Title: Topological Holography Course - Lecture 9

Speakers: Kevin Costello

Collection: Topological Holography Course (Costello)

Date: April 12, 2019 - 2:00 PM

URL: http://pirsa.org/19040042

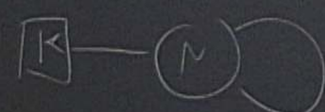
Last Time

$\mathbb{R}\times\underline{\mathbb{R}}\times\mathbb{C}^2$

D1 N

D4 K

On the D1, ADHM quantum mechanics



$X, Y \in \mathfrak{gl}_N$

$I : \mathbb{C}^K \to \mathbb{C}^N$

$J : \mathbb{C}^N \to \mathbb{C}^K$

$A \in \Omega^1(\mathbb{R}, \mathfrak{gl}_N)$

$$\int X d_A Y + I d_A J + c\int A$$

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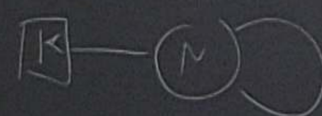

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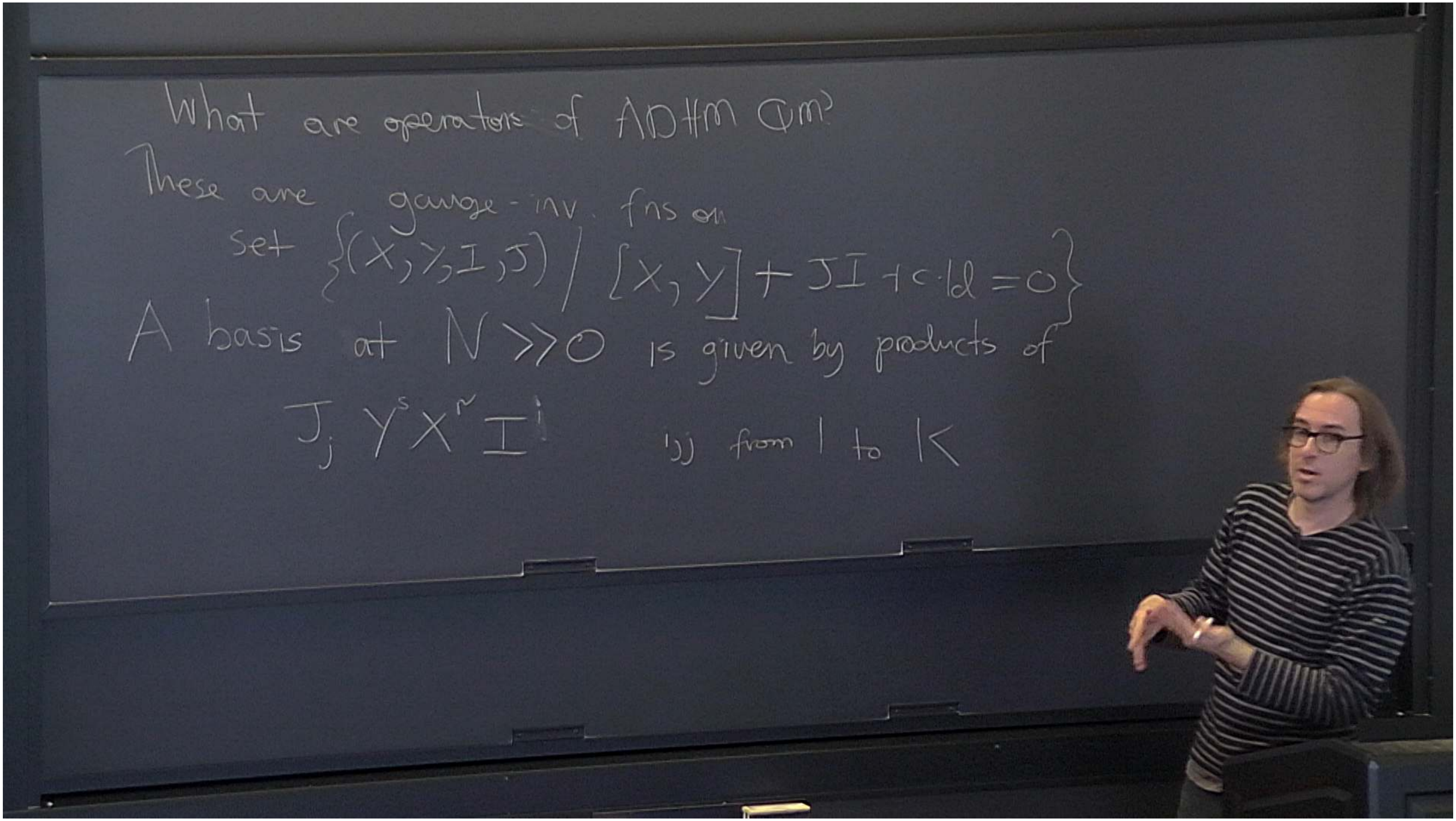
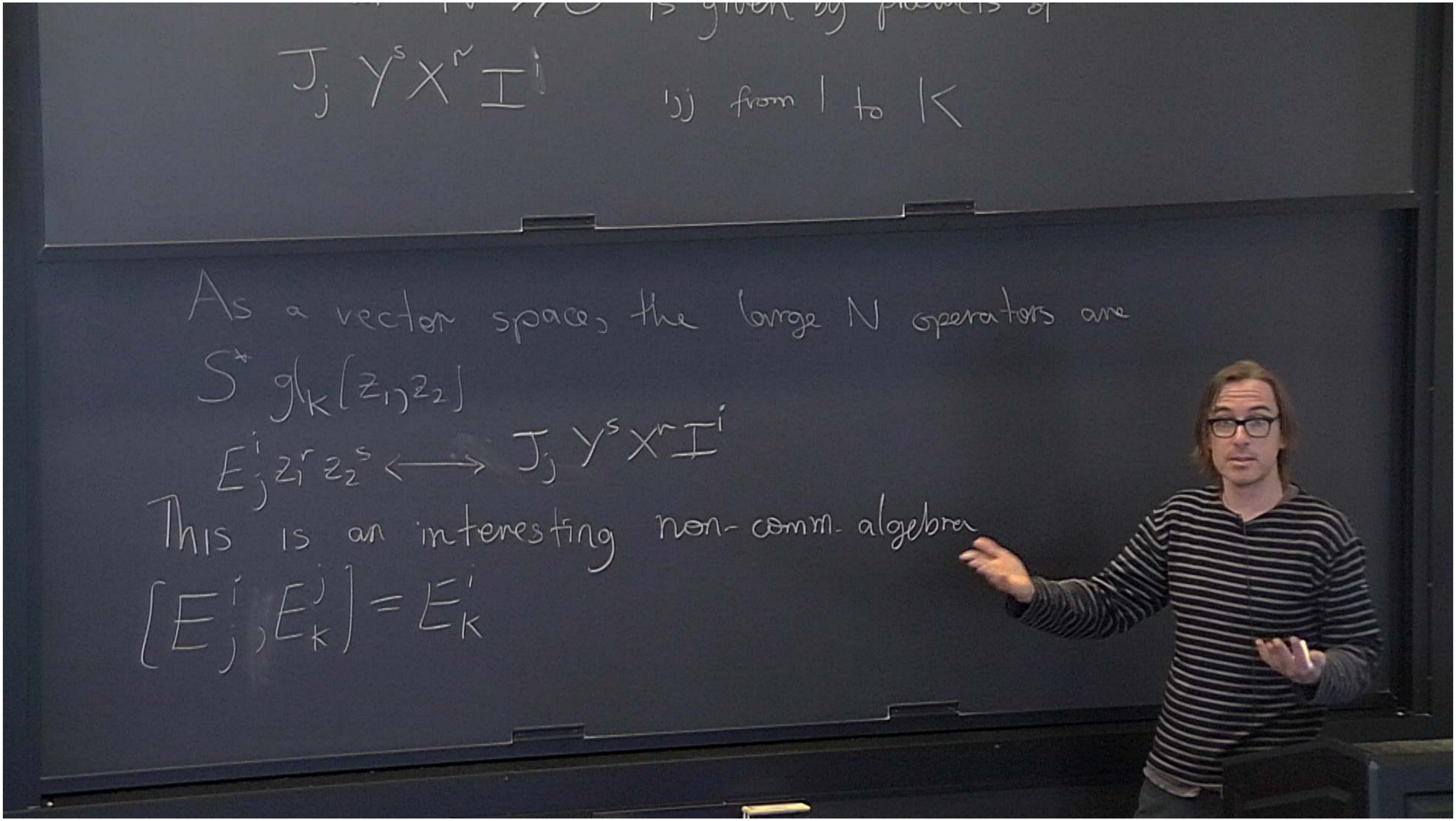

