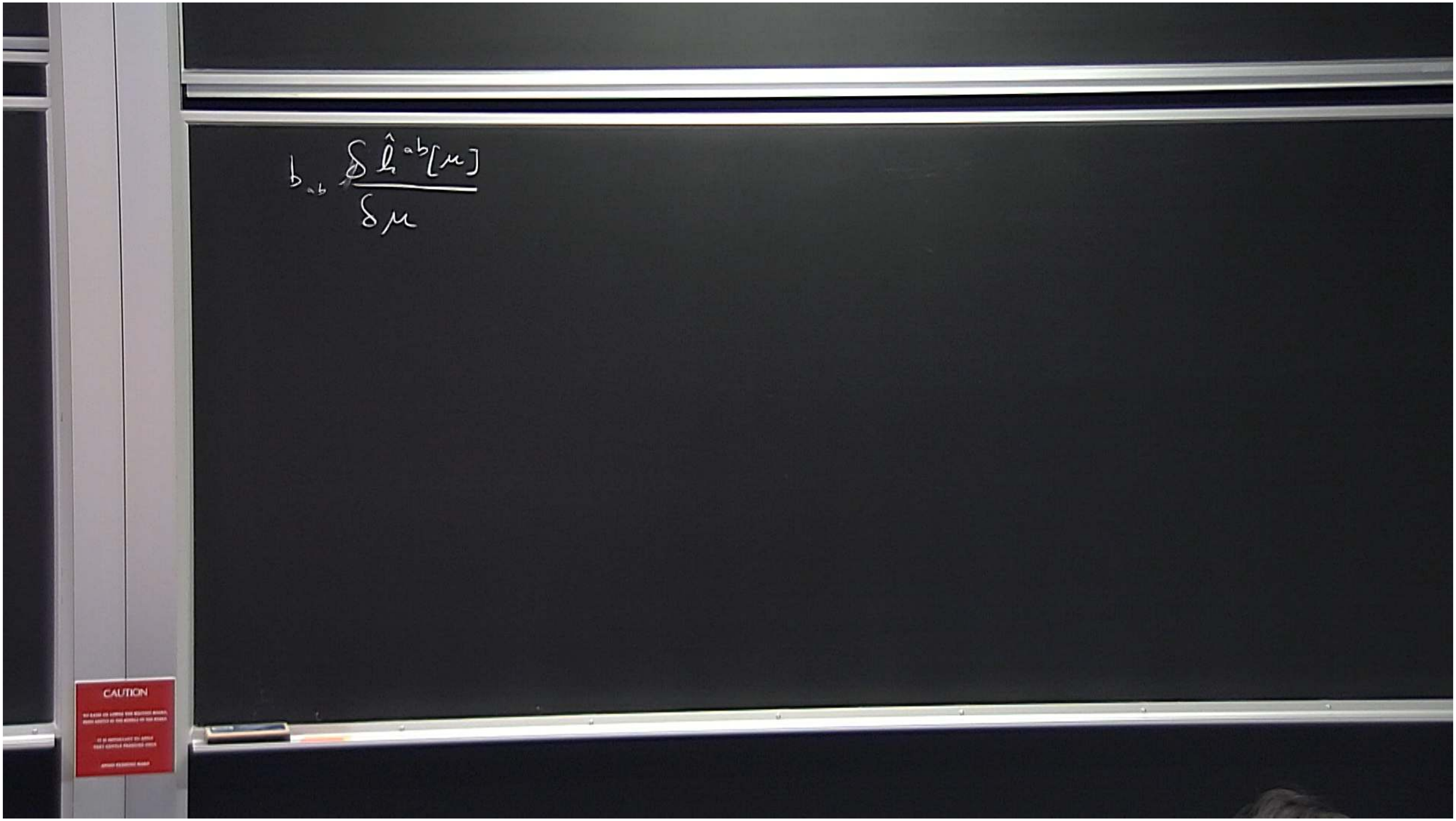
Title: PSI 2018/2019 - String Theory Review - Lecture 9

Speakers: Davide Gaiotto

Collection: PSI 2018/2019 - String Theory Review (Gaiotto)

