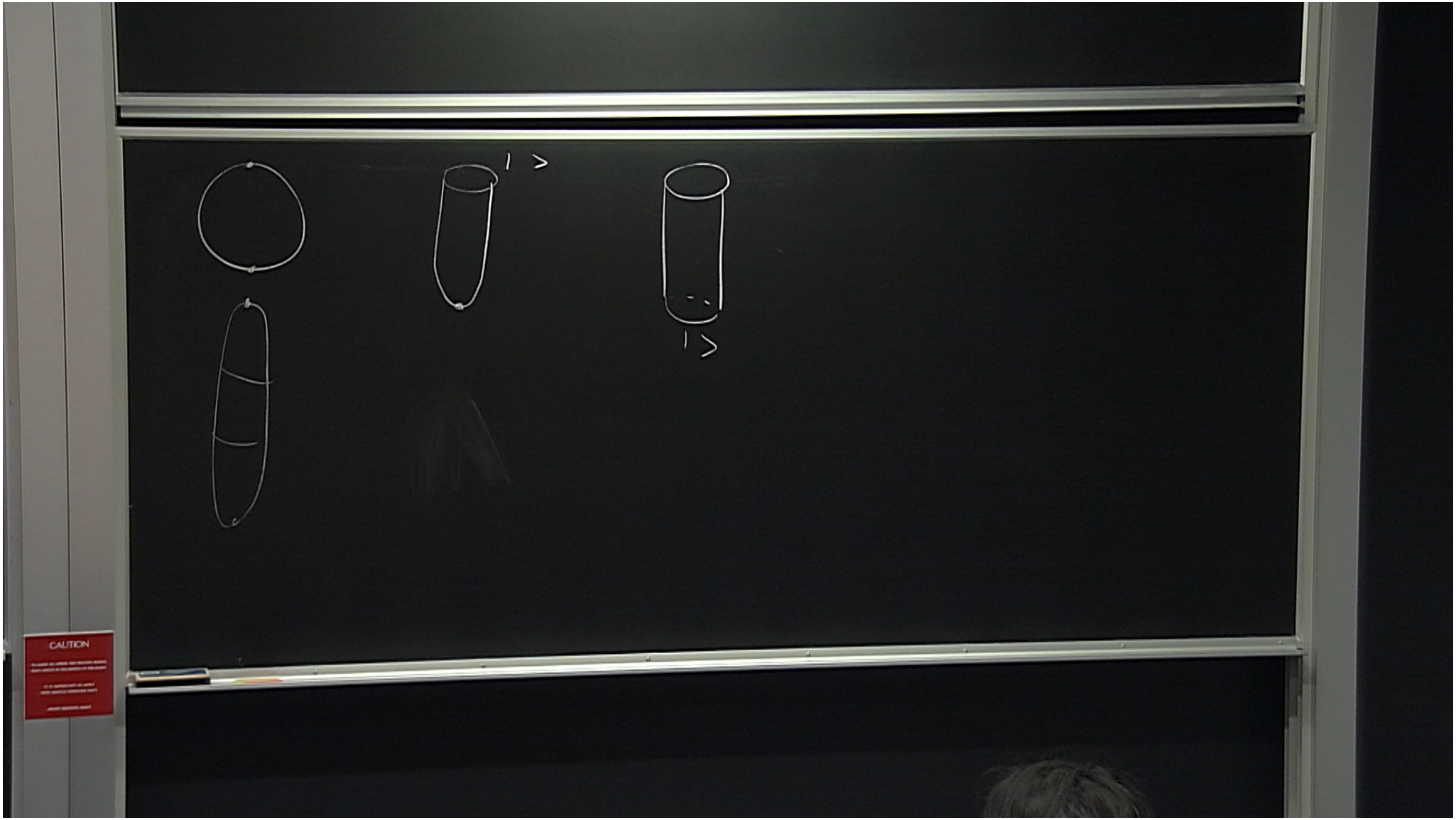
Title: PSI 2018/2019 - String Theory Review - Lecture 8

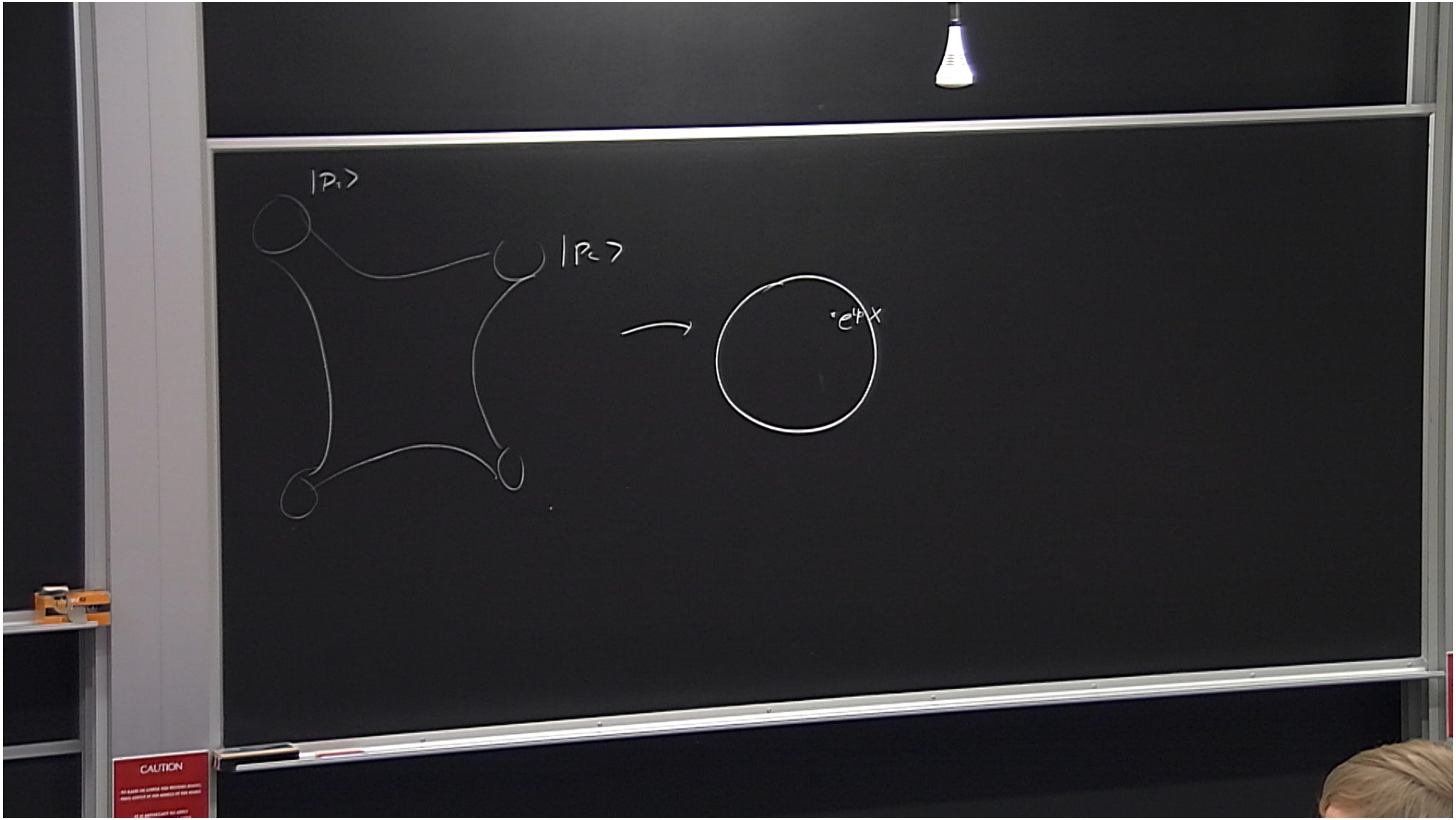
Speakers: Davide Gaiotto

Collection: PSI 2018/2019 - String Theory Review (Gaiotto)

Date: April 03, 2019 - 9:00 AM

URL: http://pirsa.org/19040033


CAUTION


|P1>
|P2>
•e^{ip·X}
CAUTION

$$\langle e^{ip_1 X(z_1,\bar{z}_1)} e^{ip_2 X(z_2,\bar{z}_2)} \cdots e^{ip_m X(z_m,\bar{z}_m)} \rangle_{S^2} =$$