What are operators of ADHM QM?
These are gauge-inv. fns on
set $\{(X, Y, I, J) \,/\, [X, Y] + JI + c \cdot \mathrm{Id} = 0\}$
A basis at $N >> 0$ is given by products of
$J_j Y^s X^r I^i$ i,j from 1 to K


is given by products of
$J_j Y^s X^r I^i$ i,j from 1 to K
As a vector space, the large N operators are
$S^* gl_K[z_1, z_2]$
$E^i_j z_1^r z_2^s \longleftrightarrow J_j Y^s X^r I^i$
This is an interesting non-comm. algebra
$[E^i_j, E^j_k] = E^i_k$

... is given by products of

$$J_j Y^s X^r I^i \qquad i,j \text{ from } 1 \text{ to } K$$

As a vector space, the large N operators are

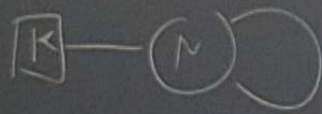
$$S^* gl_K(z_1, z_2)$$

$$E^i_j z_1^r z_2^s \longleftrightarrow J_j Y^s X^r I^i$$

This is an interesting non-comm. algebra

$$[E^i_j, E^j_k] = E^i_k \qquad [J_j I^i, J_k I^j] = J_k I^i$$

On the D1, ADHM quantum mechanics



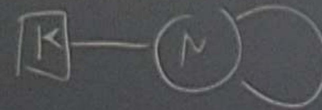
$+ q\int A$

If i, j, k, l are distinct

$$[E^i_j Z_1, E^k_l Z_2] = -E^i_l E^k_j$$

On the D1, ADHM quantum mechanics

$+ c\oint A$



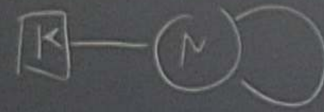
If i, j, k, l are distinct

$$[E^i_j z_1, E^k_l z_2] = -E^i_l E^k_j$$

Let $\alpha, \beta \ldots$ be gl_N indices

$$[J_j^\alpha X^\beta_\alpha I^i_\beta, J_l^\gamma Y^\delta_\gamma I^k_\gamma] = (J_l^\alpha I^i_\alpha)(J_j^\beta I^k_\beta)$$

On the D1, ADHM quantum mechanics



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$$[X^\beta_\alpha, Y^\delta_\gamma] = \delta^\delta_\alpha \delta^\beta_\gamma$$

$$[J_j^\alpha X_\alpha^\beta I_\beta^i, J_l^\gamma Y_\gamma^\delta I_\delta^k] = (J_l^\alpha I_\alpha^i)(J_j^\beta I_\beta^k)$$

$$[X_\alpha^\beta, Y_\gamma^\delta] = \delta_\alpha^\delta \delta_\gamma^\beta$$

5d Gauge Theory

$$A \in \Omega^1(\mathbb{R} \times \mathbb{C}^2) / \langle dz_1, dz_2 \rangle \otimes \mathfrak{gl}_K$$

$$\int dz_1 dz_2 \left(\frac{1}{2} A\, dA + \frac{1}{3} A^3 + \underbrace{\cdots\cdots}_{\text{non-commutativity}} \right)$$

Consider universal line defect placed at $z_1 = 0$, where $t_j^i[k,l]$ are coupled to $\partial_{z_1}^k \partial_{z_2}^k A_j^i$

$$[J^\alpha_j X^\beta_\alpha I^i_\beta, J^\gamma_l Y^\delta_\gamma I^k_\delta] = (J^\alpha_l I^i_\alpha)(J^\beta_j I^k_\beta)$$