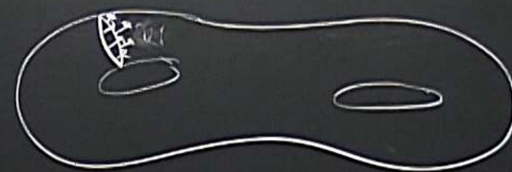
Date: April 04, 2019 - 10:15 AM

URL: http://pirsa.org/19040034


$b_{ab}\,\frac{\delta \hat{h}^{ab}[\mu]}{\delta \mu}$
CAUTION

$$b_{ab} \frac{\delta \hat{l}^{ab}[\mu]}{\delta \mu_{|||}}$$

$$\oint_\gamma b(z) v(z)$$



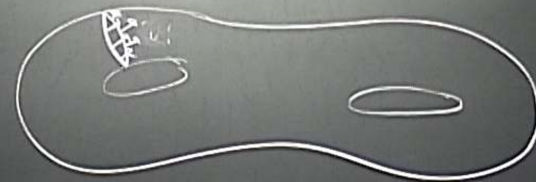
$\gamma \in \Sigma$ , $v$

$$z' = z + v(z) f(z, \bar{z})$$

CAUTION

$$b_{ab} \frac{\delta \hat{h}^{ab}[\mu]}{\delta \mu_{|||}}$$

$$\oint_\gamma b(z) v(z)$$



$$\gamma \in \Sigma, \quad v$$

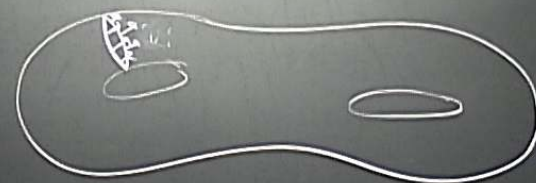
$$z' = z + v(z) f(z, \bar{z})$$

$$F^i(\phi^a, \mu^u) = 0$$

$$e^{b \frac{\partial F}{\partial g} c} \quad b \cdot \frac{\partial F}{\partial \mu} \int \det\left( \frac{\partial F^i}{\partial g^j}, \frac{\partial F^i}{\partial \mu^u} \right) \prod d\mu^u$$