$$= \prod_{i<j} |z_i - z_j|^{2p_i p_j} \delta(\Sigma p_i)$$

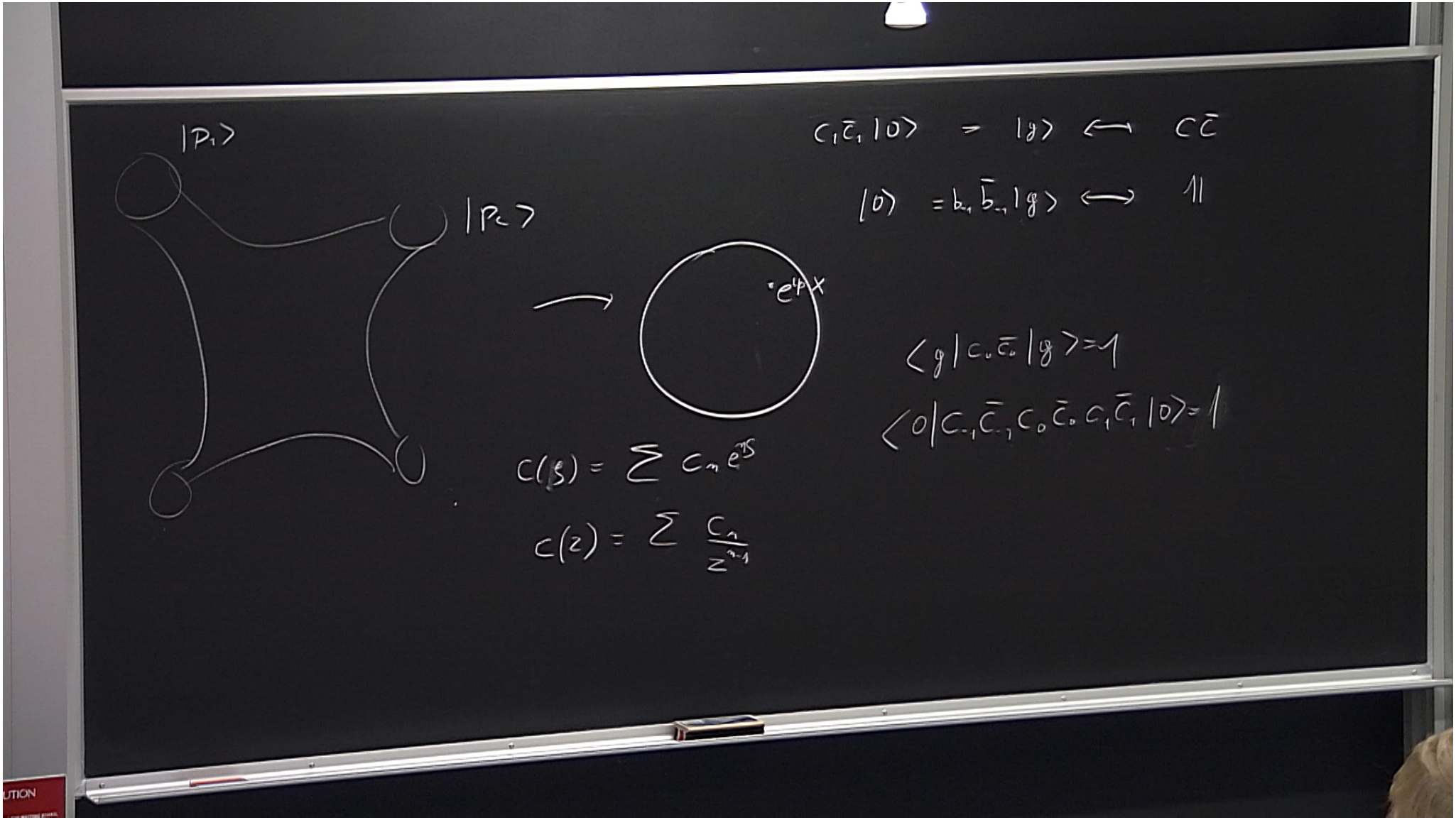
$$z \to \frac{az'+b}{cz'+d} \qquad z_i \to \frac{az_i+b}{cz_i+d} \qquad z_i - z_j = \frac{z_i' - z_j'}{(cz_i'+d)(cz_j'+d)}$$

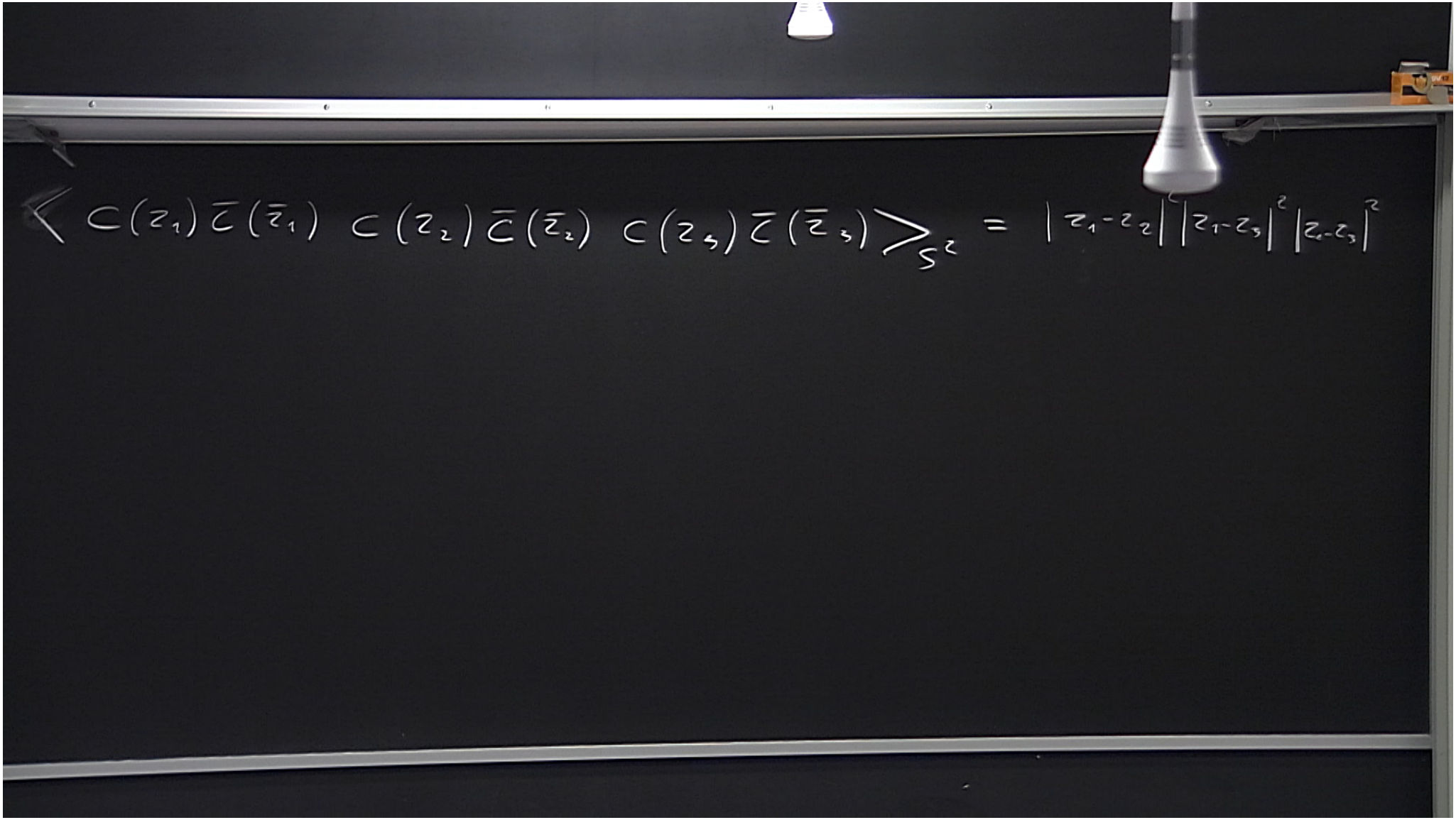
$$ad - bc = 1$$

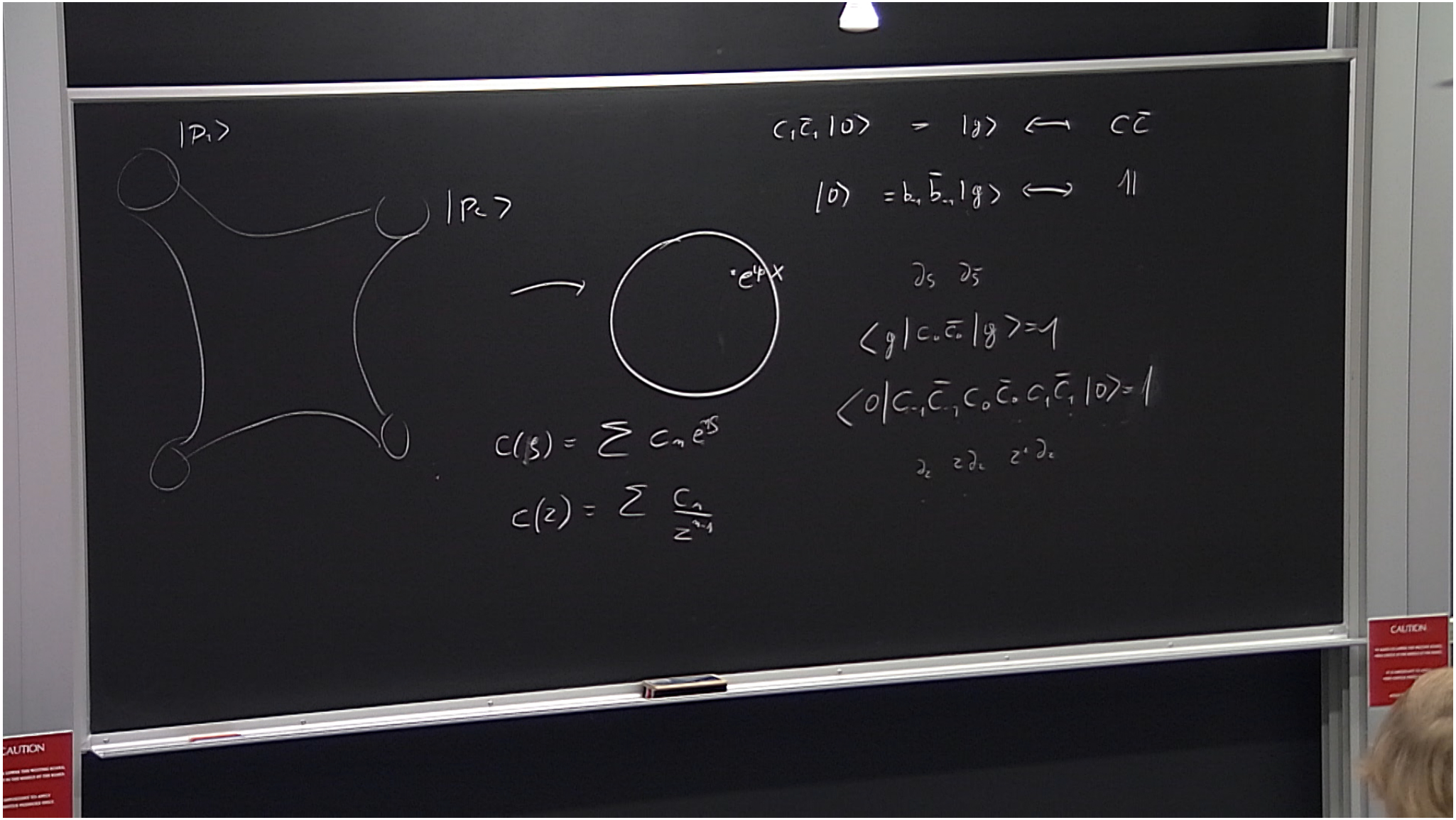
$$\frac{\partial z}{\partial z'} = \frac{1}{(cz'+d)^2}$$