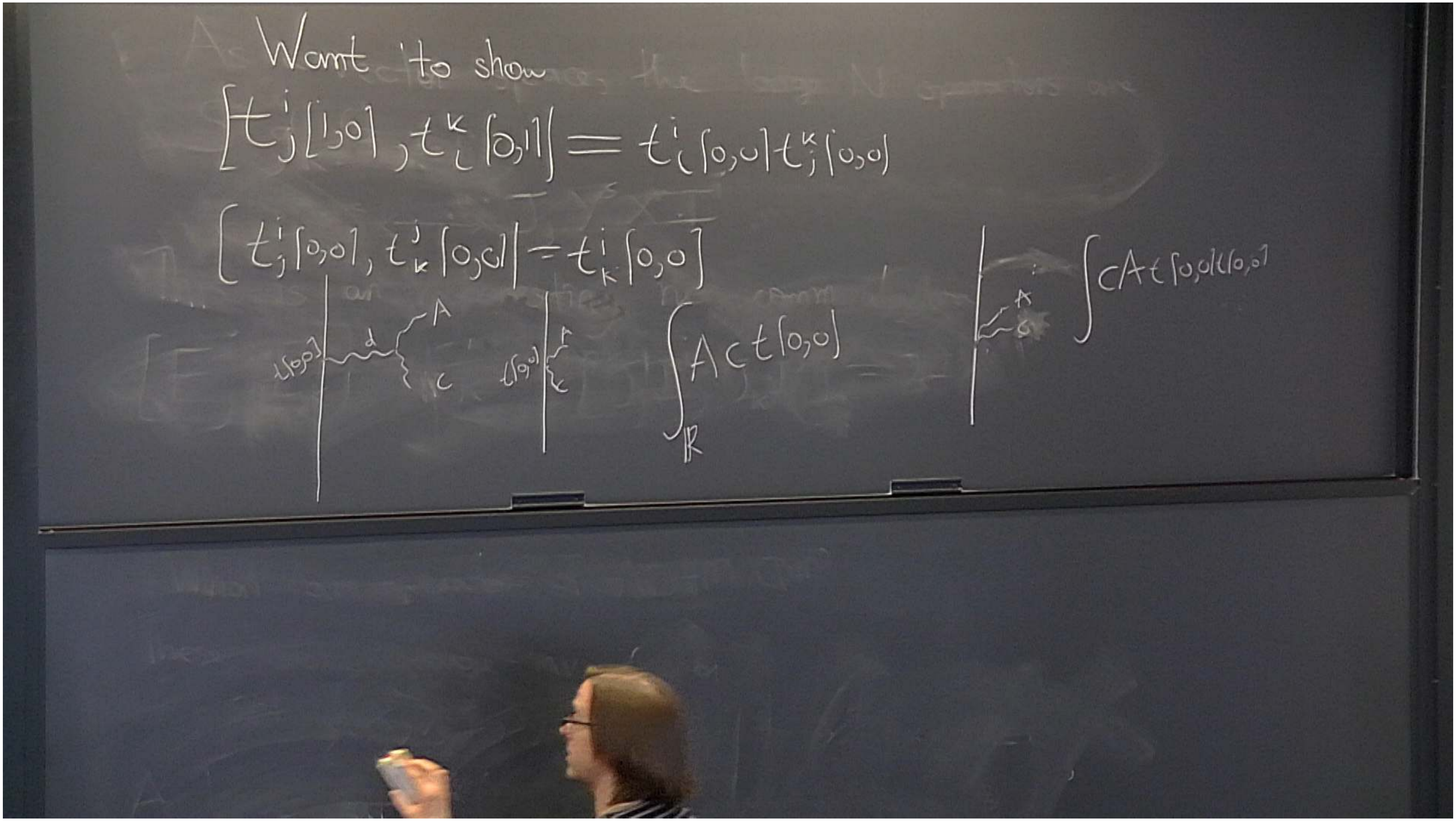
$$[X^\beta_\alpha, Y^\delta_\gamma] = \delta^\delta_\alpha \delta^\beta_\gamma$$