$$\iint_\Sigma b_{ab} \frac{\delta \hat{h}^{ab}[\mu]}{\delta \mu_{|||}}$$

$$\oint_\gamma b(z) v(z)$$



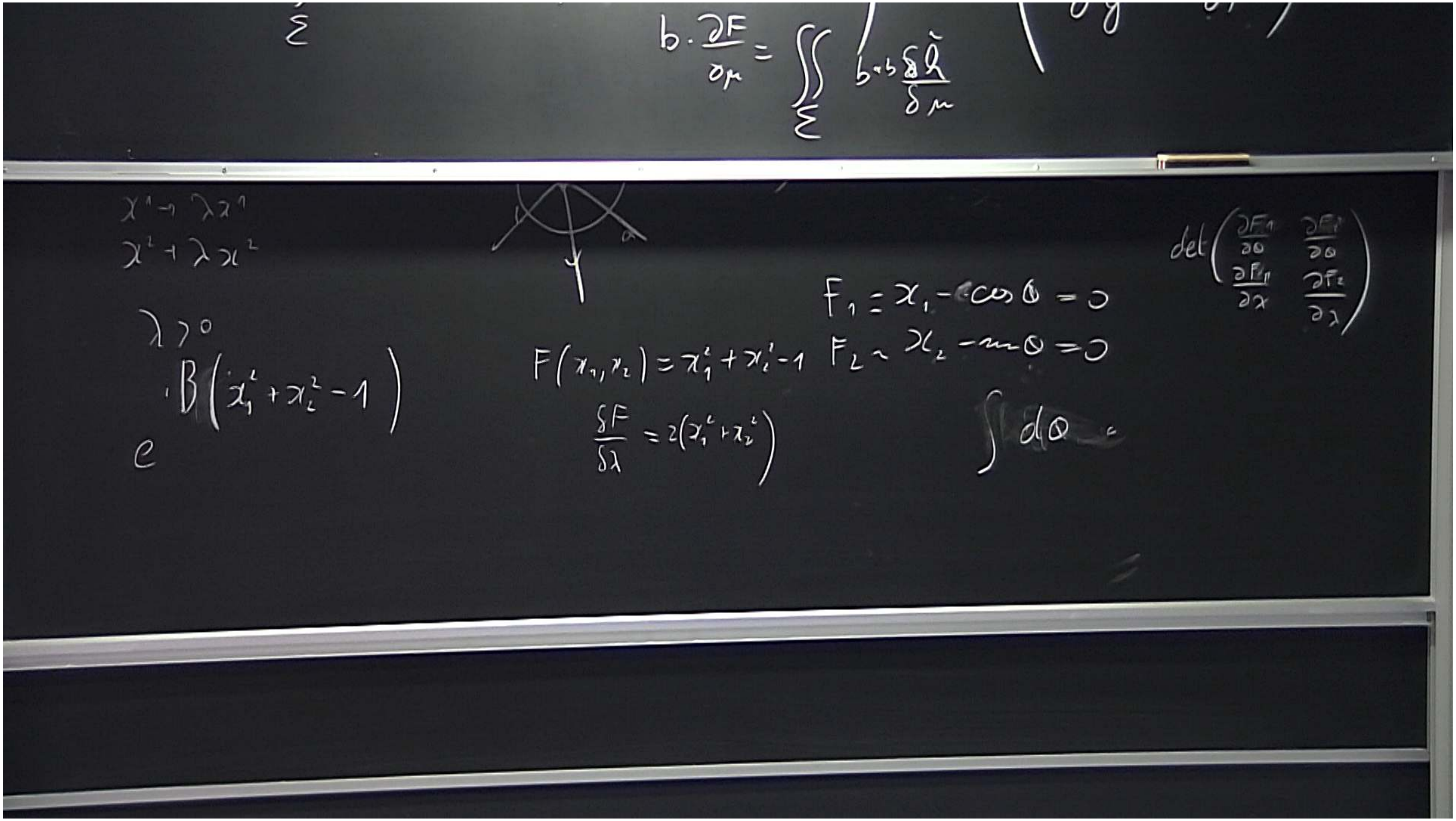
$\gamma \in \Sigma$, $v$

$$z' = z + v(z) f(z, \bar{z})$$

$$F^i(\phi^a, \mu^u) = 0$$

$$B \cdot F = \int_\Sigma B^{ab}(h_{ab} - \hat{h}_{ab}) \quad e^{b \frac{\partial F}{\partial g} c} \quad b \cdot \frac{\partial F}{\partial \mu} \int \det\left( \frac{\partial F^i}{\partial g^j}, \frac{\partial F^i}{\partial \mu^u} \right) \prod d\mu^u$$

$$b \cdot \frac{\partial F}{\partial \mu} = \iint_\Sigma b^{ab} \frac{\delta \hat{h}}{\delta \mu}$$


b · ∂F/∂μ = ∬_Σ b·b δQ/δμ
x^1 → λx^1
x^2 → λx^2
λ > 0
e^{B(x_1^2 + x_2^2 − 1)}
F(x_1, x_2) = x_1^2 + x_2^2 − 1
δF/δλ = 2(x_1^2 + x_2^2)
F_1 = x_1 − cos θ = 0
F_2 = x_2 − sin θ = 0
∫ dθ
det(∂F_1/∂θ ∂F_1/∂θ ; ∂F_1/∂λ ∂F_2/∂λ)

$\Sigma$

$$b \cdot \frac{\partial F}{\partial \mu} = \iint_{\Sigma} b \wedge b \, \frac{\delta \tilde{Q}}{\delta \mu}$$

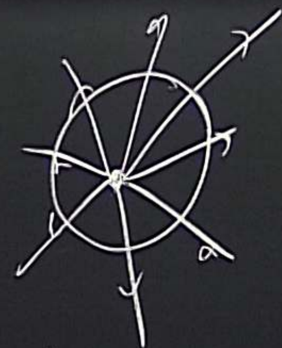
$$\int dx^1 \wedge dx^2$$

$x^1 \to \lambda x^1$

$x^2 \to \lambda x^2$

$\lambda > 0$

$$e^{B\left(x_1^2 + x_2^2 - 1\right)}$$



$$\boxed{F(x_1, x_2) = x_1^2 + x_2^2 - 1}$$

$$\frac{\delta F}{\delta \lambda} = 2\left(x_1^2 + x_2^2\right)$$

$$\int dx^1 \, dx^2 \; \delta\left((x^1)^2 + (x^2)^2 - x^2\right) \left(x_1^2 + x_2^2\right)$$

$= x_1$

$$\boxed{\begin{aligned} F_1 &= x_1 - \cos\theta = 0 \\ F_2 &= x_2 - \sin\theta = 0 \end{aligned}}$$