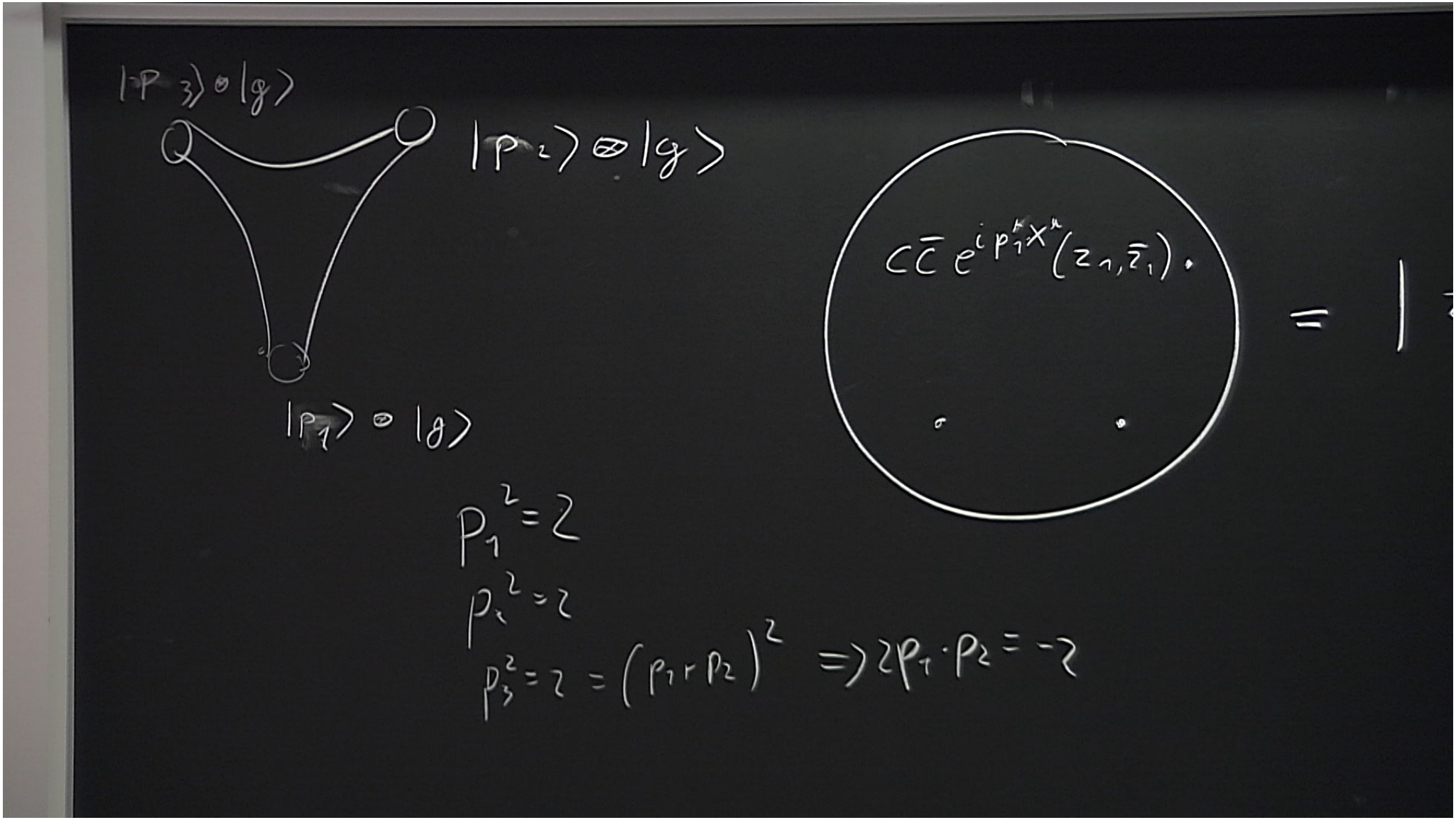
$$e^{ipX(z)} \to \left|\frac{\partial z'}{\partial z}\right|^{p^2} e^{ipX(z')}$$