5d Gauge Theory

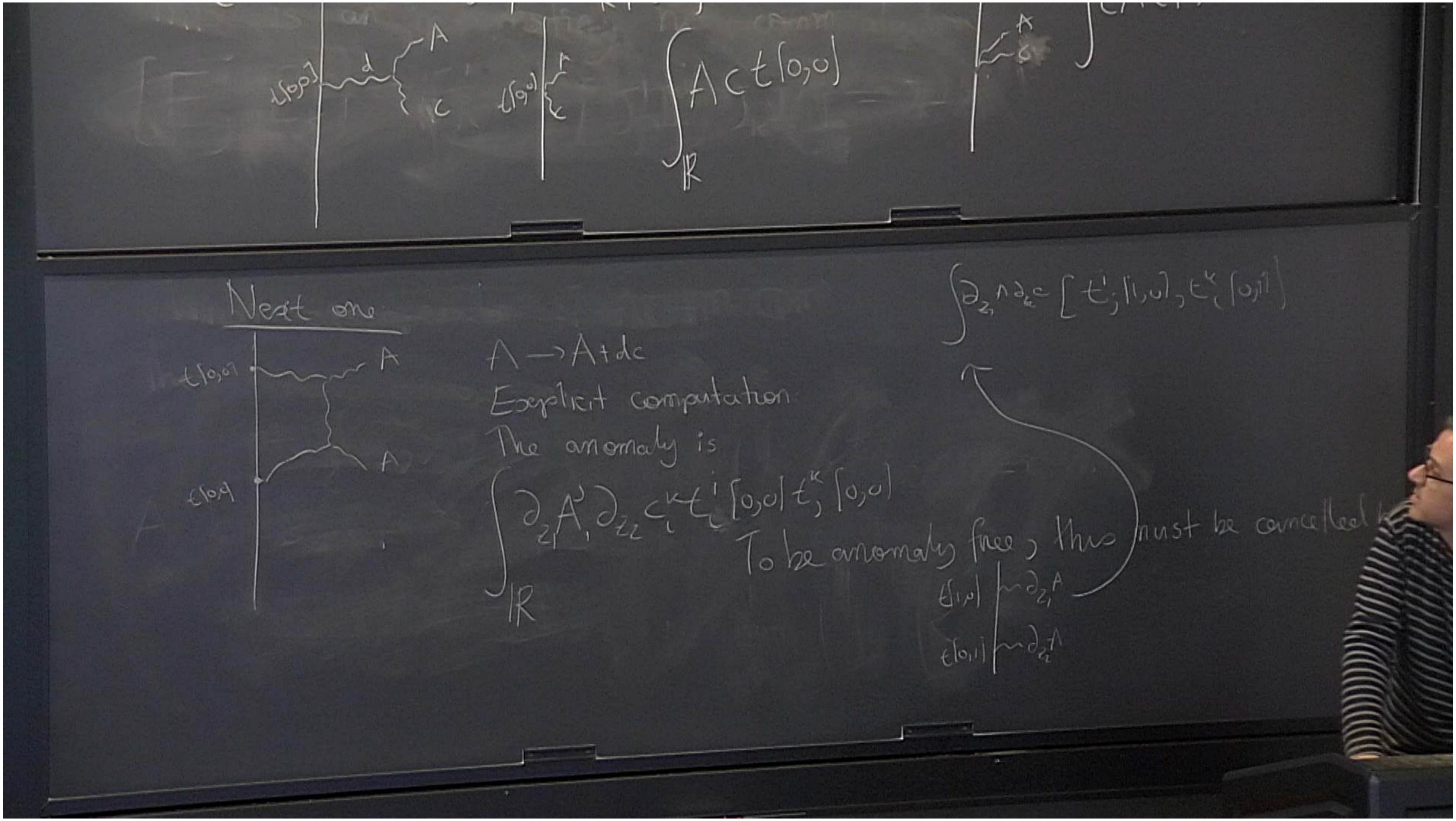
$$A \in \Omega^1(\mathbb{R} \times \mathbb{C}^2)/\langle dz_1, dz_2 \rangle \otimes \mathfrak{gl}_K \qquad Q_{BRST} \qquad \oint c$$

$$\int dz_1 dz_2 \left(\frac{1}{2} A dA + \frac{1}{3} A^3 + \underbrace{\cdots\cdots}_{\text{non-commutativity}} \right)$$