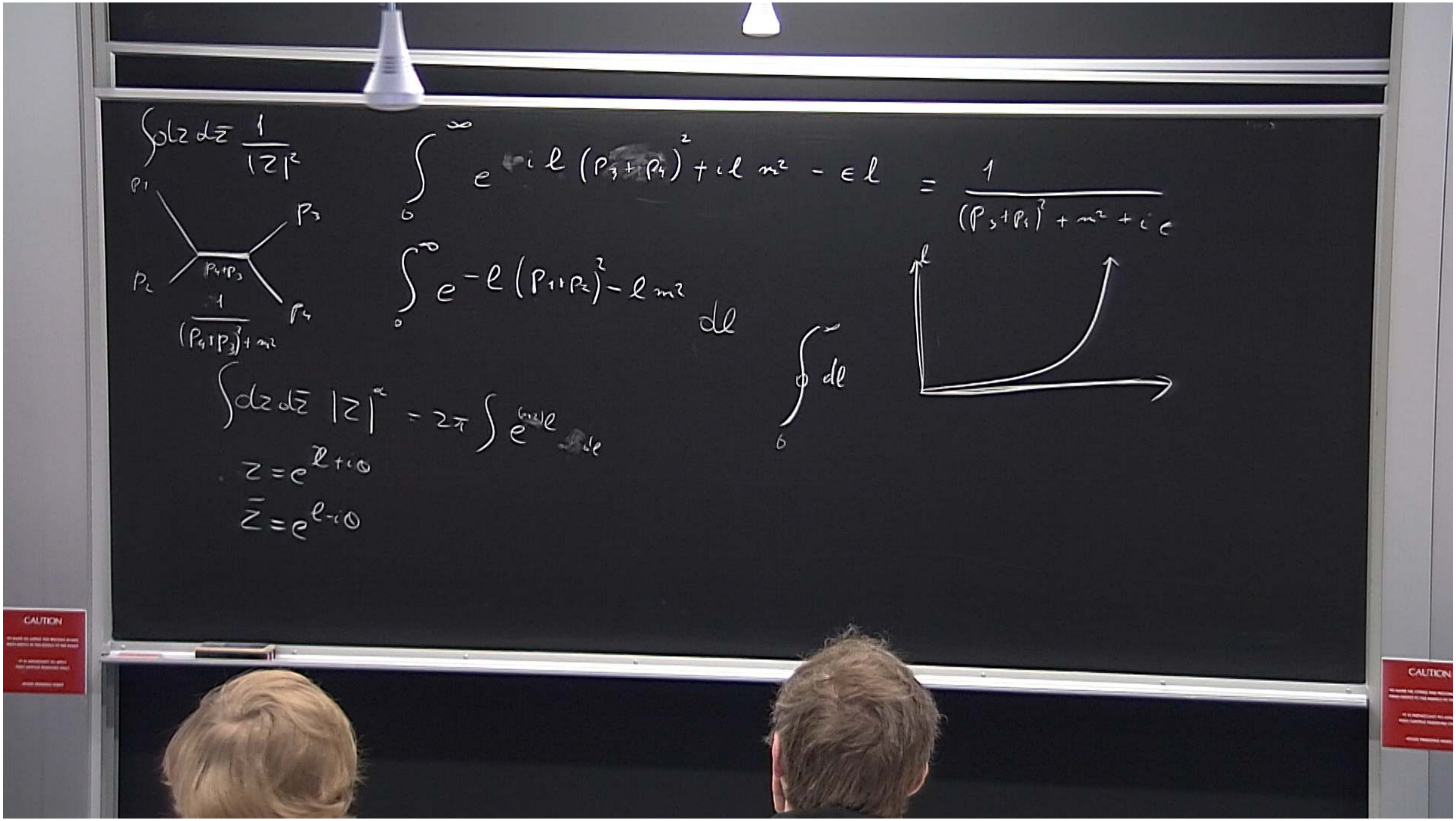
$$\int d\theta$$

$$\det \begin{pmatrix} \frac{\partial F_1}{\partial \theta} & \frac{\partial F_2}{\partial \theta} \\ \frac{\partial F_1}{\partial \lambda} & \frac{\partial F_2}{\partial \lambda} \end{pmatrix}$$

$$A_4 = \iint dz_4\, d\bar{z}_4 \left\langle c\bar{c}e^{ip_1 X}(z_1,\bar{z}_1)\; c\bar{c}e^{ip_2 X}(z_2,\bar{z}_2)\; c\bar{c}e^{ip_3 X}(z_3,\bar{z}_3)\; e^{ip_4 X}(z_4,\bar{z}_4)\right\rangle_{S^2}$$