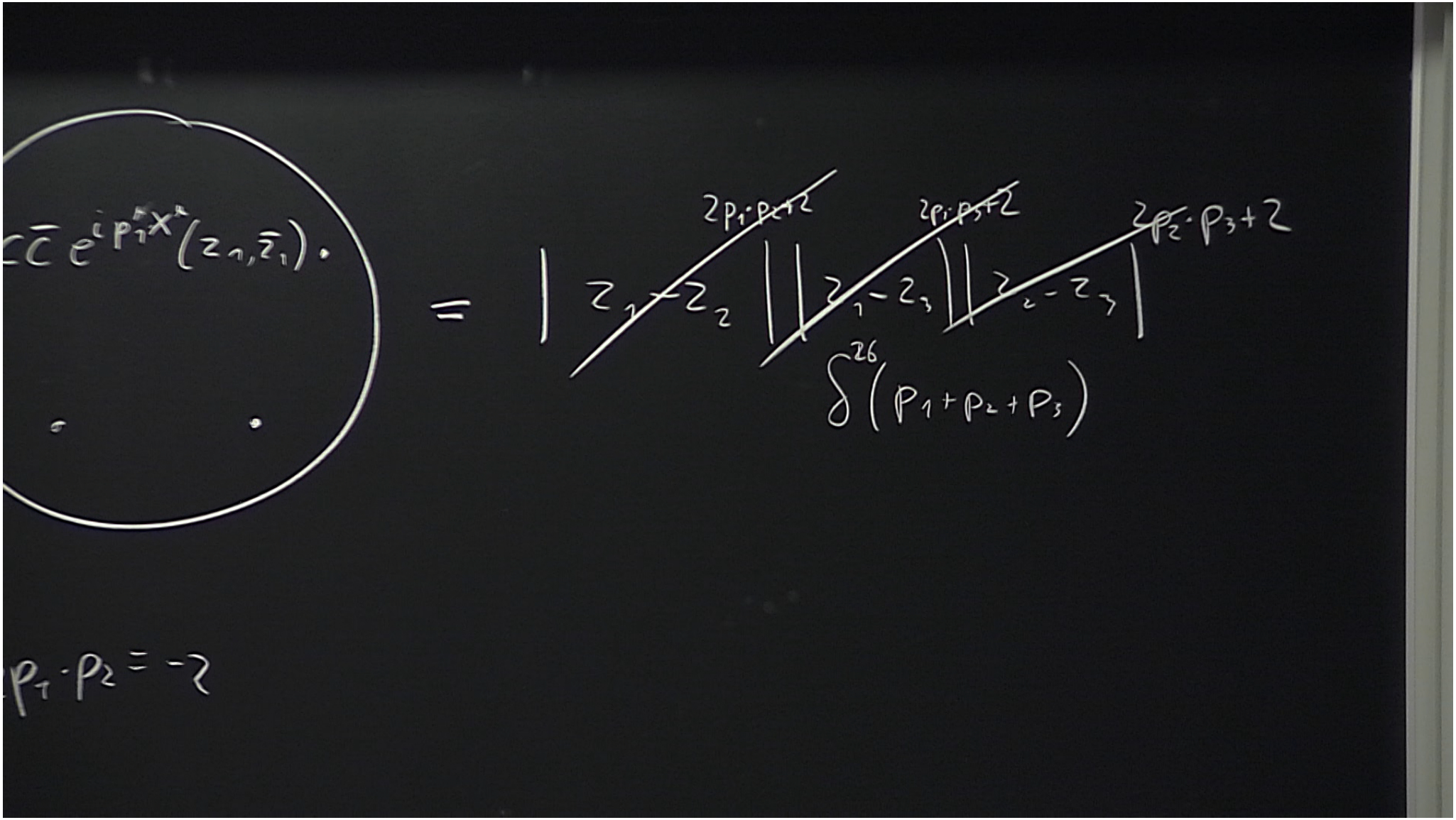
$$|cz_i+d|^{-2p_i}$$

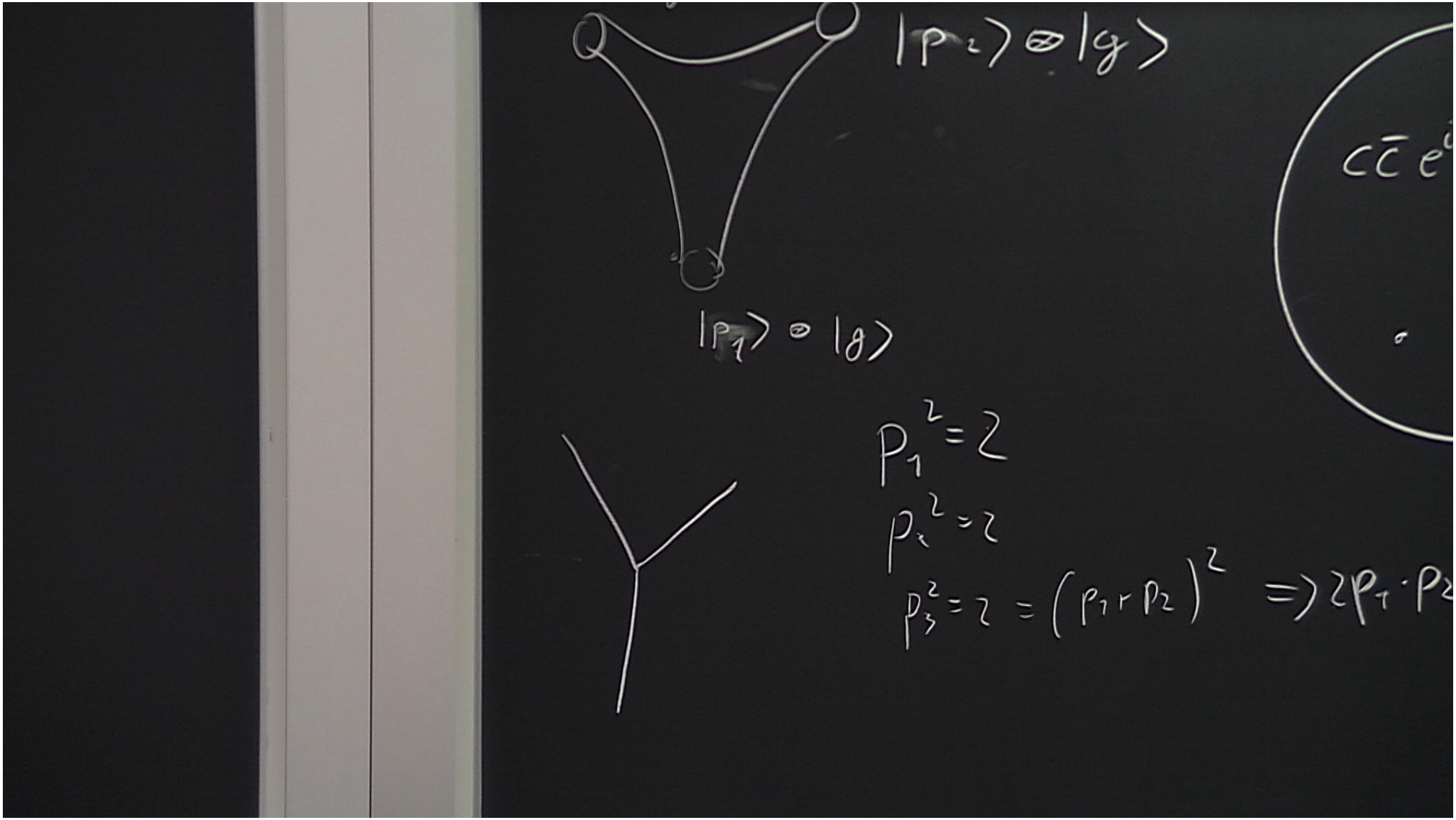

$|P_1\rangle$
$|P_2\rangle$
$e^{ip\cdot X}$
$c_1\bar{c}_1|0\rangle = |g\rangle \longleftrightarrow c\bar{c}$
$|0\rangle = b_{-1}\bar{b}_{-1}|g\rangle \longleftrightarrow \mathbb{1}$
$\langle g|c_0\bar{c}_0|g\rangle = 1$
$\langle 0|c_{-1}\bar{c}_{-1}c_0\bar{c}_0c_1\bar{c}_1|0\rangle = 1$
$c(s) = \sum c_n e^{ns}$
$c(z) = \sum \frac{c_n}{z^{n-1}}$

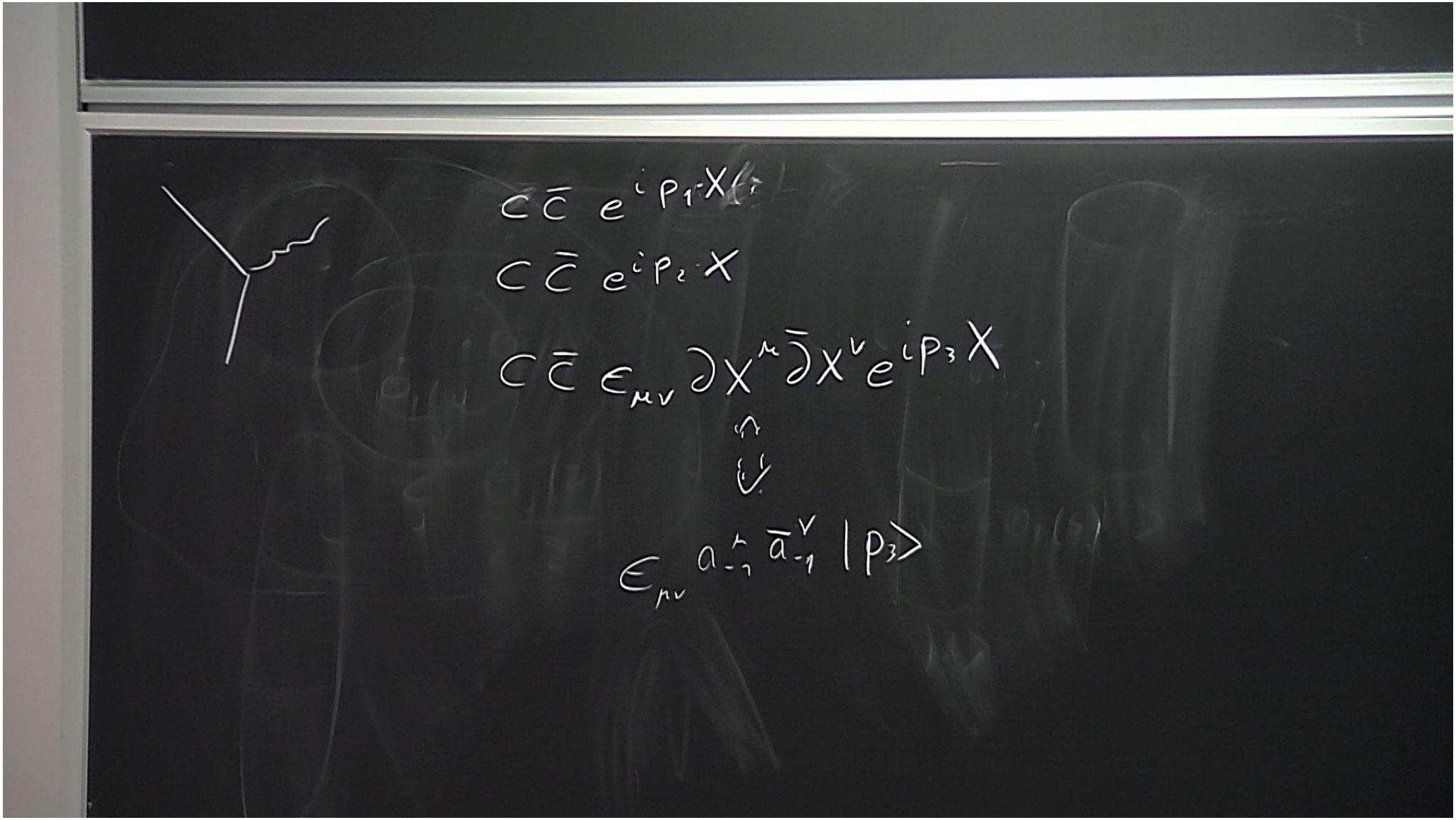

$\langle C(z_1)\bar{C}(\bar{z}_1)\ C(z_2)\bar{C}(\bar{z}_2)\ C(z_3)\bar{C}(\bar{z}_3)\rangle_{S^2} = |z_1-z_2|^2 |z_1-z_3|^2 |z_2-z_3|^2$


$|P_1\rangle$
$|P_2\rangle$
$e^{4\pi X}$
$c(\zeta) = \sum c_n e^{n\zeta}$
$c(z) = \sum \frac{c_n}{z^{n-1}}$
$c_1\bar{c}_1|0\rangle = |g\rangle \longleftrightarrow c\bar{c}$
$|0\rangle = b_{-1}\bar{b}_{-1}|g\rangle \longleftrightarrow 1\!\!1$
$\partial_\zeta \quad \partial_{\bar{\zeta}}$
$\langle g|c_0\bar{c}_0|g\rangle = 1$
$\langle 0|c_{-1}\bar{c}_{-1}c_0\bar{c}_0c_1\bar{c}_1|0\rangle = 1$
$\partial_z \quad z\partial_z \quad z^2\partial_z$


$|p_3\rangle \otimes |g\rangle$
$|p_2\rangle \otimes |g\rangle$
$|p_1\rangle \otimes |g\rangle$
$c\bar{c}\, e^{i p_1^\mu X^\mu}(z_1, \bar{z}_1)$ .
=
$p_1^2 = 2$
$p_2^2 = 2$
$p_3^2 = 2 = (p_1 + p_2)^2 \Rightarrow 2 p_1 \cdot p_2 = -2$


$c\bar{c}\, e^{ip_1 X}(z_1,\bar{z}_1)\cdot$
$= |z_1 - z_2|^{2p_1\cdot p_2+2}\,|z_1 - z_3|^{2p_1\cdot p_3+2}\,|z_2 - z_3|^{2p_2\cdot p_3+2}$
$\delta^{26}(p_1+p_2+p_3)$
$p_1\cdot p_2 = -2$


$|p_2\rangle \otimes |g\rangle$
$|p_1\rangle \otimes |g\rangle$
$c\bar{c}\,e$
$p_1^2 = 2$
$p_2^2 = 2$
$p_3^2 = 2 = (p_1 + p_2)^2 \Rightarrow 2p_1 \cdot p_2$


$c\bar{c}\, e^{ip_1\cdot X}$
$c\bar{c}\, e^{ip_2\cdot X}$
$c\bar{c}\,\epsilon_{\mu\nu}\partial X^\mu \bar{\partial} X^\nu e^{ip_3 X}$
$\epsilon_{\mu\nu}\, a_{-1}^\mu \bar{a}_{-1}^\nu |p_3\rangle$

$$\langle e^{ip_1 X(z_1,\bar{z}_1)} e^{ip_2 X(z_2,\bar{z}_2)} (\partial X \bar{\partial} X e^{ip_3 X})_{(z_3,\bar{z}_3)} \rangle$$

$$\left(\frac{p_1}{z_1 - z_3} + \frac{p_2}{z_2 - z_3}\right)\left(\frac{p_1}{\bar{z}_1 - \bar{z}_3} + \frac{p_2}{\bar{z}_2 - \bar{z}_3}\right)$$