Consider universal line defect placed at $z_1 = 0$, where $t^i_j[k,l]$ are coupled to $\partial^k_{z_1} \partial^k_{z_2} A^i_j$



Want to show
$[t^i_j[1,0], t^k_l[0,1]] = t^i_l[0,0]\, t^k_j[0,0]$
$[t^i_j[0,0], t^j_k[0,0]] = t^i_k[0,0]$
$t[0,0]$
A
c
$\int_{\mathbb{R}} A\, c\, t[0,0]$
$\int c A\, t[0,0]\, t[0,0]$



Next one
A → A + dc
Explicit computation
The anomaly is
∫_ℝ ∂_{z_1}A^j_1 ∂_{z_2}c^k_i t^i_i[0,0] t^k_j[0,0]
To be anomaly free, this must be cancelled b
∫ ∂_{z_1} ∧ ∂_{z_2}c [t^i_j[1,0], t^k_i[0,1]]
∫_ℝ A c t[0,0]

$$[J_j^{\alpha} X_{\alpha}^{\beta} I_{\beta}^{i}, J_l^{\gamma} Y_{\gamma}^{\delta} I_{\delta}^{k}] = (J_l^{\alpha} I_{\alpha}^{i})(J_j^{\beta} I_{\beta}^{k})$$

$$[X_{\alpha}^{\beta}, Y_{\gamma}^{\delta}] = \delta_{\alpha}^{\delta}\delta_{\gamma}^{\beta}$$

Backreaction in 6d

Extra lines | - - - - where - - - - was field sourced by the brane

Equivalently, we formally introduce a new element N to the chiral algebra and declare that the field it sources = field sourced by brane

$N \in$ center, $[N, t[k,l]] = 0$

we formally introduce a new element N to the chiral algebra
and declare that the field it sources = field sourced by brane
"$N \in$ center, $[N, t[k,l]]^{\mathcal{C}} = 0$"

Then the diagrams

are computing OPE in the larger algebra

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In 5d we let $N = \sum J_j I^j$