$$= \prod_{1\le i<j\le 3} |z_i - z_j|^{2+2p_i\cdot p_j} \int d^2 z_4 \prod_{i=1}^{3} |z_4 - z_i|^{2p_4\cdot p_i}$$

$$\boxed{2p_4\cdot p_i > -2} \qquad (p_4+p_i)^2 \geq 2$$


∫dz dz̄ 1/|z|²
p1
p2
p3
p4
p4+p3
1/((p4+p3)² + m²)
∫₀^∞ e^{-iℓ(p3+p4)² + iℓm² - εℓ} = 1/((p3+p4)² + m² + iε)
∫₀^∞ e^{-ℓ(p1+p2)² - ℓm²} dℓ
∫₀^∞ dℓ
ℓ
∫dz dz̄ |z|^α = 2π ∫ e^{(α+2)ℓ} dℓ
z = e^{ℓ+iθ}
z̄ = e^{ℓ-iθ}
CAUTION
CAUTION

$$= \prod_{1\le i<j\le 3} |z_i - z_j|^{2+2p_i\cdot p_j} \iint d^2 z_4 \prod_{i=1}^{3} |z_4 - z_i|^{2p_4\cdot p_i}$$

$$\boxed{2p_4\cdot p_i > -2} \qquad (p_4+p_i)^2 \geq 2$$

$$s = -(p_1+p_2)^2 \quad \cdots$$

$$A_4 = \prod_{i=1}^{3} \frac{\Gamma(1+p_4\cdot p_i)}{\Gamma(-p_4\cdot p_i)} = \frac{\Gamma\left(-1-\frac{s}{2}\right)\Gamma\left(-1-\frac{t}{2}\right)\Gamma\left(-1-\frac{u}{2}\right)}{\Gamma\left(2+\frac{s}{2}\right)\Gamma\left(2+\frac{t}{2}\right)\Gamma\left(2+\frac{u}{2}\right)}$$

$$\frac{s}{2} = -1,\ 0,\ +1,\ +2,\ +3 \cdots \qquad (p_1+p_2)^2 = 2,\ 0,\ -2,\ -4,\ -6 \cdots$$

$$\operatorname*{Res}_{p_4\cdot p_1=-n} A_4 = \prod_{k=0}^{n-2} (p_4\cdot p_2 - k)(p_4\cdot p_3 - k)$$