$$|z_1 - z_2|^{2p_1 \cdot p_2}$$

$p_3^2 = 2$



$c\bar{c}\, e^{i p_1 \cdot X(z_1)}$

$c\bar{c}\, e^{i p_2 \cdot X}$

$c\bar{c}\, \epsilon_{\mu\nu}\, \partial X^\mu \bar{\partial} X^\nu e^{i p_3 X}$

$\Downarrow$

$\epsilon_{\mu\nu}\, a_{-1}^\mu \bar{a}_{-1}^\nu |p_3\rangle$

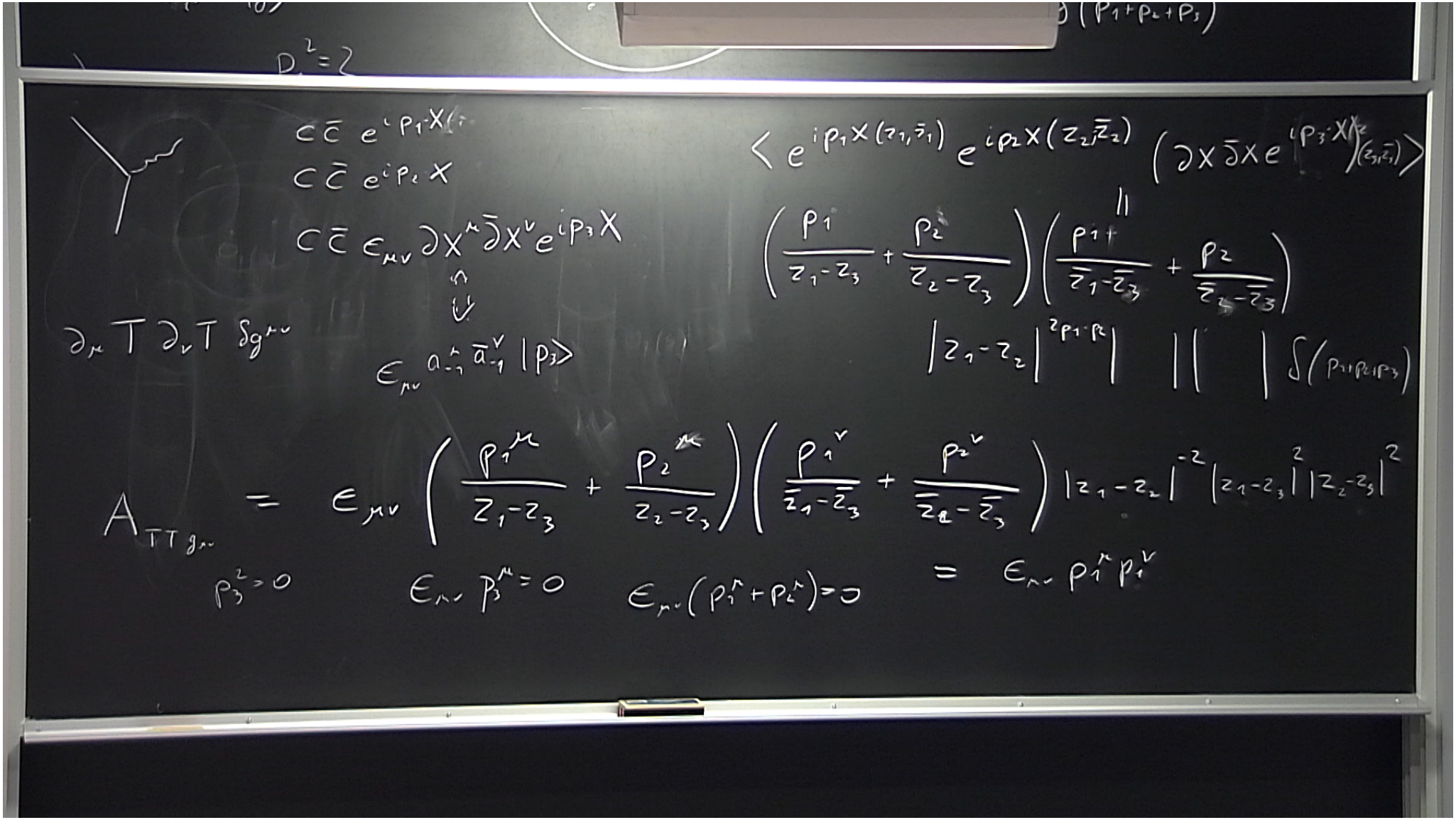
$\partial_\mu T\, \partial_\nu T\, \delta g^{\mu\nu}$

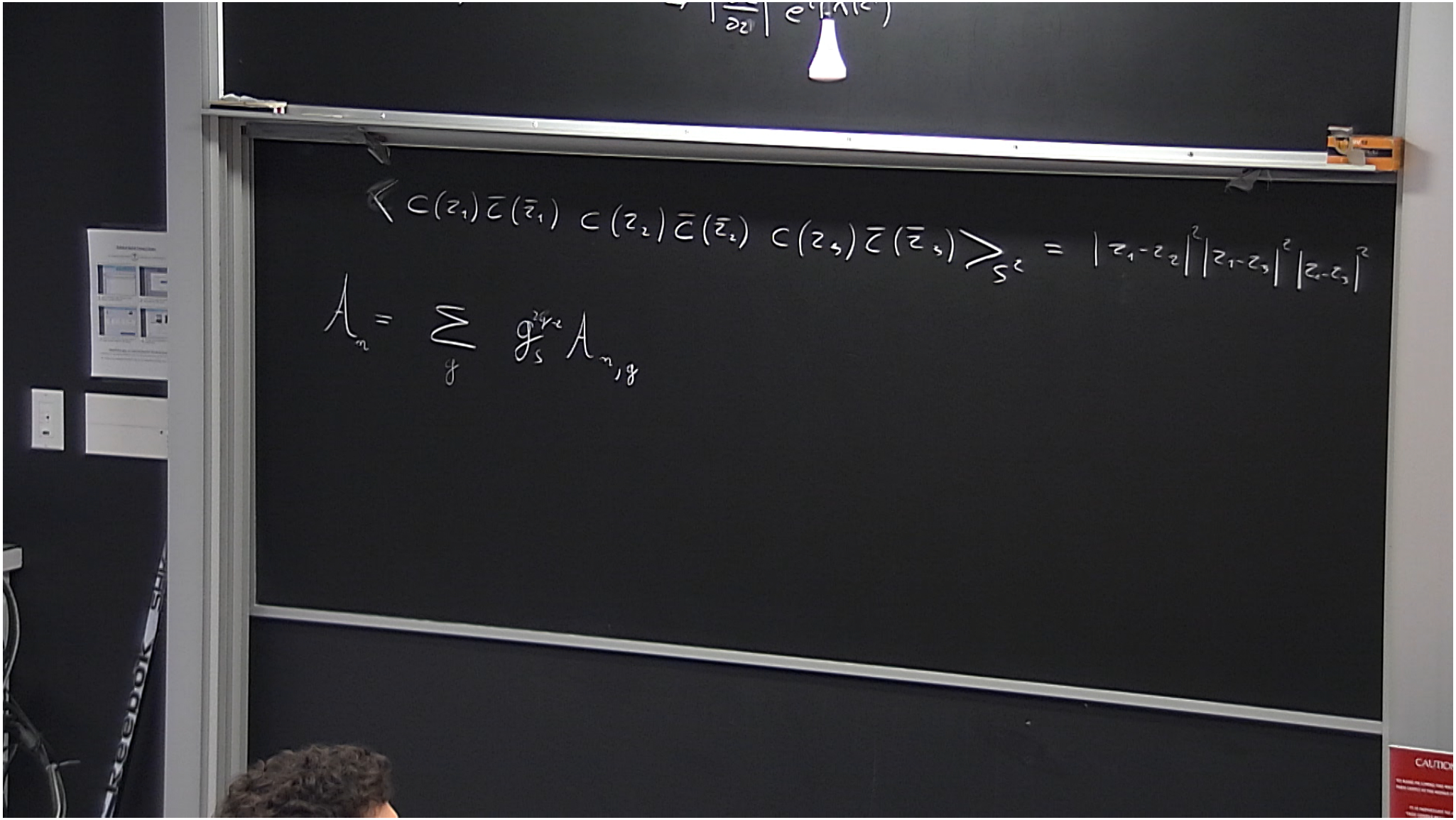
$\langle e^{i p_1 X(z_1,\bar{z}_1)}\, e^{i p_2 X(z_2,\bar{z}_2)}\, (\partial X \bar{\partial} X e^{i p_3 X})(z_3,\bar{z}_3) \rangle$

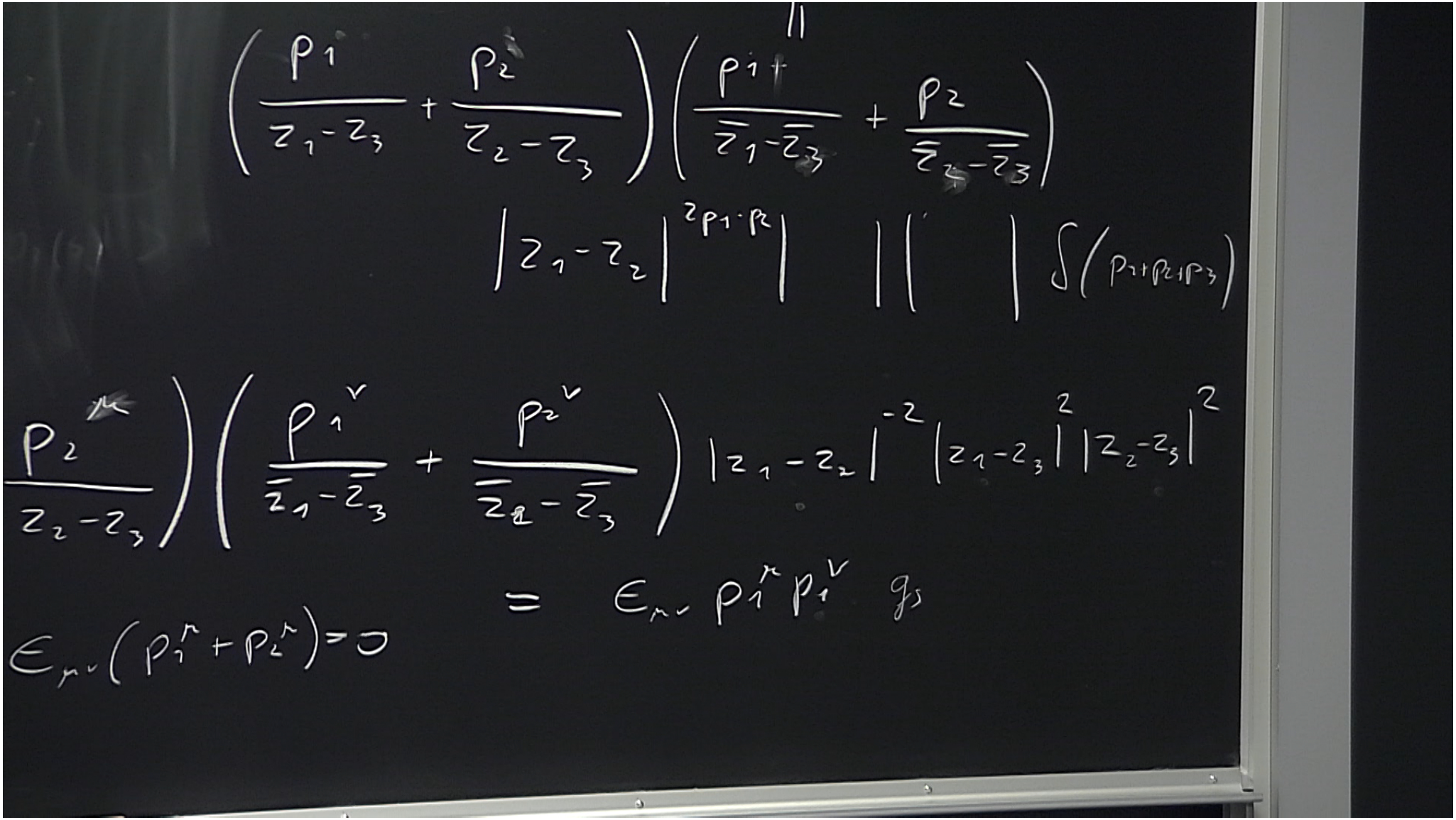
$\left(\frac{p_1}{z_1 - z_3} + \frac{p_2}{z_2 - z_3}\right)\left(\frac{p_1}{\bar{z}_1 - \bar{z}_3} + \frac{p_2}{\bar{z}_2 - \bar{z}_3}\right) |z_1 - z_2|^{2 p_1 \cdot p_2} \,||\, \delta(p_1 + p_2 + p_3)$