Then the field sourced by this operator in chiral alg
= field sourced by brane

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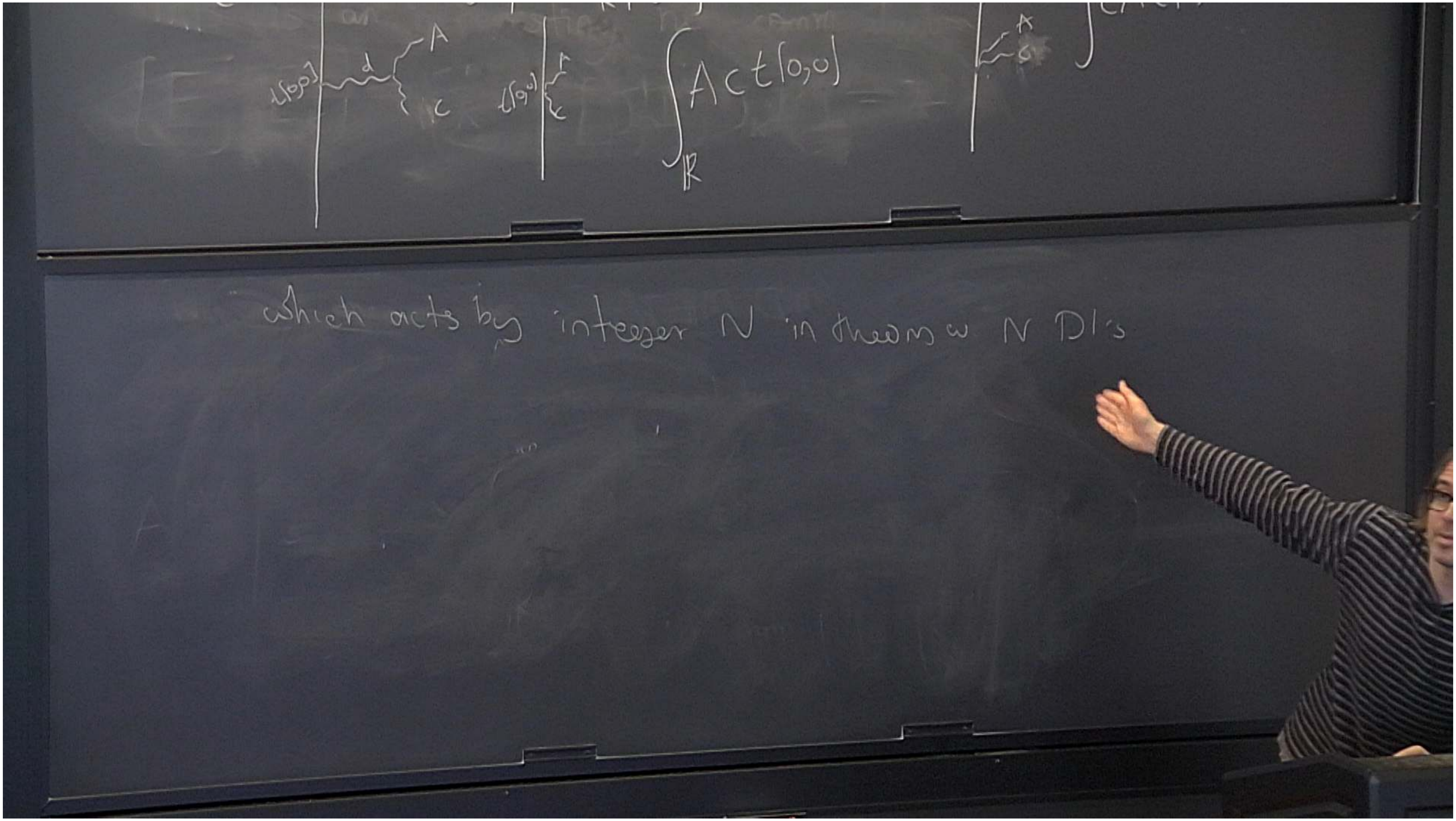
Universal algebra gen. by $t^i_j z_1^k z_2^l$ has a central element, N


which acts by integer N in theory w N D1's

Examples

(Many!)

$\mathbb{R}^2 \times \mathbb{C}^2$

but ω configurations of D1, D3, D5

$\Longrightarrow$ different quiver quantum mechanics

Then the diagrams

are computing OPE

but ω configurations of D1, D3, D5

$\Longrightarrow$ different quiver quantum mechanics

$$[X, Y] + JI = d\lambda$$

$$\sum J_j I^j = N$$

but ∞ configurations of D1, D3, D5

$\Longrightarrow$ different quiver quantum mechanics

$\mathbb{R} \times \mathbb{R} \times \mathbb{C} \times \mathbb{C}$

D1

D3

z^n superpotential

6d

Chiral algebra for

$\mathcal{N}=2$ SCFT

$Sp(N)$ gauge group

Matter in

$\Lambda^2 \mathbb{C}^N \oplus (\mathbb{C}^4 \otimes \mathbb{C}^N)$

but ω configurations of D1, D3, D5

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$\mathbb{R} \times \mathbb{R} \times \mathbb{C} \times \mathbb{C}$

D1

D3

z^n superpotential

6d

Chiral algebra for

$N=2$ SCFT

$Sp(N)$ gauge group

Matter in

$\Lambda^2 \mathbb{C}^N \oplus (\mathbb{C}^4 \otimes \mathbb{C}^N)$

This is anomaly free.

$SO(8)_R$

$t[0,0]$ d A c $t[0,0]$ A c

$$\int_{\mathbb{R}} A\, c\, t[0,0]$$

Holographic Dual

Type I B-model on $SL_2\mathbb{C}$

GS $\Longrightarrow$ this only works when we couple to $SO(8)$ hol. CS

Closed string fields: only $PV^{1,1}$ not other stuff