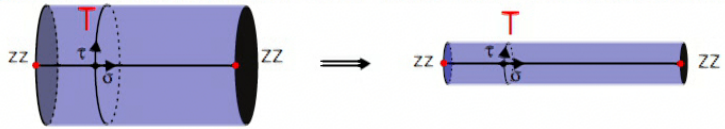
- Parabolic: Thermal AdS_2 geometry

Extension: Defects and Verlinde Loops

Liouville Perspective:

Cylindrical amplitude between ZZ-branes TM-Turiaci-Verlinde '17, TM '18



in double-scaling limit $c \to +\infty$, $T \to 0$, with $cT = C$ fixed

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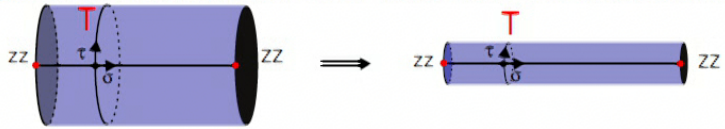

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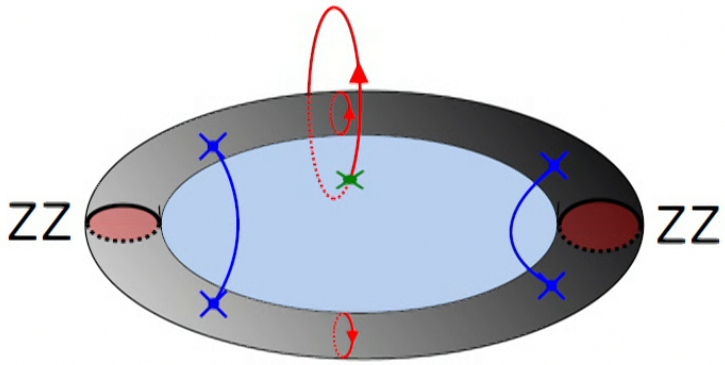
$\to$ Verlinde loops / topological defects $\to$ JT-defects

Total picture:



Verlinde loop = vertical Chern-Simons loop piercing the BF disk in a puncture (defect)

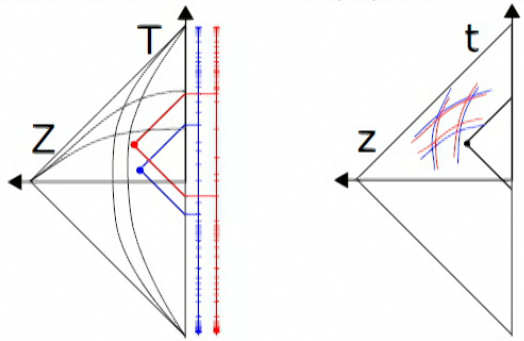
Application: Going into the bulk

Defining diff-invariant bulk operator $\mathcal{O}$

$\rightarrow$ Requires definition of bulk point

Choice: define bulk frame from the boundary reparametrization $f(t)$ as: $ds^2(f) = \frac{\dot{f}(u)\dot{f}(v)}{(f(u)-f(v))^2}(dz^2 - dt^2), \quad u, v = t \pm z$

Radar definition of bulk point $U = f(u)$, $V = f(v)$



Conclusion

Jackiw-Teitelboim gravity is toy model of quantum gravity, striking the ideal balance between relevance and solvability

- Relevance: low-energy universal gravity sector of all SYK-type models
  Dimensional reduction of 3d pure gravity
  Most basic non-trivial holographic 2d gravity model
- Solvability: gravitational dofs reduce to boundary time reparametrizations, with explicit analytic solution for correlators, non-perturbatively in G_N

Thank you!