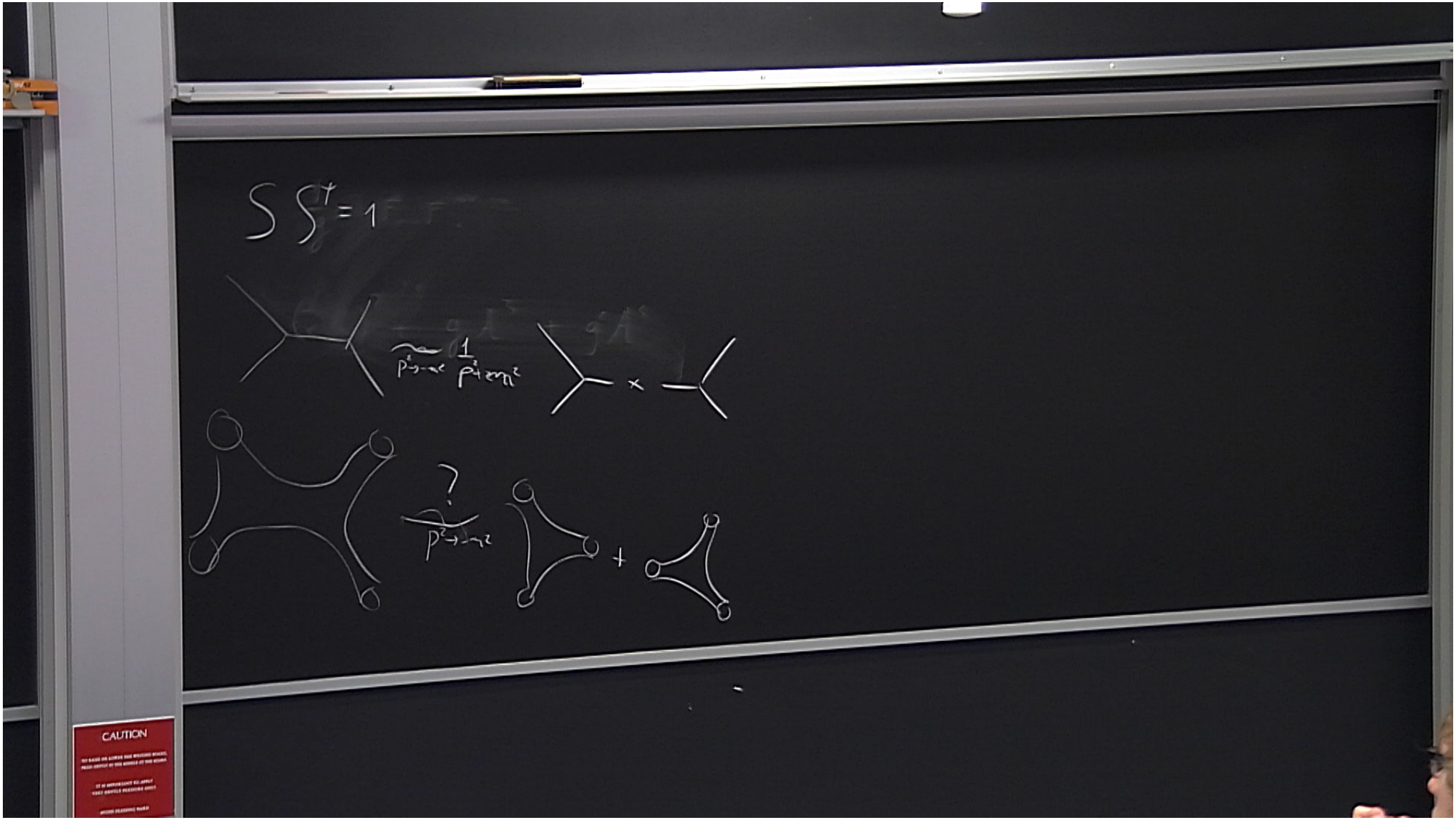
$A_{TT\, g_{\mu\nu}} = \epsilon_{\mu\nu} \left(\frac{p_1^\mu}{z_1 - z_3} + \frac{p_2^\mu}{z_2 - z_3}\right)\left(\frac{p_1^\nu}{\bar{z}_1 - \bar{z}_3} + \frac{p_2^\nu}{\bar{z}_2 - \bar{z}_3}\right) |z_1 - z_2|^{-2} |z_1 - z_3|^2 |z_2 - z_3|^2$

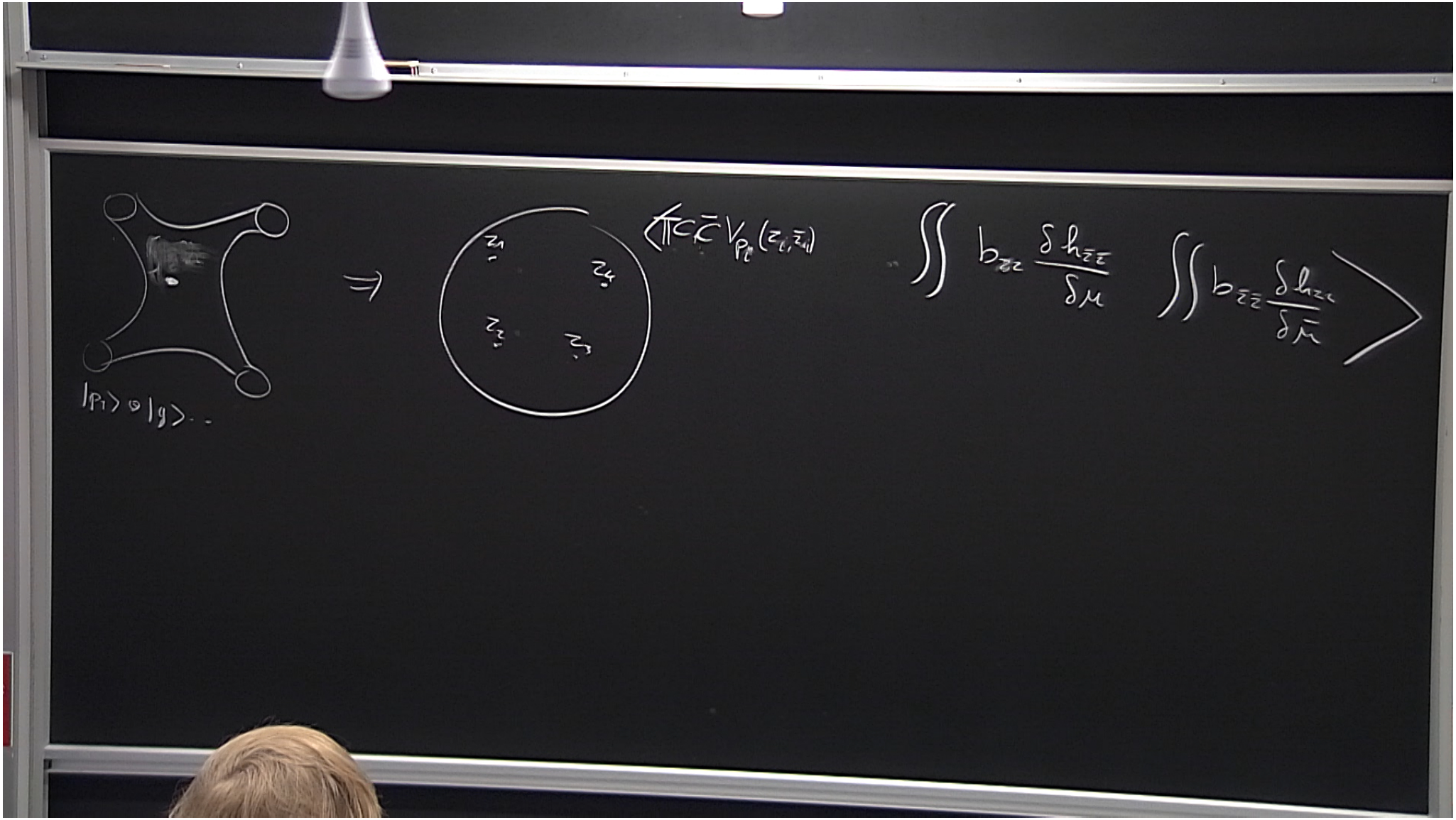
$p_3^2 = 0 \qquad \epsilon_{\mu\nu}\, p_3^\mu = 0 \qquad \epsilon_{\mu\nu}(p_1^\mu + p_2^\mu) = 0 \qquad = \epsilon_{\mu\nu}\, p_1^\mu p_1^\nu$


$D^2 = 2$
$c\bar{c}\, e^{ip_1 X}$
$c\bar{c}\, e^{ip_2 X}$
$c\bar{c}\, \epsilon_{\mu\nu} \partial X^\mu \bar{\partial} X^\nu e^{ip_3 X}$
$\partial_\mu T\, \partial_\nu T\, \delta g^{\mu\nu}$
$\epsilon_{\mu\nu}\, a_{-1}^\mu \bar{a}_{-1}^\nu |p_3\rangle$
$\langle e^{ip_1 X(z_1,\bar{z}_1)}\, e^{ip_2 X(z_2,\bar{z}_2)}\, (\partial X \bar{\partial} X e^{ip_3 X})_{(z_3,\bar{z}_3)} \rangle$
$\left(\frac{p_1}{z_1 - z_3} + \frac{p_2}{z_2 - z_3}\right)\left(\frac{p_1}{\bar{z}_1 - \bar{z}_3} + \frac{p_2}{\bar{z}_2 - \bar{z}_3}\right)$
$|z_1 - z_2|^{2p_1 \cdot p_2}$
$\delta(p_1 + p_2 + p_3)$
$A_{TT\, g_{\mu\nu}} = \epsilon_{\mu\nu} \left(\frac{p_1^\mu}{z_1 - z_3} + \frac{p_2^\mu}{z_2 - z_3}\right)\left(\frac{p_1^\nu}{\bar{z}_1 - \bar{z}_3} + \frac{p_2^\nu}{\bar{z}_2 - \bar{z}_3}\right) |z_1 - z_2|^{-2} |z_1 - z_3|^2 |z_2 - z_3|^2$
$p_3^2 = 0$
$\epsilon_{\mu\nu}\, p_3^\mu = 0$
$\epsilon_{\mu\nu}(p_1^\mu + p_2^\mu) = 0$
$= \epsilon_{\mu\nu}\, p_1^\mu p_1^\nu$