$$\int dz\, d\bar{z}\, |z| = 2\pi \int e^{(\cdot+2)\ell}\, d\ell$$

$$z = e^{\ell + i\theta}$$

$$\bar{z} = e^{\ell - i\theta}$$

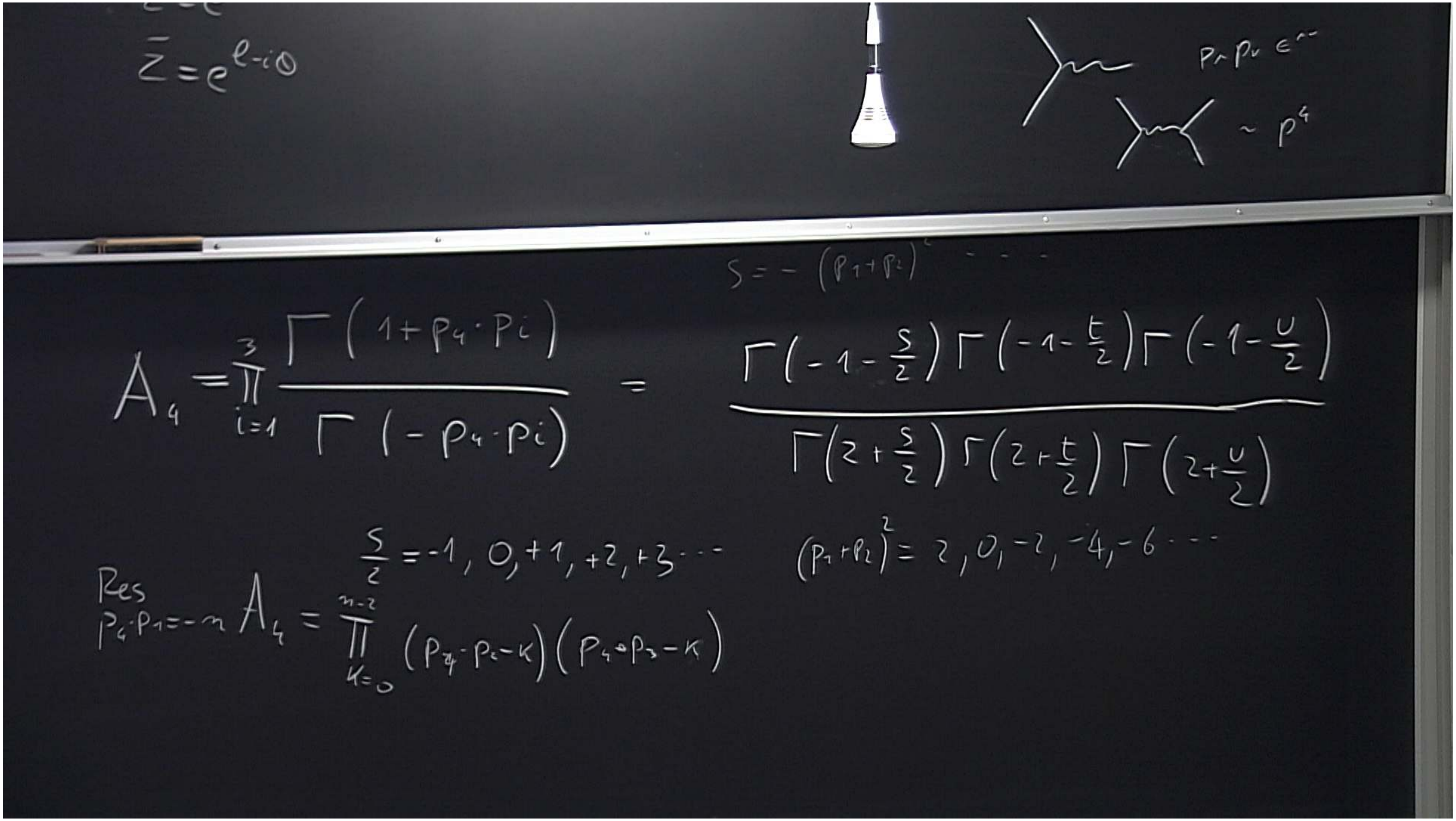
$p_\mu p_\nu \epsilon^{\mu\nu}$

$s = -(p_1+p_2)^2$

$$A_4 = \prod_{i=1}^{3} \frac{\Gamma(1 + p_4 \cdot p_i)}{\Gamma(-p_4 \cdot p_i)} = \frac{\Gamma\left(-1-\frac{s}{2}\right)\Gamma\left(-1-\frac{t}{2}\right)\Gamma\left(-1-\frac{u}{2}\right)}{\Gamma\left(2+\frac{s}{2}\right)\Gamma\left(2+\frac{t}{2}\right)\Gamma\left(2+\frac{u}{2}\right)}$$

$$\frac{s}{2} = -1, 0, +1, +2, +3 \cdots \qquad (p_1+p_2)^2 = 2, 0, -2, -4, -6 \cdots$$

$$\operatorname*{Res}_{p_4 \cdot p_1 = -n} A_4 = \prod_{k=0}^{n-2} (p_4 \cdot p_2 - k)(p_4 \cdot p_3 - k)$$


$\bar{z} = e^{\ell - i\theta}$
$p_\mu p_\nu \epsilon^{\mu\nu}$
$\sim p^4$
$s = -(p_1+p_2)^2$
$A_4 = \prod_{i=1}^{3} \frac{\Gamma(1+p_4\cdot p_i)}{\Gamma(-p_4\cdot p_i)} = \frac{\Gamma(-1-\frac{s}{2})\Gamma(-1-\frac{t}{2})\Gamma(-1-\frac{u}{2})}{\Gamma(2+\frac{s}{2})\Gamma(2+\frac{t}{2})\Gamma(2+\frac{u}{2})}$
$\frac{s}{2} = -1, 0, +1, +2, +3 \cdots$
$(p_1+p_2)^2 = 2, 0, -2, -4, -6 \cdots$
$\mathrm{Res}_{p_4\cdot p_1 = -n} A_4 = \prod_{k=0}^{n-2} (p_4\cdot p_2 - k)(p_4\cdot p_3 - k)$

$$\int_\Sigma \sqrt{\det_{(a,b)} \frac{\partial X^\mu}{\partial u^a} \frac{\partial X^\nu}{\partial u^b} G_{\mu\nu}[X]}$$

$$\int \partial_a X^\mu \partial^a X^\nu G_{\mu\nu}[X]\, du_1 du_2 + \int \partial_{[1} X^\mu \partial_{2]} X^\nu B_{\mu\nu}[X]$$

$$+ \int T(X)\, du_1 du_2$$