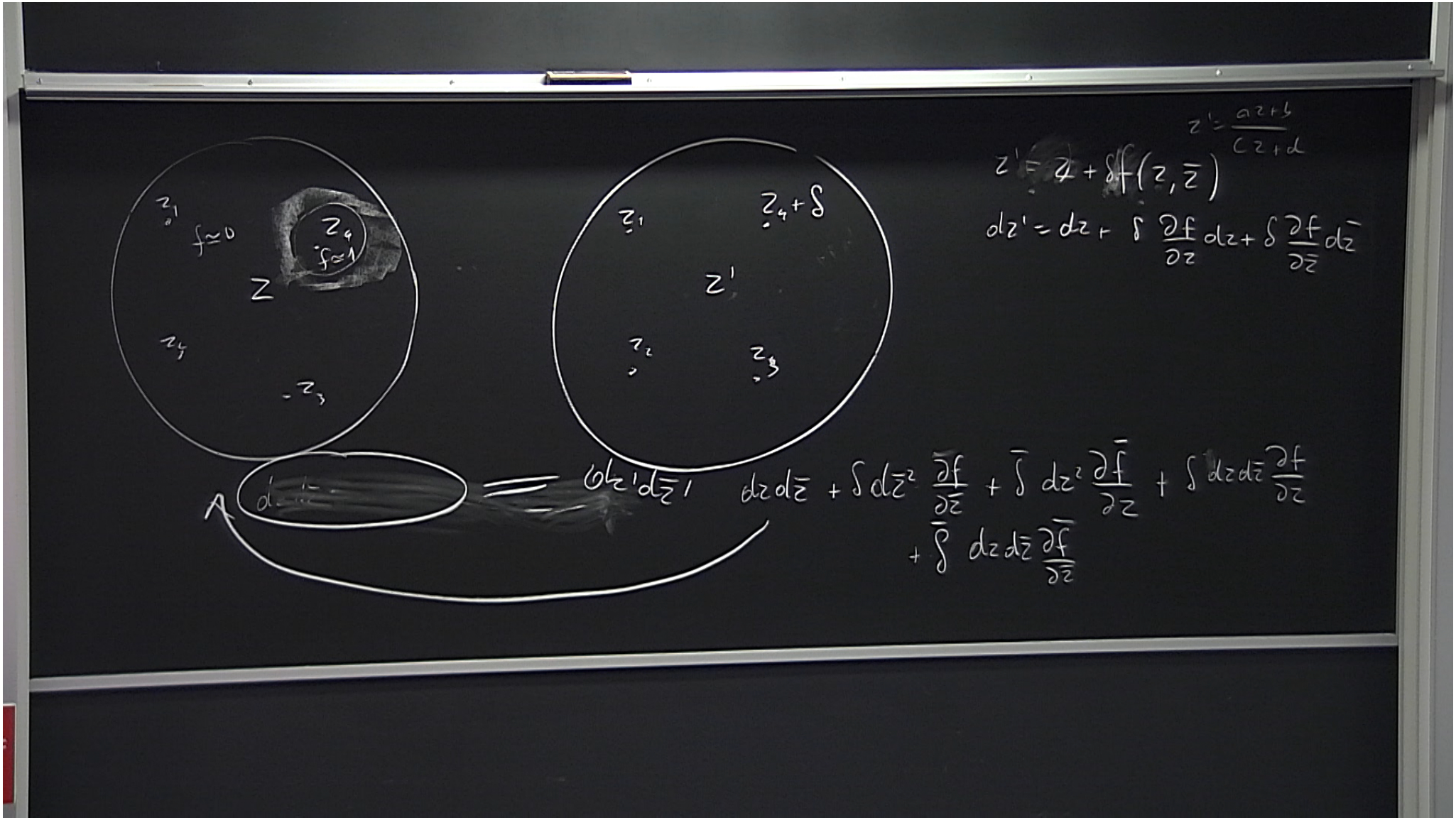

$\langle c(z_1)\bar{c}(\bar{z}_1)\, c(z_2)\bar{c}(\bar{z}_2)\, c(z_3)\bar{c}(\bar{z}_3)\rangle_{S^2} = |z_1-z_2|^2 |z_1-z_3|^2 |z_2-z_3|^2$
$\mathcal{A}_n = \sum_g g_s^{2g+n-2} \mathcal{A}_{n,g}$

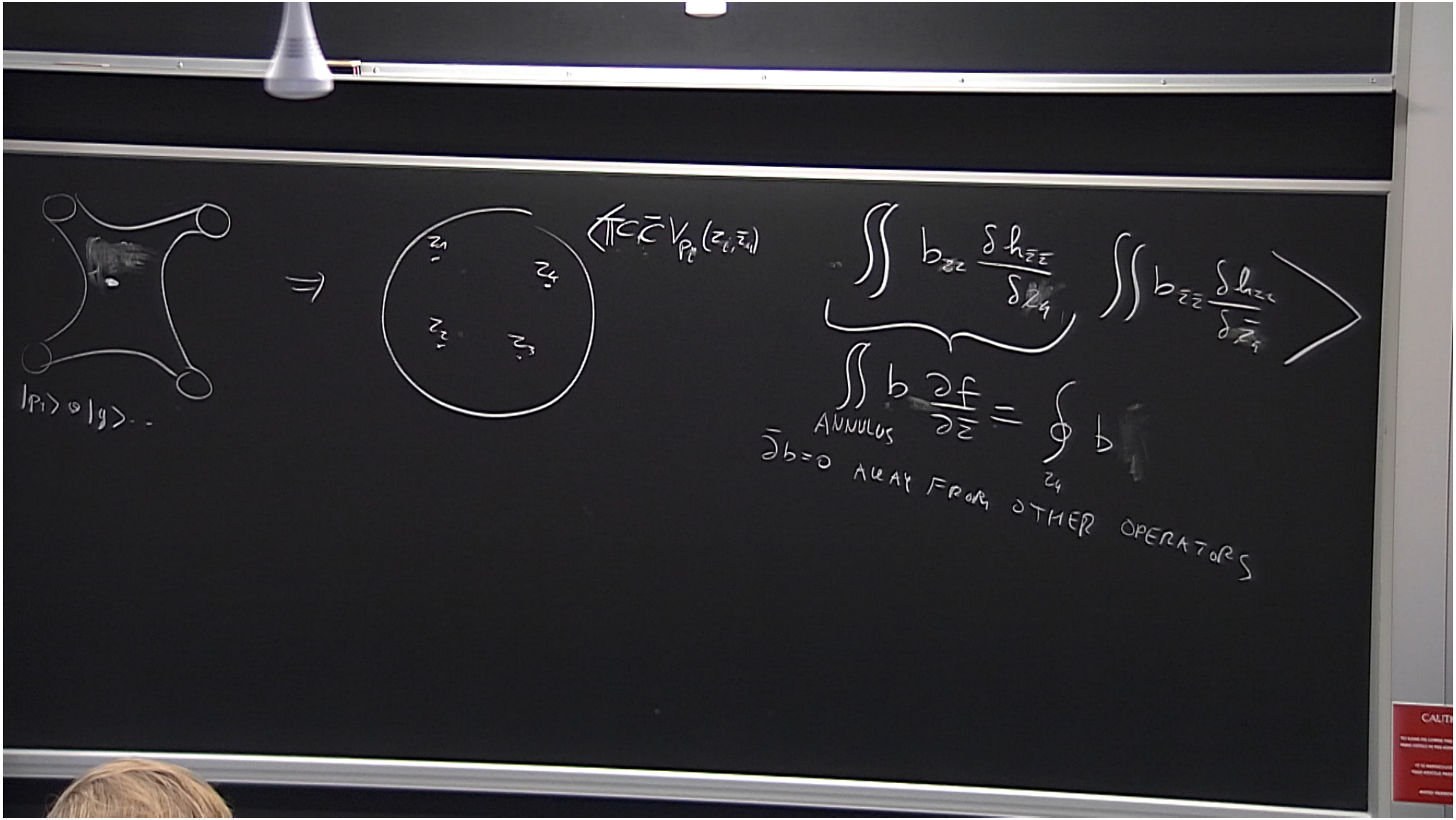

$\left(\frac{p_1}{z_1-z_3}+\frac{p_2}{z_2-z_3}\right)\left(\frac{p_1}{\bar{z}_1-\bar{z}_3}+\frac{p_2}{\bar{z}_2-\bar{z}_3}\right)$
$|z_1-z_2|^{2p_1\cdot p_2}$
$\delta(p_1+p_2+p_3)$
$\left(\frac{p_2}{z_2-z_3}\right)\left(\frac{p_1}{\bar{z}_1-\bar{z}_3}+\frac{p_2}{\bar{z}_2-\bar{z}_3}\right)|z_1-z_2|^{-2}|z_1-z_3|^2|z_2-z_3|^2$
$= \epsilon_{\mu\nu}\, p_1^\mu p_1^\nu\, g_3$
$\epsilon_{\mu\nu}(p_1^\mu+p_2^\mu)=0$

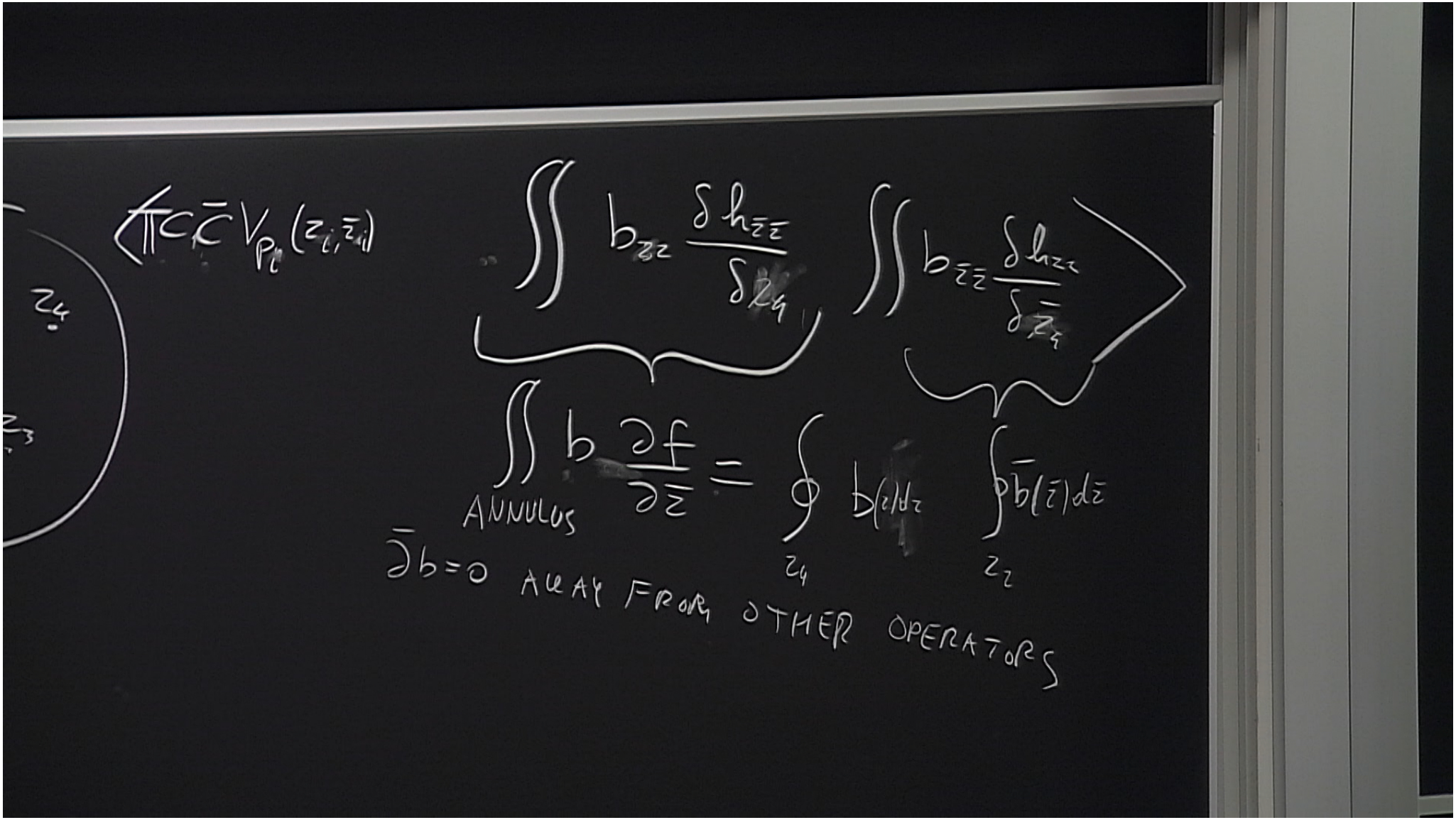

$S S^\dagger = 1$
$\frac{1}{p^2 + m^2}$
?
$p^2 + m^2$
CAUTION

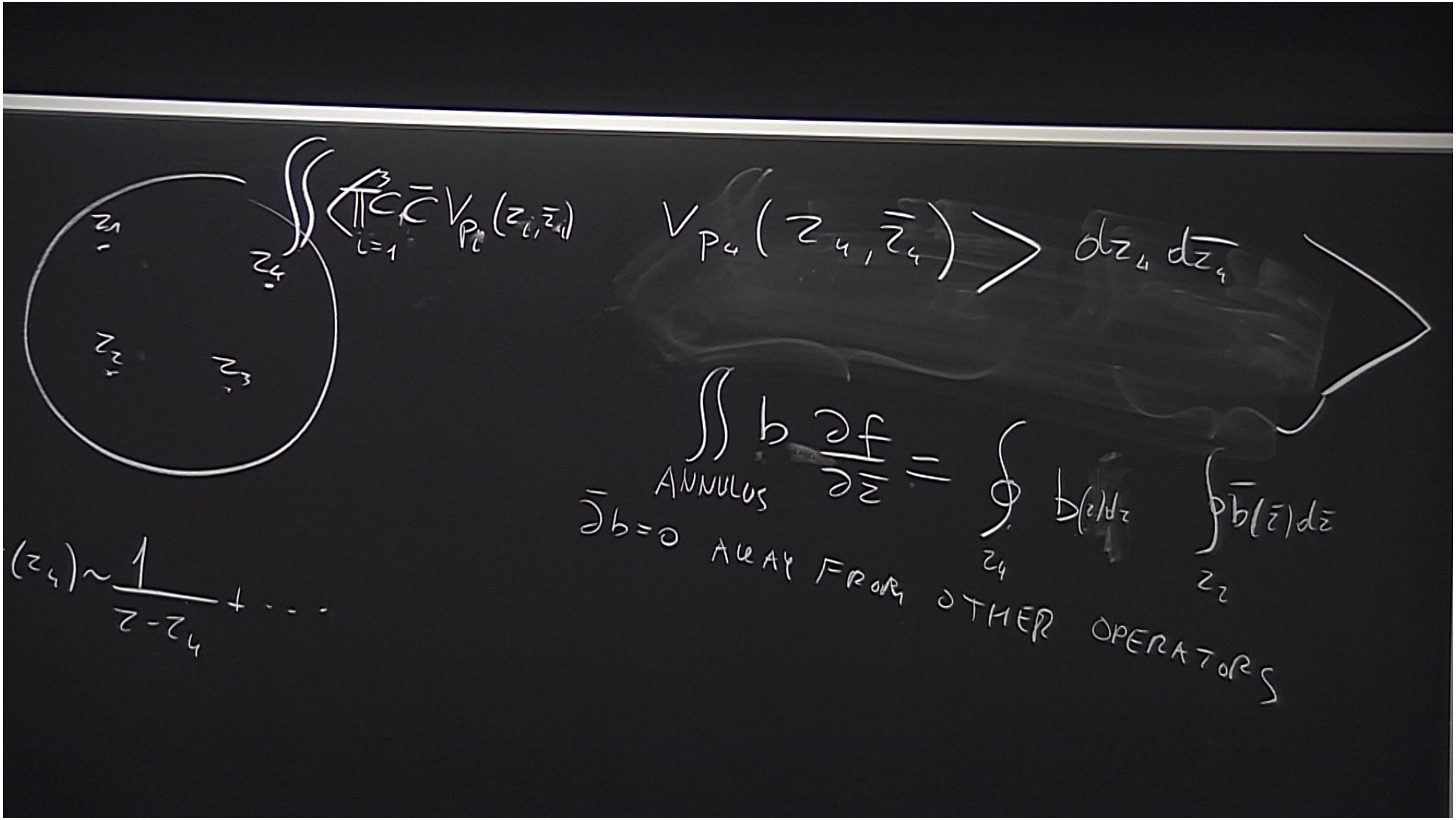

$|p_1\rangle \otimes |g\rangle \ldots$
$\Rightarrow$
$z_1$
$z_4$
$z_2$
$z_3$
$\langle \prod c\bar{c} V_{p_i}(z_i, \bar{z}_i)$
$\iint b_{zz} \frac{\delta h_{\bar{z}\bar{z}}}{\delta \mu}$
$\iint b_{\bar{z}\bar{z}} \frac{\delta h_{zz}}{\delta \bar{\mu}}$

$z' = z + \delta f(z, \bar{z})$
$dz' = dz + \delta \frac{\partial f}{\partial z} dz + \delta \frac{\partial f}{\partial \bar{z}} d\bar{z}$
$dz\, d\bar{z}$
$dz\, d\bar{z} + \delta \frac{\partial f}{\partial z} dz\, d\bar{z} + \delta \frac{\partial \bar{f}}{\partial \bar{z}} dz\, d\bar{z} + \delta \frac{\partial f}{\partial \bar{z}} d\bar{z}\, dz + \delta \frac{\partial \bar{f}}{\partial z} d\bar{z}\, dz$




$|p_1\rangle \otimes |g\rangle \ldots$
$z_1$
$z_2$
$z_3$
$z_4$
$\langle \prod c\bar{c} V_{p_i}(z_i,\bar{z}_i) |$
$\iint b_{zz} \frac{\delta h_{\bar{z}\bar{z}}}{\delta z_4}$
$\iint b_{\bar{z}\bar{z}} \frac{\delta h_{zz}}{\delta \bar{z}_4}$
$\iint_{\text{ANNULUS}} b \frac{\partial f}{\partial \bar{z}} = \oint_{z_4} b$
$\bar{\partial} b = 0$ AWAY FROM OTHER OPERATORS


$\langle \prod c\bar{c} V_{P_i}(z_i,\bar{z}_i)$
$z_4$
$\iint b_{zz} \frac{\delta h_{\bar{z}\bar{z}}}{\delta z_4}$
$\iint b_{\bar{z}\bar{z}} \frac{\delta h_{zz}}{\delta \bar{z}_4}$
$\iint_{\text{ANNULUS}} b \frac{\partial f}{\partial \bar{z}} = \oint_{z_4} b(z)dz \oint_{z_2} \bar{b}(\bar{z})d\bar{z}$
$\bar{\partial} b = 0$ AWAY FROM OTHER OPERATORS


$\int\int \langle \prod_{i=1}^{3} c\bar{c} V_{P_i}(z_i,\bar{z}_i) \; V_{P_4}(z_4,\bar{z}_4) \rangle \, dz_4 \, d\bar{z}_4$
$z_1$
$z_4$
$z_2$
$z_3$
$\int\int_{\text{ANNULUS}} b \frac{\partial f}{\partial \bar{z}} = \oint_{z_4} b(z)dz \oint_{z_2} \bar{b}(\bar{z})d\bar{z}$
$\bar{\partial} b = 0$ AWAY FROM OTHER OPERATORS
$(z_4) \sim \frac{1}{z - z_4} + \cdots$